Solutions Notebook for
Calculus – Early Transcendentals
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Contents

Chapter 1

Functions and Models

1.1 Four Ways to Represent a Function

1.1.1 Exercises

Exercise 1.1.1. If $f(x) = x + \sqrt{2-x}$ and $g(u) = u + \sqrt{2-u}$, is it true that $f = g$?

Solution. Yes. The x and u variables are free in f and g, respectively, and thus both the image and pre-image of f and g are exactly the same.

Exercise 1.1.2. If $f(x) = \frac{x^2-x}{x-1}$ and $g(x) = x$, is it true that $f = g$?

Solution. No. The two functions are not equal to one another because in order to be equal to one another, $f(x)$ must be equal to $g(x)$ for all values of x. In this particular case, f has a discontinuity at $x - 1$, and while the discontinuity is removable, f itself is defined specifically with the discontinuity included, and thus cannot be equal to g.

Exercise 1.1.3. The graph of a function f is given.

(a) State the value of $f(1)$.

(b) Estimate the value of $f(-1)$.

(c) For what values of x is $f(x) = 1$?

(d) Estimate the value of x such that $f(x) = 0$.

(e) State the domain and range of f.

(f) On what interval is f increasing?

Solution.

(a) $f(1) = 3$

(b) $f(-1) \approx 0.2$

(c) $\{0, 3\}$

(d) $f^{-1}(-1) \approx 0.7$

(e) Domain: $[-2, 4]$, Range: $[-1, 3]$

(f) $x \in [-2, 1)$

1.6 Inverse Functions and Logarithms

1.6.1 Exercises

Exercise 1.6.1.

(a) What is a one-to-one function?

(b) How can you tell from the graph of a function whether it is one-to-one?

Solution.

(a) A one-to-one function f is a function that takes a single value $f(x)$ for each value of x. In other words, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$. More formally, a one-to-one function is an injective morphism (i.e... a morphism whose kernel is trivial).

(b) The horizontal line test

1.7 Review

1.7.3 Exercises

Exercise 1.7.23. If $f(x) = 2x + \ln x$, find $f^{-1}(2)$.

Solution. The natural logarithm function ln is defined as

$$\ln x = \int_1^x \frac{1}{t}\, dt. \tag{1.1}$$

Therefore, $\ln(1) = 0$ and $f(1) = 2(1) + 0 = 2$. We thus conclude that $f^{-1}(2) = 1$.

1.8 Principles of Problem Solving

Exercise 1.8.12.

(a) Show that the function $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$ is an odd function.

(b) Find the inverse function of f.

Solution.

(a) The derivative of the function is positive, and thus f is strictly increasing over its entire domain, allowing us to conclude that f is odd.

(b) The inverse function of f is $\sinh x$.

Chapter 2

Limits and Derivatives

2.6 Limits at Infinity; Horizontal Asymptotes

Definition 2.6.8. Let f be a function defined on some interval $(-\infty, a)$. Then

$$\lim_{x\to-\infty} f(x) = L$$

means that for every $\epsilon > 0$ there is a corresponding number N such that if $x < N$ then $|f(x) - L| < \epsilon$.

2.6.1 Exercises

Exercise 2.6.71. Use Definition 8 to prove that $\lim\limits_{x\to\infty} \frac{1}{x} = 0$.

2.7 Derivatives and Rates of Change

Definition 2.7.1 (Tangent Line)**.** The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \tag{2.1}$$

2.7.1 Exercises

Chapter 5

Integrals

5.4 Indefinite Integrals and the Net Change Theorem

Theorem 5.4.1. The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a) \tag{5.1}$$

Exercise 5.4.1. Evaluate the integral.

$$\int_{-2}^{3} \left(x^2 - 3\right) dx$$

Exercise 5.4.2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq\ t \leq\ 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Solution. The net change in the volume of water in the tank during the first ten minutes is equal to the integral of the rate at which the water was flowing out of the tank, from $t = 0$ to $t = 10$.

$$\begin{aligned}
\Delta V &= \int_0^{10} r(t)\,dt \\
&= \int_0^{10} 200 - 4t dt \\
&= 200 \int_0^{10} dt - 4 \int_0^{10} t\,dt \\
&= \left[200t \Big|_0^{10} \right] - 4 \left[\frac{1}{2} t^2 \Big|_0^{10} \right] \\
&= [200(10) - 200(0)] - 4 \left[\frac{1}{2}(10)^2 - \frac{1}{2}(0)^2 \right] \\
&= 2000 - 4(50) \\
&= 2000 - 200 \\
&= 1800
\end{aligned}$$

Therefore, 1800 liters of water flowed out of the tank in the first ten minutes.

Chapter 6

Applications of Integration

6.4 Work

Exercise 6.4.1. A 360-lb gorilla climbs a tree to a height of 20 ft. Find the work done if the gorilla reaches that height in

(a) 10 seconds

(b) 5 seconds

Solution. In both cases, the answer is the same.

(a) 7200 foot-pounds

(b) 7200 foot-pounds

Force depends solely on the mass of the object and the distance it is being moved. Power, on the other hand, does depend on the amount of time it takes to do a certain amount of work.

Exercise 6.4.2. How much work is done when a hoist lifts a 200-kg rock to a height of 3 m?

Solution.

$$\begin{aligned}
\text{Work} &= \text{Force} \times \text{Distance} \\
W &= \left(200\text{kg} \cdot 9.8\frac{\text{m}}{\text{s}^2}\right) \times 3\text{m} \\
&= 600\frac{\text{kg} \cdot \text{m} \cdot \text{m}}{\text{s}^2} \\
&= 600\text{Nm}
\end{aligned}$$

Chapter 11

Infinite Sequences and Series

11.2 Series

Theorem 11.2.4. The **geometric series**

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots \tag{11.1}$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

Theorem 11.2.6. If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim\limits_{n\to\infty} a_n = 0$.

11.2.1 Exercises

Exercise 11.2.1. Answer the following questions.

(a) What is the difference between a sequence and a series?

(b) What is a convergent series? What is a divergent series?

Chapter 14

Partial Derivatives

14.1 Functions of Several Variables

Definition 14.1.1. A **function** f **of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

14.1.1 Exercises

Exercise 14.1.1. In Example 2 we considered the function $W = f(T, v)$, where W is the wind-chill index, T is the actual temperature, and v is the wind speed. A numerical representation is given in Table 1.

(a) What is the value of $f(-15, 40)$? What is its meaning?

(b) Describe in words the meaning of the question "For what value of v is $f(-20, v) = -30$?" Then answer the question.

(c) Describe in words the meaning of the question "For what value of T is $f(T, 20) = -49$?" Then answer the question.

(d) What is the meaning of the function $W = f(-5, v)$? Describe the behaviour of this function.

(e) What is the meaning of the function $W = f(T, 50)$? Describe the behaviour of this function.

Solution.

(a) $f(-15, 40) = -27$. This means that when the temperature is -15 degrees Celsius and the wind speed is 40 km/hr, the temperature outside feels more like -27 degrees Celsius, according to Table 1.

(b) The question is asking, "How hard would the wind have to be blowing (measured in terms of km/hr) to make the actual temperature of -20 degrees Celsius feel like -30 degrees Celsius?" The answer is 20 km/hr.

(c) The question is asking, "How cold would it have to be outside for 20 km/hr winds to make it feel like -49 degrees Celsius outside?" The answer is -35 degrees Celsius.

(d) This function describes the wind chill W with respect to wind speed v, as the actual temperature is held constant at -5 degrees Celsius. The function is therefore reduced to a simple 1-variable negative linear function.

(e) This function describes the wind chill W in terms of the actual temperature T, as wind speed is held constant at 50 km/hr. The function is thus a simplified 1-variable negative linear function.

Exercise 14.1.2. The *temperature-humidity index* I (or humidex, for short) is the perceived air temperature when the actual temperature is T and the relative humidity is h, so we can write $I = (T, h)$.

(a) What is the value of $f(95, 70)$? What is its meaning?

(b) For what value of h is $f(90, h) = 100$?

(c) For waht value of T is $f(T, 50) = 88$?

(d) What are the meanings of the functions $I = f(80, h)$ and $I = f(100, h)$? Compare the behavior of these two functions of h.

Solution.

(a) The value of $f(95, 70)$ is 124, meaning that when the actual temperature is 95°F and the relative humidity is at 70%, the temperature will feel like 124°F.

(b) When $h = 60\%$, $f(90, h) = 100$.

(c) When $T = 85°$, $f(T, 50) = 88$.

(d) Both $I_1 = f(80, h)$ and $I_2 = f(100, h)$ are 1-variable functions relating the relative humidity to the temperature-humidity index. Since the temperature is constant and different for I_1 and I_2, $I_1(h) \neq I_2(h)$ for any value of h. Both I_1 and I_2 are linear functions of h.

Exercise 14.1.3. A manufacturer has modeled its yearly production function P (the monetary value of its entire production in millions of dollars) as a Cobb-Douglass function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the invested capital (in millions of dollars). Find $P(120, 20)$ and interpret it.

Solution.

$$\begin{aligned} P(L, K) &= 1.47L^{0.65}K^{0.35} \\ P(120, 20) &= 1.47(120)^{0.65}(20)^{0.35} \\ P(120, 20) &\approx 94.22 \end{aligned}$$

The monetary value of the goods produced by the manufacturer after investing 120 thousand labor hours and 20 million dollars of capital is 94.22 million dollars.

Exercise 14.1.4. Verify for the Cobb-Douglas production function

$$P(L, K) = 1.01L^{0.75}K^{0.25}$$

discussed in Example 3 that the production will be doubled if both the amount of labor and the amount of capital are doubled. Determine whether this is also true for the general production function:

$$P(L, K) = bL^{\alpha}K^{1-\alpha} \tag{14.1}$$

Solution. We will analyze the general Cobb-Douglas production function to determine whether a doubling of both labor and capital results in a doubling of production.

$$\begin{aligned} 2P(L,K) &\stackrel{?}{=} b(2L)^{\alpha}(2K)^{1-\alpha} \\ &\stackrel{?}{=} b2^{\alpha}L^{\alpha}2^{1-\alpha}K^{1-\alpha} \\ &\stackrel{?}{=} \left(2^{\alpha}\cdot 2^{1-\alpha}\right)bL^{\alpha}K^{1-\alpha} \\ &\stackrel{?}{=} \left(2^{(\alpha)+(1-\alpha)}\right)bL^{\alpha}K^{1-\alpha} \\ &\stackrel{?}{=} \left(2^{1}\right)bL^{\alpha}K^{1-\alpha} \\ &\stackrel{?}{=} 2bL^{\alpha}K^{1-\alpha} \end{aligned}$$

Indeed, we can conclude that for any arbitrary Cobb-Douglas production function $P:\mathbb{R}^{+}\times\mathbb{R}^{+}\to\mathbb{R}$, a doubling of both the labor and capital input parameters will result in a doubling of the roduction output.

Exercise 14.1.79.

(a) Show that, by taking logarithms, the general Cobb-Douglas function $P=bL^{\alpha}K^{1-\alpha}$ can be expressed as

$$\ln\frac{P}{K}=\ln b+\alpha\ln\frac{L}{K}$$

(b) If we let $x=\ln\frac{L}{K}$ and $y=\ln\frac{P}{K}$, the equation in part (a) becomes the linear equation $y=\alpha x+\ln b$. Use Table 2 (in Example 3) to make a table of values of $\ln\frac{L}{K}$ and $\ln\frac{P}{K}$ for the years 1899-1922. Then use a graphing calculator or computer to find the least squares regression line through the points $\left(\ln\frac{L}{K},\ln\frac{P}{K}\right)$.

(c) Deduce that the Cobb-Douglas production function is $P=1.01L^{0.75}K^{0.25}$.

14.2 Limits and Continuity

14.2.1 Exercises

Exercise 14.2.1. Suppose that $\lim\limits_{(x,y)\to(3,1)} f(x,y)=6$. What can you say about the value of $f(3,1)$? What if f is continuous?

Solution. Unless we know that f is continuous at $(3,1)$, we cannot say anything about the value of $f(3,1)$. If we know f was continuous, then by the definition of continuity $f(3,1)=\lim\limits_{(x,y)\to(3,1)} f(x,y)$, and thus $f(3,1)=6$.

14.3 Partial Derivatives

Theorem 14.3.1 (Clairaut's Theorem)**.** Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both continuous on D, then

$$f_{xy}(x, y) = f_{yx}(x, y).$$

14.3.1 Exercises

Exercise 14.3.87. The **van der Waals equation** for n moles of a gas is

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT \tag{14.2}$$

where P is the pressure, V is the volume, and T is the temperature of the gas. The constant R is the universal gas constant and a and b are positive constants that are characteristic of a particular gas. Calculate $\partial T/\partial P$ and $\partial P/\partial V$.

Exercise 14.3.91. The kinetic energy of a body with mass m and velocity v is $K = \frac{1}{2}mv^2$. Show that

$$\frac{\partial K}{\partial m}\frac{\partial^2 K}{\partial v^2} = K$$

Solution. We begin by calculating $\frac{\partial^2 K}{\partial v^2}$.

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ \frac{\partial K}{\partial v} &= mv \\ \frac{\partial^2 K}{\partial v^2} &= m \end{aligned}$$

We now proceed by calculating $\frac{\partial K}{\partial m}$.

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ \frac{\partial K}{\partial m} &= \frac{1}{2}v^2 \end{aligned}$$

We now simply combine our two components to produce our final answer.

$$\begin{aligned}\left(\frac{\partial K}{\partial M}\right)\cdot\left(\frac{\partial^2 K}{\partial v^2}\right) &= K\\ (m)\cdot\left(\frac{1}{2}v^2\right) &= K\\ \frac{1}{2}mv^2 &= K\end{aligned}$$

Exercise 14.3.93. You are told that there is a function f whose partial derivatives are $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$. Should you believe it?

Solution. No. By Clairaut's Theorem (Theorem 14.3.1), if f_{xy} and f_{yx} are continuous, then $f_{xy}(x,y) = f_{yx}(x,y)$. However, clearly that cannot be the case if the partial derivatives we are being given are correct. We must conclude that a function with those partial derivatives does not exist.

14.4 Tangent Planes and Linear Approximations

Definition 14.4.1. The **differential** dw is defined in terms of the differentials dx, dy, and dz of the independent variables by

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz \tag{14.3}$$

14.5 The Chain Rule

Definition 14.5.1 (The Chain Rule (General Version))**.** Suppose that u is a differentiable function of the n variables x_1, x_2, …, x_n and each x_j is a differentiable function of the m variables t_1, t_2, …, t_m. Then u is a function of t_1, t_2, …, t_m and

$$\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i} \tag{14.4}$$

14.6 Directional Derivatives and the Gradient Vector

Definition 14.6.1 (Directional Derivative)**.** If $z = f(x, y)$, then the partial derivatives f_x and f_y are defined as

$$f_x(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0 + h, y_0) - f(x_0, y_0)}{h}$$

$$f_y(x_0, y_0) = \lim_{h \to 0} \frac{f(x_0, y_0 + h) - f(x_0, y_0)}{h}$$

and represents the rates of change of z in the x- and y- directions, that is, in the directions of the unit vectors $\mathbf{i}$ and $\mathbf{j}$.

Definition 14.6.2 (The Gradient Vector)**.** If f is a function of two variables x and y, then the **gradient** of f is the vector function ∇f defined by

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} \tag{14.5}$$

Definition 14.6.3. The directional derivative of a differentiable function can be written as

$$D_{\mathbf{u}} f(x, y) = \nabla f(x, y) \cdot \mathbf{u} \tag{14.6}$$

Theorem 14.6.4. Suppose f is a differentiable function of two or three variables. The maximum value of the directional derivative $D_u f(\mathbf{x})$ is $|\nabla f(\mathbf{x})|$ and it occurs when $\mathbf{u}$ has the same direction as the gradient vector $\nabla f(\mathbf{x})$.

Definition 14.6.5. If $\nabla f(x_0, y_0, z_0) \neq \mathbf{0}$, it is natural to define the **tangent plane to the level surface** $F(x, y, z) = k$ at $P(x_0, y_0, z_0)$ as the plane that passes through P and has normal vector $\nabla F(x_0, y_0, z_0)$.

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0 \tag{14.7}$$

Chapter 16

Vector Calculus

16.1 Vector Fields

Definition 16.1.1. A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is, if there exists a function f such that $\mathbf{F} = \nabla f$.

Definition 16.1.2. If there exists a vector field $\mathbf{F}$ such that it is the gradient of some function f (symbollically, $\mathbf{F} = \nabla f$), then f is called a **potential function** for $\mathbf{F}$.

16.1.1 Exercises

Exercise 16.1.21. Find the gradient vector field of f.

$$f(x, y) = xe^{xy}$$

Solution. The equation for the gradient vector field of f is

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}.$$

We begin by finding the first partial derivative with respect to x.

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x}\left(xe^{xy}\right) \\
&= f'(x)\, g(x) + f(x)\, g'(x)\,, \text{ for } f(x) = x \text{ and } g(x) = e^{xy} \\
&= (1)\left(e^{xy}\right) + (x)\left(ye^{xy}\right) \\
&= e^{xy} + xye^{xy} \\
&= e^{xy}\left(1 + xy\right)
\end{aligned}$$

We now find the first partial derivative with respect to y.

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y}\left(xe^{xy}\right) \\ &= \left(xe^{xy}\right)(x) \\ &= x^2e^{xy}\end{aligned}$$

We thus conclude that the gradient vector field ∇f of the function f defined by

$$f(x,y) = xe^{xy}$$

is

$$\nabla f = \left[e^{xy}(1+xy)\right]\mathbf{i} + \left[x^2e^{xy}\right]\mathbf{j}.$$

Exercise 16.1.22. Find the gradient vector field of f.

$$f(x,y) = \tan(3x-4y)$$

Solution. The formula for the gradient vector field of a two-dimensional function f is

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}.$$

We thus proceed by deriving the first partial derivatives of f with respect to x and y.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\tan(3x-4y)\right)$$

Let $u = 3x - 4y$.

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} \\ &= \frac{\partial}{\partial u}\left(\tan(u)\right)\cdot\frac{\partial}{\partial x}(3x-4y) \\ &= \sec^2(u)\cdot(3) \\ &= 3\sec^2(3x-4y)\end{aligned}$$

By the same reasoning, $\frac{\partial f}{\partial y} = -4\sec^2(3x-4y)$. Therefore, the gradient vector field ∇f for f is defined to be

$$\nabla f = \left[3\sec^2(3x-4y)\right]\mathbf{i} - \left[4\sec^2(3x-4y)\right]\mathbf{j}.$$

16.2 Line Integrals

Definition 16.2.1. If f is defined on a smooth curve C given by Equation 1, then the **line integral of f along C** is

$$\int_C f(x,y)\,ds = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*, y_i^*)\,\Delta s_i \tag{16.1}$$

if this limit exists.

Definition 16.2.2. If f is a continuous function, then the limit in Definition 16.2.1 always exists and the following formula can be used to evaluate the line integral.

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \tag{16.2}$$

Definition 16.2.3. Let $\mathbf{F}$ be a continuous vector field defined on a smooth curve C given by a vector function $\mathbf{r}(t)$, $a \le t \le b$. Then the **line integral of $\mathbf{F}$ along C** is

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt = \int_C \mathbf{F}\cdot\mathbf{T}\,ds \tag{16.3}$$

16.2.1 Exercises

Evaluate the line integral, where C is the given curve.

Exercise 16.2.1. $\int_C y^3 ds,\ C: x=t^3,\ y=t,\ 0\le t\le 2$

Solution. The equation for the line integral of $f(x,y)$ over C is

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt.$$

Therefore, we begin by converting $f(x,y)$ into a function of t.

$$\begin{aligned} f(x,y) &= f(x(t),y(t)) \\ &= t^3 \end{aligned}$$

We now proceed by finding the partial derivatives of $x(t)$ and $y(t)$ with respect to t.

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt}\left(x(t)\right) \\ &= \frac{d}{dt}\left(t^3\right) \\ &= 3t^2\end{aligned}$$

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt}\left(y(t)\right) \\ &= \frac{d}{dt}\left(t\right) \\ &= 1\end{aligned}$$

With the above three components in hand, we simply plug in these values into the line integral formula above and integrate.

$$\begin{aligned}\int_C f(x,y)\,ds &= \int_a^b f\left(x(t),y(t)\right)\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \\ &= \int_0^2 t^3\sqrt{\left(3t^2\right)^2+\left(1\right)^2}\,dt \\ &= \int_0^2 t^3\sqrt{9t^4+1}\,dt\end{aligned}$$

Let $u = 9t^4+1$. Then $du = 36t^3dx$ and $\frac{1}{36}du = t^3dx$. The limits of integration become $u(0) = 9(0)^4+1 = 1$ and $u(2) = 9(2)^4+1 = 145$.

$$\begin{aligned}&= \frac{1}{36}\int_1^{145}\sqrt{u}\,du \\ &= \frac{1}{36}\cdot\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{145} \\ &= \frac{1}{36}\cdot\left[\left(\frac{2}{3}(145)^{\frac{3}{2}}\right)-\left(\frac{2}{3}(1)^{\frac{3}{2}}\right)\right] \\ &= \frac{1}{54}\left(145^{\frac{3}{2}}-1\right)\end{aligned}$$

16.3 The Fundamental Theorem for Line Integrals

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