Solutions Notebook for Calculus – Early Transcendentals by James Stewart

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Chapter 5

Integrals

5.4 Indefinite Integrals and the Net Change Theorem

Theorem 5.4.1. The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a) \tag{5.1}$$

Exercise 5.4.1. Evaluate the integral.

$$\int_{-2}^{3} \left(x^2 - 3\right) dx$$

Exercise 5.4.2. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Solution. The net change in the volume of water in the tank during the first ten minutes is equal to the integral of the rate at which the water was flowing out of the tank, from $t = 0$ to $t = 10$.

$$\begin{aligned}
\Delta V &= \int_0^{10} r(t)\,dt \\
&= \int_0^{10} 200 - 4t dt \\
&= 200 \int_0^{10} dt - 4 \int_0^{10} t\,dt \\
&= \left[200t \Big|_0^{10} \right] - 4 \left[\frac{1}{2} t^2 \Big|_0^{10} \right] \\
&= [200(10) - 200(0)] - 4 \left[\frac{1}{2}(10)^2 - \frac{1}{2}(0)^2 \right] \\
&= 2000 - 4(50) \\
&= 2000 - 200 \\
&= 1800
\end{aligned}$$

Therefore, 1800 liters of water flowed out of the tank in the first ten minutes.

Chapter 11

Infinite Sequences and Series

11.2 Series

Theorem 11.2.4. The **geometric series**

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots \tag{11.1}$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

Theorem 11.2.6. If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim\limits_{n\to\infty} a_n = 0$.

11.2.1 Exercises

Exercise 11.2.1. Answer the following questions.

(a) What is the difference between a sequence and a series?

(b) What is a convergent series? What is a divergent series?