① $\theta + \varphi = \pi$

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F$$

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta$$

$$\begin{bmatrix} x \\ \dot{x} \\ \varphi \\ \dot{\varphi} \end{bmatrix}$$

$\cancel{\theta = \pi + \varphi}$

$$\ddot{x} = \frac{F - b\dot{x} - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta}{(M+m)}$$

$$\ddot{\theta} = \frac{-ml\ddot{x}\cos\theta - mgl\sin\theta}{(I+ml^2)}$$

$$\ddot{x} = \frac{F}{(M+m)} - \frac{b\dot{x}}{(M+m)} - \frac{ml}{(M+m)}\cos\theta\left[\frac{-ml\ddot{x}\cos\theta}{(I+ml^2)} - \frac{mgl\sin\theta}{(I+ml^2)}\right] + \frac{ml\dot{\theta}^2\sin\theta}{(M+m)}$$

$$\ddot{x} - \frac{m^2l^2\cos^2\theta}{(M+m)(I+ml^2)}\ddot{x} = \frac{F}{(M+m)} - \frac{b\dot{x}}{(M+m)} + \frac{m^2l^2 g\sin\theta\cos\theta}{(M+m)(I+ml^2)} + \frac{ml\dot{\theta}^2\sin\theta}{(M+m)}$$

$$\ddot{x}\left(\frac{(M+m)(I+ml^2) - m^2l^2\cos^2\theta}{(M+m)(I+ml^2)}\right) = \frac{F}{(M+m)} - \frac{b\dot{x}}{(M+m)} + \frac{m^2l^2 g\sin\theta\cos\theta}{(M+m)(I+ml^2)} + \frac{ml\dot{\theta}^2\sin\theta}{(M+m)}$$

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$$\ddot{x} = \frac{F\,(I+ml^2)}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} - \frac{\dot{x}\,b\,(I+ml^2)}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}$$

$$+ \frac{m^2l^2 g\sin\theta\cos\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} + \frac{ml\dot{\theta}^2\sin\theta\,(I+ml^2)}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}$$

$$\ddot{\theta} = -\frac{ml\cos\theta}{(I+ml^2)}\left[\frac{F}{(M+m)} - \frac{b\dot{x}}{(M+m)} - \frac{ml\ddot{\theta}\cos\theta}{(M+m)} + \frac{ml\dot{\theta}^2\sin\theta}{(M+m)}\right] - \frac{mgl\sin\theta}{(I+ml^2)}$$

$$\ddot{\theta} = \frac{-ml\cos\theta\,F}{(I+ml^2)(M+m)} + \frac{\dot{x}bml\cos\theta}{(M+m)(I+ml^2)} + \frac{m^2l^2\ddot{\theta}\cos^2\theta}{(M+m)(I+ml^2)} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{(M+m)(I+ml^2)} - \frac{mgl\sin\theta}{(I+ml^2)}$$

$$\ddot{\theta}\left(\frac{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}{(M+m)(I+ml^2)}\right) = \frac{-ml\cos\theta\,F}{(I+ml^2)(M+m)} + \frac{\dot{x}bml\cos\theta}{(M+m)(I+ml^2)} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{(M+m)(I+ml^2)} - \frac{mgl\sin\theta}{(I+ml^2)}$$

$$\ddot{\theta} = \frac{-ml\cos\theta\,F}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} + \frac{\dot{x}bml\cos\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} - \frac{mgl(M+m)\sin\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}$$

$$\ddot{\theta} = \frac{-ml\cos\theta\,F}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} + \frac{\dot{x}bml\cos\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}$$

$$-\frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta} - \frac{mgl(M+m)\sin\theta}{(M+m)(I+ml^2)-m^2l^2\cos^2\theta}$$

TEMA:

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tenemos las caracteristicas

$M = 0.696\ Kg$
$m = 0.017Kg$
$l = 0.3m$
$I = 0.0011\ Kg/m^2$

$g = 9.8m/s^2$
$b = 0.001\ Ns/m$

con las ecuaciones obtenidas del Modelo anterior Linealizaremos

$$\ddot{x} \approx \sum_{i=1}^{4}\left[\frac{\partial f_2(x,u)}{\partial x_i}\bigg|_{x=0,\,u=0} \Delta x_i\right] + \frac{\partial f_2(x,u)}{\partial u}\bigg|_{x=0,u=0}$$

$\ddot{\theta}$:

$$\frac{\partial f_2(x,u)}{\partial x}\bigg|_{x=0;\dot{x}=0,\theta=\pi,\dot{\theta}=0} = 0 \qquad = 0$$

$$\frac{\partial f_2(x,u)}{\partial \dot{x}}\bigg|_{x=0,\dot{x}=0,\theta=\pi,\dot{\theta}=0} = -\frac{b\,(I+ml^2)}{(M+m)(I+ml^2)-m^2l^2} = -0.00142225203$$

$$= -1.42\times10^{-3}$$

$$\frac{\partial f_2(x,u)}{\partial \theta}\bigg|_{x=0,\dot{x}=0,\theta=\pi,\dot{\theta}=0} = +\frac{m^2l^2g}{(M+m)(I+ml^2)-m^2l^2} = 0.13784$$

$$\frac{\partial f_2(x,u)}{\partial \dot{\theta}}\bigg|_{x=0,\dot{x}=0,\theta=\pi,\dot{\theta}=0} = 0 \qquad = 0$$

$$\frac{\partial f_2(x,u)}{\partial F}\bigg|_{x=0,\dot{x}=0,\theta=\pi,\dot{\theta}=0} = \frac{(I+ml^2)}{(M+m)(I+ml^2)-m^2l^2} = 1.42225$$

TEMA: FECHA:

$$\ddot{\theta} = \sum_{i=1}^{4} \left[\frac{\partial f_4(x,u)}{\partial x_i} \bigg|_{\substack{x=0 \\ \theta=\pi}\; u=0} \Delta x_i \right] + \frac{\partial f_4(x,u)}{\partial u} \bigg|_{x=0,\, u=0}$$

$$\frac{\partial f_4(x,u)}{\partial x} \bigg|_{x=0,\, \dot{x}=0,\, \theta=\pi,\, \dot{\theta}=0} = 0$$

$$\frac{\partial f_4(x,u)}{\partial \dot{x}} \bigg|_{x=0,\, \dot{x}=0,\, \theta=\pi,\, \dot{\theta}=0} = -\frac{b\,m\,l}{(M+m)(I+ml^2)-m^2l^2} = -0.00275 = -2.75\times10^{-3}$$

$$\frac{\partial f_4(x,u)}{\partial \theta} \bigg|_{x=0,\, \dot{x}=0,\, \theta=\pi,\, \dot{\theta}=0} = \frac{m\,g\,l\,(M+m)}{(M+m)(I+ml^2)-m^2l^2} = 19.27110$$

$$\frac{\partial f_4(x,u)}{\partial \dot{\theta}} \bigg|_{x=0,\, \dot{x}=0,\, \theta=\pi,\, \dot{\theta}=0} = 0$$

$$\frac{\partial f_4(x,u)}{\partial F} \bigg|_{x=0,\, \dot{x}=0,\, \theta=\pi,\, \dot{\theta}=0} = \frac{m\,l}{(M+m)(I+ml^2)-m^2l^2} = 2.75797$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.42\times10^{-3} & 0.1378 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.75\times10^{-3} & 19.2711 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.42225 \\ 0 \\ 2.75797 \end{bmatrix} F$$

$$y = \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

FECHA: / /

tambien compro baremos el modelo obtenido de el primer laboratorio del curso de aqui tenemos

$M = 0.696\ Kg$

$m = 0.017\ Kg$

$l = 0.3\ m$

$I = 0.0011\ Kg/m^2$

$g = 9.8\ m/s^2$

$b = 0.001\ Ns/m$

$x_1 = x$

$x_2 = \phi$

$x_3 = \dot{x}$

$x_4 = \dot{\phi}$

de aqui tenemos (extraido de nuestro informe 1)

$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = \frac{\alpha^2 g \operatorname{sen} x_2 \cos x_2}{\theta M_T - \alpha^2 \cos^2 x_2} - \frac{\alpha C x_4 \cos x_2}{\theta M_T - \alpha^2 \cos^2 x_2} - \frac{\theta \alpha x_4^2 \operatorname{sen} x_2}{\theta M_T - \alpha^2 \cos^2 x_2} - \frac{\theta b x_3}{\theta M_T - \alpha^2 \cos^2 x_2} + \frac{\theta \cdot F}{\theta M_T - \alpha^2 \cdot \cos^2 x_2}$$

$$\dot{x}_4 = \frac{\alpha M g \operatorname{sen} x_2}{\theta M_T - \alpha^2 \cos^2 x_2} - \frac{\alpha b \cos x_2}{\theta M_T - \alpha^2 \cos^2 x_2} x_3 - \frac{M C x_4}{\theta M_T - \alpha^2 \cos^2 x_2} - \frac{\alpha^2 \operatorname{sen} x_2 \cos x_2}{\theta M_T - \alpha^2 \cos^2 x_2} + \frac{\alpha \cos x_2}{\theta M_T - \alpha^2 \cos^2 x_2} F$$

donde:

$\alpha = m l \quad = 0.0051$

$M_T = M + m \quad = 0.713$

$\theta = I + m l^2 \quad = 0.00263$

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TEMA:

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Dado que la fricción seca no se nos proporciona $C=0$

Linealizamos las ecuaciones con $\beta = \theta M_T - \alpha^2 = 0.00184918$

$$\left.\frac{\partial F_3(x,u)}{\partial x_1}\right|_{x=0,u=0} = 0$$

$$\left.\frac{\partial F_3(x,u)}{\partial x_2}\right|_{x=0,u=0} = \frac{\alpha^2 g}{\beta} = 0.13784$$

$$\left.\frac{\partial F_3(x,u)}{\partial x_3}\right|_{x=0,u=0} = -\frac{\theta b}{\beta} = -0.00142$$

$$\left.\frac{\partial F_3(x,u)}{\partial x_4}\right|_{x=0,u=0} = \frac{-\alpha c}{\beta} = 0$$

$$\left.\frac{\partial F_3(x,u)}{\partial F}\right|_{x=0,u=0} = \frac{\theta}{\beta} = 1.42225$$

para x_4

$$\left.\frac{\partial F_4(x,u)}{\partial x_1}\right|_{x=0,u=0} = 0$$

$$\left.\frac{\partial F_4(x,u)}{\partial x_2}\right|_{x=0,u=0} = \frac{\alpha M_T g}{\beta} = 19.27110$$

$$\left.\frac{\partial F_4(x,u)}{\partial x_3}\right|_{x=0,u=0} = -\frac{\alpha b}{\beta} = -0.00275$$

$$\left.\frac{\partial F_4(x,u)}{\partial x_4}\right|_{x=0,u=0} = -\frac{Mc}{\beta} = 0 = 0$$

$$\left.\frac{\partial F_4(x,u)}{\partial F}\right|_{x=0,u=0} = \frac{\alpha}{\beta} = 2.75797$$

FECHA: / /

quedando

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.13784 & -14.2\times10^{-3} & 0 \\ 0 & 19.2711 & -2.75\times10^{-3} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1.42275 \\ 2.75797 \end{bmatrix} F$$

$$y = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F$$