Lexington Bikes Optimization Model

1 Introduction

This document details the mathematical specifics of the optimization model used in this project.

At the most basic level, this model should give an answer the question “Where are the best places to install new bike lanes?”.

In more detail, cycling infrastructure can be classified as low-stress or high-stress, see [1] and [2]. We model a city with network flow: people traveling from census blocks, through the bike network, to destinations. We assume people want to bike exclusively on low-stress infrastructure, but must accept use of high-stress segments due to the car-centric design of many North American cities. In this setup, we imagine a city has a budget to add new bicycle infrastructure, such as protected lanes, to existing roadway to convert high-stress segments to low-stress. For each possible arrangement of newly low-stress infrastructure, we calculate total distance traveled and keep track of the configuration which resulted in the lowest total trip distance.

2 Condensed Model

We must solve the following optimization problem:

$$\begin{aligned}
\text{minimize} \quad & w := \sum_{(i,j)\in A} d_{ij}x_{ij} + \sum_{(i,j)\in A} (f-1)d_{ij}z_{ij}, && (1)\\
\text{subject to} \quad & \sum_{(n,j)\in A} x_{nj} - \sum_{(i,n)\in A} x_{in} = b_n, \quad \forall n \in N; && (2)\\
& z_{ij} \geq x_{ij} - Py_{ij}, \quad \forall (i,j) \in H; && (3)\\
& y_{ij} \leq y_{ji}, \quad \forall (i,j) \in H_2; && (4)\\
& \sum_{(i,j)\in H_1} u_{ij}y_{ij} + \sum_{(i,j)\in H_2} v_{ij}y_{ij} \leq B; && (5)\\
& x_{ij} \in \mathbb{Z}_{\geq 0}, \quad \forall (i,j) \in A && (6)\\
& y_{ij} \in \{0,1\}, \quad \forall (i,j) \in H && (7)\\
& z_{ij} \in \{0,1\}, \quad \forall (i,j) \in H && (8)
\end{aligned}$$

Constraint 2 ensures that each node has the correct in- and out-flow. Constraint 3 adds a penalty to the objective function in the case that a non-upgraded high stress arc is used, and relies on the assumption that no trip will make a loop, and hence no arc will have more than P trips across it. Constraint 5 ensures that the money spent on upgrades will not exceed the total budget. Constraint 6 ensures that the number of trips is a nonnegative integer. Constraint 7 ensures that the upgrade status of high stress arcs is binary.

3 Explanations

3.1 Sets

N = set of nodes.

O = set of origins (census blocks) $\subset N$. Define a dummy origin node $o^* \in O$ that doesn't represent a census block, but rather is used to balance supply and demand between origin and destination nodes.

D = set of destinations (amenities of a single type) $\subset N$.

A = set of arcs, of the form (i, j) for $i, j \in N$.

L = set of low stress arcs $\subset A$.

H = set of high stress arcs $\subset A$.

Note that H and L form a partition of A.

3.2 Parameters

d_{ij} = distance of arc (i, j), for $(i, j) \in A$.

p_n = population at origin node n, for $n \in O$.

b_n = net flow at node n = (supply at n) $-$ (demand at n), for $n \in N$. In particular, define

$$b_n = \begin{cases} -P, & n = o^* \\ p_n, & n \in O \\ 0 & \text{else} \end{cases}.$$

where o^* is a dummy node.

$P = \sum_{n \in O} p_n$ total population. (Note: This is redundant to adding up the supply at each origin node.)

u_{ij} = cost to upgrade arc $(i, j) \in H$ to become low stress, for (i, j).

B = total budget.

f = the factor by which a low stress path would need to exceed a high stress route in order for someone to choose the shorter high stress route. Equivalently, we can say someone would be willing f times further in order to stay on a low stress path.

3.3 Decision Variables

x_{ij} = number of trips on arc (i, j), for $(i, j) \in A$.

$$y_{ij} = \begin{cases} 0, & (i,j) \text{ not upgraded to low stress} \\ 1, & (i,j) \text{ upgraded to low stress} \end{cases}, \text{ for } (i, j) \in H.$$

$$z_{ij} = \begin{cases} 0, & (i,j) \text{ not upgraded to low stress} \\ x_{ij}, & (i,j) \text{ upgraded to low stress} \end{cases}, \text{ for } (i, j) \in H.$$

3.4 Objective Function

Goal: Minimize the total trip distance, while including the penalty for using high stress arcs.

$$\sum_{(i,j) \in A} d_{ij} x_{ij} + \sum_{(i,j) \in H} (f - 1) d_{ij} z_{ij}.$$

3.5 Constraints

Each arc $(i, j) \in A$ has a whole, nonnegative number of trips:

$$x_{ij} \in \mathbb{Z}_{\geq 0}, \quad \forall (i, j) \in A.$$

The flow must be balanced at each node $n \in N$. That is, the number of outgoing trips $-$ the number of incoming trips must equal the net flow:

$$\sum_{(n,j) \in A} x_{nj} - \sum_{(i,n) \in A} x_{in} = b_n, \quad \forall n \in N.$$

Trips can use low stress arcs, or high stress arc that have been upgraded without penalty. If a trip uses a high stress arc, it will incur a penalty in the objective function. Assuming trips don't make loops, we get that $x_{ij} \leq P$ for all $(i, j) \in A$. Using this, we can model the above constraint with the inequality with a 'big M' constraint, where the total population P serves as our M.

$$z_{ij} \geq x_{ij} - P y_{ij}, \quad \forall (i, j) \in H.$$

Stay within budget:

$$\sum_{(i,j) \in H} u_{ij} y_{ij} \leq B.$$

4 Lexington

For simplicity, we begin with parks as the only destination type. Additionally, we limited our data to the city's Urban Service Area. Further, the bike network data from [2] has a handful of parking lots, golf course, etc. that are not connected to the larger network, and we removed these.

References

[1] Maaza C. Mekuria, Peter G. Furth, and Hilary Nixon. "Low-Stress Bicycling and Network Connectivity". In: *Mineta Transportation Institute* (2012).

[2] PeopleForBikes. *Bicycle Network Analysis: Lexington, KY, US.* URL: https://bna.peopleforbikes.org/#/places/f12b7872-ac3a-47e8-9c24-f06774b0e2a0/. (accessed: 01.19.2024).