Week 5 Homework Assignment

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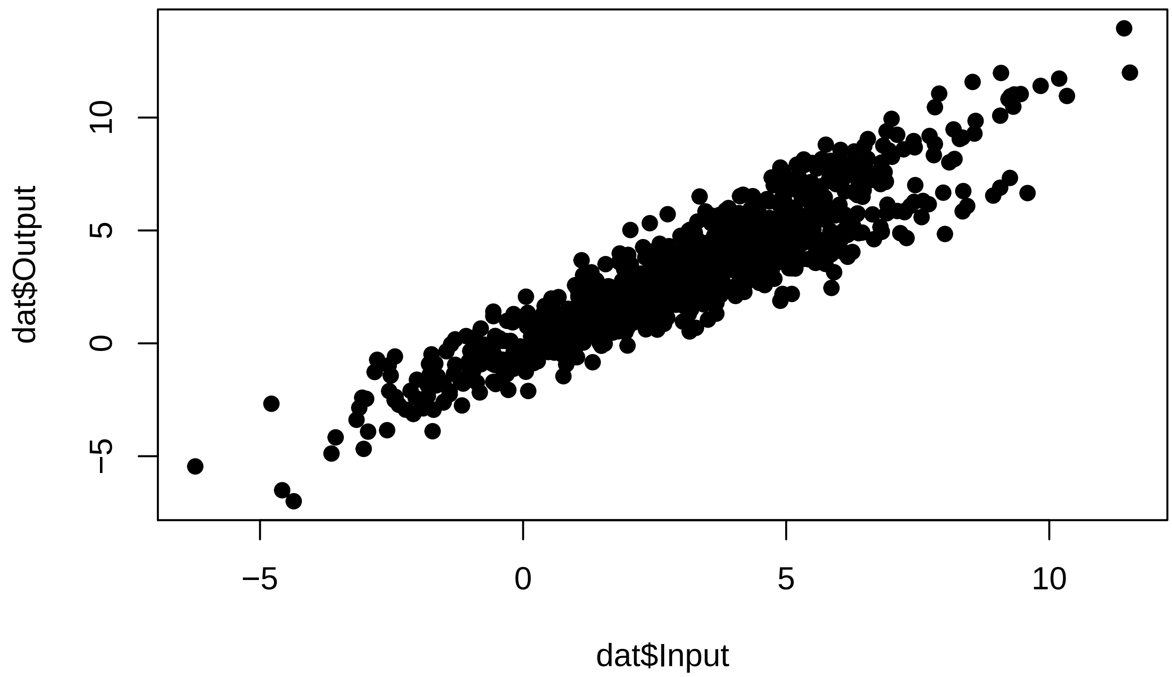
1 Method 1

1.1 Project data

```
##Download data and read into R. Print the head and plot the data. Set nSample variable.
datapath <- "C:/Users/JohntheGreat/Documents/MSCA/StatisticalAnalysis/Week5/Assignments"
dat<-read.csv(file=paste(datapath,"ResidualAnalysisProjectData_2.csv",sep="/"),header=TRUE,sep=",")
head(dat)
```

```
##      Input      Output
## 1 3.132859  4.17792255
## 2 5.561134  5.84669919
## 3 1.984543 -0.09834184
## 4 5.619160  7.84692946
## 5 6.378149  7.57941491
## 6 6.123204  5.68973137
```

```
plot(dat$Input,dat$Output, type="p",pch=19)
```



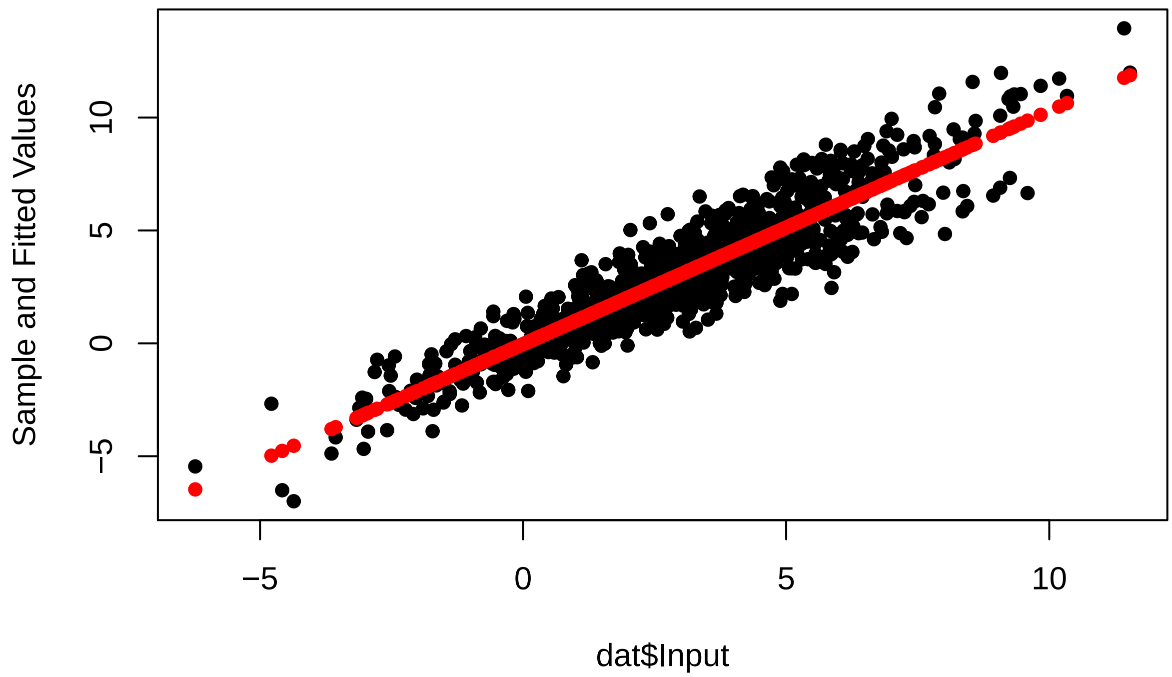

```
nSample<-length(dat$Input)
```

1.2 Estimate linear model

```
##Create a linear model, plot the points, print the summary.
m1<-lm(Output~Input,dat)
m1$coefficients
```

```
## (Intercept)       Input
## -0.03657717  1.03270579
```

```
matplot(dat$Input,cbind(dat$Output,m1$fitted.values),type="p",pch=16,ylab="Sample and Fitted Values")
```

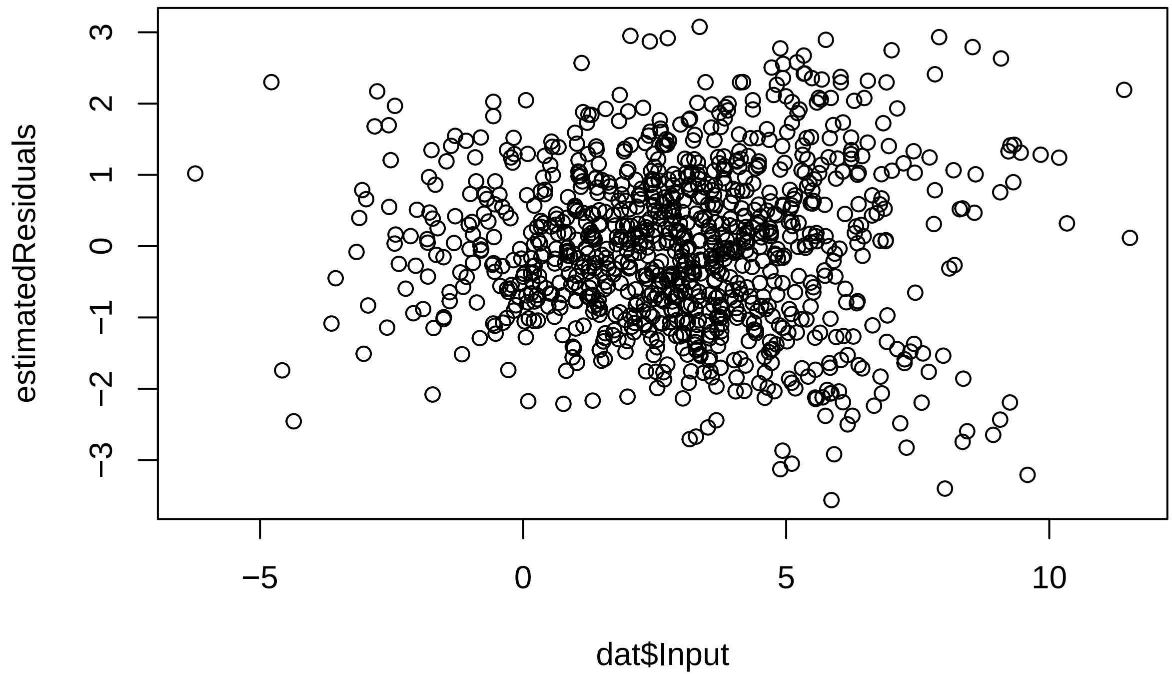



```r
summary(m1)
```

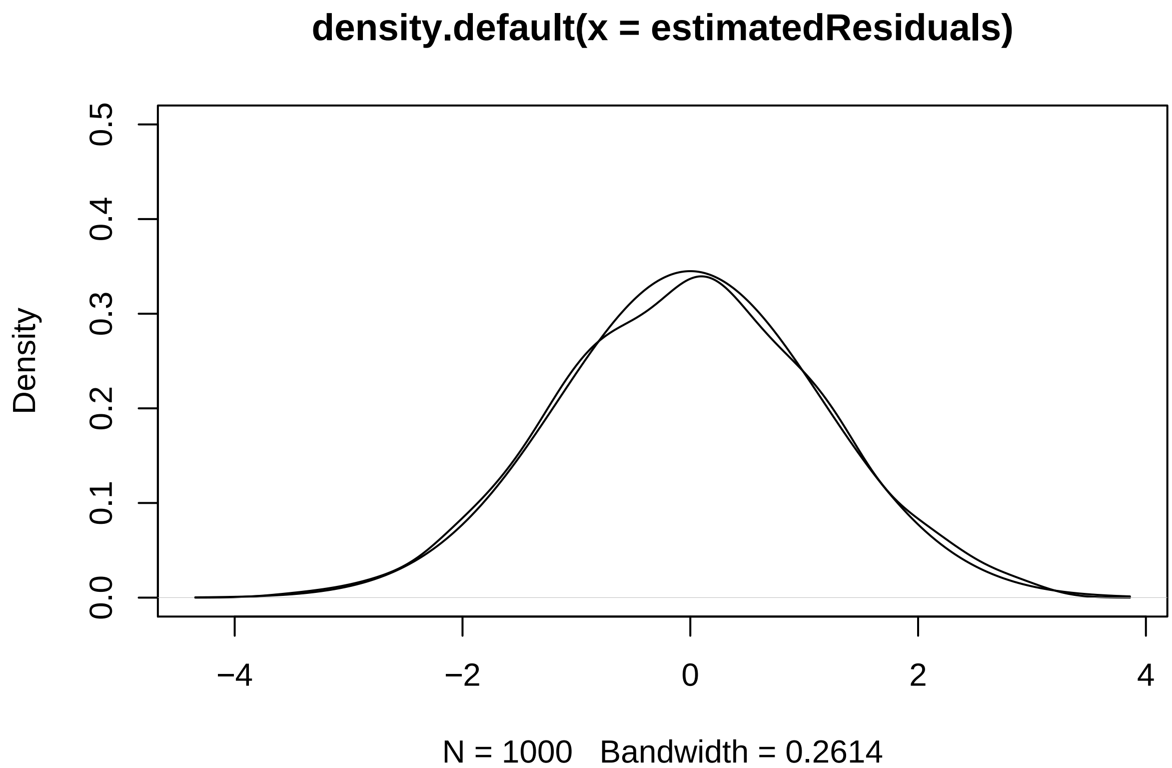
```
##
## Call:
## lm(formula = Output ~ Input, data = dat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5620 -0.7987  0.0267  0.7870  3.0759
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.03658    0.05718   -0.64    0.523
## Input        1.03271    0.01453   71.06   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.157 on 998 degrees of freedom
## Multiple R-squared:  0.835,  Adjusted R-squared:  0.8348
## F-statistic:  5050 on 1 and 998 DF,  p-value: < 2.2e-16
```

Interpret the summary of the model. It appears that the model gives a fairly high correlation. R squared is around 0.835. It seems that the slope of the model is significantly different from zero. Looking at it visually, it looks to be a good fit through most of the data, but has some separation at the tails.

```
##Plot the residuals, Plot the prob density function
estimatedResiduals<-m1$residuals
plot(dat$Input,estimatedResiduals)
```




```
Probability.Density.Residuals<-density(estimatedResiduals)
plot(Probability.Density.Residuals,ylim=c(0,.5))
lines(Probability.Density.Residuals$x,
      dnorm(Probability.Density.Residuals$x,mean=mean(estimatedResiduals),sd=sd(estimatedResiduals)))
```



What does the pattern of residuals and the pattern of the data tell you about the sample? It is close to normal, but quirky. *What kind of mixture of two models do you see in the data?* It appears that the data is a mixed distribuiton of 2 normal distributions. There appears to be a difference in the tails. *Try to separate the subsamples with different models.*

1.3 Creating training sample for separation of mixed models

```
##Create 3 data frames for Training samples
Train.Sample<-data.frame(trainInput=dat$Input,trainOutput=rep(NA,nSample))
Train.Sample.Steeper<-data.frame(trainSteepInput=dat$Input,
                                  trainSteepOutput=rep(NA,nSample))
Train.Sample.Flatter<-data.frame(trainFlatInput=dat$Input,
                                  trainFlatOutput=rep(NA,nSample))

head(cbind(dat,
           Train.Sample,
           Train.Sample.Steeper,
           Train.Sample.Flatter))
```

```
##      Input      Output trainInput trainOutput trainSteepInput
## 1 3.132859  4.17792255   3.132859          NA        3.132859
## 2 5.561134  5.84669919   5.561134          NA        5.561134
## 3 1.984543 -0.09834184   1.984543          NA        1.984543
## 4 5.619160  7.84692946   5.619160          NA        5.619160
```

```
## 5 6.378149  7.57941491   6.378149          NA        6.378149
## 6 6.123204  5.68973137   6.123204          NA        6.123204
##   trainSteepOutput trainFlatInput trainFlatOutput
## 1               NA       3.132859              NA
## 2               NA       5.561134              NA
## 3               NA       1.984543              NA
## 4               NA       5.619160              NA
## 5               NA       6.378149              NA
## 6               NA       6.123204              NA
```

```
##Select parts of the sample with Input greater than 5 and Output either above the estimated regression
Train.Sample.Selector<-dat$Input>=5
Train.Sample.Steeper.Selector<-Train.Sample.Selector&
  (dat$Output>m1$fitted.values)
Train.Sample.Flatter.Selector<-Train.Sample.Selector&
  (dat$Output<=m1$fitted.values)
##Create training samples for steep and flat slopes.
Train.Sample[Train.Sample.Selector,2]<-dat[Train.Sample.Selector,2]
Train.Sample.Steeper[Train.Sample.Steeper.Selector,2]<-dat[Train.Sample.Steeper.Selector,2]
Train.Sample.Flatter[Train.Sample.Flatter.Selector,2]<-dat[Train.Sample.Flatter.Selector,2]
head(Train.Sample)
```

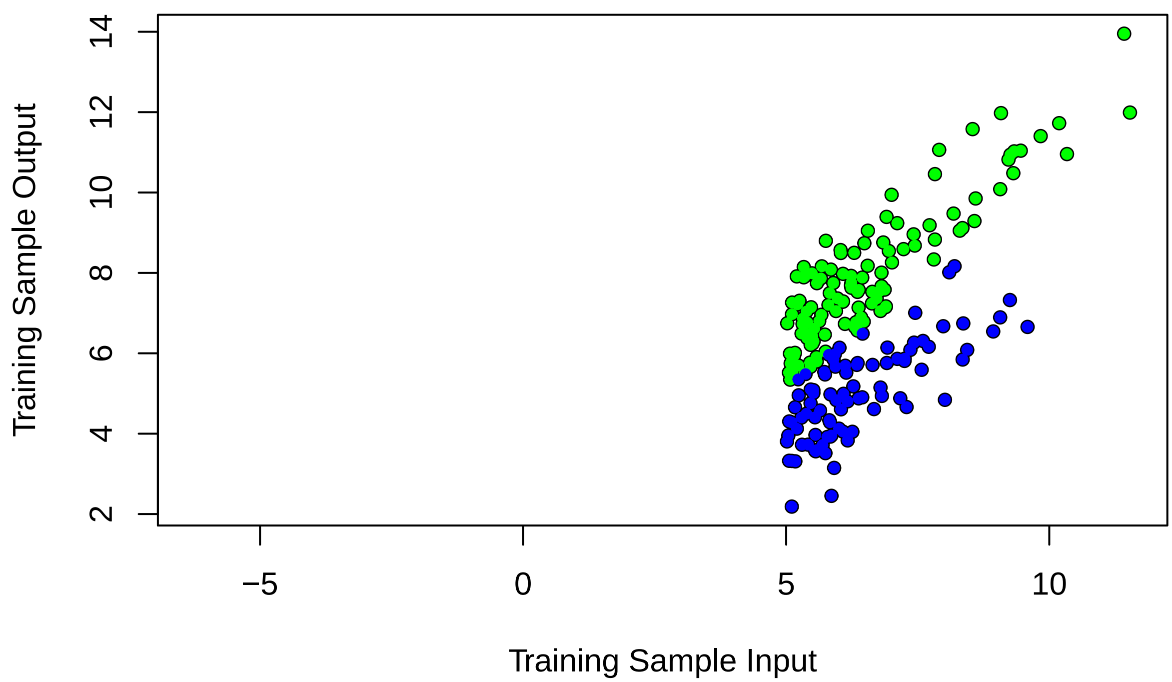
```
##   trainInput trainOutput
## 1   3.132859          NA
## 2   5.561134    5.846699
## 3   1.984543          NA
## 4   5.619160    7.846929
## 5   6.378149    7.579415
## 6   6.123204    5.689731
```

```
head(cbind(dat,
           Train.Sample,
           Train.Sample.Steeper,
           Train.Sample.Flatter),10)
```

```
##        Input      Output trainInput trainOutput trainSteepInput
## 1  3.1328589  4.17792255  3.1328589          NA       3.1328589
## 2  5.5611337  5.84669919  5.5611337    5.846699       5.5611337
## 3  1.9845429 -0.09834184  1.9845429          NA       1.9845429
## 4  5.6191601  7.84692946  5.6191601    7.846929       5.6191601
## 5  6.3781486  7.57941491  6.3781486    7.579415       6.3781486
## 6  6.1232040  5.68973137  6.1232040    5.689731       6.1232040
## 7  0.7666195 -1.45675560  0.7666195          NA       0.7666195
## 8  4.3535141  4.16746077  4.3535141          NA       4.3535141
## 9  2.3627156  2.38611901  2.3627156          NA       2.3627156
## 10 6.3272368  6.77354738  6.3272368    6.773547       6.3272368
##    trainSteepOutput trainFlatInput trainFlatOutput
## 1                NA      3.1328589              NA
## 2          5.846699      5.5611337              NA
## 3                NA      1.9845429              NA
## 4          7.846929      5.6191601              NA
## 5          7.579415      6.3781486              NA
## 6                NA      6.1232040        5.689731
```

```
## 7                 NA       0.7666195              NA
## 8                 NA       4.3535141              NA
## 9                 NA       2.3627156              NA
## 10          6.773547       6.3272368              NA
```

```r
##Plot the seperation in residuals, above 5
plot(Train.Sample$trainInput,Train.Sample$trainOutput,pch=16,ylab="Training Sample Output",
     xlab="Training Sample Input")
points(Train.Sample.Steeper$trainSteepInput,Train.Sample.Steeper$trainSteepOutput,pch=20,col="green")
points(Train.Sample.Flatter$trainFlatInput,Train.Sample.Flatter$trainFlatOutput,pch=20,col="blue")
```



#1.4 Fit linear models to train samples

```r
##Fit linear models to both samples
Train.Sample.Steep.lm <- lm(Train.Sample.Steeper$trainSteepOutput ~Train.Sample.Steeper$trainSteepInput)
Train.Sample.Flat.lm <- lm(Train.Sample.Flatter$trainFlatOutput ~Train.Sample.Flatter$trainFlatInput)

summary(Train.Sample.Steep.lm)$coefficients
```

```
##                                      Estimate Std. Error  t value
## (Intercept)                          1.021798 0.33271502  3.07109
## Train.Sample.Steeper$trainSteepInput 1.048763 0.04981059 21.05503
##                                          Pr(>|t|)
## (Intercept)                          2.647868e-03
## Train.Sample.Steeper$trainSteepInput 8.979480e-42
```

```r
summary(Train.Sample.Steep.lm)$sigma
```

```
## [1] 0.7800507
```

```r
summary(Train.Sample.Steep.lm)$df
```

```
## [1]   2 118   2
```

```r
summary(Train.Sample.Steep.lm)$r.squared
```

```
## [1] 0.7897791
```

```r
summary(Train.Sample.Steep.lm)$adj.r.squared
```

```
## [1] 0.7879975
```

```r
summary(Train.Sample.Steep.lm)$fstatistic
```

```
##    value    numdf    dendf
## 443.3142   1.0000 118.0000
```

```r
summary(Train.Sample.Flat.lm)$coefficients
```

```
##                                      Estimate Std. Error   t value
## (Intercept)                        0.08316391 0.50585815 0.1644016
## Train.Sample.Flatter$trainFlatInput 0.77825624 0.07855636 9.9069789
##                                        Pr(>|t|)
## (Intercept)                        8.698008e-01
## Train.Sample.Flatter$trainFlatInput 7.031944e-16
```

```r
summary(Train.Sample.Flat.lm)$sigma
```

```
## [1] 0.8072447
```

```r
summary(Train.Sample.Flat.lm)$df
```

```
## [1]  2 86  2
```

```r
summary(Train.Sample.Flat.lm)$r.squared
```

```
## [1] 0.5329849
```

```r
summary(Train.Sample.Flat.lm)$adj.r.squared
```

```
## [1] 0.5275545
```

```r
summary(Train.Sample.Flat.lm)$fstatistic
```

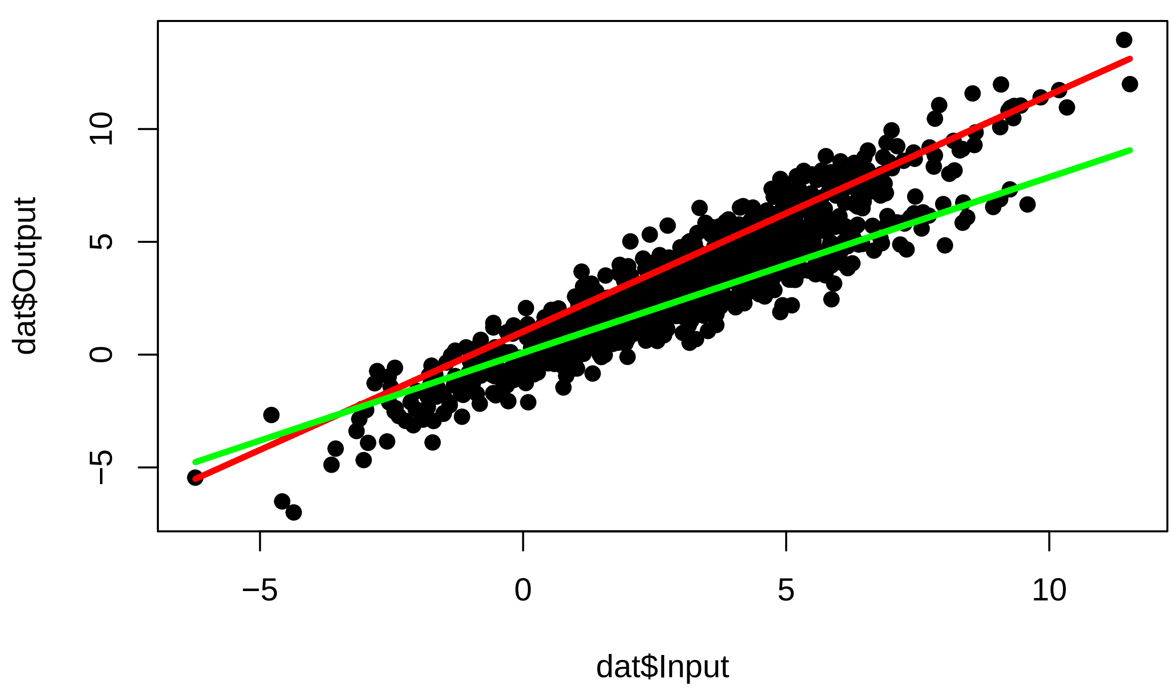
```
##    value    numdf    dendf
## 98.14823  1.00000 86.00000
```

```r
##It looks like we are getting worse models, lower correlation shown in R squared.
##Despite a slope that is significantly different from zero. Also, the standard errors
##of the residuals are smaller than the original model.

rbind(Steeper.Coefficients=Train.Sample.Steep.lm$coefficients,
      Flatter.Coefficients=Train.Sample.Flat.lm$coefficients)
```

```
##                       (Intercept) Train.Sample.Steeper$trainSteepInput
## Steeper.Coefficients  1.02179775                            1.0487634
## Flatter.Coefficients  0.08316391                            0.7782562
```

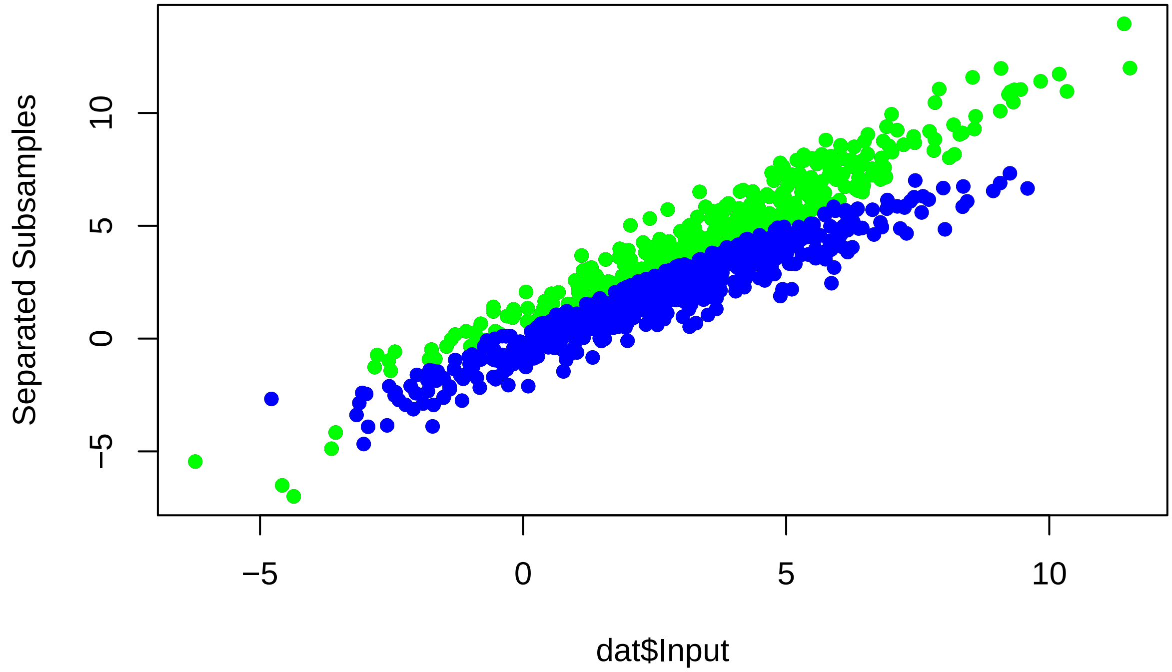
```r
plot(dat$Input,dat$Output, type="p",pch=19)
lines(dat$Input,predict(Train.Sample.Steep.lm,
                        data.frame(trainSteepInput=dat$Input),
                        interval="prediction")[,1],col="red",lwd=3)
lines(dat$Input,predict(Train.Sample.Flat.lm,data.frame(trainFlatInput=dat$Input),
                        interval="prediction")[,1],col="green",lwd=3)
```



```r
##Define distances from each point to both regression lines.
Distances.to.Steeper<-abs(dat$Output-
                            dat$Input*Train.Sample.Steep.lm$coefficients[2]-
                            Train.Sample.Steep.lm$coefficients[1])
Distances.to.Flatter<-abs(dat$Output-
                           dat$Input*Train.Sample.Flat.lm$coefficients[2]-
                           Train.Sample.Flat.lm$coefficients[1])
# Define the unscramble sequence
Unscrambling.Sequence.Steeper<-Distances.to.Steeper<Distances.to.Flatter
# Define  two subsamples with NAs in the Output columns
Subsample.Steeper<-data.frame(steeperInput=dat$Input,steeperOutput=rep(NA,nSample))
Subsample.Flatter<-data.frame(flatterInput=dat$Input,flatterOutput=rep(NA,nSample))
# Fill in the unscrambled outputs instead of NAs where necessary
Subsample.Steeper[Unscrambling.Sequence.Steeper,2]<-dat[Unscrambling.Sequence.Steeper,2]
Subsample.Flatter[!Unscrambling.Sequence.Steeper,2]<-dat[!Unscrambling.Sequence.Steeper,2]
# Check the first rows
head(cbind(dat,Subsample.Steeper,Subsample.Flatter))
```

```
##      Input      Output steeperInput steeperOutput flatterInput
## 1 3.132859  4.17792255     3.132859      4.177923     3.132859
## 2 5.561134  5.84669919     5.561134      5.846699     5.561134
## 3 1.984543 -0.09834184     1.984543            NA     1.984543
## 4 5.619160  7.84692946     5.619160      7.846929     5.619160
## 5 6.378149  7.57941491     6.378149      7.579415     6.378149
## 6 6.123204  5.68973137     6.123204            NA     6.123204
##   flatterOutput
## 1            NA
## 2            NA
## 3   -0.09834184
## 4            NA
## 5            NA
## 6    5.68973137
```

```r
# Plot the unscrambled subsamples, include the original entire sample as a check
matplot(dat$Input,cbind(dat$Output,
                        Subsample.Steeper$steeperOutput,
                        Subsample.Flatter$flatterOutput),
        type="p",col=c("black","green","blue"),
        pch=16,ylab="Separated Subsamples")
```




```
# Mixing Probability Of Steeper Slope
(Mixing.Probability.Of.Steeper.Slope<-sum(Unscrambling.Sequence.Steeper)/length(Unscrambling.Sequence.St
```

```
## [1] 0.417
```

Run binomial test for the null hypothesis p=0.5 and two-sided alternative "p is not equal to 0.5". Interpret the output of binom.test

```
binom.test(417,1000,p = 0.5)
```

```
##
##  Exact binomial test
##
## data:  417 and 1000
## number of successes = 417, number of trials = 1000, p-value =
## 1.702e-07
## alternative hypothesis: true probability of success is not equal to 0.5
## 95 percent confidence interval:
##  0.3862231 0.4482670
## sample estimates:
## probability of success
##                  0.417
```

What do you conclude from the test results?. The results show that the results are statistically significant. We can reject the null hypothesis and accept the alternative hypothesis that the probability of getting True is statistically different than 0.5.

1.5 Fitting models to separated samples

```r
##Fit linear models to the separated samples, look at estimators.
Linear.Model.Steeper.Recovered <- lm(Subsample.Steeper$steeperOutput ~ Subsample.Steeper$steeperInput)
Linear.Model.Flatter.Recovered <- lm(Subsample.Flatter$flatterOutput ~Subsample.Flatter$flatterInput)
rbind(Steeper.Coefficients=Linear.Model.Steeper.Recovered$coefficients,
      Flatter.Coefficients=Linear.Model.Flatter.Recovered$coefficients)
```

```
##                      (Intercept) Subsample.Steeper$steeperInput
## Steeper.Coefficients   0.9325475                      1.0517077
## Flatter.Coefficients  -0.3467106                      0.8630519
```

```r
summary(Linear.Model.Steeper.Recovered)$r.sq
```

```
## [1] 0.9365043
```

```r
summary(Linear.Model.Flatter.Recovered)$r.sq
```

```
## [1] 0.902158
```

1.6 Analyze the residuals

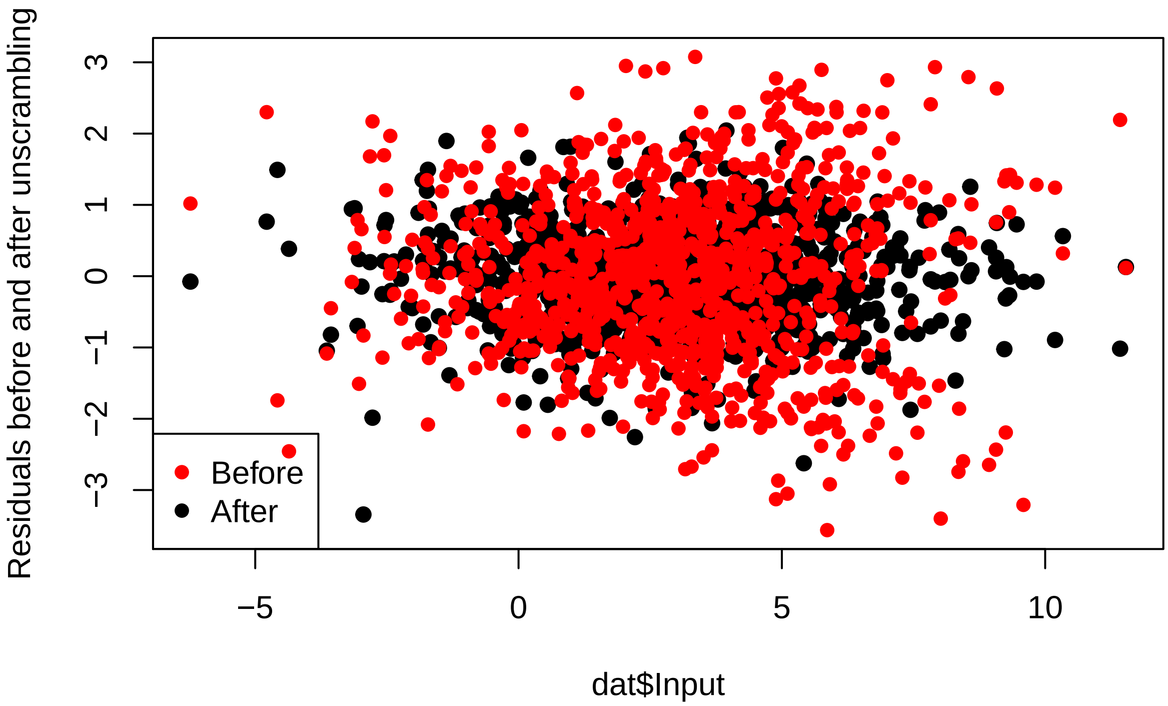
```r
# Plot residuals,compare the difference before splitting
matplot(dat$Input,cbind(c(summary(Linear.Model.Steeper.Recovered)$residuals,
                          summary(Linear.Model.Flatter.Recovered)$residuals),
                        estimatedResiduals),type="p",pch=c(19,16),ylab="Residuals before and after unsc
legend("bottomleft",legend=c("Before","After"),col=c("red","black"),pch=16)
# Estimate standard deviations
unmixedResiduals<-c(summary(Linear.Model.Steeper.Recovered)$residuals,
                                    summary(Linear.Model.Flatter.Recovered)$residuals)
apply(cbind(ResidualsAfter=unmixedResiduals,
            ResidualsBefore=estimatedResiduals),2,sd)
```

```
##  ResidualsAfter ResidualsBefore
##       0.6863947       1.1564568
```

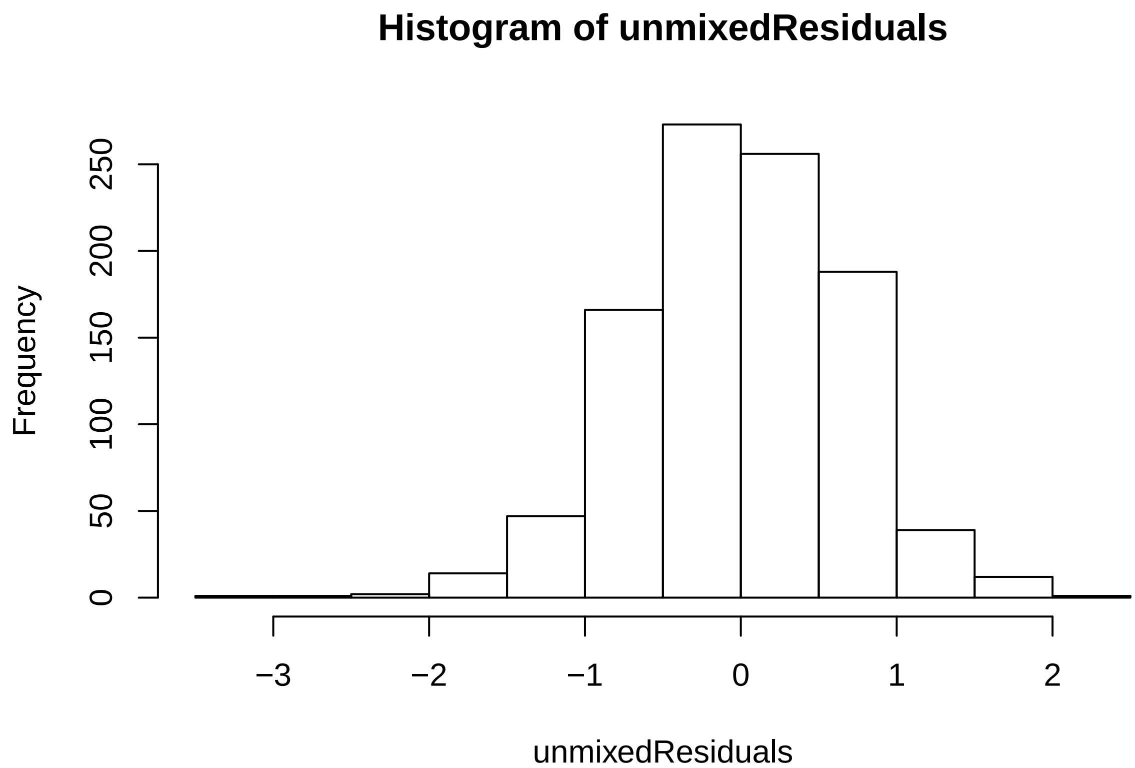
```r
suppressWarnings(library(fitdistrplus))
```

```
## Loading required package: MASS
```

```
## Loading required package: survival
```




```r
hist(unmixedResiduals)
```



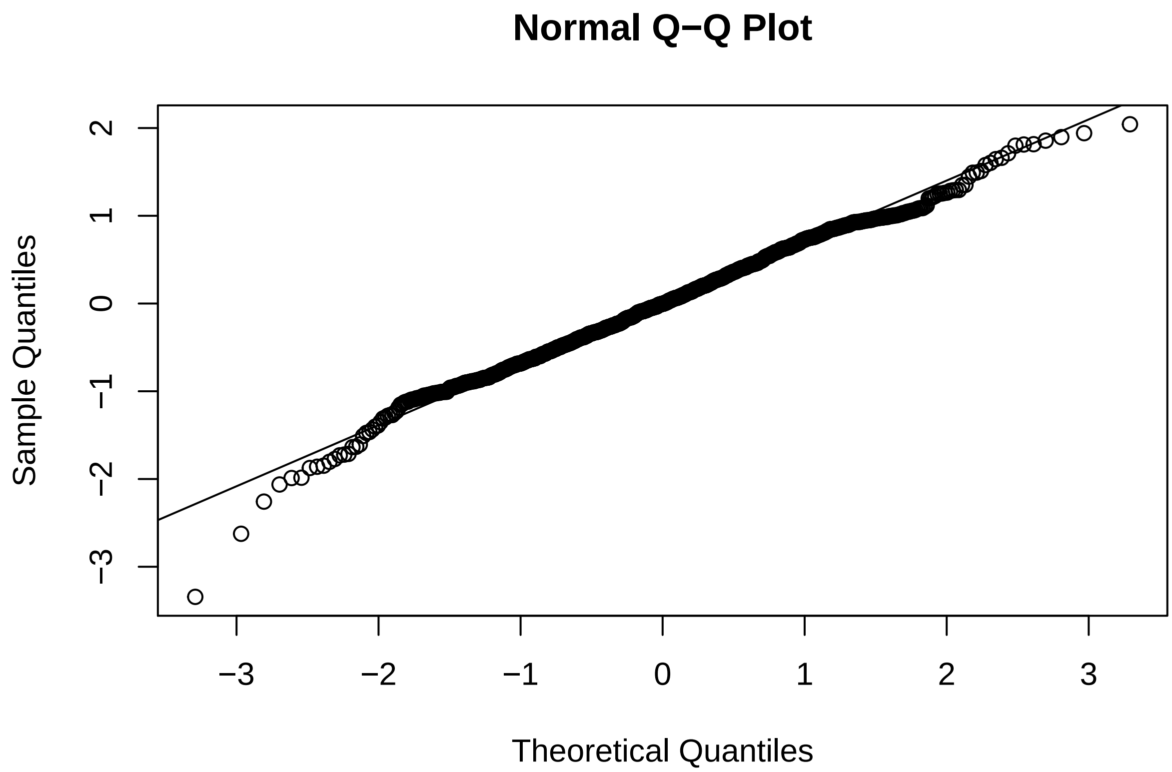

```
(residualsParam<-fitdistr(unmixedResiduals,"normal"))
```

```
##        mean             sd
##   3.242171e-18   6.860514e-01
##  (2.169485e-02) (1.534058e-02)
```

```
ks.test(unmixedResiduals,"pnorm",residualsParam$estimate[1],residualsParam$estimate[2])
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  unmixedResiduals
## D = 0.023344, p-value = 0.6471
## alternative hypothesis: two-sided
```

```
qqnorm(unmixedResiduals)
qqline(unmixedResiduals)
```




```
# Slopes
c(Steeper.SLope=Linear.Model.Steeper.Recovered$coefficients[2],Flatter.Slope=Linear.Model.Flatter.Recove
```

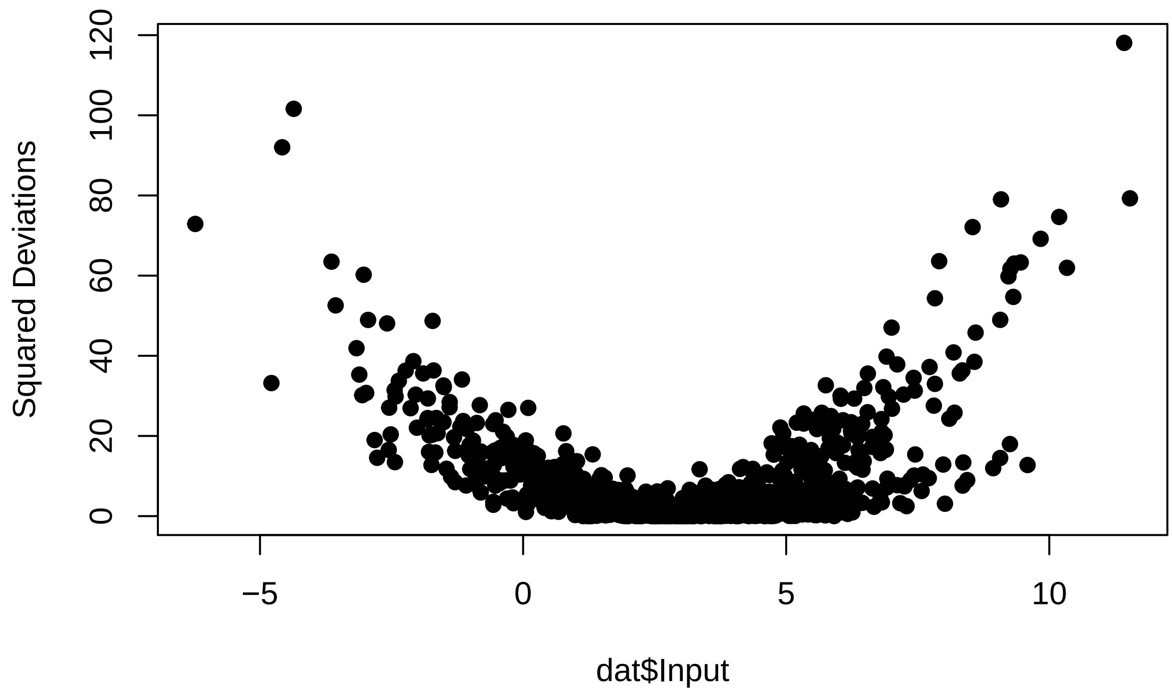
```
## Steeper.SLope.Subsample.Steeper$steeperInput
##                                    1.0517077
## Flatter.Slope.Subsample.Flatter$flatterInput
##                                    0.8630519
```

```
# Intercepts
c(Steeper.Intercept=Linear.Model.Steeper.Recovered$coefficients[1],Flatter.Intercept=Linear.Model.Flatte
```

```
## Steeper.Intercept.(Intercept) Flatter.Intercept.(Intercept)
##                     0.9325475                    -0.3467106
```

2 Alternative Method Based on Volatility Clustering

```
plot(dat$Input,(dat$Output-mean(dat$Output))^2, type="p",pch=19,
     ylab="Squared Deviations")
```

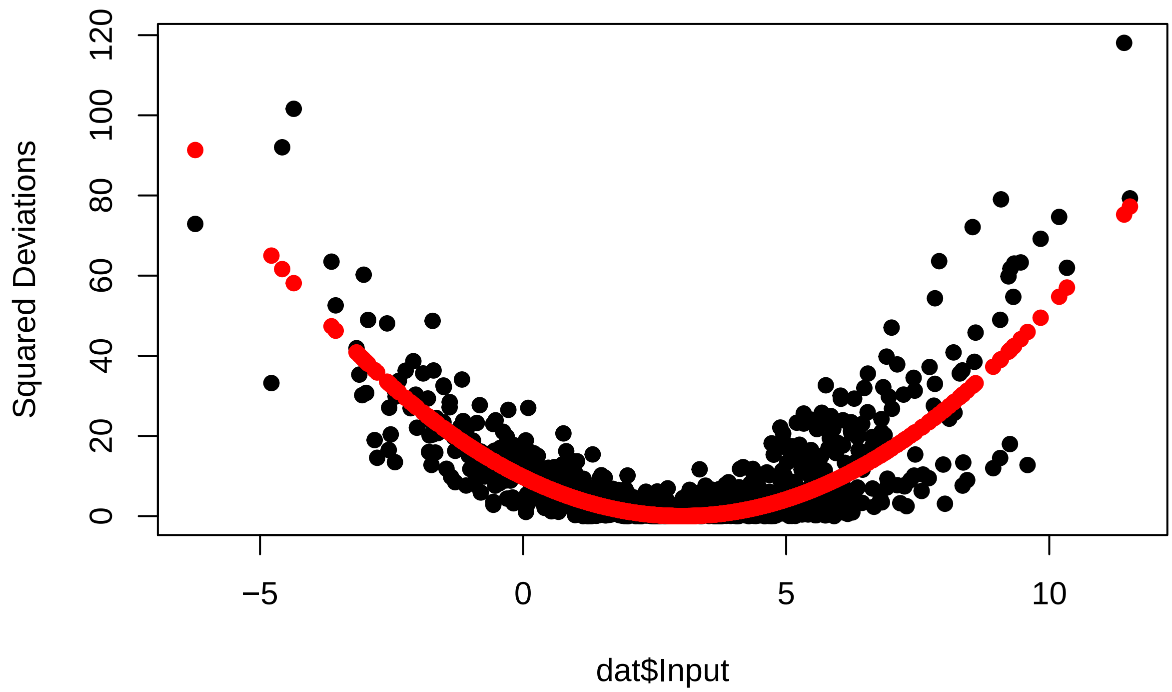


Explain how increased slope affects variance of the output and the pattern of variables zi. It appears that increasing the slope will increase the variance of the output.The pattern of zi's is steeper and farther spread apart. *What are the differences between the shapes of parabolas corresponding to a steeper slope versus flatter slope?* A steeper slope makes the parabolic output of the pattern narrower/taller, while a flatter slope makes the parabola wider/shorter.

```
##Create clustering parabola
y.bar <- mean(dat$Output)

B0.hat <- m1$coefficients[1]
B1.hat <- m1$coefficients[2]
y.i = B0.hat + B1.hat*dat$Input
clusteringParabola <- (y.i-y.bar)^2

plot(dat$Input,(dat$Output-mean(dat$Output))^2, type="p",pch=19,
     ylab="Squared Deviations")
points(dat$Input,clusteringParabola,pch=19,col="red")
```




```
##Define the separating sequence
Unscrambling.Sequence.Steeper.var <- (dat$Output-mean(dat$Output))^2 > clusteringParabola
head(Unscrambling.Sequence.Steeper.var,10)
```

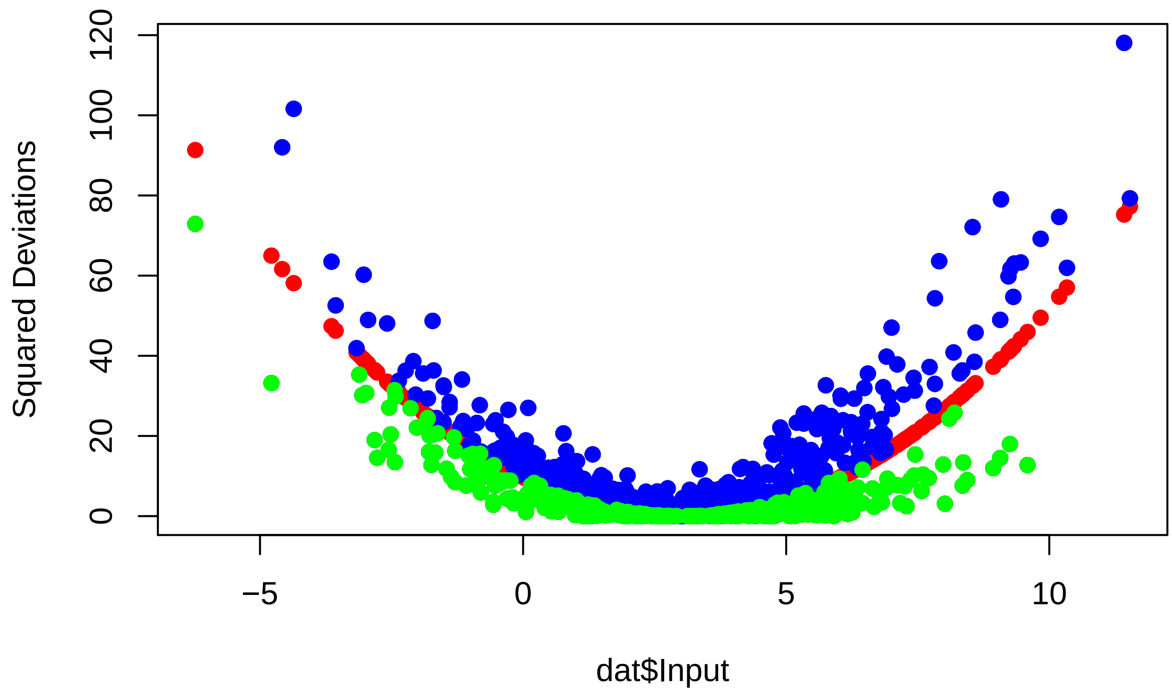
```
##  [1]  TRUE  TRUE  TRUE  TRUE  TRUE FALSE  TRUE FALSE  TRUE  TRUE
```

```
##Separate the samples into two data frames
Subsample.Steeper.var<-
  data.frame(steeperInput.var=dat$Input,steeperOutput.var=rep(NA,nSample))
Subsample.Flatter.var<-
  data.frame(flatterInput.var=dat$Input,flatterOutput.var=rep(NA,nSample))
##Fill in the unscrambled outputs instead of NAs where necessary
Subsample.Steeper.var[Unscrambling.Sequence.Steeper.var,2]<-
  dat[Unscrambling.Sequence.Steeper.var,2]
Subsample.Flatter.var[!Unscrambling.Sequence.Steeper.var,2]<-
  dat[!Unscrambling.Sequence.Steeper.var,2]
##Print head of sample
head(cbind(dat,Subsample.Steeper.var,Subsample.Flatter.var),10)
```

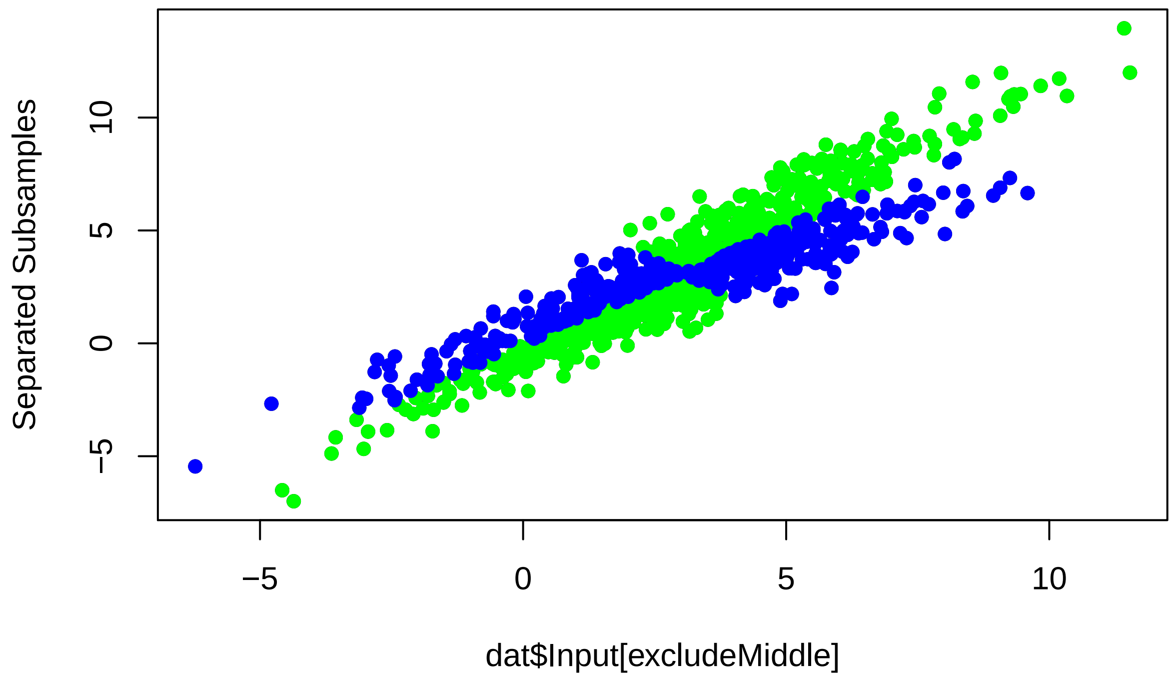
```
##        Input      Output steeperInput.var steeperOutput.var
## 1  3.1328589  4.17792255        3.1328589        4.17792255
## 2  5.5611337  5.84669919        5.5611337        5.84669919
## 3  1.9845429 -0.09834184        1.9845429       -0.09834184
## 4  5.6191601  7.84692946        5.6191601        7.84692946
## 5  6.3781486  7.57941491        6.3781486        7.57941491
```

```
## 6  6.1232040  5.68973137        6.1232040                NA
## 7  0.7666195 -1.45675560        0.7666195       -1.45675560
## 8  4.3535141  4.16746077        4.3535141                NA
## 9  2.3627156  2.38611901        2.3627156        2.38611901
## 10 6.3272368  6.77354738        6.3272368        6.77354738
##    flatterInput.var flatterOutput.var
## 1         3.1328589                NA
## 2         5.5611337                NA
## 3         1.9845429                NA
## 4         5.6191601                NA
## 5         6.3781486                NA
## 6         6.1232040          5.689731
## 7         0.7666195                NA
## 8         4.3535141          4.167461
## 9         2.3627156                NA
## 10        6.3272368                NA
```

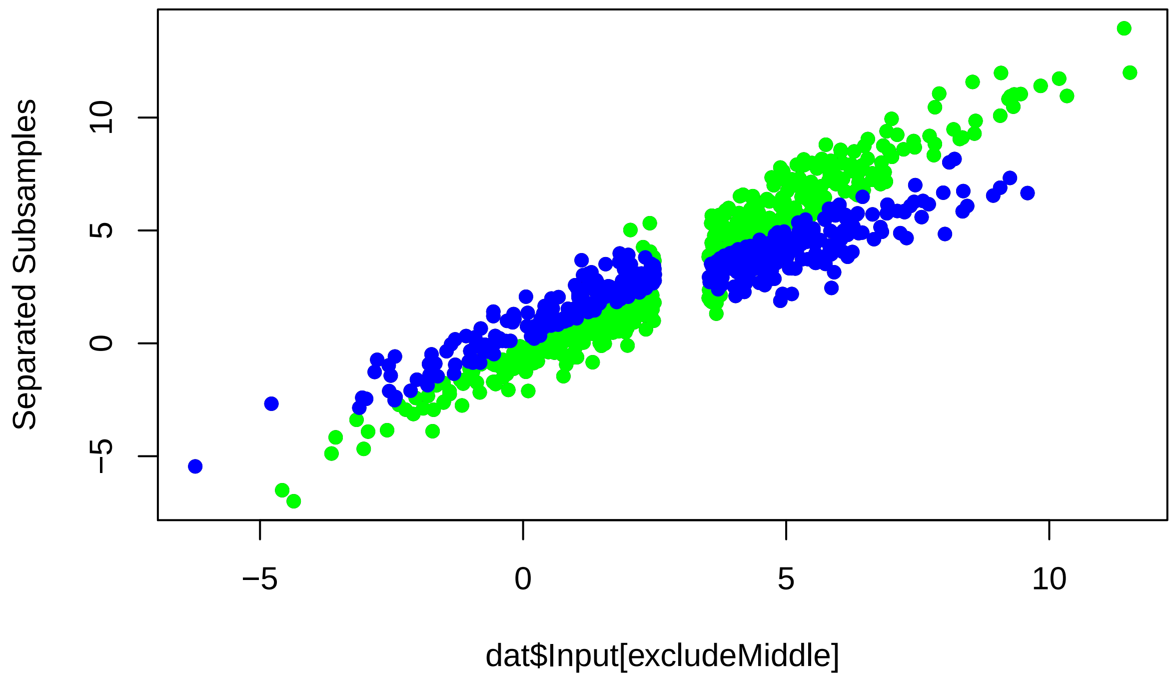
```r
##Plot clusters of variance data and the separating parabola
plot(dat$Input,
     (dat$Output-mean(dat$Output))^2,
     type="p",pch=19,ylab="Squared Deviations")
points(dat$Input,clusteringParabola,pch=19,col="red")
points(dat$Input[Unscrambling.Sequence.Steeper.var],
       (dat$Output[Unscrambling.Sequence.Steeper.var]-
          mean(dat$Output))^2,
       pch=19,col="blue")
points(dat$Input[!Unscrambling.Sequence.Steeper.var],
       (dat$Output[!Unscrambling.Sequence.Steeper.var]-
          mean(dat$Output))^2,
       pch=19,col="green")
```




```
## Plot the unscrambled samples
excludeMiddle<-(dat$Input<=mean(dat$Input)-0)|
                 (dat$Input>=mean(dat$Input)+0)
matplot(dat$Input[excludeMiddle],cbind(dat$Output[excludeMiddle],
                                       Subsample.Steeper.var$steeperOutput.var[excludeMiddle],
                                       Subsample.Flatter.var$flatterOutput.var[excludeMiddle]),
        type="p",col=c("black","green","blue"),
        pch=16,ylab="Separated Subsamples")
```




```
##Omit interval
excludeMiddle<-(dat$Input<=mean(dat$Input)-0.5)|
                 (dat$Input>=mean(dat$Input)+0.5)
matplot(dat$Input[excludeMiddle],cbind(dat$Output[excludeMiddle],
                                       Subsample.Steeper.var$steeperOutput.var[excludeMiddle],
                                       Subsample.Flatter.var$flatterOutput.var[excludeMiddle]),
        type="p",col=c("black","green","blue"),
        pch=16,ylab="Separated Subsamples")
```




```
##fit linear models to the separated samples

dat.Steep.var <- lm(Subsample.Steeper.var$steeperOutput.var[excludeMiddle]~Subsample.Steeper.var$steepe
dat.Flat.var <- lm(Subsample.Flatter.var$flatterOutput.var[excludeMiddle]~Subsample.Flatter.var$flatter


##Plot the data and regression lines
##plot(dat$Input,dat$Output, type="p",pch=19)
##lines(dat$Input,predict(dat.Steep.var,
##                          data.frame(trainSteepInput=dat$Input),
##                          interval="prediction")[,1],col="red",lwd=3)
##lines(dat$Input,predict(dat.Flat.var,data.frame(trainFlatInput=dat$Input),
##                          interval="prediction")[,1],col="green",lwd=3)

##Print estimated parameters and summaries of both models
rbind(Steeper.Coefficients.var=dat.Steep.var$coefficients,
      Flatter.Coefficients.var=dat.Flat.var$coefficients)
```

```
##                          (Intercept)
## Steeper.Coefficients.var  -0.6944529
## Flatter.Coefficients.var   0.7454309
##                          Subsample.Steeper.var$steeperInput.var[excludeMiddle]
## Steeper.Coefficients.var                                             1.3045575
## Flatter.Coefficients.var                                             0.6950256
```

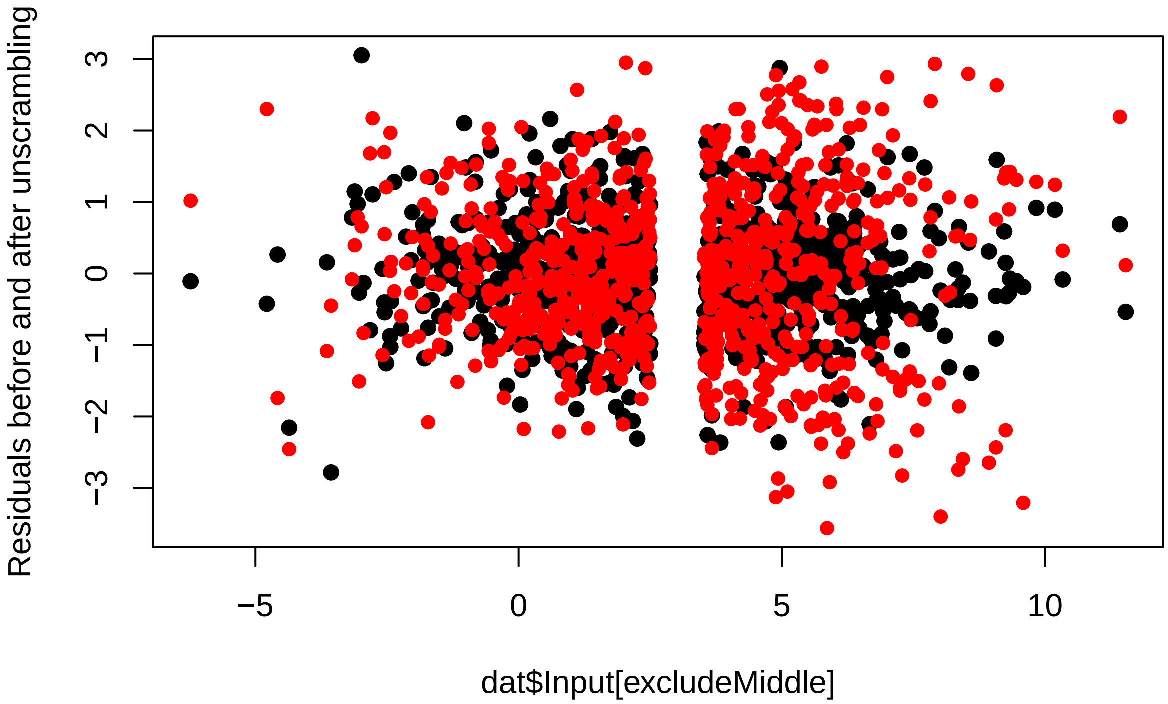
```r
summary(dat.Steep.var)
```

```
##
## Call:
## lm(formula = Subsample.Steeper.var$steeperOutput.var[excludeMiddle] ~
##     Subsample.Steeper.var$steeperInput.var[excludeMiddle])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.78292 -0.45940 -0.04679  0.48595  3.05298
##
## Coefficients:
##                                                       Estimate Std. Error
## (Intercept)                                           -0.69445    0.06083
## Subsample.Steeper.var$steeperInput.var[excludeMiddle]  1.30456    0.01443
##                                                       t value Pr(>|t|)
## (Intercept)                                            -11.42   <2e-16 ***
## Subsample.Steeper.var$steeperInput.var[excludeMiddle]   90.42   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8488 on 441 degrees of freedom
##   (382 observations deleted due to missingness)
## Multiple R-squared:  0.9488, Adjusted R-squared:  0.9487
## F-statistic:  8176 on 1 and 441 DF,  p-value: < 2.2e-16
```

```r
summary(dat.Flat.var)
```

```
##
## Call:
## lm(formula = Subsample.Flatter.var$flatterOutput.var[excludeMiddle] ~
##     Subsample.Flatter.var$flatterInput.var[excludeMiddle])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.3651 -0.4251  0.0361  0.5039  2.1618
##
## Coefficients:
##                                                       Estimate Std. Error
## (Intercept)                                            0.74543    0.05434
## Subsample.Flatter.var$flatterInput.var[excludeMiddle]  0.69503    0.01369
##                                                       t value Pr(>|t|)
## (Intercept)                                             13.72   <2e-16 ***
## Subsample.Flatter.var$flatterInput.var[excludeMiddle]   50.75   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7309 on 380 degrees of freedom
##   (443 observations deleted due to missingness)
## Multiple R-squared:  0.8714, Adjusted R-squared:  0.8711
## F-statistic:  2576 on 1 and 380 DF,  p-value: < 2.2e-16
```

```r
##Plot the residuals from the combined model and the separated sample models
matplot(dat$Input[excludeMiddle],
        cbind(c(summary(dat.Steep.var)$residuals,
                summary(dat.Flat.var)$residuals),
              estimatedResiduals[excludeMiddle]),
        type="p",pch=c(19,16),ylab="Residuals before and after unscrambling")
```



3 Answer the Question on Slide 10 of the Lecture Notes.

The statement is false. The two expressions are equal. The LHS numerator (cov(y,x))can be rewritten as sum (yi-meany)(xi-meanx). If we expand this expression, we can factor yi outside the first summation and factor out mean y outside the 2nd summation. Then in the 2nd expression, we can apply the summation to both terms xi and mean x. these terms both reduce to n*meanx* - *m*meanx, which cancels itself out and reduces to zero. So we are only left with yi(sum(xi-meanx)), which matches the expression on the RHS numerator