Homework #4(Week 5) Vector Spaces II – MSCA 32010

John Navarro

1. U0 = $\begin{matrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \end{matrix}$ R0 = $\begin{matrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$

   R0 * X = 0 $\begin{matrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} * \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$ so $\begin{matrix} x_1 + 2x_2 = 0 \\ x_3 = 0 \end{matrix}$ Then x = $\begin{matrix} 2 \\ -1 \\ 0 \end{matrix}$

   Check it by U*x = 0

   $$\begin{matrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{matrix} * \begin{matrix} 2 \\ -1 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

   Uc = $\begin{matrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 4 & 8 \end{matrix}$ Rd = $\begin{matrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{matrix}$

   Rd * x = d $\begin{matrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} * \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} = \begin{matrix} -1 \\ 2 \end{matrix}$ so $\begin{matrix} x_1 + 2x_2 = -1 \\ x_3 = 2 \end{matrix}$ Then x = $\begin{matrix} -3 \\ 1 \\ 2 \end{matrix}$

   Check it by U*x = c

   $$\begin{matrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{matrix} * \begin{matrix} -3 \\ 1 \\ 2 \end{matrix} = \begin{matrix} 5 \\ 8 \end{matrix}$$

2. Yes it is true. If Ax = b = Cx, then Ax = Cx and we can say that A = C, for a given x
3. There can be 3 independent vectors

   V2 = v1 + v4 and is not independent

   V3 = v1 + v4 + v6 and is not independent

   V5 = v4 + v6 and is not independent

   That leaves us with 3 independent vectors, v1, v4 and v6.

   You can also put the 6 vectors in a matrix and use Gauss Jordan elimination, which gives us 3 pivots, and 3 free variables, so we have 3 independent vectors.

4a. Basis for the plane x-2y+3z=0, we can use 2 special solutions to find a basis for this plane. Using (0,1) and (1,0) for y and z, we get

$$s_1 = \begin{matrix} 3 \\ 0 \\ 1 \end{matrix} \qquad s_2 = \begin{matrix} 2 \\ 1 \\ 0 \end{matrix}$$ These two vectors form a basis for the plane.

4b. The intersection of the plane and the xy plane is a line. If we set z=0, x-2y = 0 -> y =x/2

Then this entire line, which passes through the origin forms the basis of the intersection. If we set x =2, a special solution would be

$$s_1 = \begin{matrix} 2 \\ 1 \end{matrix}$$

4c. If 2 vectors are perpendicular, then their dot product equals 0. Since the original equation, is a linear combination of 2 vectors (x, y, z) and (1,-2, 3) that equals 0. Then we can say that these two vectors are perpendicular. The Span of {1,-2,3} are all the vectors perpendicular to the plane. And the vector (1,-2,3) forms a basis for those vectors.