

Homework #3(Week 4) – MSCA 32010

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1. The missing components are 12,3,1.

$$\begin{array}{rcl} x & 12 & 3 & 1 \\ y = & 0 & + y & 1 + z & 0 \\ z & 0 & 0 & 1 \end{array}$$

2. $N(C) = N(A) \cap N(B)$

3a. $A = \begin{array}{cccc} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 2 & -2 & 11 & -3 \end{array}$ The reduced row form of this matrix is

$$R = \begin{array}{cccc} 1 & 0 & -23/4 & 1/4 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & 0 & 0 & 0 \end{array}$$

- 3b. Since it has 2 pivots, this matrix is rank 2.

3c. $S_1 = \begin{array}{c} 23/4 \\ -1/4 \end{array} \quad S_2 = \begin{array}{c} -1/4 \\ -7/4 \end{array}$

$$\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

Bonus

Since B has a rank 1, then we can multiply it by the Identity matrix to so A_1B has rank 1

$$A_1B * B = A_1B \quad A_1 * \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} = \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}$$

$$\text{Therefore, } A_1 = \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

A_2 will be the zero vector, since A_2B is a matrix with rank 0, it has no pivots

$$A_2 * B = A_2B \quad A_2 * \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} = \begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}$$

$$\text{Therefore, } A_2 = \begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}$$

