Homework week #2- Solving Linear Equations – Due 10/17

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1

$$x_1 \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} + x_2 * \begin{matrix} 4 \\ 5 \\ 6 \end{matrix} + x_3 * \begin{matrix} 7 \\ 8 \\ 9 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$x_1 + 4x_2 + 7x_3 = 0$

$2x_1 + 5x_2 + 8x_3 = 0$

$3x_1 + 6x_2 + 9x_3 = 0$

$\begin{matrix} 1 & 4 & 7 & 0 \\ 2 & 5 & 8 \,| & 0 \\ 3 & 6 & 9 & 0 \end{matrix}$ Multiply 1st row by 2, and subtract that from the second row and replace

$\begin{matrix} 1 & 4 & 7 & 0 \\ 0 & -3 & -6 \,| & 0 \\ 3 & 6 & 9 & 0 \end{matrix}$ Multiply 1st row by 3 and subtract that from 3rd row and replace

$\begin{matrix} 1 & 4 & 7 & 0 \\ 0 & -3 & -6 \,| & 0 \\ 0 & -6 & -12 & 0 \end{matrix}$ Multiply 2nd row by 2 and subtract that from 3rd row and replace

$\begin{matrix} 1 & 4 & 7 & 0 \\ 0 & -3 & -6 \,| & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ Since there are no pivots in the 3rd column, these vectors are dependent.

These three vectors lie on a line.

2

$$\begin{matrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 4 & 1 & 1 \end{matrix} * \begin{matrix} 3 \\ -2 \\ 1 \end{matrix} = \begin{matrix} 1*3+2*-2+0*1 \\ 2*3+0*-2+3*1 \\ 4*3+1*-2+1*1 \end{matrix} = \begin{matrix} -1 \\ 9 \\ 11 \end{matrix}$$

3 False. A 5x3 matrix multiplied by a 3x4 matrix will give a 5x4 matrix.

4 A multiple of 3 should be used.

$\begin{matrix} 2 & 3 \\ 6 & 15 \end{matrix} \,|\, \begin{matrix} 5 \\ 12 \end{matrix}$ The first pivot is in location (1,1) value of 2. The second pivot is in location (2,2)

Multiply the first row by 3 and subtract it from the second row and replace.

$$\begin{matrix} 2 & 3 \\ 0 & 6 \end{matrix} \,|\, \begin{matrix} 5 \\ -3 \end{matrix}$$

Now we can use the bottom row and say 6y = -3 therefore y = $-\frac{1}{2}$

Using that in the first equation, we can now solve for x

2x +3($-\frac{1}{2}$) = 5

$2x = \frac{13}{2}$ therefore, $x = \frac{13}{4}$ $y = -\frac{1}{2}$

5 Use Gauss-Jordan elimination on [U I] to find the upper triangular U^{-1}:

$$\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$ multiply row 3 by c and subtract from row 2

$$\begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$ multiply row 2 by a and subtract from row 1

$$\begin{array}{ccc|ccc} 1 & 0 & b & 1 & -a & ac \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$ multiply row 3 by b and subtract from row 1

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} = U^{-1}$$

Bonus Find the conditions on a and b that makes matrix A invertible and find A^{-1}:

$$\begin{array}{ccc} a & b & b \\ a & a & b \\ a & a & a \end{array} = A$$

$$\begin{array}{ccc} a^2-ab & a^2-ab & a^2-a^2 \\ ab-ab & a^2-ab & a^2-ab \\ b^2-ab & ab-ab & a^2-ab \end{array}$$ Matrix of minors

$$\begin{array}{ccc} a^2-ab & -a^2+ab & a^2-a^2 \\ -ab+ab & a^2-ab & -a^2+ab \\ b^2-ab & -ab+ab & a^2-ab \end{array}$$ Matrix of cofactors

$$\begin{array}{ccc} a^2-ab & 0 & b^2-ab \\ -a^2+ab & a^2-ab & 0 \\ 0 & -a^2+ab & a^2-ab \end{array}$$ Adjudicate

$a(a^2-ab)-b(a^2-ab)+b(a^2-a^2) = a^3-2a^2b+ab^2$ Determinant

$$\begin{array}{ccc} \frac{a^2-ab}{a^3-2a^2b+ab^2} & 0 & \frac{b^2-ab}{a^3-2a^2b+ab^2} \\ \frac{-a^2+ab}{a^3-2a^2b+ab^2} & \frac{a^2-ab}{a^3-2a^2b+ab^2} & 0 \\ 0 & \frac{-a^2+ab}{a^3-2a^2b+ab^2} & \frac{a^2-ab}{a^3-2a^2b+ab^2} \end{array} = A^{-1}$$