4: Linear Regression 1

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4.2 Review of least squares method

- Least Squares Estimator

$$\hat{\beta}_{LS} = \hat{\beta}_{LS}(\mathbf{X}, \mathbf{y}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

- Full rank: X must be full rank, otherwise collinear
- Collinear: we one variable depends on another
- Identifiable: model is identifiable if $\beta_1 \neq \beta_2$ then $X\beta_1 \neq X\beta_2$ for all β_1, β_2
- Desirable properties: allow us to know how the estimator change under transformations
  a. Regression

$$\hat{\beta}_{LS}(\mathbf{X}, \mathbf{y} + \mathbf{X}\gamma) = \hat{\beta}_{LS}(\mathbf{X}, \mathbf{y}) + \boldsymbol{\gamma} \qquad \text{for all } \boldsymbol{\gamma} \in R^p$$

  b. Scale

$$\hat{\beta}_{LS}(\mathbf{X}, \lambda\mathbf{y}) = \lambda\hat{\beta}_{LS}(\mathbf{X}, \mathbf{y}) \qquad \text{for all } \lambda \in R$$

  c. Affine Equivariant

$$\hat{\beta}_{LS}(\mathbf{XA}, \mathbf{y}) = A^{-1}\hat{\beta}_{LS}(\mathbf{X}, \mathbf{y})$$

for all nonsingular $p \times p$ matrices A

- Distribution of estimators

If the u_i are normal and $\mathbf{X}$ is of full rank, then $\hat{\beta}_{LS}$ is multivariate normal

$$\hat{\beta}_{LS} \sim \mathrm{N}_p(\beta, \sigma^2(\mathbf{X}'\mathbf{X})^{-1}),$$

4.3 Classical methods for outlier detection

- Leverage of $x_1, \ldots, x_n$ are the diagnoals elements of hat matrix H which is the orthogonal projection on the image of X

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \qquad \text{and} \qquad h_i = \mathbf{x}_i'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_i.$$

  they measure the influence of one x_i on LS estimator
- Cook's distance of z_i

$$D_i = \frac{1}{p^* s^2}\|\hat{\mathbf{y}}_{(i)} - \hat{\mathbf{y}}\|^2$$

  where $p^* = rank(X)$ and s^2 is the residual standard deviation estimator

$$s^2 = \frac{1}{n - p^*}\sum_{i=1}^{n} r_i^2$$

4.4 Regression M-estimators

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4.5 Numerical computing of monotone M-estimators

4.6 BP of monoton regression estimators

4.7 Robust tests for linear hypothesis

4.8 Regression Quantiles