The 3rd Degree PHYSICAL Motion Model

**Input:**

* Jerks: , , ,
* Accelerations: ,
* Velocity:
* Time:

**Output:**

* Distance:
* Modified accelerations: , and velocity: (if applicable)

**Algorithm:**

The whole motion can be broken down into seven phases (jerked, uniformly accelerated, jerked, uniform velocity, jerked, uniformly decelerated, jerked). With low values of time, some of them may be reduced; this will result in the system never to reach its nominal acceleration or, in some cases, velocity.

First, find the harmonic mean of the jerks:

Combined jerks and , acting over and to achieve the acceleration , cannot exceed velocity . This constrains the achievable accelerations as follows:

The time for jerked motion is now given as:

The total time for jerked motion should not exceed the time :

If the total time is exceeded, the system will never reach the nominal acceleration and velocity values, the phases of jerked motion must be reduced, and there will be no uniform acceleration or uniform velocity phases ():

Now, velocity gain in each of the seven phases can be easily calculated:

Constraints on the acceleration values calculated so far guarantee that and

Also, the time values for the uniform acceleration phase may be calculated:

Again, the total time for jerked and accelerated motion should not exceed the time t:

If the total time is exceeded, the phases of uniform acceleration and deceleration must be reduced, and there will be no uniform velocity phase ():

where

With some configurations of time and acceleration, it may still happen that or . This will lead to decreasing the jerked motion phases and reducing both the maximum acceleration and the velocity. See the appendix for detailed discussion.

For the case it is:

For the case it is:

In all cases, velocity gains need to be re-calculated:

The reduced, new maximum velocity is given as:

Finally, the time of uniform velocity motion is given as:

Velocity values at the end of each phase are given as:

And, finally, the distance travelled in each phase:

The total distance is given as:

APPENDIX

After the and times are calculated, it may happen that one of them is less than zero ( or ). The calculation procedure must then include reduction to either or acceleration as well as maximum velocity .

This Appendix discusses the latter case of , the other one is similar.

As shown in the diagram on the left, the problem appears if the velocity gained in the accelerated stage of the motion cannot be compensated by decelerated phase, even if and (the maximum amount available). This may happen if the mean jerk is too slow or the acceleration is too high.

It is assumed that the jerk values remain intact, but by decreasing the times and both the acceleration, , and velocity become reduced. It is also assumed that to achieve optimal performance there should be no uniform velocity period, therefore the uniform deceleration phase extends, accordingly increasing deceleration effectiveness: .

This leads to following equations:

Substituting:

we get:

One can easily find solving this quadratic equation, and subsequently finding and values as inversely proportional to the consecutive jerks. These results are presented in the main text.