Robot's trajectories based on the speed of the wheels

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ω_l is the left wheel's angular velocity expressed in radians/seconds.

ω_r is the right wheel's angular velocity expressed in radians/seconds.

r is the wheel's radius expressed in meters.

L is the distance between wheels expressed in meters.

T is the movement time.

The most general trajectory that a robot can describe when the speed of the wheels are different (different in magnitude and sign), and remain constant for T seconds, is a circumference of radius R. This trajectory can be simplified depending of how these speeds are related.

In the next section the equations for the most general case are derived. A graph is provided for clarity purposes. In the graph it has been considered:

$$\omega_l \; != \; \omega_r$$

with ω_l and ω_r positive, so the robot moves forwards.

(NOTE: These equations works for every possible value of ω_l and ω_r, but for clarity issues I chose positive values for the speed of the wheels in order to get the robot moves forwards in the graph.)

Linear velocity of each wheel.

$$v_l = \omega_l \cdot r \tag{1}$$

$$v_r = \omega_r \cdot r \tag{2}$$

Turned angle for each wheel after T seconds.

$$\theta_l = \omega_l \cdot T \tag{3}$$

$$\theta_r = \omega_r \cdot T \tag{4}$$

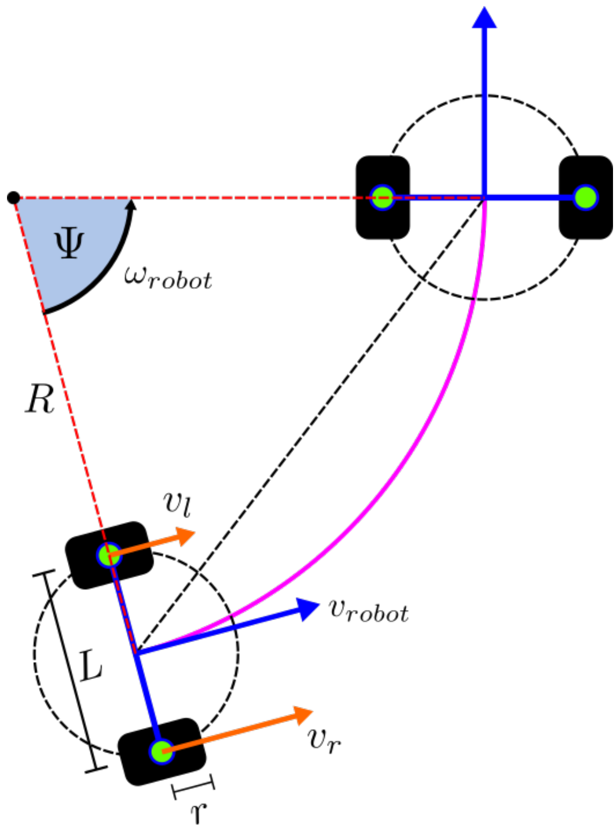

Figura 1: General trajectory

Traveled distance for each wheel after T seconds.

$$D_l = \theta_l \cdot r = \omega_l \cdot T \cdot r = v_l \cdot T \tag{5}$$

$$D_r = \theta_r \cdot r = \omega_r \cdot T \cdot r = v_r \cdot T \tag{6}$$

Turned angle by the robot after T seconds.

$$\psi = \omega_{robot} \cdot T \tag{7}$$

Arcs subtending the ψ angle for each wheel after T seconds.

$$A_l = \psi \cdot \left(R - \frac{L}{2}\right) = \omega_{robot} \cdot T \cdot \left(R - \frac{L}{2}\right) \tag{8}$$

$$A_r = \psi \cdot \left(R + \frac{L}{2}\right) = \omega_{robot} \cdot T \cdot \left(R + \frac{L}{2}\right) \tag{9}$$

Each wheel has traveled a distance D_x which is associated to the corresponding arc A_x, where $x = \{l, r\}$

$$A_l = D_l \tag{10}$$

$$A_r = D_r \tag{11}$$

$$\omega_{robot} \cdot T \cdot \left(R + \frac{L}{2}\right) = \omega_r \cdot T \cdot r \tag{12}$$

$$\omega_{robot} \cdot T \cdot \left(R - \frac{L}{2}\right) = \omega_l \cdot T \cdot r \tag{13}$$

If 12 and 13 are added:

$$2 \cdot \omega_{robot} \cdot R = (\omega_r + \omega_l) \cdot r \tag{14}$$

Using the following expression

$$v_{robot} = \omega_{robot} \cdot R \tag{15}$$

the equation for the robot's linear velocity is obtained:

$$v_{robot} = \frac{(\omega_r + \omega_l) \cdot r}{2} = \frac{v_r + v_l}{2} \tag{16}$$

If 12 and 13 are subtracted:

$$\omega_{robot} \cdot L = (\omega_r - \omega_l) \cdot r \tag{17}$$

$$\omega_{robot} = \frac{(\omega_r - \omega_l) \cdot r}{L} = \frac{v_r - v_l}{L} \tag{18}$$

An expression for the turn radius is derived using the previous results:

$$R = \frac{v_{robot}}{\omega_{robot}} = \frac{L}{2} \cdot \frac{(\omega_r + \omega_l)}{(\omega_r - \omega_l)} \tag{19}$$

Finally, an expression for ψ is obtained using the turned angle for each wheel after T seconds.

$$\psi = \omega_{robot} \cdot T = \frac{r}{L} (\omega_r - \omega_l) \cdot T = \frac{r}{L} (\theta_r - \theta_l) \tag{20}$$

Equations summary:

$$v_{robot} = \frac{v_r + v_l}{2} = \frac{(\omega_r + \omega_l) \cdot r}{2} \tag{21}$$

$$\omega_{robot} = \frac{v_r - v_l}{L} = \frac{(\omega_r - \omega_l) \cdot r}{L} \tag{22}$$

$$R = \frac{v_{robot}}{\omega_{robot}} = \frac{L}{2} \cdot \frac{(\omega_r + \omega_l)}{(\omega_r - \omega_l)} \tag{23}$$

$$\psi = \omega_{robot} \cdot T = \frac{r}{L} (\theta_r - \theta_l) = \frac{r}{L} (\omega_r - \omega_l) \cdot T \tag{24}$$

Reverse equations summary:

$$\omega_r = \frac{2 \cdot v_{robot} + L \cdot \omega_{robot}}{2 \cdot r} \tag{25}$$

$$\omega_l = \frac{2 \cdot v_{robot} - L \cdot \omega_{robot}}{2 \cdot r} \tag{26}$$

Examples:

- The robot moves in straight line if $\omega_l = \omega_r$. If $\omega_l > 0$ rad/sec the robot moves forwards. If $\omega_l < 0$ rad/sec the robot moves backwards. A straight line is a circumference with infinite radius.

  $\omega_l = \omega_r = 0{,}2$ rad/sec. $T = 2$ sec. $r = 10$ cm and $L = 30$ cm.

$$v_{robot} = 2 \text{ cm/s} \tag{27}$$

$$\omega_{robot} = 0 \text{ rad/sec} \tag{28}$$

$$R_{robot} = \infty \text{ cm} \tag{29}$$

$$\psi = 0 \text{ cm} \tag{30}$$

- The robot rotates around itself if $\omega_l = -\omega_r$. If $\omega_l > 0$ rad/sec the robot rotates to the right (clockwise). If $\omega_l < 0$ rad/sec the robot rotates to the left (counterclockwise).

$\omega_l = 0{,}2$ rad/sec. $\omega_r = -0{,}2$ rad/sec. $T = 2$ sec. $r = 10$ cm and $L = 30$ cm.

$$v_{robot} = 0 \text{ cm/s} \tag{31}$$
$$\omega_{robot} = -0{,}13 \text{ rad/sec } (-7{,}44^o/seg) \tag{32}$$
$$R_{robot} = 0 \text{ cm} \tag{33}$$
$$\psi = -0{,}26 \text{ rad } (-14{,}89^o) \tag{34}$$

- The robot describes a circumference of radius R if $\omega_l \; != \; \omega_r$. If $\omega_l > \omega_r > 0$ rad/sec the robot rotates to the right (clockwise). If $\omega_r > \omega_l > 0$ rad/sec the robot rotates to the left (counterclockwise).

  $\omega_l = 0{,}1$ rad/sec. $\omega_r = 0{,}2$ rad/sec. $T = 2$ sec. $r = 10$ cm and $L = 30$ cm.

$$v_{robot} = 1{,}5 \text{ cm/s} \tag{35}$$
$$\omega_{robot} = 0{,}03 \text{ rad/sec } (1{,}94^o/seg) \tag{36}$$
$$R_{robot} = 45 \text{ cm} \tag{37}$$
$$\psi = 0{,}06 \text{ rad } (3{,}81^o) \tag{38}$$

```
d_lEnc=p3dx.leftEncoder-self.lEnc
d_rEnc=p3dx.rightEncoder-self.rEnc

d_th = (d_rEnc-d_lEnc)*R3DX_WHEEL_RADIUS/R3DX_WHEEL_BASE
d_S=(d_rEnc+d_lEnc)*R3DX_WHEEL_RADIUS/2

self.x=self.x+d_S*math.cos(self.th+d_th/2)
self.y=self.y+d_S*math.sin(self.th+d_th/2)

self.th=self.th+d_th
if self.th>math.pi*2:
    self.th=self.th-math.pi*2
elif self.th<-math.pi*2:
    self.th=self.th+math.pi*2
```

Figura 2: Odometry equations