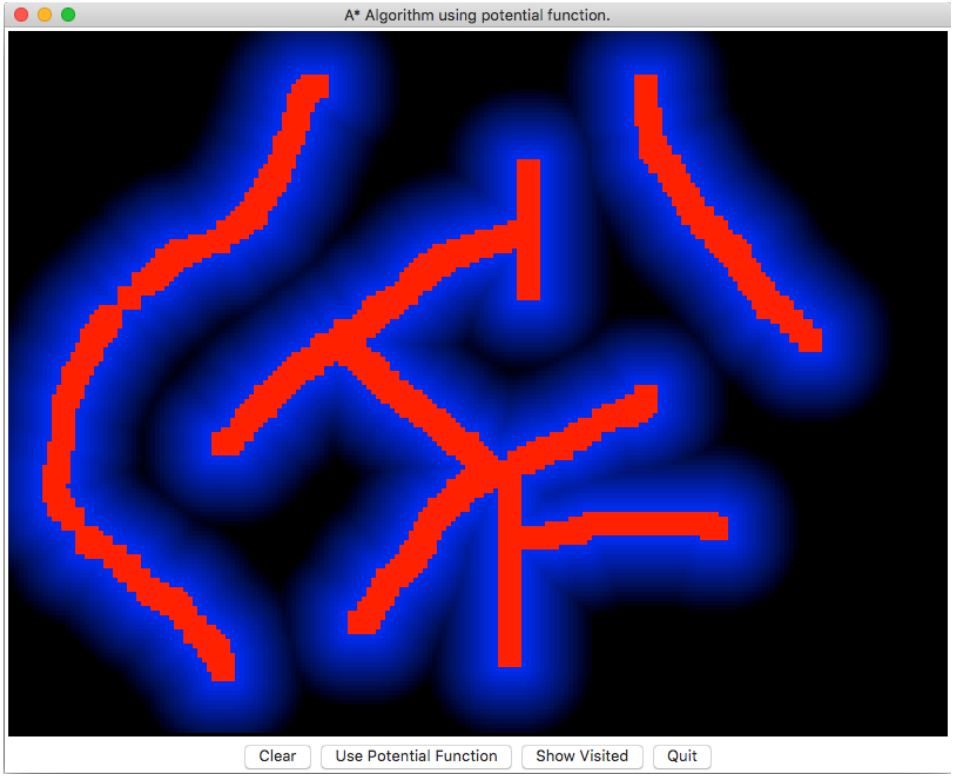
THE POTENTIAL FIELD AROUND THE OBSTACLES



Distance transform[1]

The distance transform is an operator normally only applied to **binary images**(so, binary matrices) . The result of the transform is a **graylevel image** of the **pixels** that are **inside** the **foreground regions**. The resulting image looks similar to the input image. Those graylevel intensities of the pixels that are inside the foreground regions show the **distance** to the **closest boundary** from each point (*in our case the distance from each free space node to the closest obstacle node*).

One way to think about the distance transform is to first imagine that foreground regions in the input binary image are made of some uniform slow burning inflammable material. Then consider simultaneously starting a fire at all points on the boundary of a foreground region and letting the fire burn its way into the interior. If we then label each point in the interior with the amount of time that the fire took to first reach that point, then we have effectively computed the distance transform of that region.

There are several different sorts of distance transform, depending upon which distance metric is being used to determine the distance between pixels. The most common distance metrics are: the **chessboard** distance metric, the **Euclidean** distance metric and the **city block** distance metric (aka **Manhattan** distance metric).

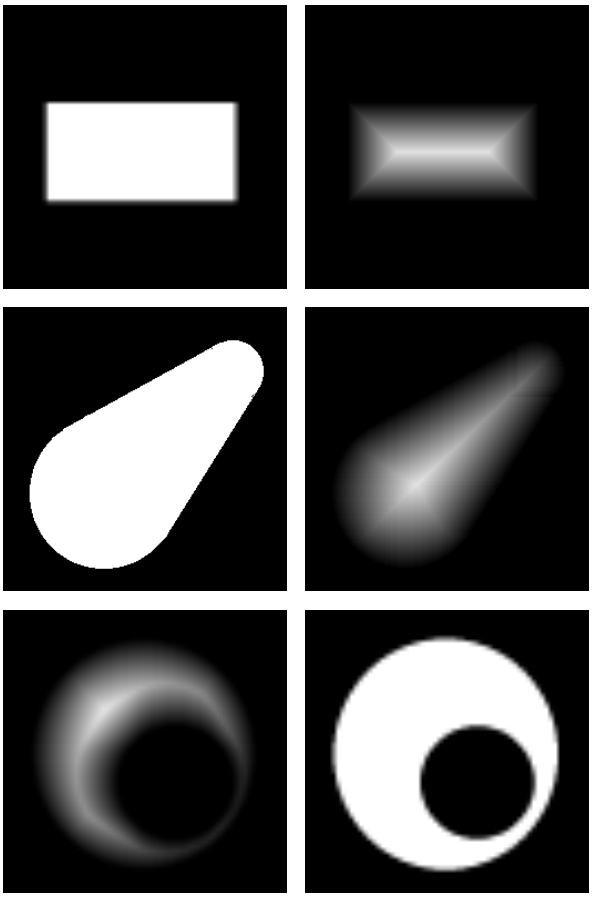


Figure 1: Euclidean distance transforms

There is a dual to the distance transform described above which produces the distance transform for the background region rather than the foreground region. It can be considered as a process of inverting the original image and then applying the standard transform as above.

[1]The previous information was gathered form: https://homepages.inf.ed.ac.uk/rbf/HIPR2/distance.htm

How it works

The distance transform can be calculated efficiently using clever algorithms in only two passes (e.g. Rosenfeld and Pfaltz 1968).

In python you should use:

```
scipy.ndimage.morphology.distance_transform_edt(input_binary_matrix)
```

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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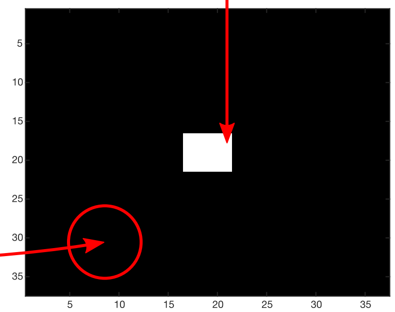
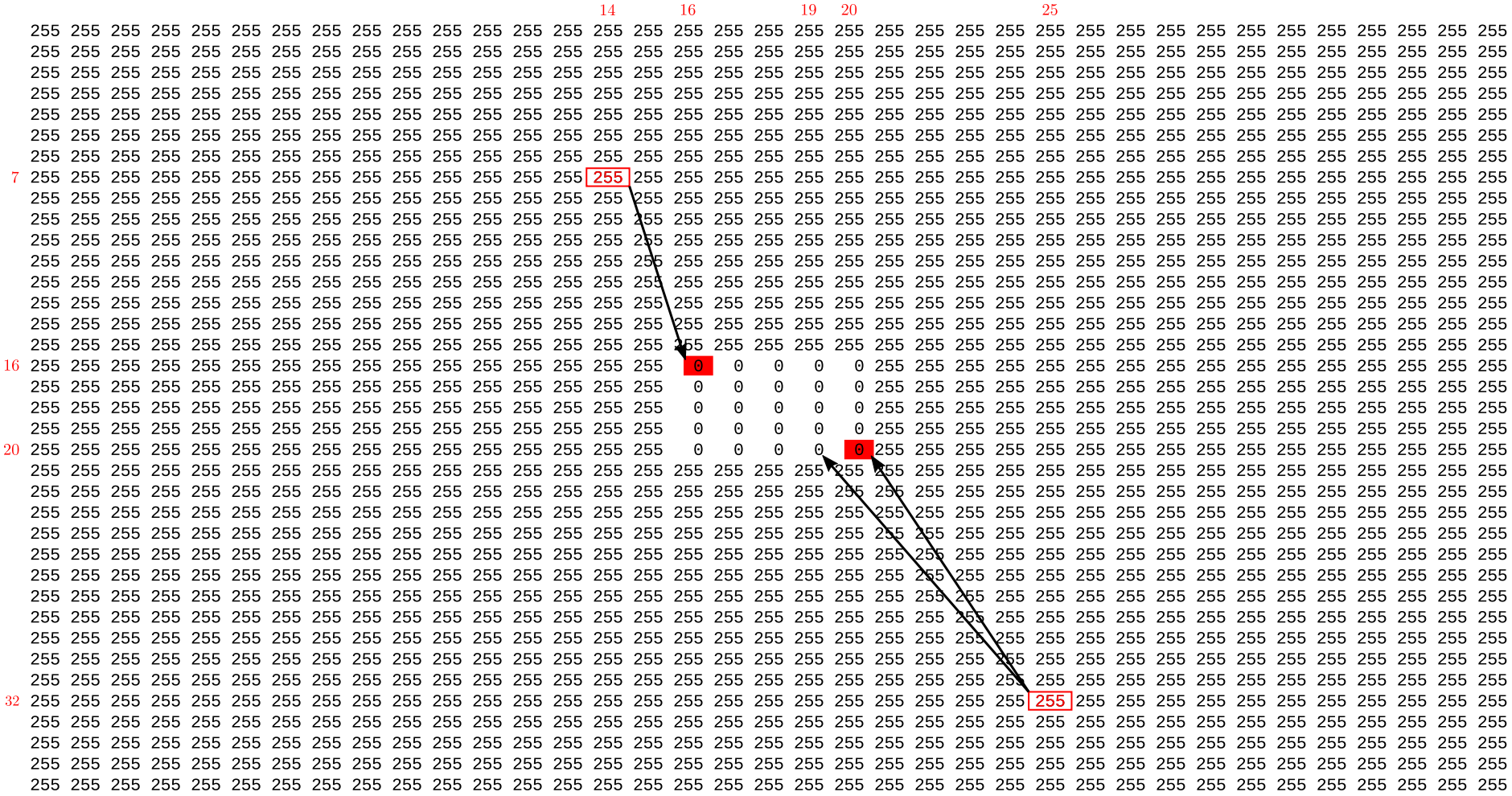
`world_obstacles`

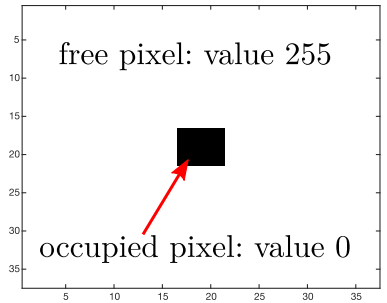


The EDT, that is going to be used, requires that a free pixel is represented with the value 255 and an occupied pixel is represented with the value 0.

But in this lecture a free pixel is represented with the value 0 and the occupied pixel is represented with the value 255.



We invert the image to use the EDT. → `255-world_obstacles`



Example of EDT for two pixels: (7, 14) and (32, 25)

$$\sqrt{(25-20)^2+(32-20)^2} = 13$$

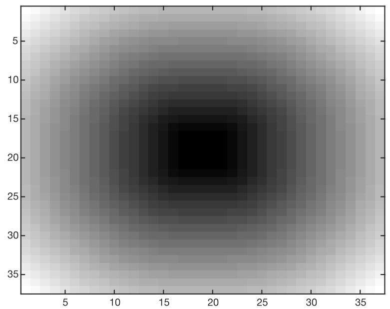
~~$$\sqrt{(25-19)^2+(32-20)^2} = 13.146$$~~

$$\sqrt{(16-14)^2+(16-7)^2} = 9.220$$

RESULT OF THE EDT

22.627	21.932	21.260	20.616	20.000	19.416	18.868	18.358	17.889	17.464	17.088	16.763	16.492	16.279	16.125	16.031	16.000	16.000	16.000	16.000	16.000	16.031	16.125	16.279	16.492	16.763	17.088	17.464	17.889	18.358	18.868	19.416	20.000	20.616	21.260	21.932	22.627
21.932	21.213	20.518	19.849	19.209	18.601	18.028	17.493	17.000	16.553	16.155	15.811	15.524	15.297	15.133	15.033	15.000	15.000	15.000	15.000	15.000	15.033	15.133	15.297	15.524	15.811	16.155	16.553	17.000	17.493	18.028	18.601	19.209	19.849	20.518	21.213	21.932
21.260	20.518	19.799	19.105	18.439	17.804	17.205	16.643	16.125	15.652	15.232	14.866	14.560	14.318	14.142	14.036	14.000	14.000	14.000	14.000	14.000	14.036	14.142	14.318	14.560	14.866	15.232	15.652	16.125	16.643	17.205	17.804	18.439	19.105	19.799	20.518	21.260
20.616	19.849	19.105	18.385	17.692	17.029	16.401	15.811	15.264	14.765	14.318	13.928	13.601	13.342	13.153	13.038	13.000	13.000	13.000	13.000	13.000	13.038	13.153	13.342	13.601	13.928	14.318	14.765	15.264	15.811	16.401	17.029	17.692	18.385	19.105	19.849	20.616
20.000	19.209	18.439	17.692	16.971	16.279	15.620	15.000	14.422	13.892	13.416	13.000	12.649	12.369	12.166	12.042	12.000	12.000	12.000	12.000	12.000	12.042	12.166	12.369	12.649	13.000	13.416	13.892	14.422	15.000	15.620	16.279	16.971	17.692	18.439	19.209	20.000
19.416	18.601	17.804	17.029	16.279	15.556	14.866	14.213	13.601	13.038	12.530	12.083	11.705	11.402	11.180	11.045	11.000	11.000	11.000	11.000	11.000	11.045	11.180	11.402	11.705	12.083	12.530	13.038	13.601	14.213	14.866	15.556	16.279	17.029	17.804	18.601	19.416
18.868	18.028	17.205	16.401	15.620	14.866	14.142	13.454	12.806	12.207	11.662	11.180	10.770	10.440	10.198	10.050	10.000	10.000	10.000	10.000	10.000	10.050	10.198	10.440	10.770	11.180	11.662	12.207	12.806	13.454	14.142	14.866	15.620	16.401	17.205	18.028	18.868
18.358	17.493	16.643	15.811	15.000	14.213	13.454	12.728	12.042	11.402	10.817	10.296	9.849	9.487	9.220	9.055	9.000	9.000	9.000	9.000	9.000	9.055	9.220	9.487	9.849	10.296	10.817	11.402	12.042	12.728	13.454	14.213	15.000	15.811	16.643	17.493	18.358
17.889	17.000	16.125	15.264	14.422	13.601	12.806	12.042	11.314	10.630	10.000	9.434	8.944	8.544	8.246	8.062	8.000	8.000	8.000	8.000	8.000	8.062	8.246	8.544	8.944	9.434	10.000	10.630	11.314	12.042	12.806	13.601	14.422	15.264	16.125	17.000	17.889
17.464	16.553	15.652	14.765	13.892	13.038	12.207	11.402	10.630	9.899	9.220	8.602	8.062	7.616	7.280	7.071	7.000	7.000	7.000	7.000	7.000	7.071	7.280	7.616	8.062	8.602	9.220	9.899	10.630	11.402	12.207	13.038	13.892	14.765	15.652	16.553	17.464
17.088	16.155	15.232	14.318	13.416	12.530	11.662	10.817	10.000	9.220	8.485	7.810	7.211	6.708	6.325	6.083	6.000	6.000	6.000	6.000	6.000	6.083	6.325	6.708	7.211	7.810	8.485	9.220	10.000	10.817	11.662	12.530	13.416	14.318	15.232	16.155	17.088
16.763	15.811	14.866	13.928	13.000	12.083	11.180	10.296	9.434	8.602	7.810	7.071	6.403	5.831	5.385	5.099	5.000	5.000	5.000	5.000	5.000	5.099	5.385	5.831	6.403	7.071	7.810	8.602	9.434	10.296	11.180	12.083	13.000	13.928	14.866	15.811	16.763
16.492	15.524	14.560	13.601	12.649	11.705	10.770	9.849	8.944	8.062	7.211	6.403	5.657	5.000	4.472	4.123	4.000	4.000	4.000	4.000	4.000	4.123	4.472	5.000	5.657	6.403	7.211	8.062	8.944	9.849	10.770	11.705	12.649	13.601	14.560	15.524	16.492
16.279	15.297	14.318	13.342	12.369	11.402	10.440	9.487	8.544	7.616	6.708	5.831	5.000	4.243	3.606	3.162	3.000	3.000	3.000	3.000	3.000	3.162	3.606	4.243	5.000	5.831	6.708	7.616	8.544	9.487	10.440	11.402	12.369	13.342	14.318	15.297	16.279
16.125	15.133	14.142	13.153	12.166	11.180	10.198	9.220	8.246	7.280	6.325	5.385	4.472	3.606	2.828	2.236	2.000	2.000	2.000	2.000	2.000	2.236	2.828	3.606	4.472	5.385	6.325	7.280	8.246	9.220	10.198	11.180	12.166	13.153	14.142	15.133	16.125
16.031	15.033	14.036	13.038	12.042	11.045	10.050	9.055	8.062	7.071	6.083	5.099	4.123	3.162	2.236	1.414	1.000	1.000	1.000	1.000	1.000	1.414	2.236	3.162	4.123	5.099	6.083	7.071	8.062	9.055	10.050	11.045	12.042	13.038	14.036	15.033	16.031
16.000	15.000	14.000	13.000	12.000	11.000	10.000	9.000	8.000	7.000	6.000	5.000	4.000	3.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000	15.000	16.000
16.000	15.000	14.000	13.000	12.000	11.000	10.000	9.000	8.000	7.000	6.000	5.000	4.000	3.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000	15.000	16.000
16.000	15.000	14.000	13.000	12.000	11.000	10.000	9.000	8.000	7.000	6.000	5.000	4.000	3.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000	15.000	16.000
16.000	15.000	14.000	13.000	12.000	11.000	10.000	9.000	8.000	7.000	6.000	5.000	4.000	3.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000	15.000	16.000
16.000	15.000	14.000	13.000	12.000	11.000	10.000	9.000	8.000	7.000	6.000	5.000	4.000	3.000	2.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	11.000	12.000	13.000	14.000	15.000	16.000
16.031	15.033	14.036	13.038	12.042	11.045	10.050	9.055	8.062	7.071	6.083	5.099	4.123	3.162	2.236	1.414	1.000	1.000	1.000	1.000	1.000	1.414	2.236	3.162	4.123	5.099	6.083	7.071	8.062	9.055	10.050	11.045	12.042	13.038	14.036	15.033	16.031
16.125	15.133	14.142	13.153	12.166	11.180	10.198	9.220	8.246	7.280	6.325	5.385	4.472	3.606	2.828	2.236	2.000	2.000	2.000	2.000	2.000	2.236	2.828	3.606	4.472	5.385	6.325	7.280	8.246	9.220	10.198	11.180	12.166	13.153	14.142	15.133	16.125
16.279	15.297	14.318	13.342	12.369	11.402	10.440	9.487	8.544	7.616	6.708	5.831	5.000	4.243	3.606	3.162	3.000	3.000	3.000	3.000	3.000	3.162	3.606	4.243	5.000	5.831	6.708	7.616	8.544	9.487	10.440	11.402	12.369	13.342	14.318	15.297	16.279
16.492	15.524	14.560	13.601	12.649	11.705	10.770	9.849	8.944	8.062	7.211	6.403	5.657	5.000	4.472	4.123	4.000	4.000	4.000	4.000	4.000	4.123	4.472	5.000	5.657	6.403	7.211	8.062	8.944	9.849	10.770	11.705	12.649	13.601	14.560	15.524	16.492
16.763	15.811	14.866	13.928	13.000	12.083	11.180	10.296	9.434	8.602	7.810	7.071	6.403	5.831	5.385	5.099	5.000	5.000	5.000	5.000	5.000	5.099	5.385	5.831	6.403	7.071	7.810	8.602	9.434	10.296	11.180	12.083	13.000	13.928	14.866	15.811	16.763
17.088	16.155	15.232	14.318	13.416	12.530	11.662	10.817	10.000	9.220	8.485	7.810	7.211	6.708	6.325	6.083	6.000	6.000	6.000	6.000	6.000	6.083	6.325	6.708	7.211	7.810	8.485	9.220	10.000	10.817	11.662	12.530	13.416	14.318	15.232	16.155	17.088
17.464	16.553	15.652	14.765	13.892	13.038	12.207	11.402	10.630	9.899	9.220	8.602	8.062	7.616	7.280	7.071	7.000	7.000	7.000	7.000	7.000	7.071	7.280	7.616	8.062	8.602	9.220	9.899	10.630	11.402	12.207	13.038	13.892	14.765	15.652	16.553	17.464
17.889	17.000	16.125	15.264	14.422	13.601	12.806	12.042	11.314	10.630	10.000	9.434	8.944	8.544	8.246	8.062	8.000	8.000	8.000	8.000	8.000	8.062	8.246	8.544	8.944	9.434	10.000	10.630	11.314	12.042	12.806	13.601	14.422	15.264	16.125	17.000	17.889
18.358	17.493	16.643	15.811	15.000	14.213	13.454	12.728	12.042	11.402	10.817	10.296	9.849	9.487	9.220	9.055	9.000	9.000	9.000	9.000	9.000	9.055	9.220	9.487	9.849	10.296	10.817	11.402	12.042	12.728	13.454	14.213	15.000	15.811	16.643	17.493	18.358
18.868	18.028	17.205	16.401	15.620	14.866	14.142	13.454	12.806	12.207	11.662	11.180	10.770	10.440	10.198	10.050	10.000	10.000	10.000	10.000	10.000	10.050	10.198	10.440	10.770	11.180	11.662	12.207	12.806	13.454	14.142	14.866	15.620	16.401	17.205	18.028	18.868
19.416	18.601	17.804	17.029	16.279	15.556	14.866	14.213	13.601	13.038	12.530	12.083	11.705	11.402	11.180	11.045	11.000	11.000	11.000	11.000	11.000	11.045	11.180	11.402	11.705	12.083	12.530	13.038	13.601	14.213	14.866	15.556	16.279	17.029	17.804	18.601	19.416
20.000	19.209	18.439	17.692	16.971	16.279	15.620	15.000	14.422	13.892	13.416	13.000	12.649	12.369	12.166	12.042	12.000	12.000	12.000	12.000	12.000	12.042	12.166	12.369	12.649	13.000	13.416	13.892	14.422	15.000	15.620	16.279	16.971	17.692	18.439	19.209	20.000
20.616	19.849	19.105	18.385	17.692	17.029	16.401	15.811	15.264	14.765	14.318	13.928	13.601	13.342	13.153	13.038	13.000	13.000	13.000	13.000	13.000	13.038	13.153	13.342	13.601	13.928	14.318	14.765	15.264	15.811	16.401	17.029	17.692	18.385	19.105	19.849	20.616
21.260	20.518	19.799	19.105	18.439	17.804	17.205	16.643	16.125	15.652	15.232	14.866	14.560	14.318	14.142	14.036	14.000	14.000	14.000	14.000	14.000	14.036	14.142	14.318	14.560	14.866	15.232	15.652	16.125	16.643	17.205	17.804	18.439	19.105	19.799	20.518	21.260
21.932	21.213	20.518	19.849	19.209	18.601	18.028	17.493	17.000	16.553	16.155	15.811	15.524	15.297	15.133	15.033	15.000	15.000	15.000	15.000	15.000	15.033	15.133	15.297	15.524	15.811	16.155	16.553	17.000	17.493	18.028	18.601	19.209	19.849	20.518	21.213	21.932
22.627	21.932	21.260	20.616	20.000	19.416	18.868	18.358	17.889	17.464	17.088	16.763	16.492	16.279	16.125	16.031	16.000	16.000	16.000	16.000	16.000	16.031	16.125	16.279	16.492	16.763	17.088	17.464	17.889	18.358	18.868	19.416	20.000	20.616	21.260	21.932	22.627

```
distance_transform_edt(255-world_obstacles)
// Image normalize by Matlab from 0 to 255
// imagesc(matrix); colormap(gray)
```



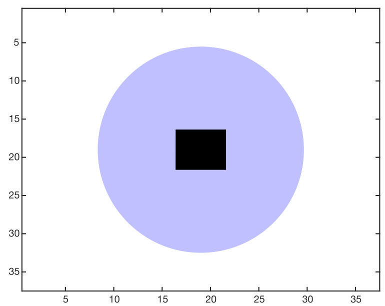
The previous matrix is so dark that if the range of gray colors is not expanded from 0 to 255 we won't see any difference when plotting that matrix.

$$\frac{\text{matrix}[i,j]}{max\,(\text{matrix})}\,255$$

Important coefficient: This coefficient controls how big the potential field is around the obstacles

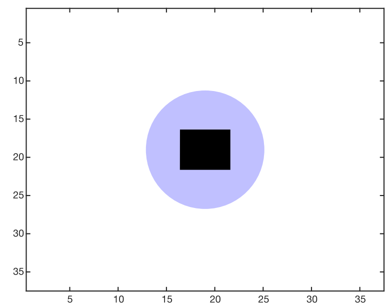
```
16*distance_transform_edt(255-world_obstacles)
```

362.039	350.907	340.165	329.848	320.	310.664	301.887	293.721	286.217	279.428	273.408	268.209	263.879	260.461	257.992	256.5	256.	256.	256.	256.	256.	256.5	257.992	260.461	263.879	268.209	273.408	279.428	286.217	293.721	301.887	310.664	320.	329.848	340.165	350.907	362.039
350.907	339.411	328.293	317.591	307.35	297.617	288.444	279.886	272.	264.847	258.488	252.982	248.387	244.753	242.124	240.533	240.	240.	240.	240.	240.	240.533	242.124	244.753	248.387	252.982	258.488	264.847	272.	279.886	288.444	297.617	307.35	317.591	328.293	339.411	350.907
340.165	328.293	316.784	305.68	295.025	284.872	275.274	266.293	257.992	250.44	243.705	237.857	232.964	229.085	226.274	224.571	224.	224.	224.	224.	224.	224.571	226.274	229.085	232.964	237.857	243.705	250.44	257.992	266.293	275.274	284.872	295.025	305.68	316.784	328.293	340.165
329.848	317.591	305.68	294.156	283.069	272.47	262.42	252.982	244.229	236.237	229.085	222.854	217.624	213.467	210.447	208.614	208.	208.	208.	208.	208.	208.614	210.447	213.467	217.624	222.854	229.085	236.237	244.229	252.982	262.42	272.47	283.069	294.156	305.68	317.591	329.848
320.	307.35	295.025	283.069	271.529	260.461	249.928	240.	230.755	222.279	214.663	208.	202.386	197.909	194.648	192.666	192.	192.	192.	192.	192.	192.666	194.648	197.909	202.386	208.	214.663	222.279	230.755	240.	249.928	260.461	271.529	283.069	295.025	307.35	320.
310.664	297.617	284.872	272.47	260.461	248.902	237.857	227.403	217.624	208.614	200.479	193.329	187.275	182.428	178.885	176.726	176.	176.	176.	176.	176.	176.726	178.885	182.428	187.275	193.329	200.479	208.614	217.624	227.403	237.857	248.902	260.461	272.47	284.872	297.617	310.664
301.887	288.444	275.274	262.42	249.928	237.857	226.274	215.258	204.9	195.305	186.59	178.885	172.325	167.045	163.169	160.798	160.	160.	160.	160.	160.	160.798	163.169	167.045	172.325	178.885	186.59	195.305	204.9	215.258	226.274	237.857	249.928	262.42	275.274	288.444	301.887
293.721	279.886	266.293	252.982	240.	227.403	215.258	203.647	192.666	182.428	173.066	164.73	157.582	151.789	147.513	144.886	144.	144.	144.	144.	144.	144.886	147.513	151.789	157.582	164.73	173.066	182.428	192.666	203.647	215.258	227.403	240.	252.982	266.293	279.886	293.721
286.217	272.	257.992	244.229	230.755	217.624	204.9	192.666	181.019	170.082	160.	150.944	143.108	136.704	131.939	128.996	128.	128.	128.	128.	128.	128.996	131.939	136.704	143.108	150.944	160.	170.082	181.019	192.666	204.9	217.624	230.755	244.229	257.992	272.	286.217
279.428	264.847	250.44	236.237	222.279	208.614	195.305	182.428	170.082	158.392	147.513	137.637	128.996	121.852	116.482	113.137	112.	112.	112.	112.	112.	113.137	116.482	121.852	128.996	137.637	147.513	158.392	170.082	182.428	195.305	208.614	222.279	236.237	250.44	264.847	279.428
273.408	258.488	243.705	229.085	214.663	200.479	186.59	173.066	160.	147.513	135.765	124.964	115.378	107.331	101.193	97.324	96.	96.	96.	96.	96.	97.324	101.193	107.331	115.378	124.964	135.765	147.513	160.	173.066	186.59	200.479	214.663	229.085	243.705	258.488	273.408
268.209	252.982	237.857	222.854	208.	193.329	178.885	164.73	150.944	137.637	124.964	113.137	102.45	93.295	86.163	81.584	80.	80.	80.	80.	80.	81.584	86.163	93.295	102.45	113.137	124.964	137.637	150.944	164.73	178.885	193.329	208.	222.854	237.857	252.982	268.209
263.879	248.387	232.964	217.624	202.386	187.275	172.325	157.582	143.108	128.996	115.378	102.45	90.51	80.	71.554	65.97	64.	64.	64.	64.	64.	65.97	71.554	80.	90.51	102.45	115.378	128.996	143.108	157.582	172.325	187.275	202.386	217.624	232.964	248.387	263.879
260.461	244.753	229.085	213.467	197.909	182.428	167.045	151.789	136.704	121.852	107.331	93.295	80.	67.882	57.689	50.596	48.	48.	48.	48.	48.	50.596	57.689	67.882	80.	93.295	107.331	121.852	136.704	151.789	167.045	182.428	197.909	213.467	229.085	244.753	260.461
257.992	242.124	226.274	210.447	194.648	178.885	163.169	147.513	131.939	116.482	101.193	86.163	71.554	57.689	45.255	35.777	32.	32.	32.	32.	32.	35.777	45.255	57.689	71.554	86.163	101.193	116.482	131.939	147.513	163.169	178.885	194.648	210.447	226.274	242.124	257.992
256.5	240.533	224.571	208.614	192.666	176.726	160.798	144.886	128.996	113.137	97.324	81.584	65.97	50.596	35.777	22.627	16.	16.	16.	16.	16.	22.627	35.777	50.596	65.97	81.584	97.324	113.137	128.996	144.886	160.798	176.726	192.666	208.614	224.571	240.533	256.5
256.	240.	224.	208.	192.	176.	160.	144.	128.	112.	96.	80.	64.	48.	32.	16.	0.	0.	0.	0.	0.	16.	32.	48.	64.	80.	96.	112.	128.	144.	160.	176.	192.	208.	224.	240.	256.
256.	240.	224.	208.	192.	176.	160.	144.	128.	112.	96.	80.	64.	48.	32.	16.	0.	0.	0.	0.	0.	16.	32.	48.	64.	80.	96.	112.	128.	144.	160.	176.	192.	208.	224.	240.	256.
256.	240.	224.	208.	192.	176.	160.	144.	128.	112.	96.	80.	64.	48.	32.	16.	0.	0.	0.	0.	0.	16.	32.	48.	64.	80.	96.	112.	128.	144.	160.	176.	192.	208.	224.	240.	256.
256.	240.	224.	208.	192.	176.	160.	144.	128.	112.	96.	80.	64.	48.	32.	16.	0.	0.	0.	0.	0.	16.	32.	48.	64.	80.	96.	112.	128.	144.	160.	176.	192.	208.	224.	240.	256.
256.	240.	224.	208.	192.	176.	160.	144.	128.	112.	96.	80.	64.	48.	32.	16.	0.	0.	0.	0.	0.	16.	32.	48.	64.	80.	96.	112.	128.	144.	160.	176.	192.	208.	224.	240.	256.
256.5	240.533	224.571	208.614	192.666	176.726	160.798	144.886	128.996	113.137	97.324	81.584	65.97	50.596	35.777	22.627	16.	16.	16.	16.	16.	22.627	35.777	50.596	65.97	81.584	97.324	113.137	128.996	144.886	160.798	176.726	192.666	208.614	224.571	240.533	256.5
257.992	242.124	226.274	210.447	194.648	178.885	163.169	147.513	131.939	116.482	101.193	86.163	71.554	57.689	45.255	35.777	32.	32.	32.	32.	32.	35.777	45.255	57.689	71.554	86.163	101.193	116.482	131.939	147.513	163.169	178.885	194.648	210.447	226.274	242.124	257.992
260.461	244.753	229.085	213.467	197.909	182.428	167.045	151.789	136.704	121.852	107.331	93.295	80.	67.882	57.689	50.596	48.	48.	48.	48.	48.	50.596	57.689	67.882	80.	93.295	107.331	121.852	136.704	151.789	167.045	182.428	197.909	213.467	229.085	244.753	260.461
263.879	248.387	232.964	217.624	202.386	187.275	172.325	157.582	143.108	128.996	115.378	102.45	90.51	80.	71.554	65.97	64.	64.	64.	64.	64.	65.97	71.554	80.	90.51	102.45	115.378	128.996	143.108	157.582	172.325	187.275	202.386	217.624	232.964	248.387	263.879
268.209	252.982	237.857	222.854	208.	193.329	178.885	164.73	150.944	137.637	124.964	113.137	102.45	93.295	86.163	81.584	80.	80.	80.	80.	80.	81.584	86.163	93.295	102.45	113.137	124.964	137.637	150.944	164.73	178.885	193.329	208.	222.854	237.857	252.982	268.209
273.408	258.488	243.705	229.085	214.663	200.479	186.59	173.066	160.	147.513	135.765	124.964	115.378	107.331	101.193	97.324	96.	96.	96.	96.	96.	97.324	101.193	107.331	115.378	124.964	135.765	147.513	160.	173.066	186.59	200.479	214.663	229.085	243.705	258.488	273.408
279.428	264.847	250.44	236.237	222.279	208.614	195.305	182.428	170.082	158.392	147.513	137.637	128.996	121.852	116.482	113.137	112.	112.	112.	112.	112.	113.137	116.482	121.852	128.996	137.637	147.513	158.392	170.082	182.428	195.305	208.614	222.279	236.237	250.44	264.847	279.428
286.217	272.	257.992	244.229	230.755	217.624	204.9	192.666	181.019	170.082	160.	150.944	143.108	136.704	131.939	128.996	128.	128.	128.	128.	128.	128.996	131.939	136.704	143.108	150.944	160.	170.082	181.019	192.666	204.9	217.624	230.755	244.229	257.992	272.	286.217
293.721	279.886	266.293	252.982	240.	227.403	215.258	203.647	192.666	182.428	173.066	164.73	157.582	151.789	147.513	144.886	144.	144.	144.	144.	144.	144.886	147.513	151.789	157.582	164.73	173.066	182.428	192.666	203.647	215.258	227.403	240.	252.982	266.293	279.886	293.721
301.887	288.444	275.274	262.42	249.928	237.857	226.274	215.258	204.9	195.305	186.59	178.885	172.325	167.045	163.169	160.798	160.	160.	160.	160.	160.	160.798	163.169	167.045	172.325	178.885	186.59	195.305	204.9	215.258	226.274	237.857	249.928	262.42	275.274	288.444	301.887
310.664	297.617	284.872	272.47	260.461	248.902	237.857	227.403	217.624	208.614	200.479	193.329	187.275	182.428	178.885	176.726	176.	176.	176.	176.	176.	176.726	178.885	182.428	187.275	193.329	200.479	208.614	217.624	227.403	237.857	248.902	260.461	272.47	284.872	297.617	310.664
320.	307.35	295.025	283.069	271.529	260.461	249.928	240.	230.755	222.279	214.663	208.	202.386	197.909	194.648	192.666	192.	192.	192.	192.	192.	192.666	194.648	197.909	202.386	208.	214.663	222.279	230.755	240.	249.928	260.461	271.529	283.069	295.025	307.35	320.
329.848	317.591	305.68	294.156	283.069	272.47	262.42	252.982	244.229	236.237	229.085	222.854	217.624	213.467	210.447	208.614	208.	208.	208.	208.	208.	208.614	210.447	213.467	217.624	222.854	229.085	236.237	244.229	252.982	262.42	272.47	283.069	294.156	305.68	317.591	329.848
340.165	328.293	316.784	305.68	295.025	284.872	275.274	266.293	257.992	250.44	243.705	237.857	232.964	229.085	226.274	224.571	224.	224.	224.	224.	224.	224.571	226.274	229.085	232.964	237.857	243.705	250.44	257.992	266.293	275.274	284.872	295.025	305.68	316.784	328.293	340.165
350.907	339.411	328.293	317.591	307.35	297.617	288.444	279.886	272.	264.847	258.488	252.982	248.387	244.753	242.124	240.533	240.	240.	240.	240.	240.	240.533	242.124	244.753	248.387	252.982	258.488	264.847	272.	279.886	288.444	297.617	307.35	317.591	328.293	339.411	350.907
362.039	350.907	340.165	329.848	320.	310.664	301.887	293.721	286.217	279.428	273.408	268.209	263.879	260.461	257.992	256.5	256.	256.	256.	256.	256.	256.5	257.992	260.461	263.879	268.209	273.408	279.428	286.217	293.721	301.887	310.664	320.	329.848	340.165	350.907	362.039



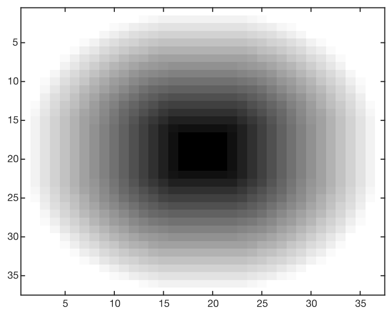
coefficient ↑ ⟶ potential field expansion ↓ (it shrinks)
(potential field closer to the obstacles)

coefficient ↓ ⟶ potential field expansion ↑ (it expands)
(potential field further away from to the obstacles)

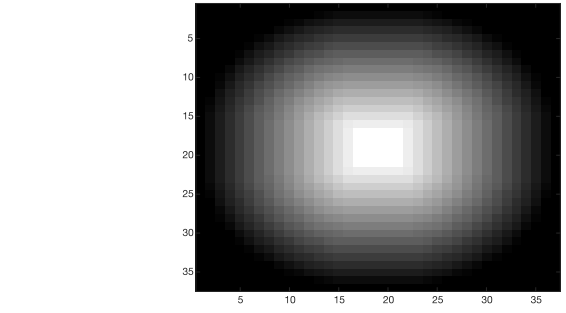


np.minimum(16*distance_transform_edt(255-world_obstacles), 255)

255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	252.982	248.387	244.753	242.124	240.533	240.	240.	240.	240.	240.	240.533	242.124	244.753	248.387	252.982	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	255.	255.	250.44	243.705	237.857	232.964	229.085	226.274	224.571	224.	224.	224.	224.	224.	224.571	226.274	229.085	232.964	237.857	243.705	250.44	255.	255.	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	252.982	244.229	236.237	229.085	222.854	217.624	213.467	210.447	208.614	208.	208.	208.	208.	208.	208.614	210.447	213.467	217.624	222.854	229.085	236.237	244.229	252.982	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	249.928	240	230.755	222.279	214.663	208.	202.386	197.909	194.648	192.666	192.	192.	192.	192.	192.	192.666	194.648	197.909	202.386	208.	214.663	222.279	230.755	240.	249.928	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	248.902	237.857	227.403	217.624	208.614	200.479	193.329	187.275	182.428	178.885	176.726	176.	176.	176.	176.	176.	176.726	178.885	182.428	187.275	193.329	200.479	208.614	217.624	227.403	237.857	248.902	255.	255.	255.	255.	255.
255.	255.	255.	255.	249.928	237.857	226.274	215.258	204.9	195.305	186.59	178.885	172.325	167.045	163.169	160.798	160.	160.	160.	160.	160.	160.798	163.169	167.045	172.325	178.885	186.59	195.305	204.9	215.258	226.274	237.857	249.928	255.	255.	255.	255.
255.	255.	255.	252.982	240.	227.403	215.258	203.647	192.666	182.428	173.066	164.73	157.582	151.789	147.513	144.886	144.	144.	144.	144.	144.	144.886	147.513	151.789	157.582	164.73	173.066	182.428	192.666	203.647	215.258	227.403	240.	252.982	255.	255.	255.
255.	255.	255.	244.229	230.755	217.624	204.9	192.666	181.019	170.082	160.	150.944	143.108	136.704	131.939	128.996	128.	128.	128.	128.	128.	128.996	131.939	136.704	143.108	150.944	160.	170.082	181.019	192.666	204.9	217.624	230.755	244.229	255.	255.	255.
255.	255.	250.44	236.237	222.279	208.614	195.305	182.428	170.082	158.392	147.513	137.637	128.996	121.852	116.482	113.137	112.	112.	112.	112.	112.	113.137	116.482	121.852	128.996	137.637	147.513	158.392	170.082	182.428	195.305	208.614	222.279	236.237	250.44	255.	255.
255.	255.	243.705	229.085	214.663	200.479	186.59	173.066	160.	147.513	135.765	124.964	115.378	107.331	101.193	97.324	96.	96.	96.	96.	96.	97.324	101.193	107.331	115.378	124.964	135.765	147.513	160.	173.066	186.59	200.479	214.663	229.085	243.705	255.	255.
255.	252.982	237.857	222.854	208.	193.329	178.885	164.73	150.944	137.637	124.964	113.137	102.45	93.295	86.163	81.584	80.	80.	80.	80.	80.	81.584	86.163	93.295	102.45	113.137	124.964	137.637	150.944	164.73	178.885	193.329	208.	222.854	237.857	252.982	255.
255.	248.387	232.964	217.624	202.386	187.275	172.325	157.582	143.108	128.996	115.378	102.45	90.51	80.	71.554	65.97	64.	64.	64.	64.	64.	65.97	71.554	80	90.51	102.45	115.378	128.996	143.108	157.582	172.325	187.275	202.386	217.624	232.964	248.387	255.
255.	244.753	229.085	213.467	197.909	182.428	167.045	151.789	136.704	121.852	107.331	93.295	80.	67.882	57.689	50.596	48.	48.	48.	48.	48.	50.596	57.689	67.882	80.	93.295	107.331	121.852	136.704	151.789	167.045	182.428	197.909	213.467	229.085	244.753	255.
255.	242.124	226.274	210.447	194.648	178.885	163.169	147.513	131.939	116.482	101.193	86.163	71.554	57.689	45.255.	35.777	32.	32.	32.	32.	32.	35.777	45.255.	57.689	71.554	86.163	101.193	116.482	131.939	147.513	163.169	178.885	194.648	210.447	226.274	242.124	255.
255.	240.533	224.571	208.614	192.666	176.726	160.798	144.886	128.996	113.137	97.324	81.584	65.97	50.596	35.777	22.627	16.	16.	16.	16.	16.	22.627	35.777	50.596	65.97	81.584	97.324	113.137	128.996	144.886	160.798	176.726	192.666	208.614	224.571	240.533	255.
255.	240.	224.	208.	192.	176.	160.	144	128.	112.	96.	80.	64.	48.	32.	16	0.	0.	0.	0.	0.	16.	32.	48	64.	80.	96.	112.	128.	144.	160.	176	192.	208.	224.	240.	255.
255.	240.	224.	208.	192.	176.	160.	144	128.	112.	96.	80.	64.	48.	32.	16	0.	0.	0.	0.	0.	16.	32.	48	64.	80.	96.	112.	128.	144.	160.	176	192.	208.	224.	240.	255.
255.	240.	224.	208.	192.	176.	160.	144	128.	112.	96.	80.	64.	48.	32.	16	0.	0.	0.	0.	0.	16.	32.	48	64.	80.	96.	112.	128.	144.	160.	176	192.	208.	224.	240.	255.
255.	240.	224.	208.	192.	176.	160.	144	128.	112.	96.	80.	64.	48.	32.	16	0.	0.	0.	0.	0.	16.	32.	48	64.	80.	96.	112.	128.	144.	160.	176	192.	208.	224.	240.	255.
255.	240.	224.	208.	192.	176.	160.	144	128.	112.	96.	80.	64.	48.	32.	16	0.	0.	0.	0.	0.	16.	32.	48	64.	80.	96.	112.	128.	144.	160.	176	192.	208.	224.	240.	255.
255.	240.533	224.571	208.614	192.666	176.726	160.798	144.886	128.996	113.137	97.324	81.584	65.97	50.596	35.777	22.627	16.	16.	16.	16.	16.	22.627	35.777	50.596	65.97	81.584	97.324	113.137	128.996	144.886	160.798	176.726	192.666	208.614	224.571	240.533	255.
255.	242.124	226.274	210.447	194.648	178.885	163.169	147.513	131.939	116.482	101.193	86.163	71.554	57.689	45.255.	35.777	32.	32.	32.	32.	32.	35.777	45.255.	57.689	71.554	86.163	101.193	116.482	131.939	147.513	163.169	178.885	194.648	210.447	226.274	242.124	255.
255.	244.753	229.085	213.467	197.909	182.428	167.045	151.789	136.704	121.852	107.331	93.295	80.	67.882	57.689	50.596	48.	48.	48.	48.	48.	50.596	57.689	67.882	80.	93.295	107.331	121.852	136.704	151.789	167.045	182.428	197.909	213.467	229.085	244.753	255.
255.	248.387	232.964	217.624	202.386	187.275	172.325	157.582	143.108	128.996	115.378	102.45	90.51	80.	71.554	65.97	64.	64.	64.	64.	64.	65.97	71.554	80	90.51	102.45	115.378	128.996	143.108	157.582	172.325	187.275	202.386	217.624	232.964	248.387	255.
255.	252.982	237.857	222.854	208.	193.329	178.885	164.73	150.944	137.637	124.964	113.137	102.45	93.295	86.163	81.584	80.	80.	80.	80.	80.	81.584	86.163	93.295	102.45	113.137	124.964	137.637	150.944	164.73	178.885	193.329	208.	222.854	237.857	252.982	255.
255.	255.	243.705	229.085	214.663	200.479	186.59	173.066	160.	147.513	135.765	124.964	115.378	107.331	101.193	97.324	96.	96.	96.	96.	96.	97.324	101.193	107.331	115.378	124.964	135.765	147.513	160.	173.066	186.59	200.479	214.663	229.085	243.705	255.	255.
255.	255.	250.44	236.237	222.279	208.614	195.305	182.428	170.082	158.392	147.513	137.637	128.996	121.852	116.482	113.137	112.	112.	112.	112.	112.	113.137	116.482	121.852	128.996	137.637	147.513	158.392	170.082	182.428	195.305	208.614	222.279	236.237	250.44	255.	255.
255.	255.	255.	244.229	230.755	217.624	204.9	192.666	181.019	170.082	160.	150.944	143.108	136.704	131.939	128.996	128.	128.	128.	128.	128.	128.996	131.939	136.704	143.108	150.944	160.	170.082	181.019	192.666	204.9	217.624	230.755	244.229	255.	255.	255.
255.	255.	255.	252.982	240.	227.403	215.258	203.647	192.666	182.428	173.066	164.73	157.582	151.789	147.513	144.886	144.	144.	144.	144.	144.	144.886	147.513	151.789	157.582	164.73	173.066	182.428	192.666	203.647	215.258	227.403	240.	252.982	255.	255.	255.
255.	255.	255.	255.	249.928	237.857	226.274	215.258	204.9	195.305	186.59	178.885	172.325	167.045	163.169	160.798	160.	160.	160.	160.	160.	160.798	163.169	167.045	172.325	178.885	186.59	195.305	204.9	215.258	226.274	237.857	249.928	255.	255.	255.	255.
255.	255.	255.	255.	255.	248.902	237.857	227.403	217.624	208.614	200.479	193.329	187.275	182.428	178.885	176.726	176.	176.	176.	176.	176.	176.726	178.885	182.428	187.275	193.329	200.479	208.614	217.624	227.403	237.857	248.902	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	249.928	240	230.755	222.279	214.663	208.	202.386	197.909	194.648	192.666	192.	192.	192.	192.	192.	192.666	194.648	197.909	202.386	208.	214.663	222.279	230.755	240.	249.928	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	252.982	244.229	236.237	229.085	222.854	217.624	213.467	210.447	208.614	208.	208.	208.	208.	208.	208.614	210.447	213.467	217.624	222.854	229.085	236.237	244.229	252.982	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	255.	255.	250.44	243.705	237.857	232.964	229.085	226.274	224.571	224.	224.	224.	224.	224.	224.571	226.274	229.085	232.964	237.857	243.705	250.44	255.	255.	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	252.982	248.387	244.753	242.124	240.533	240.	240.	240.	240.	240.	240.533	242.124	244.753	248.387	252.982	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.
255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.	255.

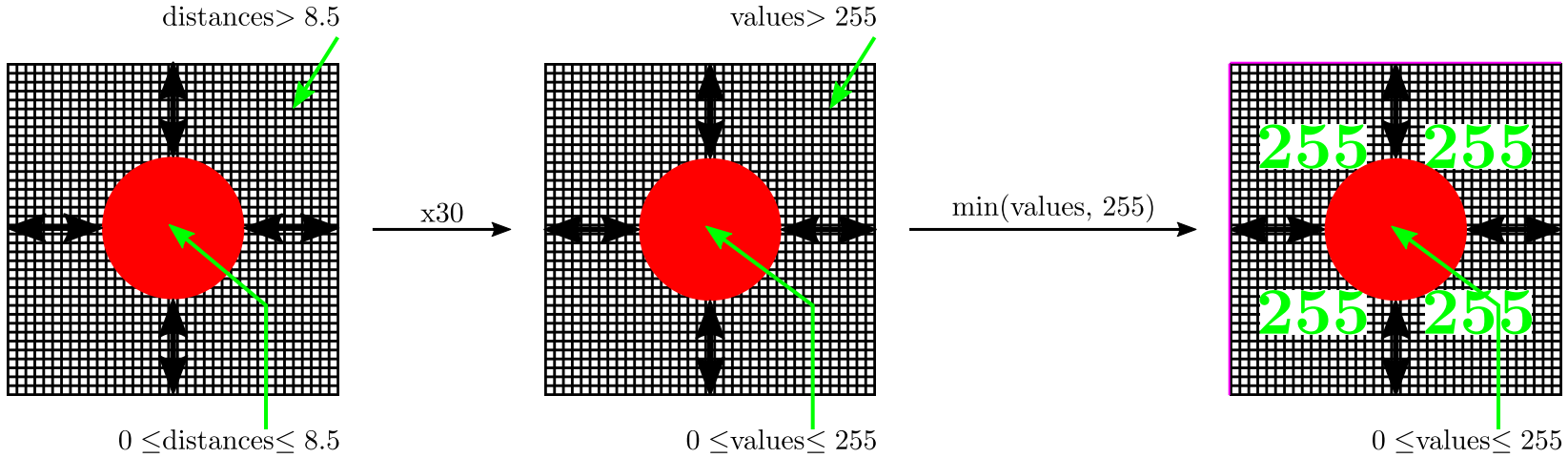


```
dist_transform = 255-np.minimum(16*distance_transform_edt(255-world_obstacles), 255)
```

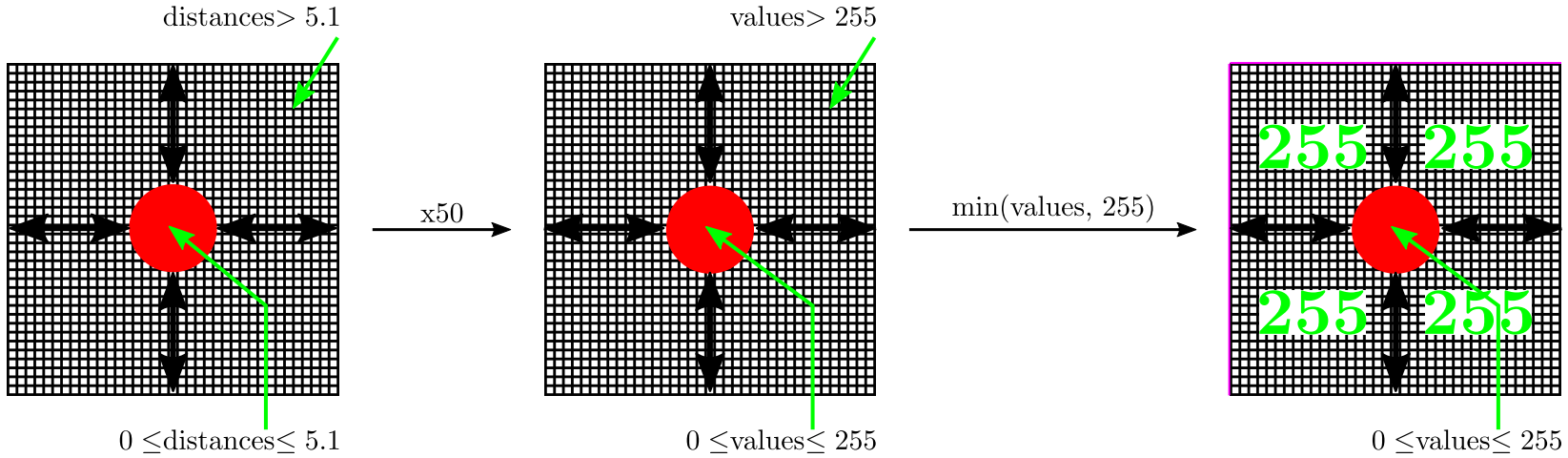



```
m = max(np.max(dist_transform), 1)  # Prevent m==0.
world_obstacles = np.uint8((dist_transform * 255) / m)
```

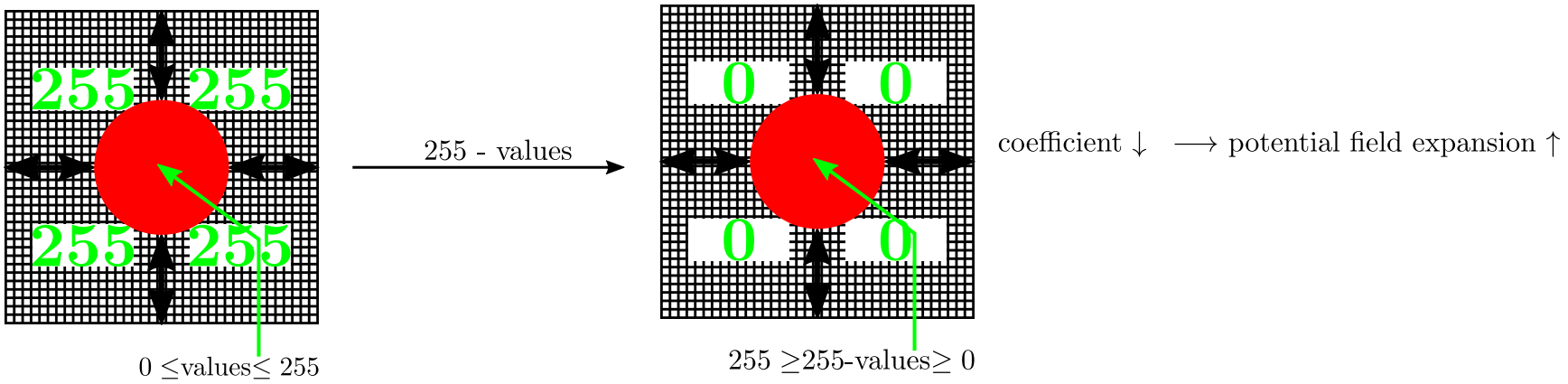
If coefficient = 30, then 255/30 = 8.5.
All the distances greater or equal 8.5 give values greater or equal 255.



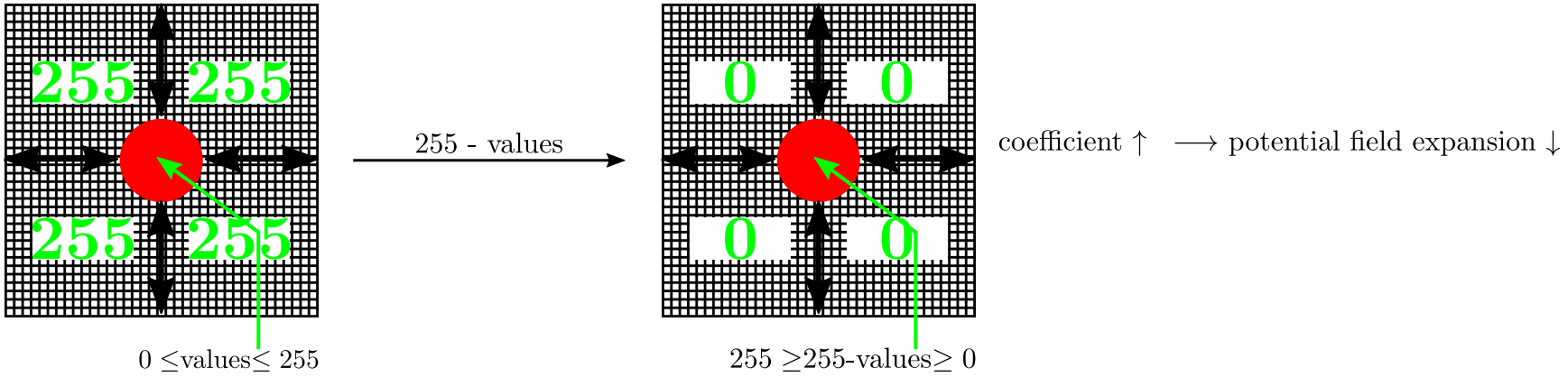
If coefficient = 50, then 255/50 = 5.1.
All the distances greater or equal 5.1 give values greater or equal 255.



If coefficient = 30, then 255/30 = 8.5.
All the distances greater or equal 8.5 give values greater or equal 255.

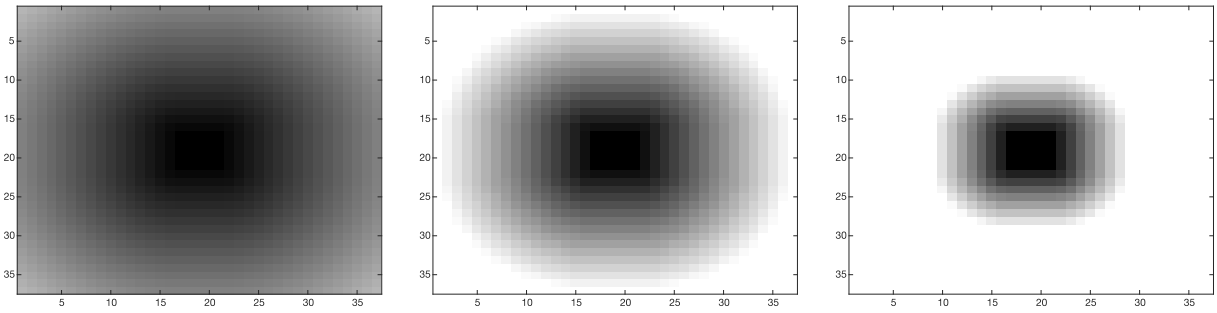


If coefficient = 50, then 255/50 = 5.1.
All the distances greater or equal 5.1 give values greater or equal 255.



np.minimum(coeff*distance_transform_edt(255-world_obstacles), 255)

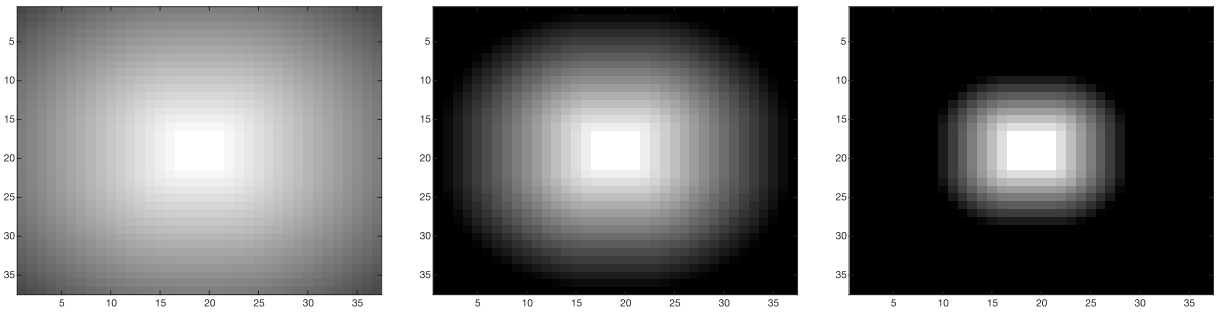
x8 x16 x32

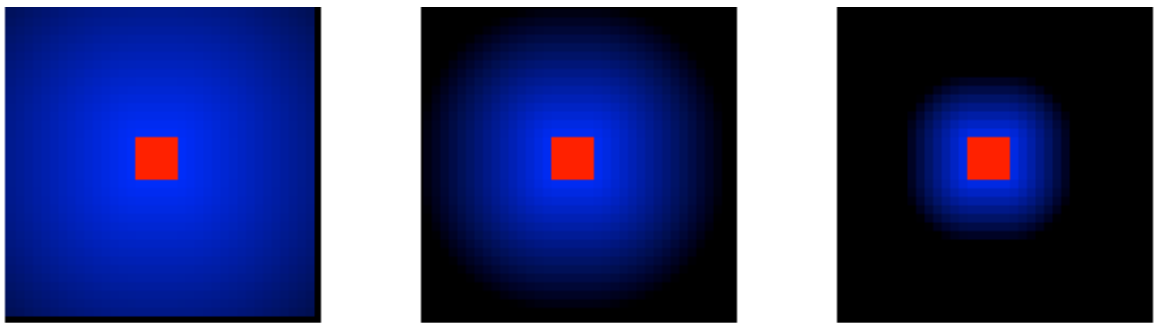


Matrix with obstacles, free spaces and the potential field

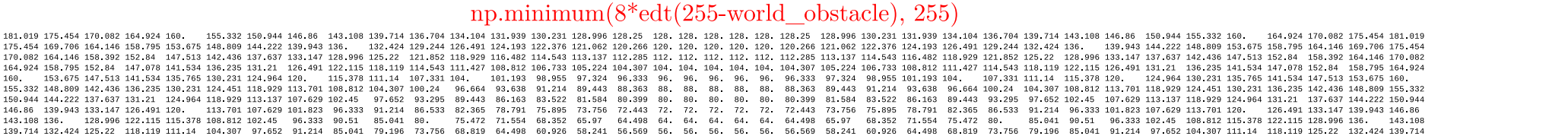
255 - np.minimum(coeff*distance_transform_edt(255-world_obstacles), 255)

x8 x16 x32

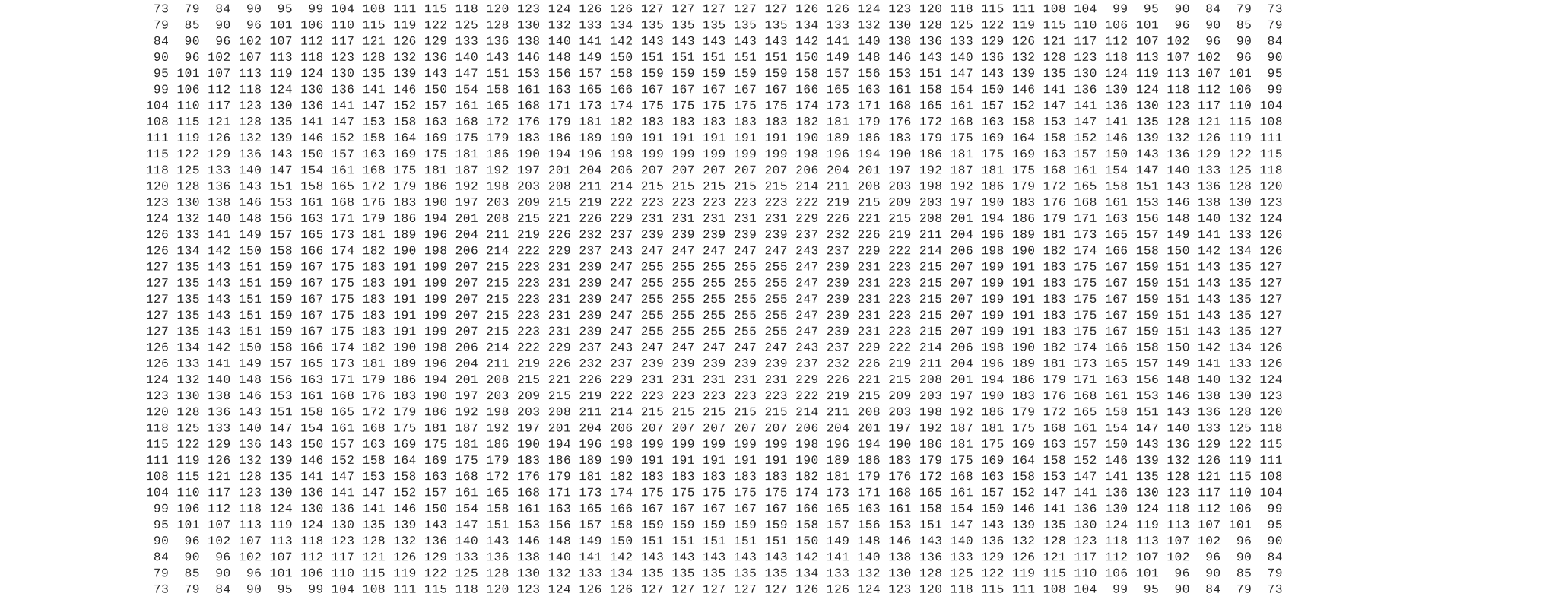




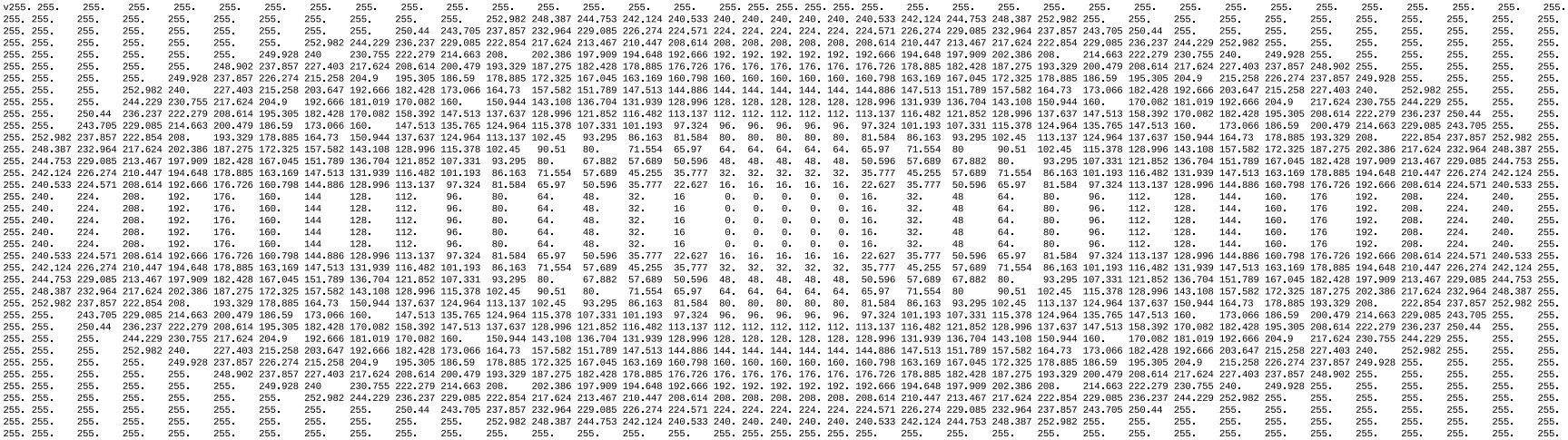
np.minimum(8*edt(255-world_obstacle), 255)



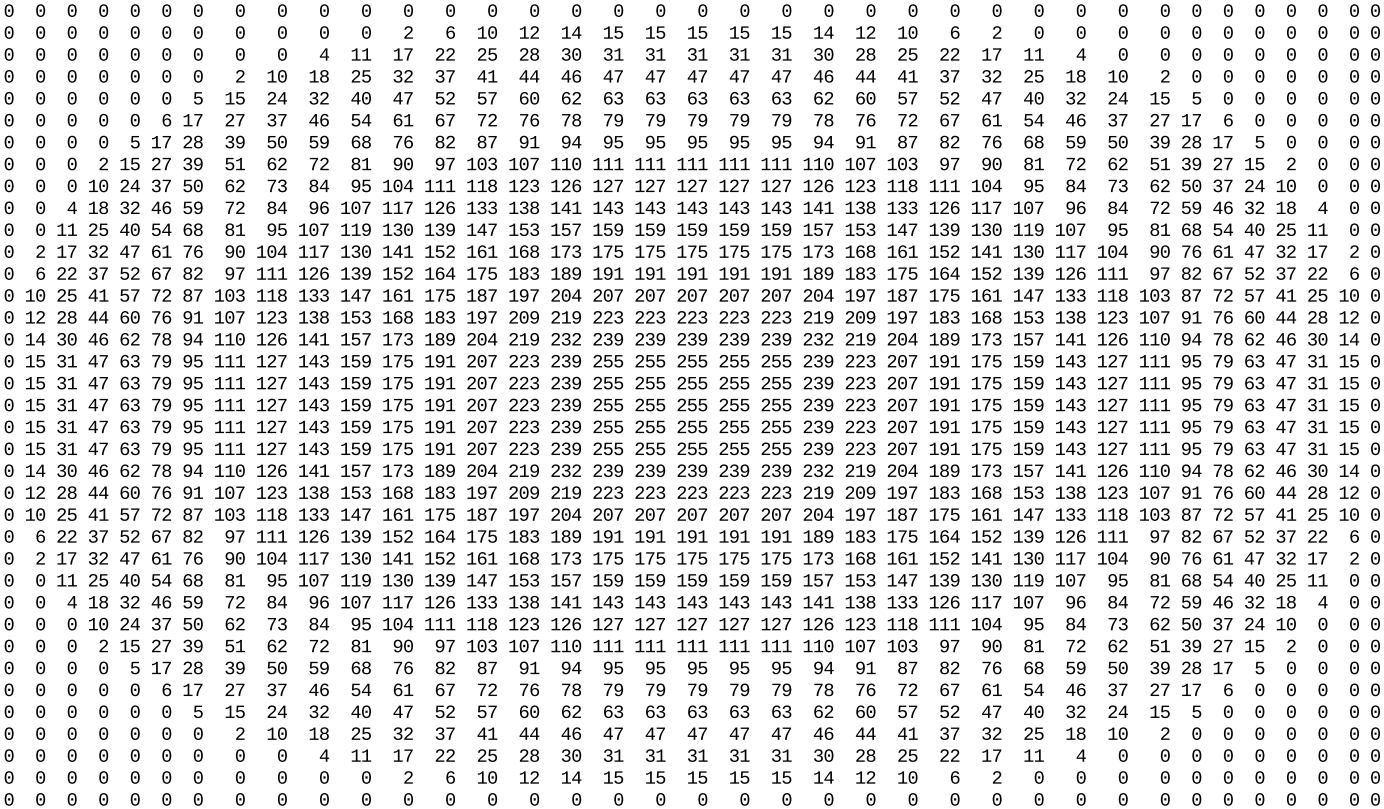
255 - np.minimum(8*edt(255-world_obstacle), 255)



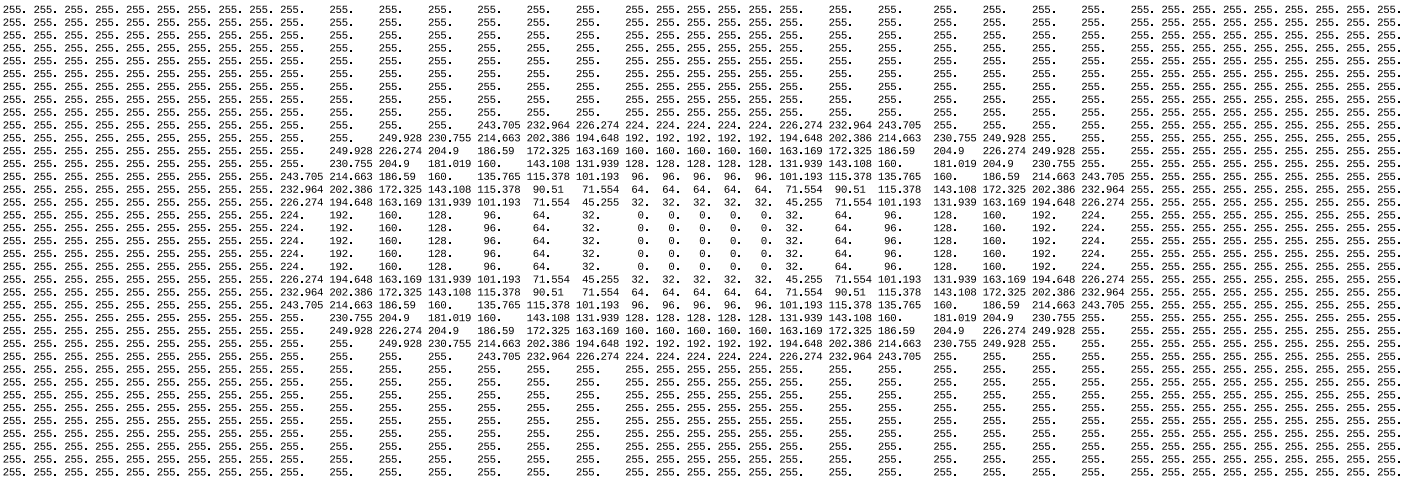
np.minimum(16*edt(255 - world_obstacles), 255)



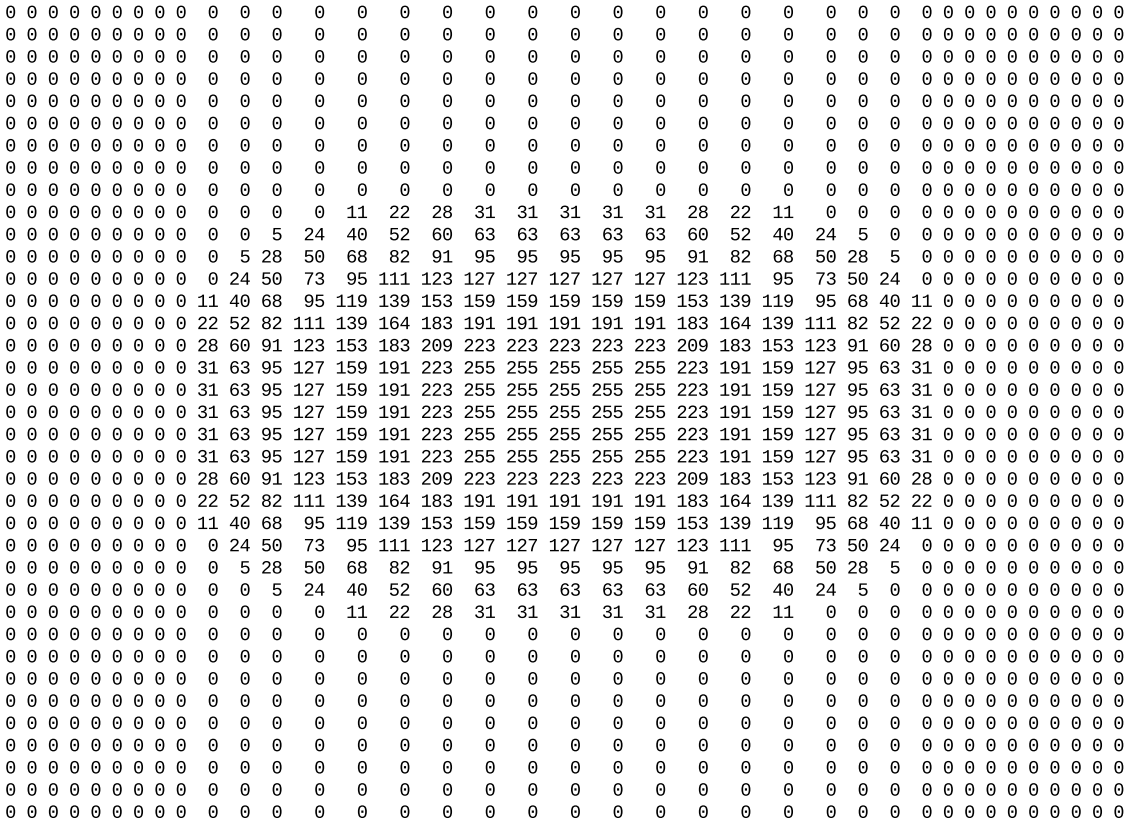
255 - np.minimum(16*edt(255-world_obstacles), 255)



np.minimum(32*edt(255-world_obstacles), 255)



255 - np.minimum(32*edt(255-world_obstacles), 255)



The cost associated to the node $world\_map\,(i,\,j)$ due to the presence of a potential field around the obstacles is:

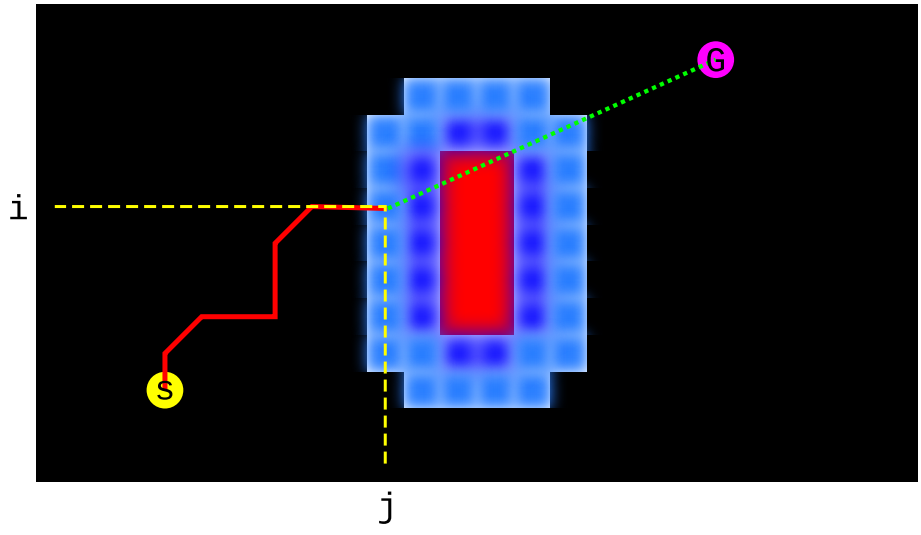
$$cost_{PF}(i,\,\mathrm{j}) = \frac{world\_map(i,j)}{64}$$

where (i, j) are the i-th row and the j-th column in the `world_map` matrix. The above equation uses values of the `world_map` between 0 and 254.
Remember:

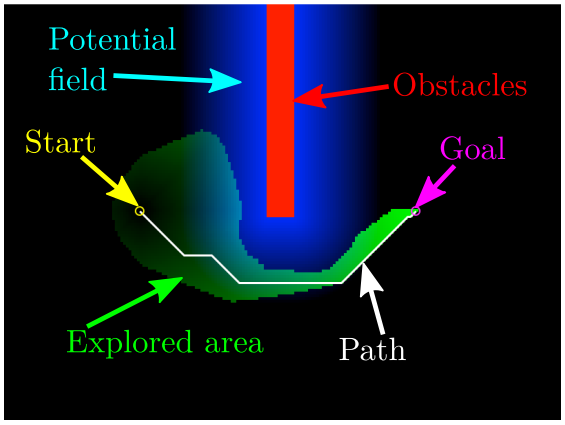
- The value $world\_map\,(i,\,j) = 0$, in these path planning lectures, represents a free space node and any path can pass through that node.
- The value $world\_map\,(i,\,j) = 255$, in these path planning lectures, represents an obstacle node and no one path can pass through that node. This value can't be used in the above equation.

The total cost associated to the node $world\_map\,(i,\,j)$ is:
$total\_cost\,(i,\,j) = A + B + C + D$
where:

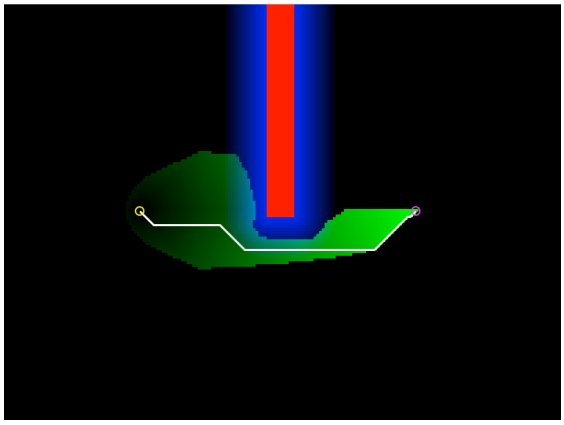
- A is the total cost of the previous node in the path.
- B is the cost of the transition between the previous node in the path and the current node $(world\_map\,(i,\,j))$.
- C is the cost associated to the node $world\_map\,(i,\,j)$ due to the presence of a potential field around the obstacles, $cost_{PF}\,(i,\,j)$.
- D is the estimated Euclidean distance between the current node $(world\_map\,(i,\,j))$ and the goal node.



x8



x16



x32