1st tutorial

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1. Missing Number We are given an unsorted array A which contains n pairwise different numbers from set $\{0, \ldots, n\}$. One number from this set is missing in A. Describe an efficient algorithm that finds the missing number. Try to describe an algorithm that uses only a constant amount of additional memory.

2. Sorted Array Fixed Point We are a given an array A of n integers that are sorted in increasing order. The range of the integers is not bounded in this case, in particular, numbers can be negative. Describe an efficient algorithm which finds an index i such that $A[i] = i$.

Bonus. Does your approach need to change if A contains some duplicate entries?

3. Egg testing The Empire State Building is a 102-story skyscraper, and we know that if we throw an egg from the K-th floor or higher, it will break. Unless the egg breaks, it can be collected and reused. We want to determine K, but use as few attempts (throws) as possible. What's the best strategy (minimizing the number of throws) if we have

a) one egg,

b) unlimited eggs,

c) two eggs,

d) **Bonus.** three eggs, or in general $e \in \mathbb{N}$ eggs?

(The eggs can be special, so nothing can be *a priori* assumed about K.)

4. Laser There is a row of N buildings with $h_1, \ldots, h_n$ floors, and we need to demolish all of them. To that end, you found at home a demolition laser which is capable of firing *vertically* to destroy an arbitrary building or *horizontally* to destroy a given floor in all buildings (i.e., if you choose to destroy floor L, then the number of floors decreases by 1 for all buildings whose number of floors is $\geq L$). What is the minimum number of firings necessary to eliminate all buildings? (Beware that the maximum number of floors can be much more than N.)

5. Submatrix search We are given an integer matrix A with n columns and m rows. Describe an efficient algorithm that finds a maximum submatrix of A consisting only of values 0 (i.e. the submatrix with largest area). Can you achieve complexity $O(nm)$?

6. Fibonacci sequence Fibonacci sequence is defined using $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$.

a) What is the value of F_n for a given n?

b) Describe an algorithm for computing F_n using only $\Theta(\log_2 n)$ arithmetic operations.