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Asymptotics

1. Comparing functions Determine the asymptotic relationships between each pair of functions f and g in the following table (i.e. whether $f \in \mathcal{O}(g)$, $f \in \Omega(g)$ or $f = \Theta(g)$).

	$f(n)$	$g(n)$	$\mathcal{O}$	Ω	Θ
(1)	$n^2 - 30n + 5$	$0.7n^2 - 20n + 15$	◯	◯	◯
(2)	$100n^3 + 40n^2 - n$	$0.5n^4 - 1000n^3$	◯	◯	◯
(3)	$5n^2 - n$	$30n + 4$	◯	◯	◯
(4)	n	$\sqrt{n}$	◯	◯	◯
(5)	$n^{\frac{3}{4}}$	$\sqrt{n}$	◯	◯	◯
(6)	$\log_2 n$	$\ln n$	◯	◯	◯
(7)	$n(\log_2 n)^5$	$n\sqrt{n}$	◯	◯	◯
(8)	2^n	2^{2n}	◯	◯	◯
(9)	e^n	2^n	◯	◯	◯
(10)	$n!$	n^n	◯	◯	◯
(11)	$n!$	2^n	◯	◯	◯
(12)	2^n	2^{n+1}	◯	◯	◯

2. Properties of asymptotic notation Let $f(n)$ and $g(n)$ be asymptotically positive functions (which means that $\lim_{n\to\infty} f(n) > 0$ and $\lim_{n\to\infty} g(n) > 0$). Prove or disprove each of the following conjectures

(1) $f(n) + g(n) \in \Theta(\min(f(n), g(n)))$.

(2) $f(n) + g(n) \in \Theta(\max(f(n), g(n)))$.

(3) $f(n) \in \mathcal{O}((f(n))^2)$.

(4) $f(n) \in \Omega((f(n))^2)$.

(5) $f(n) \in \mathcal{O}(g(n))$ implies $g(n) \in \Omega(f(n))$.

Interlude: Bridges. Jamie will use SYGA to demo an algorithm for finding "bridges" in undirected graphs. See page 122 of https://iuuk.mff.cuni.cz/ koutecky/pruvodce-en-wip.pdf for a reference. Note that translation of this document into English is still a work in progress.
This also serves as our introduction to the SYGA (See Your Graph Algorithm) visualizer.

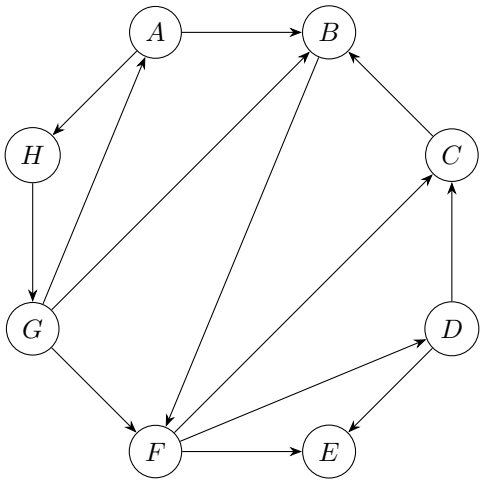
Depth First Search

3. DFS Runs — SYGA Starting at vertex A, perform depth-first search on the following graph in SYGA:

syga.space/exercises/week-2/practice-depth-first-search.

Whenever there's a choice of vertices, proceed in alphabetical order. First, find the tree edges, then classify the non-tree edges into back, forward and cross edges.

Finally you can compare your solution to one generated algorithmically. Feel free to change the graph (at the top of the script) or explore the algorithm.



4. Singly Connected Graphs — SYGA A directed graph G is *singly connected* if for every pair of vertices u and v there is at most one path from u to v in G. Follow this link to the SYGA exercise:

syga.space/exercises/week-2/singly-connected-graphs

Adjust the algorithm to determine whether or not a directed graph is singly connected. Your finished algorithm should set `flag = 1` whenever you find a counterexample.

There are three graphs at the top of the script for you to try your solution on. The first two are *not* singly connected.

5. Longest Path In a DAG Given a directed, acyclic graph (DAG) $G = (V, E)$ and two of its vertices s and t, we want to compute the length of a longest path from s to t.

6. Count Paths In a DAG Given a DAG $G = (V, E)$ and two of its vertices s and t, design an algorithm that calculates the number of distinct paths from s to t.