Anéis

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MAT-UnB



Exercício

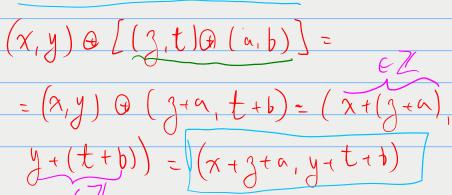
Considere o conjunto $\mathbb{Z} \times \mathbb{Z}$ com as operações

$$(a,b) \oplus (c,d) = (a+c,b+d)$$
$$(a,b) \otimes (c,d) = (ac,ad+bc).$$

para todos (a, b), $(c, d) \in \mathbb{Z} \times \mathbb{Z}$. Verifique se $(\mathbb{Z} \times \mathbb{Z}, \oplus, \otimes)$ é um anel. Caso afirmativo, esse anel é comutativo? Possui unidade?

Solucio: SETM
$$(x,y)$$
, (g,t) $(a,b) \in \mathbb{Z} \times \mathbb{Z}$. TEMDS
$$(x,y) \oplus (g,t) \oplus (a,b) = (x+y,y+t) \oplus$$

$$(x,y) \oplus (z,t) \oplus (a,b) = (x+z,y+b) = (x+z)+b = -(x+z)+a$$



ASSim

 $[(x,y)e(z,t)] \circ (a,b) = (x,y)o((z,t)e$

(a, b) .

AGONA

$$(x,y) \circ (3,t) = (x + 3, y + t)$$

$$= (3 + x, t + y) = (3,t) \circ (x,y)$$

$$2060$$

 $(x,y) \circ (x,t) = (x,y) \cdot (x,y)$.

$$(x, p) \in \mathbb{Z} \times \mathbb{Z}$$

$$(x, y) \circ (x, \beta) = (x, y)$$

$$(x + \alpha, y + \beta) = (x, y)$$

$$x + \alpha = x$$

A 60M TOMANDO (0,0) € Z/xZ/

TEMOS

 $(x,y) \oplus (0,0) = (x+0,y+0) = (x,y)$

PAM TODO (x,y) E Z/x Z/. 1060

$$\frac{(x,y) \in \mathbb{Z} \times \mathbb{Z}}{1 \text{ At ONE}} = \frac{(x,\beta) \in \mathbb{Z} \times \mathbb{Z}}{(x,\beta) \in \mathbb{Z} \times \mathbb{Z}}$$

$(\wedge, y) \oplus (\alpha, \beta) = (0, 0)$	
$(\chi \leftrightarrow \chi + (3) = (0, 0)$	

DADO
$$(x,y) \in \mathbb{Z} \times \mathbb{Z}$$
 to ME O
E CENENTO $(-x,-y) \in \mathbb{Z} \times \mathbb{Z}$. ASSIM
 $(x,y) \oplus (-x,-y) = (x+(-x),y+(-y))$

LOGO O POSTO DE $(x,y) \in \mathbb{Z}$ \mathbb{Z} \mathbb{Z}

460M DOOS (x,y),(g,t),(a,b) e

$$(x,y) \circ (z,t) = (xz,xt+yz)$$

$$(x,y) \circ (z,t) = (xz,xt+yz)$$

$$(x,y) \circ (z,t) = (xz,xt+yz)$$

= (xza, xzb + xta + yza)

 $(x,y) \otimes [(3,t) \otimes (a,b)] =$

=
$$(x,y) \otimes (3a, 3b+ta) =$$

= $(x3a, x(3b+ta) + y3a)$
= $(x3a, x3b+xta + y3a)$

AGONA.

$$[(x,y) \circ (3,t)] \circ (a,b) = (x+3,y+t) \circ (a,b)$$

= $((x+3)a,(x+3)b+(y+t)a)$

=(xa+3a, xb+3b+ya+ta)

· (x,y) @ (a,b) @ (3,t) @ (a,b) =

$$= (xa, xb+ya) \otimes (ya, yb+ta)$$

$$= (xa+ya) \otimes (ya+yb+ta)$$

con ISO, (x,y) & (3,t] o (a,b) = (x,y) @ (a,b) @ (3,t) @ (a,b).

Find Interest $(x,y) \otimes [(3,t) \otimes (a,b)) = (x,y) \otimes (3+a,t+b)$ = (x(3+a), x(t+b)+y(3+a))

= (x3 + xa, xt + xb + y3 + ya)

$$(x,y)\Theta(z,t)\Theta(x,y)\Theta(\alpha,b) =$$

$$=(xz,xt+yz)\Theta(xa,xb+ya)$$

- (xz + xa, xt + yz + xb + ya)

$$(x,y) \otimes [(3,t) \otimes (a,b)] = (x,y) \otimes (3,t)$$

$$\otimes (x,y) \otimes (a,b).$$

PORTANTO (Z12 D, Q) E un ANEL.

$$(x,y) \otimes (z,t) = (xz, xt + yz)$$

 $(3, t) \circ (x, y) = (3x, 3y + tx)$

$$(x,y) \otimes (3,t) = (3,t) \otimes (x,y).$$

OU SECA, (ZxZ, ø, &) É um

ANEL OMUTATIVO.

$$(x,y) \in \mathbb{Z} \mathbb{Z}$$
; $\text{Exist}((x,\beta) \in \mathbb{Z} \mathbb{Z})$
 $(x,y) \otimes (x,\beta) = (x,y)$

TOMANDO O PAR (1,0) EZIXZI

TEMOS

PAM TODO (x,y) c Zx Z.

 $(x,y) \otimes (1,0) = (x,y)$

PORTANTO (ZxZ,O,O) POSSUI

AJEL É O PAR (1,0).

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