$f: \mathbb{Z}_5 \times \mathbb{Z}_9 \to \mathbb{Z}_5 \times \mathbb{Z}_9$ É INJETORA? É SOBREJETORA?

$$f(x,y) = (\overline{2}x + \overline{3}, \overline{4}y + \overline{5}), \quad x \in \mathbb{Z}_5, \ y \in \mathbb{Z}_9$$

$(x,y), (z,t) \in \mathbb{Z}_5 \times \mathbb{Z}_9$ TAIS QUE

$$f(x,y) = f(z,t) \overset{?}{\Rightarrow} (x,y) = (z,t)$$

$$(\overline{2}x + \overline{3}, \overline{4}y + \overline{5}) = (\overline{2}z + \overline{3}, \overline{4}t + \overline{5})$$

$\Rightarrow [\overline{2}x + \cancel{\overline{3}} = \overline{2}z + \cancel{\overline{3}}] \in \mathbb{Z}_5$ $\quad +\overline{2}$ $\quad \Rightarrow [\overline{4}y + \overline{5} = \overline{4}t + \overline{5}] \in \mathbb{Z}_9$ $\quad +\overline{4}$

$$2x + 3 = 2z + 3 \quad + (-3)$$

$\mathbb{Z}_m$, $\Rightarrow \overline{0} \oplus \overline{x} = \overline{x}$

$\Rightarrow \overline{x} \in \mathbb{Z}_m$, EXISTE $\overline{y} \in \mathbb{Z}_m$ T.Q.

$\Rightarrow \overline{x} \oplus \overline{y} = \overline{0}$.

$\Rightarrow \overline{x} \otimes \overline{1} = \overline{x}$

$$\overline{2}x + \overline{0} = \overline{2}z + \overline{0}$$

$\Rightarrow \overline{2}x = \overline{2}z \ (\times \overline{3})$ EM $\mathbb{Z}_5$; $\overline{x} \neq \overline{0}$; $mdc(x,5) = 1$

$\overline{x} = \overline{z}$ e $\overline{y} = \overline{z}$

$$2x = 2z \quad \times \left(\frac{1}{2}\right) \Rightarrow \boxed{2 \cdot \frac{1}{2}}\, x = \boxed{2 \cdot \frac{1}{2}}\, z$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \underset{1}{\parallel} \quad\quad\quad\quad \underset{1}{\parallel}$$

$$x = \frac{2}{2} z \Rightarrow x = z$$

$\mathbb{Z}_m$, $\bar{x} \in \mathbb{Z}_m$, $\bar{x} \neq \bar{0}$; EXISTE $\bar{y} \in \mathbb{Z}_m$

TAL QUE

$\Rightarrow \bar{x} \otimes \bar{y} = \bar{1}$

SE, E SÓ SE, $mdc(x, m) = 1$.

$\bar{2} \otimes \bar{3} = \overline{2 \cdot 3} = \bar{6} = \bar{1}$

LOGO, $(x, y) = (z, t)$ E $f$ É INJETORA.

$g : \mathbb{Z}_5 \times \mathbb{Z}_9 \to \mathbb{Z}_5 \times \mathbb{Z}_9$

$g(x, y) = (\bar{2}x + \bar{3}, \bar{3}y + \bar{5})$

$g(x, y) = g(z, t)$

$\bar{3}y + \bar{5} = \bar{3}t + \bar{5} \quad + \bar{4}$

$\Rightarrow \bar{3}y = \bar{3}t \qquad mdc(3, 9) = 3 \neq 1$

$\boxed{y \neq t} \qquad \bar{3} \otimes \bar{y} \neq \bar{1}, \bar{y} \in \mathbb{Z}_9$

$$\left.\begin{array}{l} g(\bar{1}, \bar{0}) = (\bar{0}, \bar{5}) \\ g(\bar{1}, \bar{3}) = (\bar{0}, \bar{5}) \end{array}\right] \quad g(\bar{1}, \bar{0}) = g(\bar{1}, \bar{3})$$

$$\text{E} \quad (\bar{1}, \bar{0}) \neq (\bar{1}, \bar{3})$$

LOGO $g$ NÃO É INJETORA.

$\to \overline{3} \otimes \overline{3} = \overline{0}$ ; $\underbrace{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}}$

$\overline{3} \neq \overline{0}$, $\overline{3} \neq \overline{0}$

$\to xy = 0 \Rightarrow x = 0$ OU $y = 0$

$\mathbb{Z}_m$

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LISTA 8: $f : A \to B$.

9) e) $f$ É BIJETORA SE, E SÓ SE,

$[\underbrace{f(P^c) = (f(P))^c}$, PARA TODO $P \subseteq A]$

SE $f(P^c) = [f(P)]^c$, PARA TODO $P \subseteq A$, ENTÃO $f$ É INJETORA.

~~$[x_1, x_2 \in A ; f(x_1) = f(x_2) \Rightarrow x_1 = x_2]$~~

$\{\Rightarrow (\underline{x_1 \neq x_2} \Rightarrow \underline{f(x_1)} \neq \underline{f(x_2)}) \Leftarrow\}$

SEJAM $x_1, x_2 \in A$ TAIS QUE $\boxed{x_1 \neq x_2}$.

$P = \{x_2\}$ $\qquad \underline{f(P^c)} = [f(P)]^c = \underline{\{f(x_2)\}^c}$

$f(P) = \{f(t) \mid t \in P\}$

COMO $x_1 \neq x_2$ E $P = \{x_2\}$, ENTÃO $x_1 \notin P$.

DAÍ $\underline{x_1} \in P^C$. E ENTÃO $\underline{f(x_1)} \in \underline{f(P^C)}$.

$$f(x_1) \in f(P^C) = \{ f(z) \mid z \in P^C \}$$

COMO $\underline{f(P^C) = [f(P)]^C = \boxed{\{f(x_2)\}^C}}$, TEMOS

$\underline{f(x_1)} \in (\underbrace{\{f(x_2)\}}^C)$. LOGO, $f(x_1) \notin \{f(x_2)\}$

$f(x_1) \notin E$

PORTANTO, $\boxed{f(x_1) \neq f(x_2)}$, OU SEJA,

$f$ É INJETORA.

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SEMANA 7: $f: \mathbb{R} \to \mathbb{R}$; $g: \mathbb{R} \to \mathbb{R}$

2) $f(x) = 3x + m$ ; $g(x) = \underline{ax + 2}$

$$[(f \circ g)(x) = (g \circ f)(x)] \; \forall$$

$$f(g(x)) = g(f(x))$$

$$f(ax + 2) = g(\underline{3x + m})$$

$$3(ax+2)+m = a(3x+m)+2$$

$$\cancel{3ax}+6+m = \cancel{3ax}+\underline{am}+2$$

$$m - am = -4$$

$$\underline{m}(\underline{1-a}) = -4$$

$$i:\ m \neq 0 \ \text{e} \ 1-a \neq 0$$

$$m \neq 0 \ \text{e} \ a \neq 1$$

$$\underline{m} = \frac{-4}{1-a}$$