Grupo Simétrico

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MAT-UnB

Exercício

Considere o grupo S_4.

(a) *Determine elementos $f, g \in S_4$ tais que*

$$(f \circ g)^4 \neq f^4 \circ g^4.$$

$$f^4 = \underbrace{f \circ f \circ f \circ f}$$

(b) *Para o elemento*

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$

encontre o menor $\boxed{l \geq 0}$ tal que $\boxed{\sigma^l = 1}$, onde

$$1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

$$\sigma^l = \underbrace{\sigma \circ \sigma \circ \sigma \ldots}_{l}$$

$S_4 = \{ f : A \to A \mid f$ É BIJETORA

E $A = \{ \underline{1, 2, 3, 4} \} \}$

$(S_4, \circ)$ É UM <u>GRUPO</u>

SOLUÇÃO: a) TOME

$$\underline{f} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \in S_4$$

$$\underline{g} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \in S_4$$

$$\begin{cases} (f\circ g)(1) = f(g(1)) = f(2) = 3 \\ (f\circ g)(2) = f(g(2)) = f(4) = 1 \end{cases}$$

ASSIM

$$f\circ g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} = h$$

AGORA

$(f \circ g)^4 = h^4 = h \circ h \circ h \circ h$

DAÍ,

$$h \circ h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} =$$

$$= \underbrace{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}}_{} = \underline{h}^2$$

$$h^2 \circ h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & \underline{4} \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & \underline{4} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = h^3 = Id = 1$$

Assim

$$h^4 = h^3 \circ h = 1 \circ h = h$$

$$\boxed{h^4 = h}$$

AGORA

$f^4 = f \circ f \circ f \circ f \quad ; \quad g^4 = g \circ g \circ g \circ g$

Assim

$f \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} =$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} = f^2$$

$$f^2 \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = f^3$$

$$f^3 \circ f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = 1 = Id$$

$$f^4 = Id$$

$$g \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} = g^2$$

$$g^2 \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} = g^3$$

$$g^3 \circ g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = 1 = \mathrm{Id}$$

$g^4 = Id$

PORTANTO

$$(f \circ g)^4 = h^4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \neq \overset{Id}{\overset{\|}{f^4}} \circ \overset{Id}{\overset{\|}{g^4}} = Id$$

b) PRIMEIRO NOTE QUE $\sigma^1 \neq 1$,
LOGO $\ell > 1$.

$$\sigma^{\boxed{2}} = \sigma \circ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = 1 = Id.$$

LOGO $\ell = 2$ É TAL QUE

$\sigma^2 = 1$. #