# 3.3 Normal Random Variables

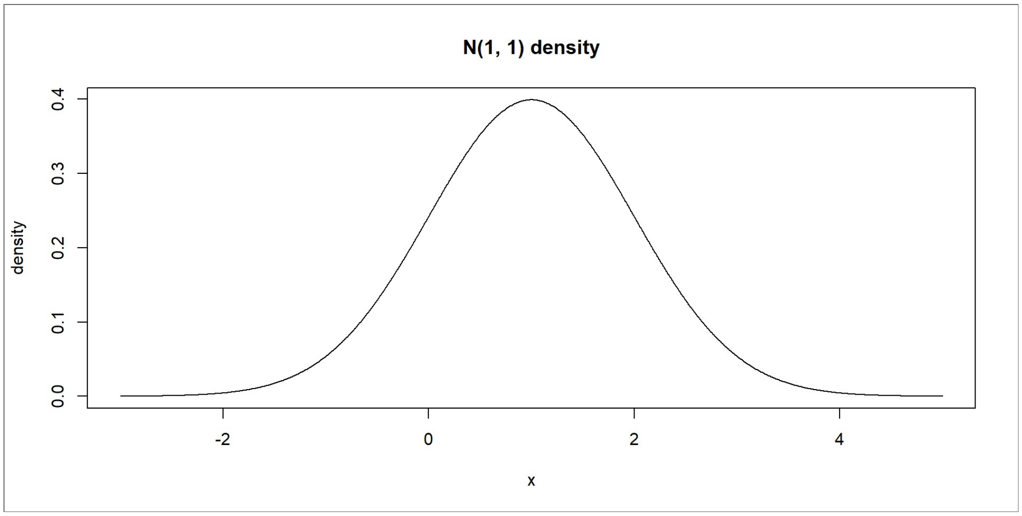
## Normal pdf

A continuous random variable $X$ is **normal** or **Gaussian** with mean $\mu$ and variance $\sigma^2$ if its pdf is

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}.$$

We write $X \sim N(\mu, \sigma^2)$.

## Normal pdf




## Mean of a normal

## Variance of a normal

## The normal is special

The standard normal random variable has mean 0 and variance 1.

**Normality is preserved by linear transformation**

If $X \sim N(0,1)$ and $Y = \mu + \sigma X$, then
$Y \sim N(\mu, \sigma^2)$.

The pdf of a standard normal is denoted by $\phi(x)$ and the cdf by $\Phi(x)$.

## Example 3.7

The annual snowfall at a location is modeld as a $N(60, 20^2)$ random variable.
What is the probability that this year's snowfall will be at least 80 inches?

## Example 3.8 Signal detection

A binary message is transmitted as a signal $s$, which is either -1 or +1. The communication channel corrupts the transmission with additive normal noise with mean 0 and variance $\sigma^2$. The receive concludes that the signal -1 was transmitted if $x < 0$ or +1 if $x \geq 0$.
What is the probability of error if $\sigma = 1$?

## Example 3.8 (cont)

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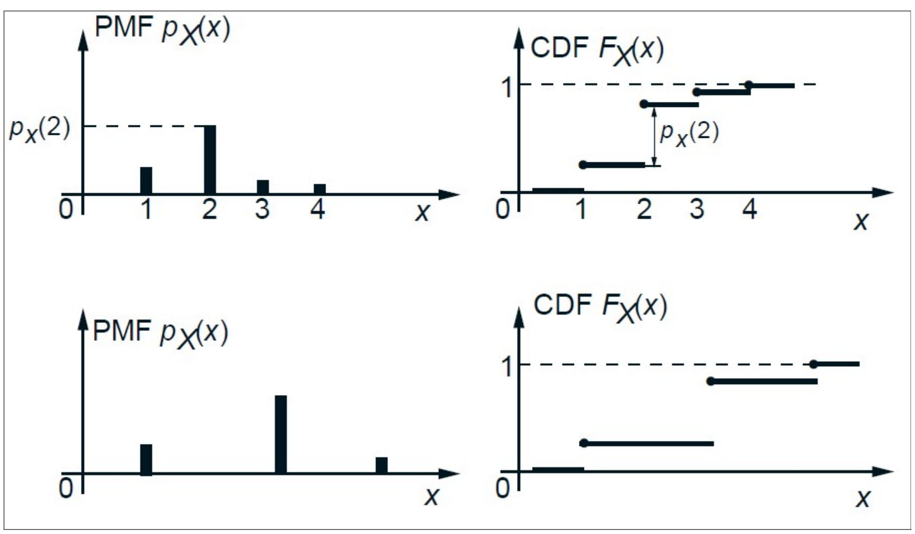


The signal detection scheme.