

2.3 Functions of random variables

Context

Applying a function to a random variable generates another random variable.

Example: Temperature

- Let X denote today's temperature in degrees Celsius.
- $Y = 1.8X + 32$ is a new random variable that gives the temperature in degrees Fahrenheit.

Linear and non-linear functions

A transformation of X is linear if it has the form

$$Y = aX + b, \quad a, b \in \mathbb{R}.$$

Any function that is not linear is **non-linear**, e.g., $Y = \log(X)$.

Transformation relationship

If X is a discrete random variable, then $Y = g(X)$ is also a discrete random variable with pmf

$$p_Y(y) = \sum_{\{x:g(x)=y\}} p_X(x).$$

Example 2.1

Let X be a discrete random variable with pmf

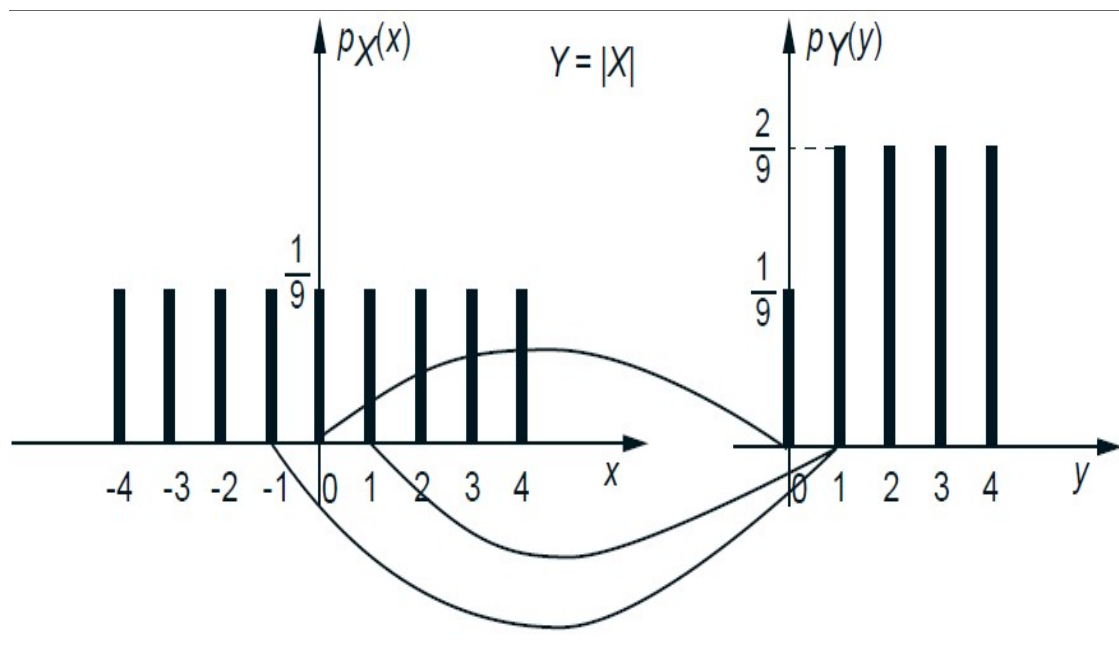
$$p_X(x) = \begin{cases} \frac{1}{9} & \text{if } x \text{ is an integer in } [-4, 4], \\ 0 & \text{otherwise.} \end{cases}$$

Let $Y = |X|$.

Determine the pmf of Y .

Example 2.1 (cont)

Visualization of pmf calculation



The pmfs of X and $Y = |X|$.

Example 2.1 (cont)

Also, obtain the pmf of $Z = X^2$.