2.2 Probability Mass Functions

Definition

The **probability mass function (pmf)** of a discrete random variable X is a function $p_X : \mathbb{R} \to [0, 1]$ defined by:
$p_X(x) = P(X = x)$
for each x in the range of X, denoted R_X, where R_X has a finite or countable infinite number of values.

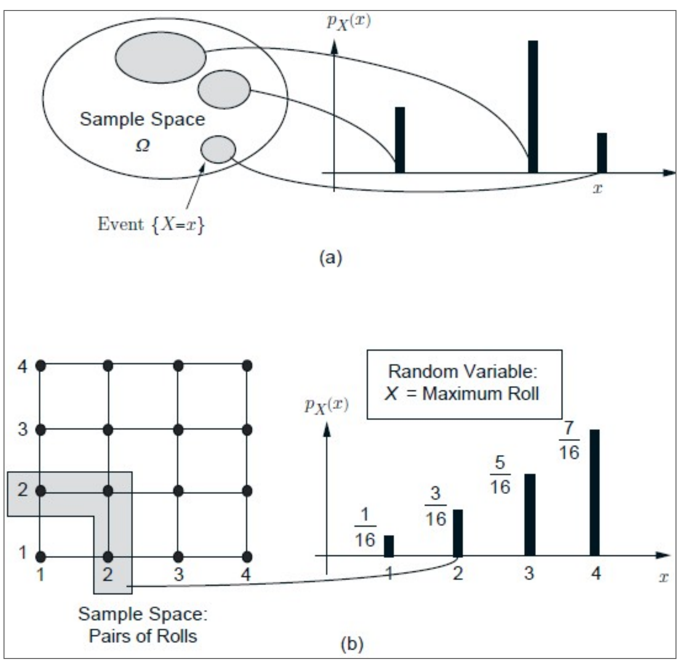
- Capital X will denote a random variable.
- Lowercase x will denote a specific numeric value.

Calculation of the pmf

For each possible value x of X:

1. Collect all the possible outcomes that give rise to the event ${X = x}.
2. Add their probabilities to obtain $p_X(x)$.

Visualization of pmf calculation



Calculating a pmf

Simple example

Suppose we toss a fair coin twice in a row and count the number of heads.

$$p_X(x) = \begin{cases} \frac{1}{4} & \text{if } x = 0 \\ \frac{1}{2} & \text{if } x = 1 \\ \frac{1}{4} & \text{if } x = 2. \end{cases}$$

Properties of pmfs

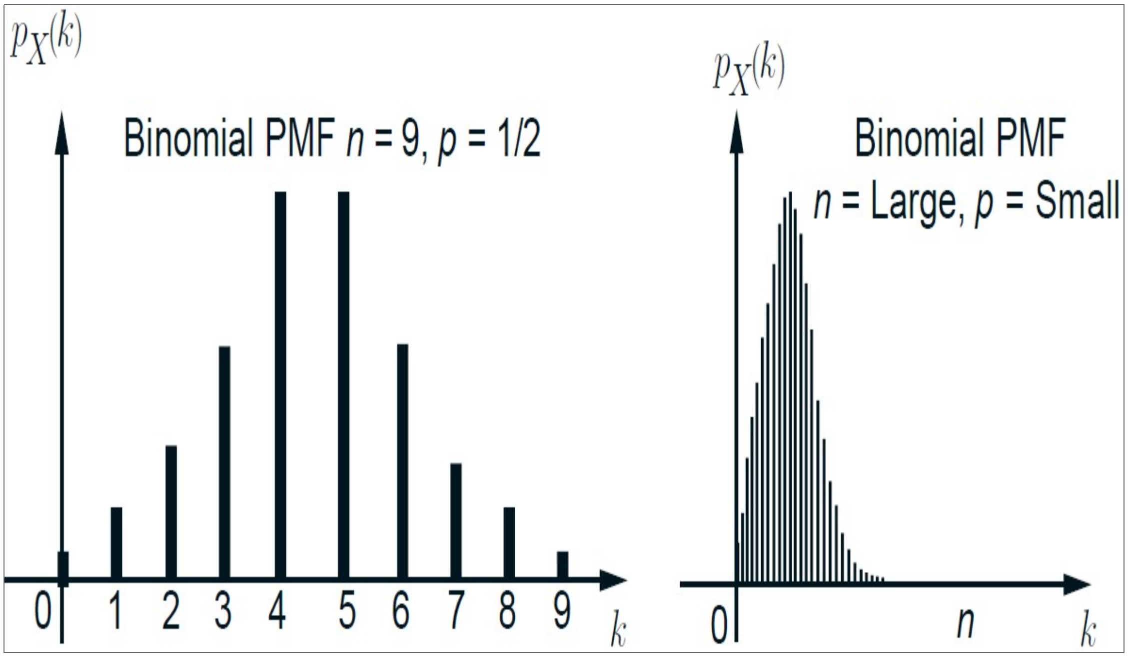
The pmf of a random variable $X, p_X(x)$, satisfies the following properties:

- $p_X(x) \geq 0$ for all x
- $\sum_{x \in R_X} p_X(x) = 1$, where R_X is the range of X.
- If $A \subset R_X$, then $P(A) = \sum_{x \in A} p_X(x)$.

E.g., continuing our simple coin flipping example,

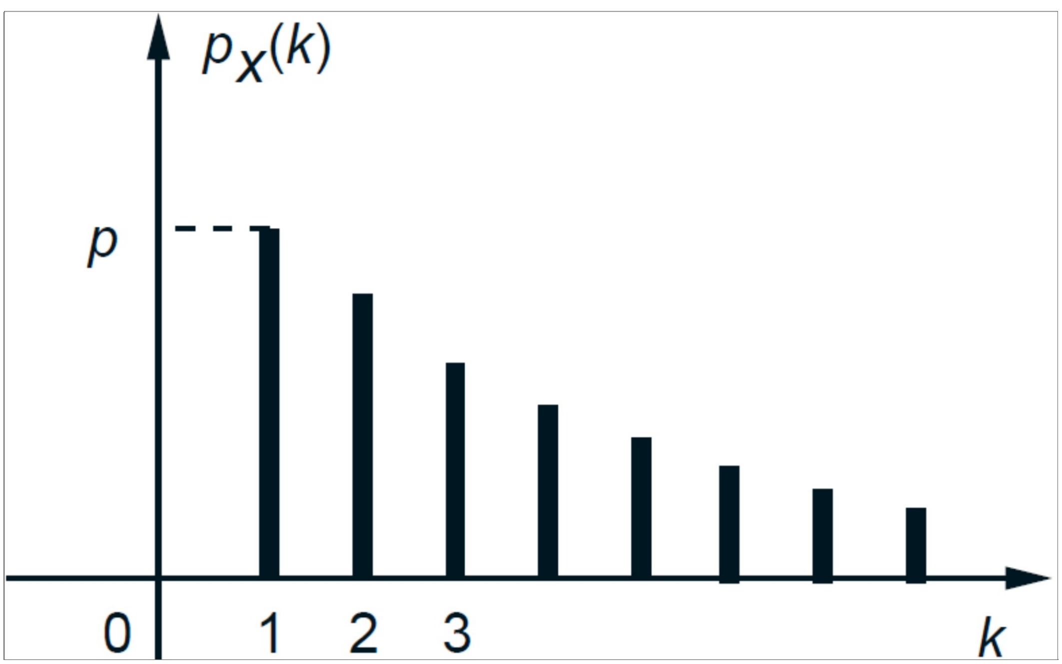
$$P(X > 0) = p_X(1) + p_X(2) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}.$$

Visualizing a binomial pmf



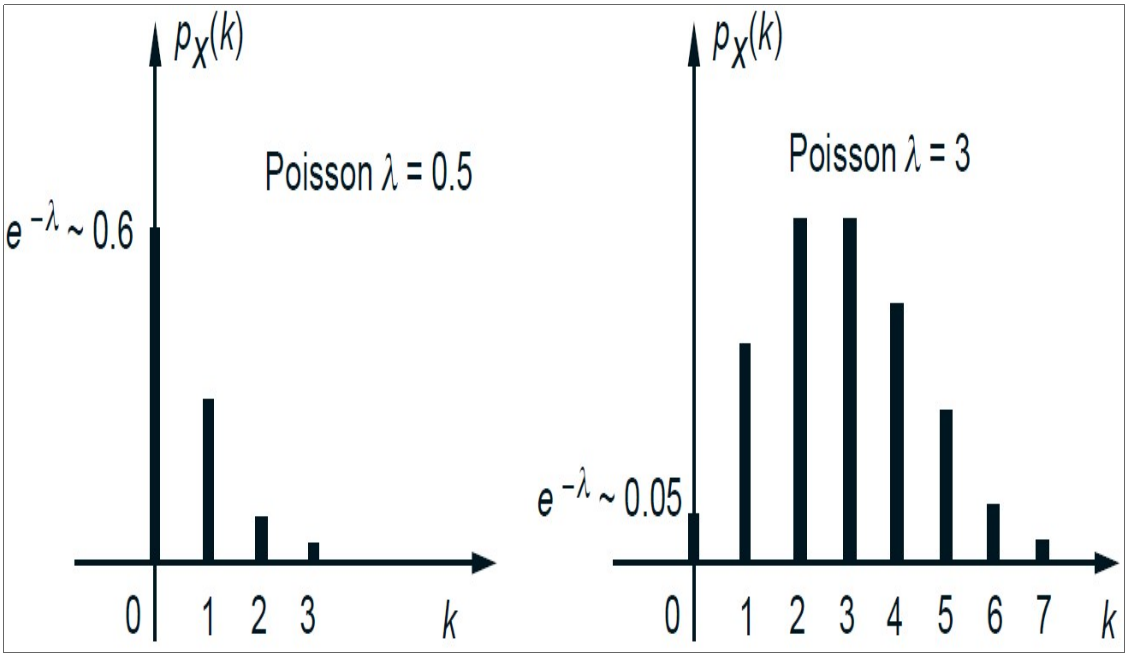
The pmf of a binomial random variable

Visualizing a geometric pmf



The pmf of a geometric random variable

Visualizing a Poisson pmf



The pmf of a Poisson random variable