Zero-Mean Pressure

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1 Zero-Mean Pressure Calculation

Since ∇p isn't distinguishable from $\nabla(p+c)$ for c a constant, I'm going to investigate since I'm fairly certain what I did in my implementation was correct.
By the divergence theorem, since $\mathbf{v} = 0$ on the boundary:

$$\int \nabla \cdot ((p+c)\,\mathbf{v})\, d\Omega = \oint (p+c)\,\mathbf{v} \cdot \hat{\mathbf{n}} d\Gamma = 0$$

$$\begin{aligned}
\int \nabla \cdot ((p+c)\,\mathbf{v})\, d\Omega &= \int \nabla \cdot (p\mathbf{v})\, d\Omega + \int \nabla \cdot (c\mathbf{v})\, d\Omega \\
&= \int \nabla p \mathbf{v} d\Omega + \int p \nabla \cdot \mathbf{v} d\Omega + \cancelto{0}{\int \mathbf{v} \nabla c d\Omega} + \int c \nabla \cdot \mathbf{v} d\Omega \\
&= \int \nabla p \mathbf{v} d\Omega + \int p \nabla \cdot \mathbf{v} d\Omega + c \int \nabla \cdot \mathbf{v} d\Omega
\end{aligned}$$

$$\int \nabla p \mathbf{v} d\Omega = -\int p \nabla \cdot \mathbf{v} d\Omega - c \int \nabla \cdot \mathbf{v} d\Omega$$

If I assert that $\int \nabla p \, \mathbf{v} \, d\Omega \equiv -\int p \nabla \cdot \mathbf{v} d\Omega$ in my implementation, aren't I then fixing $c = 0$ and calculating a unique pressure? That's what I did in my implementation.