Math 324 D - Winter 2018
Midterm exam 2
Friday, February 16, 2017

Name: ____________________________________

Problem 1	15	
Problem 2	13	
Problem 3	11	
Problem 4	11	
Total	50	

- There are 4 problems on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2}+\frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (15 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a function with $f(1,0,1) = 2, \nabla f(1,0,1) = \langle 3,-2,1\rangle$, and $f(5,0,0) = 4, \nabla f(5,0,0) = \langle 1,3,0\rangle$.

   (a) Let G be the vector field defined by $G(x,y,z) = \nabla f(5,0,0)$ for all $(x,y,z) \in \mathbb{R}^3$. Is G conservative? If so, give a potential function for G; if not, explain why.

   (b) Find $D_u f(1,0,1)$, where $u = \frac{1}{\sqrt{3}}\langle 1,1,-1\rangle$.

   (c) Consider the curve C parameterized by $r(t) = (4t+1, t-t^2, 1-t^4)$ for $0 \le t \le 1$. What is $\int_C \nabla f \cdot dr$? (Make sure to justify your answer.)

2. (13 points) Consider the vector field $F = \langle x^2y, -2x \rangle$, and let C denote the part of the parabola $y = 1 - x^2$ starting at $(-1, 0)$ and ending at $(1, 0)$.

   (a) Evaluate $\int_C F \cdot dr$.

   (b) Use the fundamental theorem for line integrals to evaluate the integral from part (a), or explain why it cannot be applied.

3. (11 points) Use the transformation $x = \frac{u}{v+1}, y = \frac{uv}{v+1}$ to set up the integral

$$\iint_D (x+y)\, dA$$

in $u - v$ coordinates, where D is the region in the $x - y$ plane bounded by the lines

$$y = 2x, y = x, y = 6 - x, y = 3 - x.$$

Your integral should have explicit bounds in terms of u and v, and the integrand should be a function of u's and v's. *You do not need to evaluate the integral.*

4. (11 points) Use Green's theorem to evaluate

$$\int_C (y^2\hat{\imath} + x^2\hat{\jmath}) \cdot dr,$$

where C is the boundary of the unit square $[0,1] \times [0,1]$, oriented counter-clockwise.