The University of British Columbia

Math 101-302 — Introduction to Probability

2018, October 24

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Name: _____ Student ID: _____

Instructions

- This exam consists of **4 questions** worth a total of 34 points.
- Make sure this exam has **4 pages** excluding this cover page.
- Note that there is a **table of discrete distributions** at the bottom of this page.
- **Explain** your reasoning thoroughly, and **justify** all answers (even if the question does not specifically say so). No credit might be given for unsupported answers.
- Final answers should be **simplified** and **clearly marked**.
- No calculators, notes, or other aids are allowed.
- If you need more space, use the back of the pages.
- Duration: **50** minutes.

Question	Points	Score
1	6	
2	10	
3	10	
4	8	
Total:	34	

Common Discrete Distributions

Random Variable X	$\mathbb{P}(X=k)$	Mean	Variance
$\mathrm{Ber}(p)$	$P(X=0)=1-p,\ P(X=1)=p$	p	$p(1-p)$
$\mathrm{Bin}(n,p)$	$\binom{n}{k}p^k(1-p)^{n-k}$	np	$np(1-p)$
$\mathrm{Geom}(p)$	$p(1-p)^{k-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$\mathrm{NegBin}(r,p)$	$\binom{k-1}{r-1}p^r(1-p)^{k-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
$\mathrm{Poisson}(\lambda)$	$\frac{\lambda^k}{k!}e^{-\lambda}$	λ	λ

6 marks 1. Let X be a Poisson(1) random variable.

(a) Show that the events $\{X \leq 1\}$ and $\{X \geq 1\}$ are not independent.

(b) Let Y be the indicator of the event $\{X \leq 1\}$ (that is, $Y = 1$ if $X \leq 1$, and $Y = 0$ otherwise). Let Z be the indicator of $\{X \geq 1\}$. Compute the variance of $Y + Z$.

10 marks

2. Your friend has a bag with two types of dice: "boring" dice with 10 sides labeled 1 to 10, and "magic" dice with 10 sides labeled 1,2,2,3,3,3,4,4,4,4. Ten percent of the dice in the bag are magic and the rest are boring. You mix the bag, your friend takes out a die and rolls it.

   (a) What is the probability that the die lands on 3?

   (b) If the die lands on 3, what is the probability that it is a magic die?

   (c) Your friend returns the die to the bag, takes out another die and rolls it twice. What is the probability that the die lands on the same number in both rolls?

10 marks

3. In a lottery, there is a ball machine containing 25 colored balls: 1 gold, 9 silver and 15 white. A gold ball is worth \$200, a silver \$15 and a white \$1.

   (a) You take out two balls. What is the probability that you win at least \$30?

   (b) You take out one ball. What is the expected amount you win?

   (c) The lottery owner replaces the ball machine. The lottery owner does not tell you the worth of the balls in the new machine, but promises that the expected value of a single ball has not changed, and also that the standard deviation is \$4. You take out one ball. Give a lower bound on the probability that you win at least \$6.

8 marks

4. You have a mini-deck of 16 cards numbered 1 to 8, with two cards of each number. You also have two 8-sided dice (whose sides are numbered 1 to 8). You draw three cards and roll the two dice. Find the probability that:

   (a) The two numbers on the dice are the same, whereas all three numbers on the cards are different.

   (b) The *set* of numbers on the two dice is the same as the *set* of numbers on the three cards.