Math 324 B, Homework 8

For problems 1-4, verify Stokes' theorem for the given oriented surface and vector field. That is, show that $\int_C F \cdot dr = \iint_S \nabla \times F \cdot dS$ where $C = \partial S$ is the boundary curve of S. You may use whatever orientation you like, but be sure to specify which orientation you are using.

1. $F = \langle 2xy, x, y+z \rangle$, S is the surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$.
2. $F = \langle yz, 0, x \rangle$, S is the portion of the plane $x/2 + y/3 + z = 1$ where $x, y, z \geq 0$.
3. $F = \langle e^{y-z}, 0, 0 \rangle$, S is the square with vertices $(1,0,1), (1,1,1), (0,1,1)$, and $(0,0,1)$.
4. $F = \langle yz^2, 0, 0 \rangle$, S is the surface of the cylinder $x^2 + y^2$ of radius 2 for $1 \leq z \leq 6$ (not including the top and bottom disks).
5. Use Stokes' theorem to evaluate

$$\iint_S \nabla \times \langle 0, x, xz \rangle \cdot dS,$$

where S is the spherical cap $x^2 + y^2 + z^2 = 1$ for $z \geq 1/2$ oriented towards the positive z-axis.

For problems 6-7, verify the divergence theorem for the given vector field F and region E. That is, show that $\iint_S F \cdot dS = \iiint_E \nabla \cdot F dV$ where $S = \partial E$ is the boundary surface of E oriented outward.

6. $F = \langle y, x, z \rangle$, E is the interior of the sphere of radius 2.
7. $F = \langle x, y^2, z+y \rangle$, E is the region bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = x, z = 8$.
8. Suppose E is a region in $\mathbb{R}^3$ such that

$$\iint_{\partial E} \langle x + 2xy + z, e^x - 3z^2 - y^2, 4z \rangle \cdot dS = 85,$$

where ∂E is the boundary of E oriented outward. Find the volume of E.