Math 324 A, Spring 2017, pop quiz!

You have 10 minutes to complete the quiz. Make sure to explain your reasoning. This quiz is out of 5 points. Your score will be added to your second midterm grade.

Name: ____

1. (For all the marbles.) Let S denote the upper hemisphere of the sphere of radius 1 centered at the origin, i.e. the points (x, y, z) satisfying $x^2 + y^2 + z^2 = 1$ and $z \geq 0$. Also, let E be the inside of the top hemisphere, i.e. all (x, y, z) with $x^2 + y^2 + z^2 \leq 1$ and $z \geq 0$. Give S the outward orientation, so the normal vector $\widehat{n}$ to S has positive z component. Consider the vector field $F = x^2yz\hat{\imath} + (x+2)\sin(z)\hat{\jmath} - xyz^2\hat{k}$. Suppose we want to compute the flux of F through S.

   Henry, Jacob's evil twin brother, reasons as follows. Note that

   $$\nabla \cdot F = \frac{d}{dx}x^2yz + \frac{d}{dy}(x+2)\sin(z) - \frac{d}{dz}xyz^2 = 2xyz + 0 - 2xyz = 0,$$

   so by the divergence theorem,

   $$\iint_S F \cdot dS = \iiint_E \nabla \cdot F \, dV = \iiint_E 0 \, dV = 0.$$

   It happens to be true that

   $$\iint_S F \cdot dS = 0,$$

   but Henry's reasoning is flawed. Explain where Henry went wrong, and why he got the right answer anyway.

   [Hint: what is ∂E?]