April ()
I Transition probabilités and Kolmogorov equations
·
Recall: We characterize a CTTC by  Pij: Proba. next jump (when it occus) is to state j, given current state is i
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to state j, given current state is i
· Vi: Exp. rate at which jump occurs.
Goal: Study P; (+) = P(x(+)=; ) X(0)=i)
Rocall: For discrete hure M-C, me saw (chap. 1)
ON+M OM OM
Rocall: For discrete hur M-C, we saw (chap. 1)  C-Keg: Pij = EPik. Pkj
In the conhinuous-time setting, we have the analog
Prop (Chapman - Kolmagorov eg.)
H(s, E) 70 Pi; (s+t) = & Pix(s). Ph; (t)
Proof = #s, t >0
Pij(s+t) = P(X(t)=jlX(0)=i)
$= \mathbb{Z} P(X(s+t)=j, X(s)=k(X(0)=i)$
RES Narkov Property
- CANON Property

$$= \sum_{k} P\left(X(s+t)=j \mid X(s)=k, X(0)=i\right) \otimes P\left(X(s)=k \mid X(0)=i\right)$$

$$= \sum_{k} P\left(X(s+t)=j \mid X(s)=k\right) \cdot P_{i,k}(s)$$

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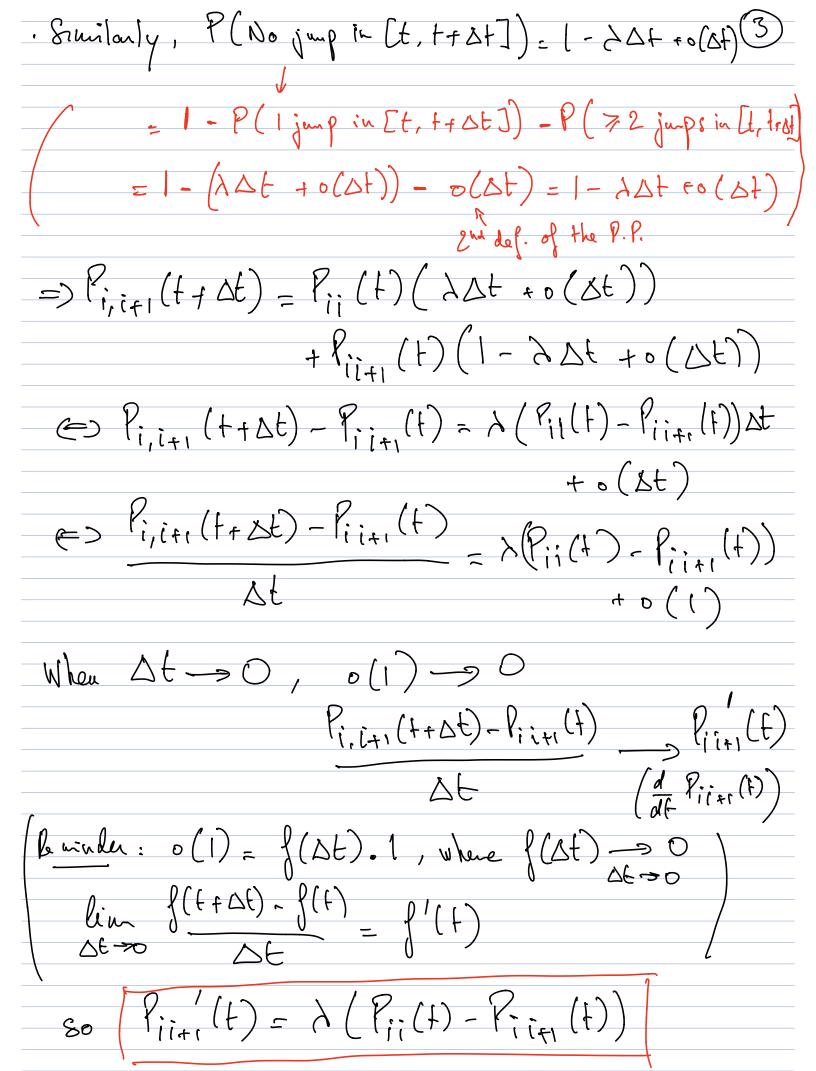
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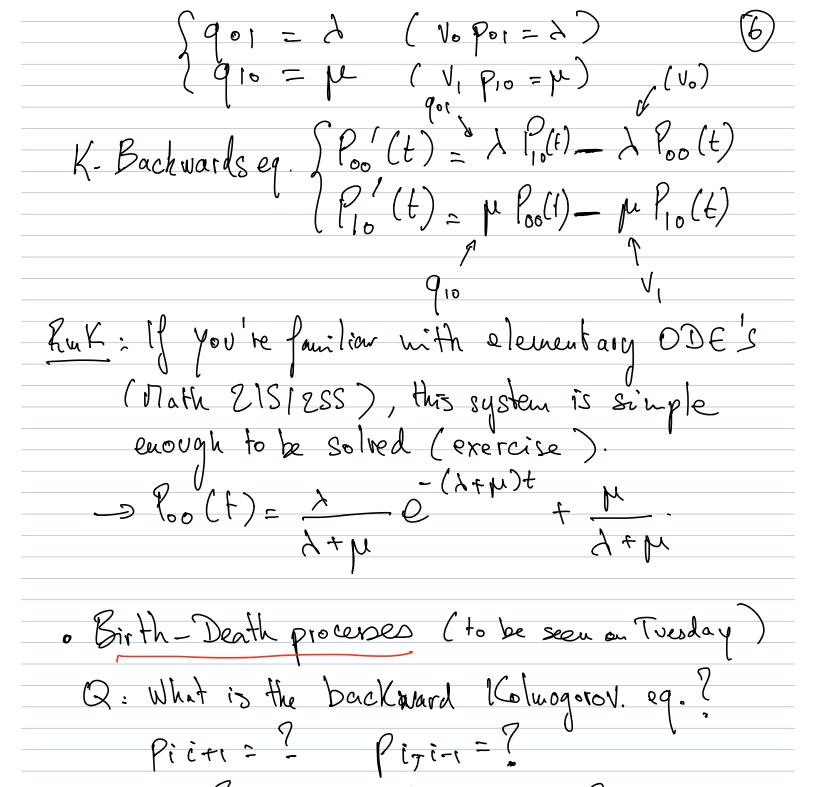
$$= P\left(X(t)=k\right)$$

$$= P\left(X($$



and similarly $P_{ii}(f) = \lambda (P_{i-1,i}(f) - P_{ii}(f))$ (exercise)
(exercise)
Cc: Troumlion probabilitiens Salisfy a system
of linear differential equations. This is
Cc: Trouvilien probabilities Salisfy a System of linear differential equations. This is also the case in general for any given CTMC.
First, let's devote (Dimportant notation)
qij: = Vi. Pij
Ruk (important): If we know the qijs, this is
Ruk (important): If we know the qijs, this is sufficient to define the CTMC, as we can deduce v; and pij;
· Vi = Vi \( \frac{1}{4i} \) pi = \( \frac{1}{4i} \) p
o Pij - Pij - Pij - Sigik k + i Pik
Then, we have (similarly as above, consider
the process at t+25t, using the Chapman Kolvegorov
Then, we have (similarly as above, consider the process at t+1st, using the Chapman Kolvoyorov eq., and $\Delta t \rightarrow 0$ )
V

Thue: (Kolmogorov backwards eq.) Pil(t) = \( \frac{1}{2} \) qik - \( \frac{1}{2} \) \( \frac{1}{2} hu: (Kolmogorov forward eg.) (i)(t)= = Fix(t) qxi - J';(t) HE, i, j we look forward from i - ske Ruk: These thus hold for any CTMC defined one has to be careful that the vi's are "small enough" so the probability that an interval of time contains infinitely many transitions is (otherwise the process is explosive in fine, and may not be defined for all t 70) -> In this diapter, ne will assume this never CTNC (D)=(1) No = >, N, = pe Por=



Vi 9 cier = ? - 9 i, i-1 = ?