Math 324 B - Winter 2017
Midterm exam 2
Friday, February 17, 2017

Name: ______________________________

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| Problem 1 | 10 | |
| Problem 2 | 14 | |
| Problem 3 | 16 | |
| Problem 4 | 10 | |
| Total | 50 | |

- There are 4 problems on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2}+\frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 points) Let $f(x, y, z) = x^2 \sin(z) - \frac{x^2+y^2}{z^2+1}$.

   (a) Find $\nabla f(x, y, z)$, and evaluate $\nabla f(1, -1, 0)$.

   (b) Find the directional derivative of $f$ in the $y-$direction at the point $(3, 2, -1)$: that is, evaluate $D_u f(3, 2, -1)$, where $u = \langle 0, 1, 0 \rangle$.

2. (14 points) Consider the vector field $F = \langle x^2y, y \rangle$, and the closed curve $C$ consisting of the parabola $y = x^2$ for $|x| \leq 1$ and the line segment connecting $(-1, 1)$ to $(1, 1)$. Give $C$ the clockwise orientation – so $C$ traverses the parabola from right to left, and the line segment from left to right. Draw a picture of $C$, and use Green's theorem to evaluate $\int_C F \cdot dr$.

3. (16 points) For each of the statements below, circle **True** if you think it is true, and circle **False** if you think it is false. You may use the space to do scratch work, but no partial credit will be awarded on this question. (4 points for each statement.)

(a) **True** **False** The vector field $F = \langle x^3, 3x^2y \rangle$, defined on all of $\mathbb{R}^2$, is conservative.

(b) **True** **False** The vector field $F = \frac{y\hat{\imath} - x\hat{\jmath}}{x^2+y^2}$, defined on $\mathbb{R}^2$ minus the origin, is conservative.

(c) **True** **False** A conservative vector field $F$ defined on any region $D \subset \mathbb{R}^2$ has $\int_C F \cdot dr = 0$ for any closed curve $C$ lying inside $D$.

(d) **True** **False** Consider the function $F(x, y, z) = 2x^4 + y^4 - z^4$. Let $S$ be the surface defined by the equation $F(x, y, z) = -14$, and consider the point $p = (1, 0, 2)$. Then $p$ lies on $S$, and the tangent plane to $S$ at the point $p$ has normal vector parallel to $\langle -1, 0, -4 \rangle$.

4. (10 points) Let $f(x, y) = 2x + y$, and consider the line segment $C$ starting at $(-2, 4)$ and ending at $(1, 3)$.

(a) Evaluate $\int_C \nabla f \cdot dr$ using the fundamental theorem for line integrals.

(b) Here's a parameterization of $C$: $r(t) = \langle 3t - 2, 4 - t \rangle$ for $0 \leq t \leq 1$. Use it to evaluate $\int_C \nabla f \cdot dr$ directly.