Math 324, Homework 3

For problems 1-3, use cylindrical coordinates to evaluate $\iiint_E f(x,y,z)\,dV$ for the given function and region.

1. $f(x,y,z) = x$, and E is the region enclosed by the planes $z = 0, z = x + y + 5$ and the cylinders $x^2 + y^2 = 4, x^2 + y^2 = 9$.

2. $f(x,y,z) = z\sqrt{x^2 + y^2}$ over the region $x^2 + y^2 \leq z \leq 8 - (x^2 + y^2)$.

3. $f(x,y,z) = z$ over the region $0 \leq z \leq x^2 + y^2 \leq 9$.

4. Express this triple integral in cylindrical coordinates: $\int_0^1 \int_0^{\sqrt{2x-x^2}} \int_0^{\sqrt{x^2+y^2}} f(x,y,z)\,dzdydx$.

For problems 5-8, use spherical coordinates to evaluate $\iiint_E f(x,y,z)\,dV$ for the given function and region.

5. $f(x,y,z) = x^2 + y^2$ over the region $4 \leq x^2 + y^2 + z^2 \leq 9$.

6. $f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$, over the region $x^2 + y^2 + z^2 \leq 2z$.

7. $f(x,y,z) = (1 + x^2 + y^2 + z^2)^{-2}$, over all of $\mathbb{R}^3$. [Do the integral on the ball of radius R, and then take the limit $R \to \infty$.]

8. $f(x,y,z) = xe^{x^2+y^2+z^2}$ over the part of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies in the octant $x \leq 0, z \leq 0$, and $y \geq 0$.

9. (Stewart 15.10.12) Find equations for a transformation T that maps a rectangular region S in the $u - v$ plane onto the parallelogram R in the $x - y$ plane with vertices $(0,0), (4,3), (2,4), (-2,1)$.

10. (15.10.15) Evaluate the integral $\iint_R (x - 3y)\,dA$, where R is the triangle with vertices $(0,0), (1,2), (2,1)$, by using the transformation $x = 2u + v, y = u + 2v$.

11. (Stewart 15.10.27) Evaluate $\iint_R e^{x+y}\,dA$, where R is the region $\{(x,y) : |x| + |y| \leq 1\}$.