Math 308 O
Midterm exam 2
Wednesday, February 26, 2020

Name: _____

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Total	40	

- There are 4 questions on this exam. Make sure you have all four.
- Always explain your reasoning clearly and concisely.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam.

GOOD LUCK!

1. For each of the following, either give an example that meets the requirements or explain why it is impossible.

   (a) A linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ that is onto.

   (b) A 2 by 2 matrix A with $A^2 = I_2$, but $A \neq I_2$.

   (c) A 3 by 5 matrix with nullity 2 and rank 1.

   (d) An invertible 3 by 3 matrix with rank 1.

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 8 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

(a) Find a basis for the null space of A.

(b) What are the rank and nullity of A?

3. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by $T(x) = Ax$.

(a) Is the vector $v = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ in the null space of A?

(b) Find a vector other than 0 in the range of A.

(c) Find a basis for range(A). What is the *dimension* of range(A)?

4. Circle **True** or **False** for each of the statements below. No justification is needed.

(a) **True** **False** A linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$ is not onto if $m < n$.

(b) **True** **False** For any 4 by 2 matrices A, B and C, the set of vectors

$$\{v \in \mathbb{R}^2 : Av - 3v = Bv + 2Cv\}$$

is a subspace of $\mathbb{R}^2$.

(c) **True** **False** The set of vectors $v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ such that $2v_1 + v_3 = 0$ and $2v_2 - 4v_3 = 1$ is a subspace of $\mathbb{R}^3$.

(d) **True** **False** If D is a 4 by 6 matrix with rank 2, the column space and the row space of D both have dimension 2.