Formulas you should know

Math 324

Change of variables

To change to polar coordinates, we have the identities $r^2 = x^2 + y^2, x = r\cos\theta, y = r\sin\theta$, and the determinant of the change of coordinate map from cartesian to polar is r. That is,

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(r,\theta) r\,dr\,d\theta,$$

where S is the image of R under the polar coordinate change.

In general, given a change of coordinates $x = g(u,v), y = h(u,v)$, we have

$$\iint_R f(x,y)\,dx\,dy = \iint_S f(u,v)\left|\det\begin{pmatrix} dx/du & dx/dv \\ dy/du & dy/dv \end{pmatrix}\right| du\,dv,$$

where S is the image of R under the coordinate change.

For spherical coordinates, we have

$$\iiint_E f(x,y,z)\,dx\,dy\,dz = \iiint_G f(\rho,\theta,\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi,$$

where G is the image of E under the coordinate change.

Similarly, for cylindrical coordinates, we have

$$\iiint_E f(x,y,z)\,dx\,dy\,dz = \iiint_G f(r,\theta,z) r\,dr\,d\theta\,dz,$$

where G is the image of E under the coordinate change.

Integral formulas

(Surface area) For a surface S defined by a function $z = f(x,y)$, where $(x,y) \in D$ is the parameterization domain, the surface area of S is given by

$$\iint_D \sqrt{1 + \left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2}\,dA.$$

In general, if S is parameterized by $r(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$ for $(u,v) \in D$, the surface area of S is given by the formula

$$\iint_D |r_u \times r_v| dA,$$

where r_u and r_v are the vector fields $r_u = \langle \frac{dx}{du}, \frac{dy}{du}, \frac{dz}{du}\rangle$, and $r_v = \langle \frac{dx}{dv}, \frac{dy}{dv}, \frac{dz}{dv}\rangle$.

(Green's Theorem) For a curve C bounding a region D with the positive orientation (region on the left), and a vector field $F = \langle P, Q \rangle$,

$$\int_C F \cdot d\mathbf{r} = \iint_D \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA.$$

(Stokes' Theorem) For a curve C bounding an oriented surface S with the positive orientation, and a vector field F,

$$\int_C F \cdot d\mathbf{r} = \iint_S \operatorname{curl}(F) \cdot d\mathbf{S}.$$

(Divergence Theorem) For a surface S bounding a region E with the outward pointing normal orientation, and a vector field F,

$$\iint_S F \cdot d\mathbf{S} = \iiint_E \operatorname{div}(F)\, dV.$$

For a surface $z = g(x, y)$ over a domain D in the $xy-$plane, an oriented surface S with the positive orientation (positive z direction) and a vector field $F = \langle P, Q, R \rangle$,

$$\iint_S F \cdot d\mathbf{S} = \iint_D (-P\frac{dg}{dx} - Q\frac{dg}{dy} + R)\, dA.$$

Trig Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$.

$\sin\theta = \cos(\frac{\pi}{2} - \theta)$.

$\sin^2\theta + \cos^2\theta = 1$.

$\tan^2\theta + 1 = \sec^2\theta$.

$1 + \cot^2\theta = \csc^2\theta$.

$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$: a useful special case is $\sin(2\theta) = 2\sin\theta\cos\theta$.

$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$: a useful special case is $\cos(2\theta) = \cos^2\theta - \sin^2\theta$.

$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$.

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$.

Integral formulas

$\int x^r\, dx = \frac{1}{r+1}x^{r+1} + C$, if $r \neq -1$.

$\int \frac{1}{x}\, dx = \ln|x| + C$

$\int \sin x\, dx = -\cos x + C$

$\int \cos x\, dx = \sin x + C$

$\int \frac{1}{1+x^2}\,dx = \arctan x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc^2 x\,dx = -\cot x + C$