HW 7:

a) $P(X=K) = \frac{1}{10} \cdot \left(\frac{2}{3}\right)^K$ ~ to verify we need to prove that for $0 \le K \le \infty$

this fxn $= 1$

$P(X=0) + \sum_{K=1}^{\infty} P(X=K) = 1$ is what we need to prove

$= \frac{4}{5} + \frac{1}{10} \cdot \underbrace{\sum_{K=1}^{\infty} \left(\frac{2}{3}\right)^K}$ — used wolfram to simplify

$= \frac{4}{5} + \frac{1}{10} \cdot 2$

$= 1$ ✓

b) $P(X \ge K \mid X \ge 1) = \frac{P(X \ge K \cap X \ge 1)}{P(X \ge 1)} = \frac{P(X \ge K)}{P(X \ge 1)}$, $K = 1, 2, \ldots$

$$\Rightarrow \frac{\cancel{\frac{1}{10}} \sum_{x=K}^{\infty} \left(\frac{2}{3}\right)^x}{\cancel{\frac{1}{10}} \sum_{x=1}^{\infty} \left(\frac{2}{3}\right)^x} = \frac{\left(\frac{2}{3}\right)^K \cdot \cancel{\left(1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \ldots\right)}}{\frac{2}{3} \cdot \cancel{\left(1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \ldots\right)}} = \frac{\left(\frac{2}{3}\right)^K}{\left(\frac{2}{3}\right)} = \left(\frac{2}{3}\right)^{K-1}$$

HW8:

a) $f_x(x) = \frac{1}{b-a} = \frac{1}{10-4} = \frac{1}{6}$

$P(X<6) = \int_a^b f_x(x)\,dx = \int_4^6 \frac{1}{6}\,dx = \frac{1}{6}[6-4] = \frac{1}{3}$

b) $P(|X-7|>1) = 1 - P(|X-7| \le 1) = 1 - P(-1 \le X-7 \le 1) = 1 - P(6 \le X \le 8)$

$\Rightarrow 1 - \int_6^8 \frac{1}{6}\,dx = \frac{2}{3}$

c) $P(X<t \mid X<6) = \frac{P(\{X<t\} \cap \{X<6\})}{P(X<6)} = \frac{P(X<t)}{P(X<6)} \Rightarrow \frac{\int_4^t \frac{1}{6}\,dx}{\frac{1}{3}} = \frac{\frac{t-4}{6}}{\frac{1}{3}} = \frac{t-4}{2}$