Math 324 C - Summer 2016
Final exam
Friday, August 19th, 2016

Name: ______________________________

Problem 1	10	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	10	
Total	50	

- There are 5 questions on this exam. Make sure you have all five.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $3\sqrt{3}+\frac{1}{\sqrt{3}}$ should be reduced to $\frac{10\sqrt{3}}{3}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 60 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 pts) Let S be the boundary surface of the cone region $x^2 + y^2 \leq z^2 \leq 1$, so S consists of the cone $x^2 + y^2 = z^2$ for $0 \leq z \leq 1$ and the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. Equip S with the positive orientation, i.e. the outward pointing normal. Use the divergence theorem to evaluate the surface integral

$$\iint_S \langle x^3, xy^2 z, -xyz^2 \rangle \cdot d\mathbf{S}.$$

2a. (3 pts) Show that $\nabla \times \langle 3y, -2yz, \log z\rangle = \langle 2y, 0, -3\rangle$.

2b. (7 pts) Let S be the part of the ellipsoid $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 5$ lying above the plane $z = 2$ oriented downward. Use Stokes' theorem and the first part of this problem to evaluate

$$\iint_S \langle 2y, 0, -3\rangle \cdot d\mathbf{S}.$$

3. Let $F = \langle xy^2, x+y \rangle$ be a vector field in the $xy-$plane, and let C denote the upper half of the unit circle $x^2+y^2 = 1, y \geq 0$ oriented counter-clockwise. Also, let D be the upper half of the unit disk $x^2+y^2 \leq 1, y \geq 0$.

   a. (4 pts) Draw a picture of D and C, and evaluate $\int_C F \cdot d\mathbf{r}$.

   b. (2 pts) Henry, Jacob's evil twin brother, claims he is able to use Green's theorem to solve part *a*. He reasons as follows: "The curve C bounds the region D with the positive orientation, so by Green's theorem,

$$\int_C F \cdot d\mathbf{r} = \iint_D \left(\frac{d}{dx}(x+y) - \frac{d}{dy}(xy^2) \right) dA = \int_0^{\pi} \int_0^1 (1 - 2\sin\theta\cos\theta) r\,dr\,d\theta = \frac{\pi}{2}."$$

   What is wrong with Henry's argument?

   c. (4 pts) Use Green's theorem correctly to relate a double integral over D to a line integral. Explain why Henry got the right answer, even though his reasoning is flawed.

4a. (5 pts) Show that $\nabla \cdot (\nabla \times F) = 0$ for any smooth vector field F on $\mathbb{R}^3$.

4b. (5 pts) Use the result from part a to show that for any oriented surface S enclosing a region E,

$$\iint_S \text{curl}(F) \cdot d\mathbf{S} = 0.$$

[If you don't know how to use the result from part a, but have some other way of showing this fact, you can receive partial credit on this problem.]

5. (10 pts) Let S be the surface parameterized by

$$r(\phi,\theta) = \sin^2\phi\cos^2\theta\hat{\imath} + \frac{1}{2}\sin^2\phi\sin^2\theta\hat{\jmath} - \cos^2\phi\hat{k}$$

for $0 \le \phi \le \frac{\pi}{2}, 0 \le \theta \le \frac{\pi}{2}$. Describe the surface S, and find the surface area of S. [Hint: try to write down a linear equation $ax + by + cz = d$ relating the x, y and z components of the parameterization.]