Exercise 6.5. Suppose X, Y have joint density function

$$f(x,y) = \begin{cases} \frac{12}{7}(xy+y^2), & 0 \le x \le 1 \text{ and } 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Check that f is a genuine joint density function.
(b) Find the marginal density functions of X and Y.
(c) Calculate the probability $P(X < Y)$.
(d) Calculate the expectation $E[X^2 Y]$.

Exercise 6.10. Let X and Y be independent uniform random variables on $(0, 1)$. Find their joint density function $f(x,y)$. Use the joint density function to calculate the probability $P(X < Y)$.