Math 324 B - Spring 2017
Final exam
Wednesday, June 7th, 2017

Name: ______________________________

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Total	100	

- There are 7 problems (8 pages) in this exam. Make sure you have them all.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2} + \frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 2 hours to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (15 pts) Let C be the cone $4y^2 + 4z^2 = x^2$ for $0 \leq x \leq 4$, oriented inward (i.e. normal points toward the positive $x-$axis). Use Stokes' theorem to evaluate

$$\iint_C \nabla \times \langle z^2, 0, 1 + xy \rangle \cdot dS.$$

Make sure to indicate how you are orienting the boundary.

2. (15 pts) Evaluate the surface integral

$$\iint_S \langle x^2, 2z, y^2 \rangle \cdot dS,$$

where S is the boundary of the quarter sphere region $E = \{(x, y, z) : x^2 + y^2 + z^2 \leq 9, x, y \leq 0\}$ oriented inward, i.e. towards E.

3. (14 pts) Let D be the ellipsoidal cylinder defined by the equation $x^2+3z^2 = 4$, for any $-1 \leq y \leq 1$.

(a) (5 pts) Give a parameterization of D in terms of the coordinates θ and y.

(b) (4 pts) Compute $r_\theta \times r_y$.

(c) (5 pts) Find a vector that is normal to D at the point $(x, y, z) = (1, \sqrt{3}/2, 1)$.

4. (16 pts; 4 pts each) For each of the statements below, circle **True** if you think it is true and **False** if you think it is false.

(a) **True** **False** If F is a conservative vector field and C is any curve, then

$$\int_C F \cdot dr = 0.$$

(b) **True** **False** Let S denote the surface of the sphere of radius 17 centered at $(1, 0, \sqrt{2})$, oriented inward. For any vector field F,

$$\iint_S \nabla \times F \cdot dS = 0.$$

(c) **True** **False** For any vector field G, $\nabla \times (\nabla \times G) = 0$.

(d) **True** **False** If $R \subset \mathbb{R}^3$ is a region in space, and $S = \partial R$ is the boundary surface of S, then

$$\iiint_R (2z + 2y)\, dV = \iint_S (2xz + y^2 - 3)\, dS.$$

5. (12 pts; 4pts each) Consider the vector field $F = \langle 2y+1, x+y, 0\rangle$ defined on all of $\mathbb{R}^3$.

(a) Use Green's theorem to compute $\int_C F \cdot dr$, where C is the curve in $\mathbb{R}^3$ parameterized by $r(t) = \cos(t)\hat{\imath} + \sin(t)\hat{\jmath}$ for $t \in [0, 2\pi]$.

(b) Find $\nabla \cdot F$ and $\nabla \times F$.

(c) Let S be the surface of the box $[0,3] \times [2,3] \times [-1,1] \subset \mathbb{R}^3$, oriented outward: that is, S is the boundary of the region $\{(x,y,z) : 0 \le x \le 3, 2 \le y \le 3, -1 \le z \le 1\}$. What is $\iint_S F \cdot dS$?

Let S denote the parabaloid $2x^2 + y + z^2 = 1$. Both problems 6 and 7 are about the surface S.

6. (15 pts) Find a point (x, y, z) where the normal vector to S at (x, y, z) is parallel to the vector $\langle 4, 1, 2 \rangle$. Are there other points where the normal vector is parallel to $\langle 4, 1, 2 \rangle$? Explain.

7. (10 pts) Suppose S represents an infinitely large sheet of charged material, with charge density

$$f(x,y,z) = \frac{1}{4\pi\epsilon_0} \frac{e^{-x^2-z^2}}{\sqrt{1+16x^2+4z^2}},$$

where ϵ_0 is a constant. Compute the total charge in the plate S by evaluating the integral

$$\iint_S f(x,y,z)\, dS.$$