Math 324 E - Fall 2017

Midterm exam 2

Wednesday, November 8, 2017

Name: ______________________________________

Problem 1	10	
Problem 2	14	
Problem 3	16	
Problem 4	10	
Total	50	

- There are 4 problems on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2} + \frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 points) Let $f(x, y, z) = xz^2 - 2yz$.

   a) Compute the directional derivative of f at the point $(-1, 0, 2)$ in the direction $u = \frac{1}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$.

   b) What is the maximum value of directional derivative at $(-1, 0, 2)$, and which direction does it occur in?

2. (14 points) For each pair of conservative vector field F and curve C, first find a potential function for F, and then use the fundamental theorem of line integrals to evaluate $\int_C F \cdot dr$.

a) $F(x, y) = xy\hat{\imath} + \frac{1}{2}x^2\hat{\jmath}$, and C is the part of the hyperbola $y = \frac{3}{x}$ between the points $(1, 3)$ and $(3, 1)$, traversed from left to right.

b) $F(x, y) = x^{-2}y^{-1}\hat{\imath} + x^{-1}y^{-2}\hat{\jmath}$, and C is the infinite(!) ray along the line $x = 2y$ for $x \geq 1$, with initial point $(1, 2)$. (Hint: do the problem with the part of the ray out to the point $(n, 2n)$, and then let $n \to \infty$.)

3. (16 points) For each of the statements below, circle **True** if you think it is true, and circle **False** if you think it is false. You may use the space to do scratch work, but no partial credit will be awarded on this question. Note that for a vector field $F = P\hat{\imath} + Q\hat{\jmath}$ and a function g, we define gF as the vector field $gP\hat{\imath} + gQ\hat{\jmath}$. (4 points for each statement.)

   (a) **True** **False** The function $f(x, y) = e^{x^2-2y^2}$ satisfies $\nabla f = f(2x\hat{\imath} - 4y\hat{\jmath})$.

   (b) **True** **False** The vector field $F = \frac{-y\hat{\imath}+x\hat{\jmath}}{x^2+y^2}$ satisfies $\int_C F \cdot dr = 0$ where C is the unit circle traversed counter clockwise.

   (c) **True** **False** Let $G(x, y) = 6y\cos(2x)\hat{\imath} + 6\sin(x)\cos(x)\hat{\jmath}$. G is a conservative vector field.

   (d) **True** **False** Let C denote the part of the parabola $y = 1 - x^2$ between the points $(0, 1)$ and $(4, -15)$, traversed from right to left. Then the vector $r(t) = 2t\hat{\imath} - 4t^3\hat{\jmath}$ is tangent to C for $0 \leq t \leq 2$.

4. (10 points) Let $F(x, y) = 2x^2y\hat{\imath} - 3x\hat{\jmath}$, and let C denote the curve defined by the ellipse $x^2 + \frac{y^2}{9} = 1$, traversed clockwise. Use Green's theorem to evaluate $\int_C F \cdot dr$.