Exercise 1.26. 10 men and 5 women are meeting in a conference room. Four people are chosen at random from the 15 to form a committee.

(a) What is the probability that the committee consists of 2 men and 2 women?
(b) Among the 15 is a couple, Bob and Jane. What is the probability that Bob and Jane both end up on the committee?
(c) What is the probability that Bob ends up on the committee but Jane does not?

Exercise 1.28. We have an urn with m green balls and n yellow balls. Two balls are drawn at random. What is the probability that the two balls have the same color?

(a) Assume that the balls are sampled without replacement.
(b) Assume that the balls are sampled with replacement.
(c) When is the answer to part (b) larger than the answer to part (a)? Justify your answer. Can you give an intuitive explanation for what the calculation tells you?

Exercise 1.30. Eight rooks are placed randomly on a chess board. What is the probability that none of the rooks can capture any of the other rooks? Translation for those who are not familiar with chess: pick 8 unit squares at random from an 8×8 square grid. What is the probability that no two chosen squares share a row or a column?

Hint. You can think of placing the rooks either with or without order, both approaches work.

Exercise 1.42. Suppose $P(A) > 0.8$ and $P(B) > 0.5$. Show that $P(AB) > 0.3$.

Exercise 1.50. We flip a fair coin repeatedly, without stopping. In Examples 1.16 and 1.22 we have seen that with probability one we will eventually see tails. Prove the following generalizations.

(a) Show that with probability one we will eventually get 5 tails in a row.
Hint. It could help to group the sequence of coin flips into groups of five.

(b) Let $\bar{a} = (a_1, a_2, \ldots, a_r)$ be a fixed ordered r-tuple of heads and tails. (That is, each $a_i \in \{\mathtt{H}, \mathtt{T}\}$.) Show that with probability one the sequence $\bar{a}$ will show up eventually in the sequence of coin flips.

Exercise 1.52. Three married couples (6 guests altogether) attend a dinner party. They sit at a round table randomly in such a way that each outcome is equally likely. What is the probability that somebody sits next to his or her spouse?

Hint. Label the seats, the individuals, and the couples. There are $6! = 720$ seating arrangements altogether. Apply inclusion-exclusion to the events $A_i = \{i\text{th couple sit next to each other}\}$, $i = 1, 2, 3$. Count carefully the numbers of arrangements in the intersections of the A_is.

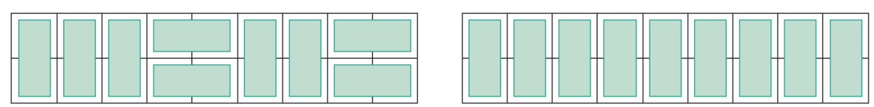
Exercise 1.56.

(a) Put k balls into n boxes at random. What is the probability that no box remains empty? (You may assume that balls and boxes are numbered and each assignment of k balls into n boxes is equally likely.)
(b) Using the result of part (a) calculate the value of

$$\sum_{j=1}^{n} (-1)^{j-1} j^k \binom{n}{j}$$

for $k \le n$.

Exercise 1.57. We tile a 2×9 board with 9 dominos of size 2×1 so that each square is covered by exactly one domino. Dominos can be placed horizontally or vertically. Suppose that we choose a tiling randomly so that each possible tiling configuration is equally likely. The figure below shows two possible tiling configurations.



(a) What is the probability that all the dominos are placed vertically?
(b) Find the probability that there is a vertical domino in the middle of the board (at the 5th position).