Math 324 E - Spring 2019

Midterm exam 2

Friday, May 24th, 2019

Name: ____________________

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- There are 5 questions on this exam. Make sure you have all five.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible.
- Use the backs of pages *for scratch work only.*
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. Let $f(x, y, z) = xz^2 - 2yz$.

a) Compute the directional derivative of f at the point $(-1, 0, 2)$ in the direction $u = \frac{1}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}+\hat{k})$.

b) What is the maximum value of directional derivative $D_u f(-1, 0, 2)$ as u ranges over all unit vectors in $\mathbb{R}^3$? What is the unit vector u for which the maximum is attained?

2. For each pair of conservative vector field F and curve C, first find a potential function for F, and then use the fundamental theorem of line integrals to evaluate $\int_C F \cdot dr$.

   a) $F(x,y) = xy\hat{\imath} + \frac{1}{2}x^2\hat{\jmath}$, and C is the part of the hyperbola $y = \frac{3}{x}$ between the points $(1,3)$ and $(3,1)$, traversed from left to right.

   b) $F(x,y,z) = z\cos(x)\hat{\imath} - \hat{\jmath} + \sin(x)\hat{k}$, and C is the curve parameterized by $r(t) = (t,t,-1)$, for $0 \leq t \leq \pi/2$.

3. For each of the statements below, circle **True** if you think it is true, and circle **False** if you think it is false. You may use the space to do scratch work, but no partial credit will be awarded on this question.

(a) **True** **False** The function $f(x,y) = e^{x^2-2y^2}$ satisfies $\nabla f = 2xf(x,y)\hat{\imath} - 4yf(x,y)$.

(b) **True** **False** The vector field $F = \frac{-y\hat{\imath}+x\hat{\jmath}}{x^2+y^2}$ satisfies $\int_C F \cdot dr = 0$ where C is the unit circle traversed counter clockwise.

(c) **True** **False** Let $G(x,y) = (9x + 2x^2y^2)\hat{\imath} + x^3y\hat{\jmath}$. G is a conservative vector field.

(d) **True** **False** Let F be a vector field defined on all of $\mathbb{R}^2$, and let C be a curve starting at $(-3,1)$ and ending at $(2,2)$. Let T denote the unit tangent vector field to C. Suppose that $F \cdot T < 0$ at every point on C. Then

$$\int_C F \cdot dr < 0.$$

4. Let $F(x, y) = 2x^2y\hat{\imath} - 3x\hat{\jmath}$, and let C denote the curve defined by the ellipse $x^2 + \frac{y^2}{9} = 1$, traversed clockwise. Use Green's theorem to evaluate $\int_C F \cdot dr$.

5. Let $f(x, y, z) = x^2 + z^2$, and consider the 3D region E of points (x, y, z) satisfying

$$z \geq 0 \text{ and } 4 \leq x^2 + y^2 + z^2 \leq 9 \text{ and } x^2 + y^2 \leq 4.$$

Use cylindrical coordinates to set up the integral

$$\iiint_E f \, dV.$$

You don't have to evaluate it.