INSTRUCTIONS: PLEASE READ ALL CAREFULLY BEFORE STARTING

After inputing your student ID, we recommend to write down the values set by the script. If you show results obtained with values different from those set by the Jupyter notebook script, your question **will not be graded.**

1. **Problem 1**: Complete the Problem 1 set in `midterm2B_students.ipynb` (there are five parts). Submit the completed notebook `midterm2B_complete.ipynb` (you don't have to finish all the questions to submit).

   Remark: If you are running the notebook on syzygy, we recommend that you save after each new answer, and take a screenshot of your work as soon as you are done. If your connection to the server broke, simply log back to the server and re-open your saved notebook.

2. **Problems 2-4**: For each problem, you will need to separately assemble a pdf file of handwritten solutions. We recommend to not use more than 2 pages per problem. **Write your name** on top of each page and do not forget to indicate which subquestion (a, b...) you are solving.

Grading

We recommend spending an equivalent amount of time on problems 1, 2, 3 and 4.

Problem 1

Complete the Problem 1 set in `midterm2B_students.ipynb` (there are five parts). Submit the completed notebook `midterm2B_complete.ipynb` (you don't have to finish all the questions to submit).

Remark: If you are running the notebook on syzygy, we recommend that you save after each new answer, and take a screenshot of your work as soon as you are done. If your connection to the server broke, simply log back into the server.

Warning: Having "Success" displayed when running the cells does not necessarily mean that the solution is correct (this will be evaluated after you submit the notebook).

Problem 2

At the grocery store, Khanh sees three cashiers serving Becca, Emmanuel and Merlin. Khanh assumes that the times they will spend at the cashier are independent and exponentially distributed, with parameters λ_B, λ_E and λ_M for Becca, Emmanuel and Merlin, respectively. He also assumes that the variance of the time spent at the cashier is proportional to the number of items each person has.

a. Run the notebook for Problem 2 with your student ID to get everyone's number of items. Find the parameters λ_M and λ_E as a function of λ_B (justify your answer).

b. Find the probability P or time T displayed in your notebook. (Justify your answer, T can depend on λ_B.)

c. Find the value of λ_B given the equation obtained from running the notebook for Problem 2 (justify your answer). (*Hint*: this equation should lead to solve a second order polynomial equation.)

Problem 3

Let $N(t)$ be a Poisson process of rate λ, and let $Z \sim \text{Exp}(\mu)$ be an independent exponential random variable.

a. Fix $n \in \{0, 1, 2, \ldots\}$. Compute $\mathbb{P}(N(Z) = n)$ as a function of n, λ and μ. [Hint: think of Z as the first arrival of a rate μ Poisson process.]

b. Compute $\mathbb{E}[N(Z)/Z]$. For what value(s) of μ is $\mathbb{E}[N(Z)/Z] = \lambda$?

Problem 4

Shooting stars appear in the sky at the times of a Poisson process with rate 1 per hour. Assume each star is visible to the human eye with probability 1/2 independently. Suppose that over the course of three hours, five total shooting stars arrived.

a. Find the probability displayed in your notebook.

b. What's the probability that two visible shooting stars occurred in succession, with no invisible stars in between?

c. Suppose four out of the five shooting stars were visible. What's the probability that exactly two occurred in the first hour, and exactly two occurred in the third hour?