Math 324 E - Fall 2017
Midterm exam 1
Wednesday, October 18, 2017

Name: ______________________________

Problem 1	14	
Problem 2	12	
Problem 3	12	
Problem 4	12	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2}+\frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (a) (7 pts) Let $B \subset \mathbb{R}^3$ be the region inside the sphere $x^2 + y^2 + z^2 = 16$, inside the half space $x \geq 0$, inside the cone $x^2 = 3y^2 + 3z^2$, and outside the sphere $x^2 + y^2 + z^2 = 1$. Set up an integral to find the volume of B. **You do not need to evaluate it.** (Hint: Use a rotated version of spherical coordinates.)

(b) (7 pts) Let S denote the sphere of radius 2 centered at $(0, 0, 0)$, and suppose S is filled with a fluid with density function $f(x, y, z) = z^3 - z + 8$. Find the total mass of fluid inside S by integrating the function f over S. (Hint: use symmetry.)

2. (12 pts) Use cylindrical coordinates to evaluate

$$\iiint_E e^z \, dV,$$

where E is the region bounded by the parabaloid $z = 4 + x^2 + y^2$, the cylinder $x^2 + y^2 = 2$, and the plane $z = 0$.

3. (a) (6 pts) Set up a double integral in polar coordinates to find the area of the region inside the circle $(x-3)^2+y^2=9$ and outside the circle $x^2+y^2=9$. **You do not need to evaluate it.**

(b) (6 pts) Consider the transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ given by $(u,v)=T(x,y)=(2x+4y, x-3y)$. Solve for the inverse of T in terms of equations $x=x(u,v)$ and $y=y(u,v)$, and find the Jacobian determinant of T.

4. (12 pts) Consider the tetrahedron $E \subset \mathbb{R}^3$ bounded by the planes $x = 0, z = 0, z = 2y$ and $2x + 2y + z = 4$. Set up the triple integral

$$\iiint_E xz \, dV$$

with the two given orders of integration. **You do not need to evaluate the integrals.**

(a) $dx\,dy\,dz$.

(b) $dy\,dz\,dx$.