HW2

Wed July 5

Exercise 1.15. An urn contains 4 balls: 1 white, 1 green and 2 red. We draw 3 balls with replacement. Find the probability that we did not see all three colors. Use two different calculations, as specified by (a) and (b) below.

(a) Define the event W = {white ball did not appear} and similarly for G and R. Use inclusion-exclusion.

(b) Compute the probability by considering the complement of the event that we did not see all three colors.

a) $P(W \cup G \cup R) = P(W) + P(G) + P(R) - P(W \cap G) - P(W \cap R) - P(G \cap R) - P(W \cap G \cap R)$

$P(W) = (\frac{3}{4})^3 \qquad P(G) = (\frac{3}{4})^3 \qquad P(R) = (\frac{1}{2})^3$

$P(W \cap G) = (\frac{2}{4})^3 \qquad P(W \cap R) = (\frac{1}{4})^3 \qquad P(G \cap R) = (\frac{1}{4})^3$

$P(W \cap G \cap R) = 0$

$P(W \cap G \cap R) = 2 \cdot (\frac{3}{4})^3 + (\frac{1}{2})^3 - 2 \cdot (\frac{1}{4})^3 - (\frac{2}{4})^3$

b) $P(\text{saw all colors}) = \frac{3! \cdot 2}{64} = \frac{3}{16}$ (permutations of picking 3 colors; $\times$ can pick either red ball)

$P(\text{didn't see all colors}) = P(\text{saw all})^c = 1 - P(\text{saw all}) = 1 - \frac{3}{16} = \frac{13}{16}$

Exercise 1.22. We pick a card uniformly at random from a standard deck of 52 cards. (If you are unfamiliar with the deck of 52 cards, see the description above Example C. 19 in Appendix C.)

(a) Describe the sample space S and the probability measure P that model this experiment.

(b) Give an example of an event in this probability space with probability 3/52.

(c) Show that there is no event in this probability space with probability 1/5.

a) $S = \{\text{all 52 cards}\}$ $\quad P(S) = \frac{|S|}{52}$

b) probability that you draw a diamond 3, 4, or 5

c) $\frac{n}{52}$ for any integer n cannot simplify to $\frac{1}{5}$