Math 324 B - Winter 2017

Final exam

Monday, March 13th, 2017

Name: ______________________________

Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	20	
Problem 5	15	
Problem 6	15	
Total	100	

- There are 6 problems (7 pages) in this exam. Make sure you have them all.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2}+\frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 2 hours to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (20 pts) Let S be the cylinder $y^2 + z^2 = 9$ for $-1 \leq x \leq 2$, oriented inward (i.e. toward the $x-$axis). Use Stokes' theorem to evaluate

$$\iint_S \nabla \times \langle z^2, 0, 1 + xy\rangle \cdot dS.$$

Make sure to indicate how you are orienting the boundary curves.

2. (15 pts) Evaluate the surface integral

$$\iint_S \langle 4xy, -2z, z^2 \rangle \cdot dS,$$

where S is the boundary of the hemisphere region $E = \{(x, y, z) : x^2 + y^2 + z^2 \leq 25, y \leq 0\}$, oriented inward, i.e. towards E.

3. (15 pts) Let P be the ellipsoid defined by the equation $x^2 + \frac{y^2}{4} + z^2 = 1$.

(a) (5 pts) Give a parameterization $r = r(\phi, \theta)$ of P in terms of the angle coordinates ϕ and θ that we usually use for spherical coordinates.

(b) (5 pts) Compute $r_\phi \times r_\theta$.

(c) (5 pts) Find a vector that is normal to P at the point $(x, y, z) = (1/4, \sqrt{3}, \sqrt{3}/4)$.

4. (20 pts; 4 pts each) For each of the statements below, circle **True** if you think it is true and **False** if you think it is false.

(a) **True** **False** Let C be the curve $r(t) = \langle 2 + 3\sin(2t), \cos^3(6t), 2\cos(2t)\rangle$ for $0 \le t \le \pi/2$. For any conservative vector field F,

$$\int_C F \cdot dr = 0.$$

(b) **True** **False** Let S denote the surface of the sphere of radius 3 centered at $(1, 0, \sqrt{2})$, oriented outward. For any vector field F,

$$\iint_S \nabla \times F \cdot dS = 0.$$

(c) **True** **False** If G is a vector field, and $\nabla \cdot G \neq 0$, then G is not conservative.

(d) **True** **False** If S is an oriented surface whose normal vector n always has negative entries (i.e. $n \cdot \hat{\imath}, n \cdot \hat{\jmath}$, and $n \cdot \hat{k}$ are always negative), then

$$\iint_S \langle 1 + \cos(x), y^2 + z^2, e^x\rangle \cdot dS \le 0.$$

(e) **True** **False** If $r(u, v)$ and $s(u, v)$ are any two parameterizations of the same surface S, over possibly different parameterization domains D, D' in the u-v plane, then

$$\iint_D (5u^2 - \sin(\pi uv))|r_u \times r_v| dA = \iint_{D'} (5u^2 - \sin(\pi uv))|s_u \times s_v| dA.$$

5. (15 pts) Consider the curve C parameterized by $r(t) = \langle t^2, t, \ln(t) \rangle$ for $0 < t < 1$. Evaluate the line integral (with respect to arclength)

$$\int_C x\sqrt{4e^{4z} + y^2 + 1}\, ds.$$

6. (15 pts) Let D denote the unit square in $\mathbb{R}^2$, so $D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Consider the change of coordinates $x = u - v, y = uv$.

(a) (8 pts) What region R in the right half of the u-v plane $\{(u, v) : u \geq 0\}$ corresponds to D under this transformation? Draw a picture of the image region, and explain how you know that $T(u, v) = \langle u - v, uv \rangle$ actually has domain R and image D.

(b) (7 pts) Use the change of coordinates formula to re-write the double integral

$$\iint_D (x^2 + 4y^2)\, dA$$

in $u - v$ coordinates, i.e. as an integral over the region R. Your answer should look like

$$\iint_R g(u, v)\, dA$$

for some function g.