MATH 303 Practice Problems - Week 9

March 19, 2021

Problem 1

Let $N(t)$ be a Poisson process of rate λ. For fixed t, $u \in [0,t]$, and $n = 1, 2, \ldots$, find the conditional distribution of $N(u)$ given $N(t) = n$, i.e. find a formula for $\mathbb{P}(N(u) = k | N(t) = n)$ for $k = 0, 1, 2, \ldots, n$.

Problem 2

Let $N(t)$ be a Poisson process of rate λ. Given that $N(t) = 3$, determine the conditional distributions of the first three arrival times S_1, S_2, S_3.

Problem 3

Customers arrive at a theme park according to a Poisson process $N(t)$ of rate λ. Each customer pays \$1 on arrival. At time t, the *discounted value* of the total sum collected so far is

$$D_t = \sum_{i=1}^{N(t)} e^{-\beta S_i},$$

where S_i is the ith arrival time, and $\beta > 0$ is the discount rate. Compute $\mathbb{E}D_t$.

Problem 4

Alpha particles are emitted by a radioactive source according to a Poisson process of rate λ. Each alpha particle independently survives for a random amount of time and then is annihilated. The lifetimes $Y_1, Y_2, \ldots$ of the particles have common distribution function $G(y) = \mathbb{P}(Y_k \leq y)$. Let $M(t)$ denote the number of alpha particles in existence at time t.

a. Determine the distribution of $M(t)$.

b. Show that as $t \to \infty$, the distribution you found in part a converges to Poisson$(\lambda\mu)$, where $\mu = \mathbb{E}Y$ is the mean lifetime of an alpha particle.