Math 324 A - Spring 2017
Midterm exam 1
Wednesday, April 19th, 2017

Name: _____

Problem 1	10	
Problem 2	12	
Problem 3	16	
Problem 4	12	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2} + \frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 pts) Consider the tetrahedron $E \subset \mathbb{R}^3$ bounded by the planes $x = 0, z = 0, z = 2y$ and $2x + 2y + z = 4$. Set up the triple integral

$$\iiint_E xz \, dV$$

with the two given orders of integration. **You do not need to evaluate the integrals.**

(a) $dx\, dy\, dz$.

(b) $dy\, dz\, dx$.

2. (12 pts) Let R be the region in the plane satisfying the polar conditions $\frac{1}{2\sin\theta} \leq r \leq \sin\theta$, and $0 \leq \theta \leq \pi/2$.

   (a) (2 pts) Draw a sketch of R.

   (b) (4 pts) Use Cartesian coordinates – x's and y's – to set up the double integral

$$\iint_R \sin^2\theta \cos\theta \, dA.$$

   You do not need to evaluate the integral.

(c) (6 pts) Use polar coordinates to set up the integral from part (b), and evaluate it.

3. (a) (8 pts) Let $B \subset \mathbb{R}^3$ be the region inside the sphere $x^2 + y^2 + z^2 = 16$, outside the sphere $x^2 + y^2 + z^2 = 1$, and inside the cone $x^2 = 3y^2 + 3z^2$. Set up an integral to find the volume of B. **You do not need to evaluate it.**

(b) (8 pts) Let S denote the sphere of radius 2 centered at $(0, 0, 0)$, and imagine that S is filled with a fluid with density function $f(x, y, z) = z^3 - z + 2$. Find the total mass of fluid inside S by integrating the function f over S.

4. (12 pts) Use the change of coordinates $x = u - 2v, y = v$ to set up the integral

$$\iint_R (x + 3y)\, dA$$

in u, v coordinates, where R is the region in the $x - y$ plane bounded by the curves

$$y = 1, y = 3, x + 2y = 10, x + 2y = 6.$$

The region R is pictured below. Sketch the image region after changing coordinates, and explain how you know what the new region looks like. **You do not need to evaluate the integral.**