Math 324 E

Final exam

Tuesday, December 10, 2019

Name: ____

Problem 1	12	
Problem 2	12	
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Problem 5	12	
Total	60	

- There are 5 questions on this exam. Make sure you have all five.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible.
- Use the backs of pages *for scratch work only.*
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 110 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. Let E be the region in $\mathbb{R}^3$ given by

$$E = \{(x, y, z) : x^2 + z^2 \leq y \leq 4 - x^2 - z^2 \text{ and } x > 0\}.$$

Use the divergence theorem to evaluate the surface integral

$$\iint_{\partial E} \left(yz^2\hat{\imath} + x^2y\hat{\jmath} + \frac{1}{3}z^3\hat{k} \right) \cdot d\mathbf{S},$$

where ∂E denotes the boundary surface of E, oriented outward (normal points away from E).

2. Let $F = x^2\hat{\imath} - xyz^2\hat{\jmath} + y^4\hat{k}$.

(a) Find $\nabla \cdot F, \nabla \times F$, and $\text{grad}(\text{div}(F))$.

(b) Is F conservative? Why or why not?

(c) Is $\text{grad}(\text{div}(F))$ conservative? Why or why not?

(d) What is $\nabla \times (\nabla \times (\nabla \times (\nabla \times (\nabla \times F))))$? Explain your reasoning.

3. Let $F = x\hat{\imath} - 2z\hat{\jmath} + \hat{k}$, and let S be the part of the plane $x - y + z = 3$ where $0 \leq y \leq 3$ and $-1 \leq z \leq 1$, oriented with the unit normal vector

$$\hat{n} = \frac{1}{\sqrt{3}}\left(-\hat{\imath} + \hat{\jmath} - \hat{k}\right).$$

Evaluate the surface integral

$$\iint_S F \cdot d\mathbf{S}.$$

4. For each of the statements below, circle **True** if you think it is true, and circle **False** if you think it is false. You may use the space to do scratch work, but no partial credit will be awarded on this question.

   (a) **True** **False** Suppose F and G are two vector fields such that

$$\iint_S F \cdot d\mathbf{S} = \iint_S G \cdot d\mathbf{S}$$

   where S is the unit sphere, oriented outward. Then $F = G$ on S.

   (b) **True** **False** If H is a vector field satisfying $\nabla \cdot H = 0$, then

$$\iint_S H \cdot d\mathbf{S} = 0$$

   for any oriented surface S.

   (c) **True** **False** $\nabla \cdot (\nabla f) = 0$ for any function $f : \mathbb{R}^3 \to \mathbb{R}$.

   (d) **True** **False** If $F, G : \mathbb{R}^3 \to \mathbb{R}$ are vector fields satisfying $F \cdot \hat{k} = 0$ and $G \cdot \hat{k} = 0$, then

$$(F \times G) \cdot \hat{\imath} = (F \times G) \cdot \hat{\jmath} = 0.$$

5. Let S be the bottom half of the unit sphere $x^2 + y^2 + z^2 = 1, z \leq 0$, oriented inward (normal points toward the origin). Also, let C denote the boundary curve of S, i.e. the circle $x^2 + y^2 = 1$, oriented clockwise (when looking down from the positive z axis). Use Stokes' theorem to evaluate the line integral

$$\int_C \left(3y\hat{\imath} + x\hat{\jmath} + 2x\hat{k}\right) \cdot d\mathbf{r}.$$