Math 324 E

Midterm exam 2

Monday, November 18th, 2019

Name: ______________________________

Problem 1	12	
Problem 2	12	
Problem 3	12	
Problem 4	14	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible.
- Use the backs of pages *for scratch work only.*
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. Let C be a curve with start point $(-2,3,1)$ and end point $(3,-5,1)$, and suppose $r(t)=(x(t),y(t),z(t))$ is a parameterization of C for $t\in[0,1]$. Also, let $f:\mathbb{R}^3\to\mathbb{R}$ be a smooth function. Assume that

$$r(1/2)=(3,0,1), r'(1/2)=\langle 1,2,4\rangle,$$

and also

$$\nabla f(1,2,4)=\langle 2,0,1\rangle, \nabla f(3,0,1)=\langle -1,0,4\rangle.$$

(a) Compute $\frac{\partial}{\partial t}f(r(t))$ at $t=1/2$.

(b) What is $\frac{\partial f}{\partial y}$ at the point $(1,2,4)$?

2. For each pair of conservative vector field F and curve C, first find a potential function for F, and then use the fundamental theorem of line integrals to evaluate $\int_C F \cdot dr$.

   a) $F(x, y) = (2y^2 - 3)\hat{\imath} + 4xy\hat{\jmath}$, and C is the part of the hyperbola $y = \frac{3}{x}$ between the points $(1, 3)$ and $(3, 1)$, traversed from left to right.

   b) $F(x, y, z) = (y+z)\hat{\imath}+(x+z)\hat{\jmath}+(x+y)\hat{k}$, and C is the curve parameterized by $r(t) = (t, t, -1)$, for $0 \leq t \leq 4$.

3. For each of the statements below, circle **True** if you think it is true, and circle **False** if you think it is false. You may use the space to do scratch work, but no partial credit will be awarded on this question.

(a) **True** **False** The function $f(x, y) = e^{x\sin(y)}$ satisfies $\nabla f = \sin(y)f(x, y)\hat{\imath} + x\cos(y)f(x, y)\hat{\jmath}$.

(b) **True** **False** The vector field $F = \frac{-y\hat{\imath}+x\hat{\jmath}}{x^2+y^2}$ satisfies $\int_C F \cdot dr = 0$ where C is the unit circle traversed counter clockwise.

(c) **True** **False** Let $G(x, y) = -4x^2y\hat{\imath} - 4xy^2\hat{\jmath}$. G is a conservative vector field.

(d) **True** **False** If C is any closed curve in $\mathbb{R}^3$ (i.e. C has the same start and end point) and F is a smooth vector field on $\mathbb{R}^3$, then

$$\int_C F \cdot dr = 0.$$

(e) **True** **False** A polite employee at a phone company is called a 'deferential operator.'

4. Let $f : [0, 1] \to [0, 1]$ be any smooth function satisfying

$$f(0) = f(1) = 0.$$

Let C be the curve given by the graph of f, starting at $(1, 0)$ and ending at $(0, 0)$. Use Green's theorem to prove that

$$\int_0^1 x f(x)\, dx = \int_C xy\, dx + x^2\, dy.$$

Make sure to justify all your steps.