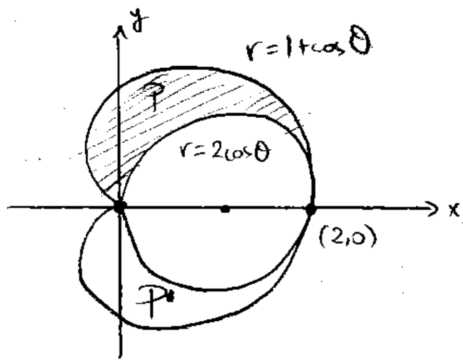
#2. $P \subset \mathbb{R}^2$ is the region bounded by $r = 1 + \cos\theta$ and $r = 2\cos\theta$. Find the area of P.



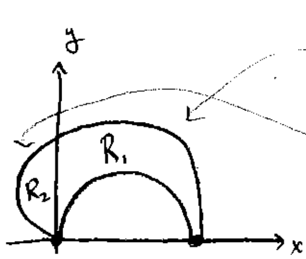
Solution: By symmetry, we'll just find the area above $y=0$, and multiply by 2.

We must split the shaded region into two smaller regions.

For $0 \le \theta \le \frac{\pi}{2}$, r is bounded by $r = 1+\cos\theta$ and $r = 2\cos\theta$; but for $\frac{\pi}{2} \le \theta \le \pi$, r is only bounded by $r = 1+\cos\theta$, (and $r \geq 0$ (*)

So

$$\text{Area}(P) = 2\Big[\underbrace{\int_0^{\pi/2}\int_{2\cos\theta}^{1+\cos\theta} r\,dr\,d\theta}_{R_1} + \underbrace{\int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta}_{R_2}\Big] = \boxed{\frac{\pi}{2}}.$$



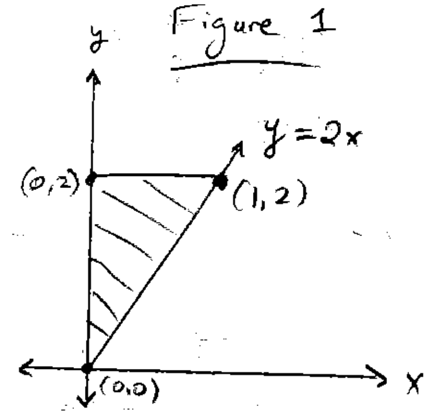
(*) Note that in R_1, $2\cos\theta \le r \le 1+\cos\theta$; but in R_2, $0 \le r \le 1+\cos\theta$.

(parabolic cylinder) (planes)

#9. $W \subset \mathbb{R}^3$ bounded by $z = 4-y^2$, $y=2x$, $z=0$, and $x=0$.

Parameterize $\iiint_W xyz\,dV$ in three orders: i) $dz\,dy\,dx$
ii) $dx\,dz\,dy$, and
iii) $dy\,dz\,dx$.

Figure 1



Solution: i) Draw W in the x-y plane:

Since $z \geq 0$, (the part for $z \leq 0$ is an infinite region!), and $x \geq 0$, $4-y^2 \geq 0$ and $y \geq 2x$, so $2x \leq y \leq 2$.

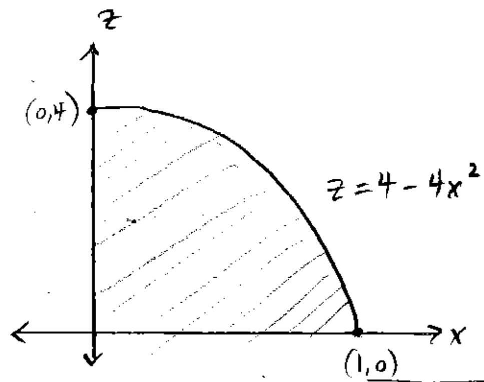
So we have $W = \{(x,y,z) : 0 \leq x \leq 1,\ 2x \leq y \leq 2,\ 0 \leq z \leq 4-y^2\}$.

So $\iiint_W xyz\,dV = \boxed{\int_0^1 \int_{2x}^{2} \int_0^{4-y^2} xyz\,dz\,dy\,dx}$

ii) For a fixed y value, $0 \leq y \leq 2$, we have $0 \leq x \leq y/2$ from Figure 1, So $\iiint_W xyz\,dV = \boxed{\int_0^2 \int_0^{4-y^2} \int_0^{y/2} xyz\,dx\,dz\,dy.}$

(and the same possible z values, $0 \leq z \leq 4-y^2$.)

#9 (continued) iii) We need to determine the projection of W in the x-z plane. Substituting $2x=y$ into $z=4-y^2$ gives $z = 4-4x^2$, so the projection looks like



Given (x,z) in the shaded region, y is bounded by $2x \le y \le \sqrt{4-z}$, so we get

$$\iiint_W xyz\, dV = \boxed{\int_0^1 \int_0^{4-4x^2} \int_{2x}^{\sqrt{4-z}} xyz\, dy\, dz\, dx.}$$

is again r (see the figure). So

$$E = k\int_{-\pi/2}^{\pi/2}\int_0^{20\cos\theta}\left(1-\tfrac{1}{20}r\right)r\,dr\,d\theta = k\int_{-\pi/2}^{\pi/2}\left[\tfrac{1}{2}r^2-\tfrac{1}{60}r^3\right]_{r=0}^{r=20\cos\theta}d\theta$$
$$= k\int_{-\pi/2}^{\pi/2}\left(200\cos^2\theta-\tfrac{400}{3}\cos^3\theta\right)d\theta = 200k\int_{-\pi/2}^{\pi/2}\left[\tfrac{1}{2}+\tfrac{1}{2}\cos 2\theta-\tfrac{2}{3}\left(1-\sin^2\theta\right)\cos\theta\right]d\theta$$
$$= 200k\left[\tfrac{1}{2}\theta+\tfrac{1}{4}\sin 2\theta-\tfrac{2}{3}\sin\theta+\tfrac{2}{3}\cdot\tfrac{1}{3}\sin^3\theta\right]_{-\pi/2}^{\pi/2} = 200k\left[\tfrac{\pi}{4}+0-\tfrac{2}{3}+\tfrac{2}{9}+\tfrac{\pi}{4}+0-\tfrac{2}{3}+\tfrac{2}{9}\right]$$
$$= 200k\left(\tfrac{\pi}{2}-\tfrac{8}{9}\right)\approx 136k$$

Therefore the risk of infection is much lower at the edge of the city than in the middle, so it is better to live at the edge.

15.6 Surface Area

1. Here $z = f(x,y) = 2+3x+4y$ and D is the rectangle $[0,5]\times[1,4]$, so by Formula 2 the area of the surface is

$$A(S) = \iint_D \sqrt{[f_x(x,y)]^2+[f_y(x,y)]^2+1}\,dA = \iint_D \sqrt{3^2+4^2+1}\,dA = \sqrt{26}\iint_D dA$$
$$= \sqrt{26}\,A(D) = \sqrt{26}\,(5)(3) = 15\sqrt{26}$$

2. $z = f(x,y) = 10-2x-5y$ and D is the disk $x^2+y^2\le 9$, so by Formula 2

$$A(S) = \iint_D \sqrt{(-2)^2+(-5)^2+1}\,dA = \sqrt{30}\iint_D dA = \sqrt{30}\,A(D) = \sqrt{30}\,(\pi\cdot 3^2) = 9\sqrt{30}\,\pi$$

3. $z = f(x,y) = 6-3x-2y$ which intersects the xy-plane in the line $3x+2y=6$, so D is the triangular region given by $\{(x,y)\mid 0\le x\le 2, 0\le y\le 3-\tfrac{3}{2}x\}$. Thus

$$A(S) = \iint_D \sqrt{(-3)^2+(-2)^2+1}\,dA = \sqrt{14}\iint_D dA = \sqrt{14}\,A(D) = \sqrt{14}\left(\tfrac{1}{2}\cdot 2\cdot 3\right) = 3\sqrt{14}$$

4. $z = f(x,y) = 1+3x+2y^2$ with $0\le x\le 2y$, $0\le y\le 1$. Thus by Formula 2,

$$A(S) = \iint_D \sqrt{1+(3)^2+(4y)^2}\,dA = \int_0^1\int_0^{2y}\sqrt{10+16y^2}\,dx\,dy = \int_0^1\sqrt{10+16y^2}\,\big[x\big]_{x=0}^{x=2y}\,dy$$
$$= \int_0^1 2y\sqrt{10+16y^2}\,dy = 2\cdot\tfrac{1}{32}\cdot\tfrac{2}{3}\left(10+16y^2\right)^{3/2}\Big]_0^1 = \tfrac{1}{24}\left(26^{3/2}-10^{3/2}\right)$$

5. $y^2+z^2=9 \;\Rightarrow\; z=\sqrt{9-y^2}$. $f_x=0$, $f_y=-y(9-y^2)^{-1/2} \;\Rightarrow$

$$A(S) = \int_0^4\int_0^2\sqrt{0^2+\left[-y(9-y^2)^{-1/2}\right]^2+1}\,dy\,dx = \int_0^4\int_0^2\sqrt{\frac{y^2}{9-y^2}+1}\,dy\,dx$$
$$= \int_0^4\int_0^2\frac{3}{\sqrt{9-y^2}}\,dy\,dx = 3\int_0^4\left[\sin^{-1}\frac{y}{3}\right]_{y=0}^{y=2}dx = 3\left[\left(\sin^{-1}\left(\tfrac{2}{3}\right)\right)x\right]_0^4 = 12\sin^{-1}\left(\tfrac{2}{3}\right)$$

6. $z = f(x,y) = 4-x^2-y^2$ and D is the projection of the paraboloid $z=4-x^2-y^2$ onto the xy-plane, that is, $D=\{(x,y)\mid x^2+y^2\le 4\}$. So $f_x=-2x$, $f_y=-2y \;\Rightarrow$

$$A(S) = \iint_D\sqrt{(-2x)^2+(-2y)^2+1}\,dA = \iint_D\sqrt{4(x^2+y^2)+1}\,dA = \int_0^{2\pi}\int_0^2\sqrt{4r^2+1}\,r\,dr\,d\theta$$
$$= \int_0^{2\pi}\left[\tfrac{1}{12}(4r^2+1)^{3/2}\right]_{r=0}^{r=2}d\theta = \int_0^{2\pi}\tfrac{1}{12}\left(17\sqrt{17}-1\right)d\theta = \tfrac{\pi}{6}\left(17\sqrt{17}-1\right)$$

7. $\int_0^{\pi/2}\int_0^y\int_0^x \cos(x+y+z)\,dz\,dx\,dy = \int_0^{\pi/2}\int_0^y \left[\sin(x+y+z)\right]_{z=0}^{z=x} dx\,dy$

$= \int_0^{\pi/2}\int_0^y \left[\sin(2x+y)-\sin(x+y)\right] dx\,dy$

$= \int_0^{\pi/2} \left[-\frac{1}{2}\cos(2x+y)+\cos(x+y)\right]_{x=0}^{x=y} dy$

$= \int_0^{\pi/2} \left[-\frac{1}{2}\cos 3y+\cos 2y+\frac{1}{2}\cos y-\cos y\right] dy$

$= \left[-\frac{1}{6}\sin 3y+\frac{1}{2}\sin 2y-\frac{1}{2}\sin y\right]_0^{\pi/2} = \frac{1}{6}-\frac{1}{2} = -\frac{1}{3}$

8. $\int_0^{\sqrt{\pi}}\int_0^x\int_0^{xz} x^2\sin y\,dy\,dz\,dx = \int_0^{\sqrt{\pi}}\int_0^x \left[-x^2\cos y\right]_{y=0}^{y=xz} dz\,dx = \int_0^{\sqrt{\pi}}\int_0^x (x^2-x^2\cos xz)\,dz\,dx$

$= \int_0^{\sqrt{\pi}} \left[x^2z - x\sin xz\right]_{z=0}^{z=x} dx = \int_0^{\sqrt{\pi}} (x^3 - x\sin x^2)\,dx$

$= \left[\frac{1}{4}x^4+\frac{1}{2}\cos x^2\right]_0^{\sqrt{\pi}} = \frac{1}{4}\pi^2-\frac{1}{2}-\frac{1}{2} = \frac{1}{4}\pi^2-1$

9. $\iiint_E y\,dV = \int_0^3\int_0^x\int_{x-y}^{x+y} y\,dz\,dy\,dx = \int_0^3\int_0^x \left[yz\right]_{z=x-y}^{z=x+y} dy\,dx = \int_0^3\int_0^x 2y^2\,dy\,dx$

$= \int_0^3 \left[\frac{2}{3}y^3\right]_{y=0}^{y=x} dx = \int_0^3 \frac{2}{3}x^3\,dx = \frac{1}{6}x^4\Big]_0^3 = \frac{81}{6} = \frac{27}{2}$

10. $\iiint_E e^{z/y}\,dV = \int_0^1\int_y^1\int_0^{xy} e^{z/y}\,dz\,dx\,dy = \int_0^1\int_y^1 \left[ye^{z/y}\right]_{z=0}^{z=xy} dx\,dy$

$= \int_0^1\int_y^1 (ye^x - y)\,dx\,dy = \int_0^1 \left[ye^x - xy\right]_{x=y}^{x=1} dy = \int_0^1 \left(ey - y - ye^y + y^2\right) dy$

$= \left[\frac{1}{2}ey^2 - \frac{1}{2}y^2 - (y-1)e^y + \frac{1}{3}y^3\right]_0^1$ [integrate by parts]

$= \frac{1}{2}e - \frac{1}{2} + \frac{1}{3} - 1 = \frac{1}{2}e - \frac{7}{6}$

11. $\displaystyle\iiint_E \frac{z}{x^2+z^2}\,dV = \int_1^4\int_y^4\int_0^z \frac{z}{x^2+z^2}\,dx\,dz\,dy = \int_1^4\int_y^4 \left[z\cdot\frac{1}{z}\tan^{-1}\frac{x}{z}\right]_{x=0}^{x=z} dz\,dy$

$= \int_1^4\int_y^4 \left[\tan^{-1}(1)-\tan^{-1}(0)\right] dz\,dy = \int_1^4\int_y^4 \left(\frac{\pi}{4}-0\right) dz\,dy = \frac{\pi}{4}\int_1^4 \left[z\right]_{z=y}^{z=4} dy$

$= \frac{\pi}{4}\int_1^4 (4-y)\,dy = \frac{\pi}{4}\left[4y-\frac{1}{2}y^2\right]_1^4 = \frac{\pi}{4}\left(16-8-4+\frac{1}{2}\right) = \frac{9\pi}{8}$

12. Here $E = \{(x,y,z) \mid 0 \le x \le \pi, 0 \le y \le \pi - x, 0 \le z \le x\}$, so

$\iiint_E \sin y\,dV = \int_0^\pi\int_0^{\pi-x}\int_0^x \sin y\,dz\,dy\,dx = \int_0^\pi\int_0^{\pi-x} \left[z\sin y\right]_{z=0}^{z=x} dy\,dx = \int_0^\pi\int_0^{\pi-x} x\sin y\,dy\,dx$

$= \int_0^\pi \left[-x\cos y\right]_{y=0}^{y=\pi-x} dx = \int_0^\pi \left[-x\cos(\pi-x)+x\right] dx$

$= \left[x\sin(\pi-x)-\cos(\pi-x)+\frac{1}{2}x^2\right]_0^\pi$ [integrate by parts]

$= 0-1+\frac{1}{2}\pi^2-0-1-0 = \frac{1}{2}\pi^2-2$

13. Here $E = \{(x,y,z) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}, 0 \le z \le 1+x+y\}$, so

$\iiint_E 6xy\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 6xy\,dz\,dy\,dx = \int_0^1\int_0^{\sqrt{x}} \left[6xyz\right]_{z=0}^{z=1+x+y} dy\,dx$

$= \int_0^1\int_0^{\sqrt{x}} 6xy(1+x+y)\,dy\,dx = \int_0^1 \left[3xy^2+3x^2y^2+2xy^3\right]_{y=0}^{y=\sqrt{x}} dx$

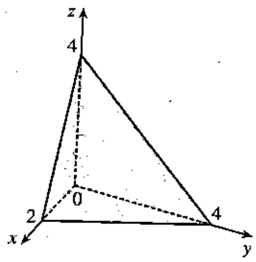
$= \int_0^1 (3x^2+3x^3+2x^{5/2})\,dx = \left[x^3+\frac{3}{4}x^4+\frac{4}{7}x^{7/2}\right]_0^1 = \frac{65}{28}$

19. The plane $2x + y + z = 4$ intersects the xy-plane when

$2x + y + 0 = 4 \quad \Rightarrow \quad y = 4 - 2x$, so

$E = \{(x, y, z) \mid 0 \le x \le 2, 0 \le y \le 4 - 2x, 0 \le z \le 4 - 2x - y\}$ and

$$V = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\,dy\,dx = \int_0^2 \int_0^{4-2x} (4 - 2x - y)\,dy\,dx$$
$$= \int_0^2 \left[4y - 2xy - \tfrac{1}{2}y^2\right]_{y=0}^{y=4-2x} dx$$
$$= \int_0^2 \left[4(4-2x) - 2x(4-2x) - \tfrac{1}{2}(4-2x)^2\right] dx$$
$$= \int_0^2 (2x^2 - 8x + 8)\,dx = \left[\tfrac{2}{3}x^3 - 4x^2 + 8x\right]_0^2 = \tfrac{16}{3}$$

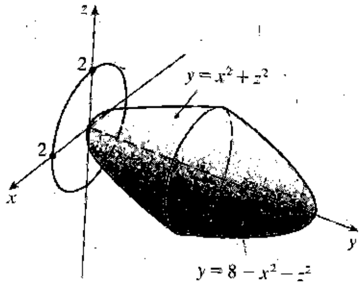


20. The paraboloids intersect when $x^2 + z^2 = 8 - x^2 - z^2 \quad \Leftrightarrow \quad x^2 + z^2 = 4$, thus the intersection is the circle $x^2 + z^2 = 4$, $y = 4$. The projection of E onto the xz-plane is the disk $x^2 + z^2 \le 4$, so

$E = \{(x, y, z) \mid x^2 + z^2 \le y \le 8 - x^2 - z^2, x^2 + z^2 \le 4\}$. Let

$D = \{(x, z) \mid x^2 + z^2 \le 4\}$. Then using polar coordinates $x = r\cos\theta$ and $z = r\sin\theta$, we have

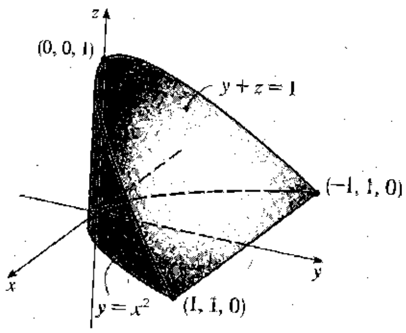
$$V = \iiint_E dV = \iint_D \left(\int_{x^2+z^2}^{8-x^2-z^2} dy\right) dA = \iint_D (8 - 2x^2 - 2z^2)\,dA$$
$$= \int_0^{2\pi} \int_0^2 (8 - 2r^2)\,r\,dr\,d\theta = \int_0^{2\pi} d\theta \int_0^2 (8r - 2r^3)\,dr$$
$$= \left[\theta\right]_0^{2\pi} \left[4r^2 - \tfrac{1}{2}r^4\right]_0^2 = 2\pi(16 - 8) = 16\pi$$



21. The plane $y + z = 1$ intersects the xy-plane in the line $y = 1$, so

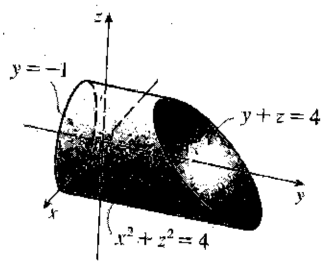
$E = \{(x, y, z) \mid -1 \le x \le 1, x^2 \le y \le 1, 0 \le z \le 1 - y\}$ and

$$V = \iiint_E dV = \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dz\,dy\,dx = \int_{-1}^1 \int_{x^2}^1 (1 - y)\,dy\,dx$$
$$= \int_{-1}^1 \left[y - \tfrac{1}{2}y^2\right]_{y=x^2}^{y=1} dx = \int_{-1}^1 \left(\tfrac{1}{2} - x^2 + \tfrac{1}{2}x^4\right) dx$$
$$= \left[\tfrac{1}{2}x - \tfrac{1}{3}x^3 + \tfrac{1}{10}x^5\right]_{-1}^1 = \tfrac{1}{2} - \tfrac{1}{3} + \tfrac{1}{10} + \tfrac{1}{2} - \tfrac{1}{3} + \tfrac{1}{10} = \tfrac{8}{15}$$



22. Here $E = \{(x, y, z) \mid -1 \le y \le 4 - z, x^2 + z^2 \le 4\}$, so

$$V = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-1}^{4-z} dy\,dz\,dx = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4 - z + 1)\,dz\,dx$$
$$= \int_{-2}^2 \left[5z - \tfrac{1}{2}z^2\right]_{z=-\sqrt{4-x^2}}^{z=\sqrt{4-x^2}} dx = \int_{-2}^2 10\sqrt{4 - x^2}\,dx$$
$$= 10\left[\tfrac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\left(\tfrac{x}{2}\right)\right]_{-2}^2 \qquad \left[\begin{array}{l}\text{using trigonometric substitution or}\\ \text{Formula 30 in the Table of Integrals}\end{array}\right]$$
$$= 10\left[2\sin^{-1}(1) - 2\sin^{-1}(-1)\right] = 20\left(\tfrac{\pi}{2} - \left(-\tfrac{\pi}{2}\right)\right) = 20\pi$$



[continued]

Then

$$\begin{aligned}\int_0^1\int_y^1\int_0^z f(x,y,z)\,dx\,dz\,dy &= \int_0^1\int_0^x\int_x^1 f(x,y,z)\,dz\,dy\,dx + \int_0^1\int_x^1\int_y^1 f(x,y,z)\,dz\,dy\,dx\\ &= \int_0^1\int_y^1\int_x^1 f(x,y,z)\,dz\,dx\,dy + \int_0^1\int_0^y\int_y^1 f(x,y,z)\,dz\,dx\,dy\\ &= \int_0^1\int_0^z\int_0^z f(x,y,z)\,dx\,dy\,dz = \int_0^1\int_x^1\int_0^z f(x,y,z)\,dy\,dz\,dx\\ &= \int_0^1\int_0^z\int_0^z f(x,y,z)\,dy\,dx\,dz\end{aligned}$$

37. The region C is the solid bounded by a circular cylinder of radius 2 with axis the z-axis for $-2 \le z \le 2$. We can write $\iiint_C (4+5x^2yz^2)\,dV = \iiint_C 4\,dV + \iiint_C 5x^2yz^2\,dV$, but $f(x,y,z) = 5x^2yz^2$ is an odd function with respect to y. Since C is symmetrical about the xz-plane, we have $\iiint_C 5x^2yz^2\,dV = 0$. Thus $\iiint_C (4+5x^2yz^2)\,dV = \iiint_C 4\,dV = 4\cdot V(E) = 4\cdot\pi(2)^2(4) = 64\pi$.

38. We can write $\iiint_B (z^3+\sin y+3)\,dV = \iiint_B z^3\,dV + \iiint_B \sin y\,dV + \iiint_B 3\,dV$. But z^3 is an odd function with respect to z and the region B is symmetric about the xy-plane, so $\iiint_B z^3\,dV = 0$. Similarly, $\sin y$ is an odd function with respect to y and B is symmetric about the xz-plane, so $\iiint_B \sin y\,dV = 0$. Thus $\iiint_B (z^3+\sin y+3)\,dV = \iiint_B 3\,dV = 3\cdot V(B) = 3\cdot\frac{4}{3}\pi(1)^3 = 4\pi$.

39. $m = \iiint_E \rho(x,y,z)\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 2\,dz\,dy\,dx = \int_0^1\int_0^{\sqrt{x}} 2(1+x+y)\,dy\,dx$

$= \int_0^1 \left[2y+2xy+y^2\right]_{y=0}^{y=\sqrt{x}}\,dx = \int_0^1\left(2\sqrt{x}+2x^{3/2}+x\right)dx = \left[\frac{4}{3}x^{3/2}+\frac{4}{5}x^{5/2}+\frac{1}{2}x^2\right]_0^1 = \frac{79}{30}$

$M_{yz} = \iiint_E x\rho(x,y,z)\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 2x\,dz\,dy\,dx = \int_0^1\int_0^{\sqrt{x}} 2x(1+x+y)\,dy\,dx$

$= \int_0^1 \left[2xy+2x^2y+xy^2\right]_{y=0}^{y=\sqrt{x}}\,dx = \int_0^1 (2x^{3/2}+2x^{5/2}+x^2)\,dx = \left[\frac{4}{5}x^{5/2}+\frac{4}{7}x^{7/2}+\frac{1}{3}x^3\right]_0^1 = \frac{179}{105}$

$M_{xz} = \iiint_E y\rho(x,y,z)\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 2y\,dz\,dy\,dx = \int_0^1\int_0^{\sqrt{x}} 2y(1+x+y)\,dy\,dx$

$= \int_0^1 \left[y^2+xy^2+\frac{2}{3}y^3\right]_{y=0}^{y=\sqrt{x}}\,dx = \int_0^1\left(x+x^2+\frac{2}{3}x^{3/2}\right)dx = \left[\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{4}{15}x^{5/2}\right]_0^1 = \frac{11}{10}$

$M_{xy} = \iiint_E z\rho(x,y,z)\,dV = \int_0^1\int_0^{\sqrt{x}}\int_0^{1+x+y} 2z\,dz\,dy\,dx = \int_0^1\int_0^{\sqrt{x}}\left[z^2\right]_{z=0}^{z=1+x+y}\,dy\,dx = \int_0^1\int_0^{\sqrt{x}}(1+x+y)^2\,dy\,dx$

$= \int_0^1\int_0^{\sqrt{x}}(1+2x+2y+2xy+x^2+y^2)\,dy\,dx = \int_0^1\left[y+2xy+y^2+xy^2+x^2y+\frac{1}{3}y^3\right]_{y=0}^{y=\sqrt{x}}\,dx$

$= \int_0^1\left(\sqrt{x}+\frac{7}{3}x^{3/2}+x+x^2+x^{5/2}\right)dx = \left[\frac{2}{3}x^{3/2}+\frac{14}{15}x^{5/2}+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{2}{7}x^{7/2}\right]_0^1 = \frac{571}{210}$

Thus the mass is $\frac{79}{30}$ and the center of mass is $(\overline{x},\overline{y},\overline{z}) = \left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right) = \left(\frac{358}{553},\frac{33}{79},\frac{571}{553}\right)$.

40. $m = \int_{-1}^1\int_0^{1-y^2}\int_0^{1-z} 4\,dx\,dz\,dy = 4\int_{-1}^1\int_0^{1-y^2}(1-z)\,dz\,dy = 4\int_{-1}^1\left[z-\frac{1}{2}z^2\right]_{z=0}^{z=1-y^2}\,dy = 2\int_{-1}^1(1-y^4)\,dy = \frac{16}{5}$,

$M_{yz} = \int_{-1}^1\int_0^{1-y^2}\int_0^{1-z} 4x\,dx\,dz\,dy = 2\int_{-1}^1\int_0^{1-y^2}(1-z)^2\,dz\,dy = 2\int_{-1}^1\left[-\frac{1}{3}(1-z)^3\right]_{z=0}^{z=1-y^2}\,dy$

$= \frac{2}{3}\int_{-1}^1(1-y^6)\,dy = \left(\frac{4}{3}\right)\left(\frac{6}{7}\right) = \frac{24}{21}$

[continued]