

INSTRUCTIONS: PLEASE READ ALL CAREFULLY BEFORE STARTING

After inputting your student ID, we recommend to write down the values for problems 2-4 set by the script. If you show results obtained with values different from those set by the Jupyter notebook script, your question **will not be graded**.

1. **Problem 1:** Complete the Problem 1 set in `Final303.ipynb`.
The problem has two independent parts: Part 1 includes questions a to e; part 2 f and g. Submit the completed notebook as `Final303_complete.ipynb` (you don't have to finish all the questions to submit).
2. **Problems 2-4:** For each problem, you will need to separately assemble a pdf file of handwritten solutions. We recommend to not use more than 2 pages per problem. **Write your name** on top of each page and do not forget to indicate which subquestion (a, b...) you are solving.

Grading

We recommend spending an equivalent amount of time on problems 1, 2, 3 and 4.

Problem 1

Submit the completed notebook as `Final303_complete.ipynb` (you don't have to finish all the questions to submit).

The problem has two independent parts: Part 1 includes questions a to e; part 2 f and g.

Remark: If you are running the notebook on syzygy, we recommend that you make a copy of the original notebook to edit it, save after each new answer, and take a screenshot of your work as soon as you are done. If your connection to the server broke, simply log back to the server and re-open your saved notebook.

Warning: Having “Success” displayed when running the cells does not necessarily mean that the solution is correct (this will be evaluated after you submit the notebook).

Problem 2

We study the weather and amount of snow every year in Seattle. A year can be either *rainy* (R) or *snowy* (S). We assume that whether a given year is S or R depends only on the previous year. Besides, if one year was S , the next year will be S or R with equal probability. Run the notebook for problem 2 with your student ID to find the transition probability P_{RR} (you can deduce all the other transition probabilities), and the average number of inches of snow associated with each state R and S . Justify all your answers. Solutions can be left as calculator ready.

- a. Find the probability that it is a rainy year two years after a rainy year.
- b. Suppose that the first year was rainy. Find the expected total number of inches of snow in the third year.
- c. What is the long run average number of inches of snow in Seattle?

Problem 3

The number of followers of Jacob's video game stream on Twitch follows a Poisson process with rate 5 per day. Assume that each follower is a subscriber with probability $1/5$, all independently. Justify all your answers. Solutions can be left as calculator ready.

- a. Run the notebook and find the probability asked for Problem 3a.
- b. Run the notebook and find the probability asked for Problem 3b.
- c. Run the notebook and find the value asked for Problem 3c.

Problem 4

Becca only watches three TV channels: *news A*, *news B*, and *sports*. While watching the news, she switches channels after an exponential amount of time with mean μ_{news} minute(s). While watching sports, she switches after an exponential amount of time with mean μ_{sports} minute(s). After watching a news channel, Becca switches to the other news channel with probability p . After watching sports, Becca switches to news A with probability p' . Run the notebook to find the values of μ_{news} , μ_{sports} , p and p' .

- a. Model Becca's news watching habits by a continuous time Markov chain, with state 1 = news A, state 2 = news B, state 3 = sports. Draw the transition diagram, and add to each arrow of the diagram the corresponding numerical value of the rates q_{ij} , as defined in the course.
- b. Write down the two forward Kolmogorov equations with initial condition being the sports channel.
- c. Is the chain reversible? Justify your answer.