Math 324 A - Summer 2017
Midterm exam 2
Friday, July 28th, 2017

Name: _____

Problem 1	10	
Problem 2	15	
Problem 3	10	
Problem 4	15	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but 3/4+1/2 should be reduced to 5/4.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 60 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 pts) Let $f(x, y)$ be a function on $\mathbb{R}^2$. Assume that the minimum value of $D_u f(1, -3)$ is attained when $u = -\frac{\sqrt{2}}{2}\widehat{i} + \frac{\sqrt{2}}{2}\widehat{j}$. Also, assume $\frac{\partial f}{\partial x}(1, -3) = +4$.

   (a) (5 pts) Find $\nabla f(1, -3)$.

   (b) (2 pts) What unit vector v maximizes $D_v f(1, -3)$?

   (c) (3 pts) Given your answer from part (a), is it possible that $f(x, y) = 10 - x^2 - 3xy + x$? Explain.

2. (15 pts) Let D be the region in the plane under the parabola $y = 4 - x^2$ and above the line $y = 3x$.

   (a) (3 pts) Draw D, and find the points where the two bounding curves meet.

   (b) (7 pts) Consider the change of coordinates $u = x, v = y - 3x$. Draw the image of D in the u-v plane, and find the Jacobian of the transformation.

   (c) (5 pts) Parameterize the double integral $\iint_D (x + y)\, dA$ in terms of u's and v's. **You do not need to evaluate it.**

3. (10 pts) Consider the vector field $F = 3x^2y\widehat{i} + x^3\widehat{j}$.

(a) (5 pts) Is F conservative? If so, find a potential function; if not, explain how you know it isn't conservative.

(b) (5 pts) Let C be the curve consisting of the part of the circle $x^2 + y^2 = 1$ below the x-axis, from $(1, 0)$ to $(-1, 0)$, followed by the line segment from $(-1, 0)$ to $(1, 1)$. Evaluate $\int_C F \cdot dr$.

4. (15 pts) Let R be the circle of radius 1 centered at $(0, 0)$, and let $C = \partial R$ be the boundary of R, oriented counter-clockwise.

   (a) (10 pts) Use Green's theorem to evaluate

$$\int_C x^3 \, dy.$$

   (b) (5 pts) Verify your answer from part (a) by evaluating the line integral directly.