Math 308 E - Spring 2018
Midterm exam 1
Friday, April 20th, 2018

Name: ____

Problem 1	15	
Problem 2	10	
Problem 3	15	
Problem 4	10	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (15 points) Consider the function $T : \mathbb{R}^4 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x - 2z + w \\ y + w \end{bmatrix}.$$

(a) (5 points) Find a matrix A such that $T(v) = A \cdot v$ for all $v \in \mathbb{R}^4$.

(b) (5 points) Is T onto? Why or why not?

(c) (5 points) Is T one-to-one? Why or why not?

2. (10 points) Consider the vectors $v, u, w, z \in \mathbb{R}^4$ given by

$$u = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \; v = \begin{bmatrix} 0 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \; w = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \; z = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 4 \end{bmatrix}$$

(a) (5 points) Is $z \in span\{u, v, w\}$? If so, find numbers x_1, x_2, x_3 such that $x_1 u + x_2 v + x_3 w = z$.

(b) (5 points) Is $\{u, v, w\}$ linearly independent? Why or why not? (Hint: use your calculation from part a.)

3. (15 points) Circle **True** or **False** for each of the statements below. No justification is needed.

(a) **True** **False** There exist six vectors in $\mathbb{R}^5$ that are linearly independent.

(b) **True** **False** A function $f : \mathbb{R} \to \mathbb{R}$ is a linear transformation if and only if $f(x) = cx$ for some $c \in \mathbb{R}$.

(c) **True** **False** A homogeneous system of equations may have infinitely many solutions.

(d) **True** **False** The matrix below is in eschelon form, and has a pivot in every column.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(e) **True** **False** There exist three vectors in $\mathbb{R}^4$ that are linearly dependent.

(f) **True** **False** There exists a 3×7 matrix A and a vector $b \in \mathbb{R}^3$ such that the equation $Ax = b$ has exactly three solutions for $x \in \mathbb{R}^7$.

4. (10 points) Find an example of a matrix A such that the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by $T(v) = A \cdot v$ satisfies

$$T\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \; T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

How many different transformations T are there that satisfy these conditions? Explain.