Math 324 B - Winter 2017
Midterm exam 1
Friday, January 27, 2017

Name: ______________________________

Problem 1	14	
Problem 2	14	
Problem 3	12	
Problem 4	10	
Total	50	

- There are 4 problems on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2} + \frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. Let C be the cone defined by $x^2 + y^2 \leq z^2 \leq R^2$, where $R > 0$ is a fixed number.

   (a) (7 pts) Draw a picture of C, and find the volume of C.

   (b) (7 pts) Recall that the surface area of a surface S defined by $z = f(x, y)$ for $(x, y) \in D \subset \mathbb{R}^2$ is given by the double integral

   $$\iint_D \sqrt{1 + \left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2}\, dA.$$

   Find the surface area of C (including the base of the cone).

2. (14 pts) Use an appropriate change of coordinates to calculate the double integral

$$\iint_D e^{8x^2+y^2}\, dA,$$

where D is the interior of the ellipse $x^2 + (\frac{y}{2\sqrt{2}})^2 = 1$.

3. (a) (6 pts) Let D be the region (with finite area) bounded by the curves $y = e^x$ and $y = e^{\sqrt{x}}$. Draw a picture of the region D, and use Cartesian coordinates to evaluate the double integral

$$\iint_D \frac{1}{\ln(y)} dA.$$

(b) (6 pts) Let $R = \{(x, y) : (x-1)^2 + y^2 \leq 1 \text{ **and** } y \geq 0 \text{ **and** } x \geq 1\}$. Draw a picture of R, and use polar coordinates to evaluate the integral

$$\iint_R \frac{x}{\sqrt{x^2+y^2}} dA.$$

You may use the fact that $\int \sec t \, dt = \ln\left|\sec t + \tan t\right| + C$.

4. (10 pts) Let $E \subset \mathbb{R}^3$ consist of all points (x, y, z) satisfying:

$$x^2 + y^2 + z^2 \le 4,$$
$$0 \le z \le \sqrt{3}(x^2 + y^2), \text{ and}$$
$$y \ge 0.$$

Consider the triple integral

$$\iiint_E \sqrt{\frac{x^2 + y^2}{x^2 + y^2 + z^2}}\, dV.$$

(a) (5 pts) Parameterize the integral in spherical coordinates. **You do not need to evaluate it.** [Hint: you will need to figure out where the surfaces $x^2 + y^2 + z^2 = 4$ and $z = \sqrt{3}(x^2 + y^2)$ intersect.]

(b) (5 pts) Set up the same integral using cylindrical coordinates. **You do not need to evaluate it.**