Math 324 A - Spring 2017
Midterm exam 2
Friday, May 12th, 2017

Name: ______________________________

Problem 1	12	
Problem 2	12	
Problem 3	14	
Problem 4	12	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $3/4+1/2$ should be reduced to $5/4$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (12 pts) Consider the function $f(x, y, z) = \frac{1}{3}x^2 \sin(y) - 2xz$.

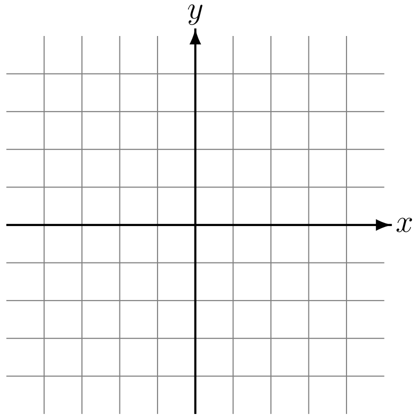
   a. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.

   b. Let v be the unit vector with tail at the origin in $\mathbb{R}^3$ and tip at the point $(1, \frac{\pi}{3}, \frac{\pi}{2})$ in spherical coordinates (ρ, θ, ϕ). Find the value of the directional derivative $D_v f(x, y, z)$ at the point $(x, y, z) = (3, -\pi, 1)$.

   c. Give the unit vector u that maximizes $D_u f(3, -\pi, 1)$.

   d. Let F be the gradient vector field of f, so $F = \nabla f$. Find the curl and divergence of F.

2a. (4 pts) Let $f(x,y) = \frac{1}{2}(x^2+y^2)$. Write down a formula for $\nabla f(x,y)$, and draw the vector $\nabla f(x,y)$ at each of the three points $(1,0), (-2,1)$ and $(2,-2)$.



2b. (4 pts) Define what it means for a vector field F defined on an *arbitrary domain* D in the plane to be conservative. (You may give any definition equivalent to the one we used in class.)

2c. (4 pts) Consider the vector field $F(x,y) = \langle y^3\cos(x), -3y^2\sin(x)\rangle$. Is F conservative? If so, give a potential function; if not, explain how you know it isn't conservative.

3a. (7 pts) State the fundamental theorem for line integrals, and use it to compute

$$\int_C \sin(y)e^{x\sin(y)}\,dx + x\cos(y)e^{x\sin(y)}\,dy,$$

where C is the line segment from $(1, \pi)$ to $(2, \pi)$ followed by the line segment from $(2, \pi)$ to $(2, 2\pi)$.

3b. (7 pts) Let F denote the force field $F(x, y, z) = \langle 3z^2 \ln(2xy - 1), x^2 + (z + 1)y^{-2}, xz - 1\rangle$. Find the work done by F on a particle that travels along the path $r(t) = (t, t^{-1}, 3), 1 \leq t \leq 2$.

4. (12 pts) Let C denote the triangle with vertices $(-2,-1), (2,-1)$, and $(0,6)$, oriented counter clockwise. Let G be the vector field $G(x,y) = \langle 3xy^2, x+y \rangle$. Evaluate

$$\int_C G \cdot dr.$$

[Hint: apply a theorem, then use symmetry to simplify the integral.]