1. a. $\frac{\pi R^3}{3}$. b. $\pi R^2(1+\sqrt{2})$.

2. $\frac{\pi\sqrt{2}(e^8-1)}{4}$.

3. a. $e-2$. b. $\frac{5\sqrt{2}}{6}-\frac{1}{2}\log(1+\sqrt{2})$.

4. a.

$$\int_0^\pi \int_{\pi/6}^{\pi/2} \int_{\frac{1}{\sqrt{3}}\csc\phi\cot\phi}^2 \rho^2 \sin^2\phi \, d\rho \, d\phi \, d\theta.$$

b.

$$\int_0^\pi \int_0^1 \int_0^{\sqrt{3}r^2} \frac{r^2}{\sqrt{r^2+z^2}} \, dz \, dr \, d\theta + \int_0^\pi \int_1^2 \int_0^{\sqrt{4-r^2}} \frac{r^2}{\sqrt{r^2+z^2}} \, dz \, dr \, d\theta,$$

or

$$\int_0^\pi \int_0^{\sqrt{3}} \int_{\sqrt{z/\sqrt{3}}}^{\sqrt{4-z^2}} \frac{r^2}{\sqrt{r^2+z^2}} \, dr \, dz \, d\theta.$$