Math 324 E - Fall 2019
Midterm exam 1
Monday, October 21, 2019

Name: ______________________________

Problem 1	12	
Problem 2	14	
Problem 3	12	
Problem 4	12	
Total	50	

- There are 4 problems on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2} + \frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (12 points) Let D be the two dimensional region defined by

$$D = \{(x, y) \in \mathbb{R}^2 : 2y \leq x + 2 \text{ and } 4y \geq x^2 + 1\}.$$

Draw a picture of D, and parameterize the integral

$$\iint_D y \, dA$$

in with order of integration $dy\, dx$. **You don't need to evaluate it.**

2. (14 pts) Suppose there is a cylindrical can full of beer occupying the region

$$C = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, 0 \leq z \leq 4\}.$$

A nervous freshman was just swirling the can, concentrating the beer near the edge. Assume the beer has density function $\rho(x, y, z) = x^2 + y^2 + (4 - z)$. Find the total volume of beer in the can by integrating ρ over the region occupied by the can.

3. (12 points) Find the volume of the "ice cream cone" region

$$E = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1, z \geq 0, z^2 \geq x^2 + y^2\}$$

using spherical coordinates. (No credit will be awarded for using any other coordinate system!)

4. (12 points) Use the change of coordinates $u = x + y, v = -x + y$ to evaluate the integral

$$\iint_R e^{x+y}\, dA,$$

where $R = \{(x, y) : |x| + |y| \le 1\}$. Your answer should include a picture of R, and a picture of the region of u and v values corresponding to R.