Math 324 A - Spring 2017
Midterm exam 1
Friday, April 21, 2017

Name: ______________________________

Problem 1	12	
Problem 2	12	
Problem 3	12	
Problem 4	14	
Total	50	

- There are 4 questions on this exam. Make sure you have all four.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $\frac{1}{2}+\frac{3}{4}$ should be simplified to $\frac{5}{4}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (12 pts) Let R be the region in the x-y plane given in polar coordinates as

$$R = \{(r, \theta) : \frac{1}{2}\csc\theta \le r \le \sin\theta, 0 \le r < \infty, 0 \le \theta \le \pi\}.$$

Sketch the region R, and evaluate the double integral

$$\iint_R \sin^2\theta \, dA.$$

2. (12 pts) Let D be the region in the x-y plane bounded by the curves $y = x, y = 3x, y = 1/x$ and $y = 3/x$. Use the change of coordinates $x = u/v, y = v$ to evaluate the integral

$$\iint_D xy\, dA.$$

3. (12 pts) Set up but **do not evaluate** the integral

$$\iiint_E xz\,dV$$

in Cartesian coordinates, where E is the solid (with finite volume) that lies in the first octant $x, y, z \geq 0$ and is bounded by the surfaces

$$x = y^4, x = 2y^2 - 1, z = 0 \text{ and } z = 3y.$$

(For example, "the surface $x = y^4$" means the set of all points $(x, y, z) \in \mathbb{R}^3$ satisfying $x = y^4$.)

4. (14 pts) Professor Stella Artois wants to know how much beer her strangely shaped beer bottle can hold. The design is obtained by rotating the curve $x = 2 + \cos \pi z, 0 \leq z \leq 5$ (in the x-z plane) around the z axis, and attaching caps at the top and bottom. Beer tends to be foamier at the top: the density of beer in the bottle is

$$d(x, y, z) = \frac{6 - z}{1 + \frac{1}{2} \cos \pi z}.$$

Set up an integral to compute the mass of the beer in the bottle when it is full: that is, choose a coordinate system use it to and parameterize the integral

$$\iiint_E d(x, y, z) \, dV$$

where E is the region inside the beer bottle. **You do not need to evaluate the integral.**