Math 324, Homework 7

1. Give a parametric description of the plane that contains the point $(-1, 0, 0)$ and is normal to the vector $\langle 3, 1, -1\rangle$.

2. Let S be the part of the surface of the unit sphere that lies above the cone $z = \sqrt{x^2 + y^2}$. Give a parametric description of S, and find the surface area of S by integrating the function 1 over S.

3. Find the area of the part of the surface $z = x^2+2y$ that lies above the triangle with vertices $(0, 0), (1, 0), (1, 2)$.

   For problems 4-6, compute the surface integral $\iint_S f\, dS$ for the given surface S and function f.

4. $f(x, y, z) = z$; S is the part of the plane $x + y + z = 1$ where $x, y, z \geq 0$.

5. $f(x, y, z) = y$; S is the surface $x^2 + y^2 + z^2 = 4$, for $0 \leq y \leq 1$.

6. $f(x, y, z) = x^2 + y^2 + z^2$; S is the part of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 2$, together with the top and bottom disks.

   For problems 4-6, evaluate the surface integral $\iint_S F \cdot dS$ for the given vector field F and oriented surface S.

7. $F(x, y, z) = \langle 0, y, -z\rangle$; S consists of the parabloid $y = x^2 + z^2, 0 \leq y \leq 1$, and the disk $x^2 + z^2 \leq 1, y = 1$ with the outward pointing normal.

8. $F(x, y, z) = \langle x, y, z\rangle$; S is the part of the unit sphere where $\frac{1}{2} \leq z \leq \frac{\sqrt{3}}{2}$ with the inward pointing normal.

9. $F(x, y, z) = \langle 0, 0, e^{y+z}\rangle$; S is the boundary of the unit cube $0 \leq x, y, z \leq 1$ with the outward pointing normal.