Math 324, Homework 4

1. Let $z = 3x\sin(y), x = 3t^2 - 1, y = \tan(t)$. Use the chain rule to find $\frac{\partial z}{\partial t}$ at the point $t = 1$.

2. Let $z = e^{x-y}, x = -t - 1, y = 1$. Use the chain rule to find $\frac{\partial z}{\partial t}$ at the point $t = -2$.

3. Find the gradient of $f(x, y) = 17y - 2xy^2$, and use it to compute the directional derivative of f at the point $(8, 1)$ in the direction of the unit vector $u = \frac{1}{\sqrt{17}}\langle -4, 1\rangle$.

4. Find the gradient of $f(x, y) = y^2 - x^3 - 2xy$, and find the value of the gradient at the point $(1, 1)$. Are there any other points (x, y) where the gradient vector is parallel to the vector $\hat{i}$? What are they?

5. Find the two points on the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$ where the tangent plane to the ellipsoid at those points is normal to the vector $v = \langle 1, 1, -2\rangle$.

6. (Stewart 16.1, 11-14). Match the given vector fields with the plots. (Pictures in the text.)

7. Does there exist a function $f(x, y, z)$ with $\nabla f = yz^2\hat{i} + xz^2\hat{j} + 2xyz\hat{k}$? Either find a function f that works, or explain why there is no such function.