Math 324 C - Summer 2016
Midterm exam 2
Wednesday, July 29th, 2016

Name: ____

Problem 1	10	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	10	
Total	50	

- There are 5 questions on this exam. Make sure you have all five.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Give exact answers, and simplify as much as possible. For example, $\frac{\pi}{\sqrt{2}}$ is acceptable, but $3\sqrt{3}+\frac{1}{\sqrt{3}}$ should be reduced to $\frac{10\sqrt{3}}{3}$.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 60 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 pts) Consider the function $f(x, y, z) = 3x\sin(y) - xz$.

    a. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.

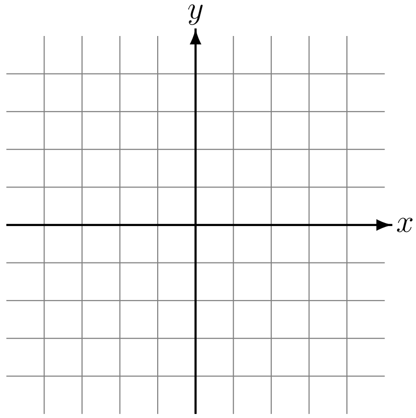
    b. Let v be the unit vector with tail at the origin and tip at the point $(1, \frac{\pi}{3}, \frac{\pi}{2})$ in spherical coordinates (ρ, θ, ϕ). Find the directional derivative $D_v f$ at the point $(3, -\pi, 1)$.

    c. Give the unit vector u that maximizes $D_u f(3, -\pi, 1)$.

2a. (3 pts) Give a definition in words of the tangent plane to a surface $F(x, y, z) = k$ at a point p in terms of the gradient of F. (Don't just write down the equation.)

2b. (7 pts) Compute the tangent plane to the implicitly defined surface $x^2z + 3y^3 - z^3 - 3z = 0$ at the point $(x, y, z) = (1, 1, 1)$.

3a. (3 pts) Let $f(x, y) = \frac{1}{2}(x^2 + y^2)$. Write down a formula for $\nabla f(x, y)$, and draw the vector $\nabla f(x, y)$ at each of the three points $(1, 0), (-2, 1)$ and $(2, -2)$.



3b. (3 pts) Define what it means for a vector field F defined on an *arbitrary domain* D in the plane to be conservative. (You may give any definition equivalent to the one we used in class.)

3c. (4 pts) Consider the vector field $F(x, y) = \langle y^3 \cos(x), -3y^2 \sin(x) \rangle$. Is F conservative? If so, give a potential function; if not, explain how you know it isn't conservative.

4a. (5 pts) State the fundamental theorem for line integrals, and use it to compute

$$\int_C \sin(y)e^{x\sin(y)}\,dx + x\cos(y)e^{x\sin(y)}\,dy,$$

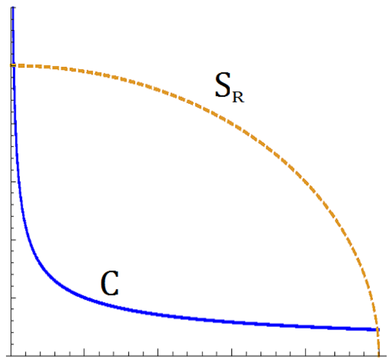
where C is the line segment from $(1,\pi)$ to $(2,\pi)$ followed by the line segment from $(2,\pi)$ to $(2,2\pi)$.

4b. (5 pts) Let F denote the force field $F(x,y) = \langle 3\ln(2xy-1), x^2 - y^2\rangle$ defined on $\{(x,y) : 2xy > 1\}$. Find the work done by F on a particle traveling along the path $r(t) = (t, t^{-1}), 1 \le t \le 2$.

5. Consider the curve C given by the equation $xy^2 = 1$ for $0 < x < \infty$. Let D be the (infinite) region above C, so $D = \{(x, y) : x > 0, y > \frac{1}{\sqrt{x}}\}$. Also, define the vector field F by $F(x, y) = \langle \frac{1}{x^2+y^2}, \frac{1}{1+y^2} \rangle$.

5a. (5 pts) Let S_R be the quarter circle of radius R centered at $(0, 0)$ in the first quadrant parameterized counter-clockwise. By evaluating the line integral directly, show that

$$\lim_{R\to\infty} \int_{S_R} F \cdot dr = \frac{\pi}{2}.$$



5b. (5 pts) Use the result from part a and Green's theorem to relate

$$\int_C F \cdot dr$$

to a double integral over the region D. (Hint: use Green's theorem on a closed curve consisting partly of C and partly of S_R, and then let $R \to \infty$. You may assume that C and S_R meet at the points $(R, 0)$ and $(0, R)$: the error in doing so is negligible.)

5b. (cont.)

5c. (Extra credit) Evaluate your integral from part b: you may use the fact that

$$\int \frac{x}{x^3+1} dx = \frac{1}{3} \log \frac{\sqrt{x^2-x+1}}{x+1} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x-1}{\sqrt{3}} + C.$$