Math 324, Homework 2

1. Use polar coordinates to parameterize the integral $\iint_D (xy-1)\,dA$, where D is the interior of the circle of radius 1 centered at $(1,1)$. You don't need to evaluate it.

2. Let P be the region in $\mathbb{R}^2$ bounded by the polar curves $r = 2\cos\theta$ and $r = 1+\cos\theta$. Find the area of P. (Hint: you will need to split P up into smaller regions.)

3. (Stewart 15.6 #5) Find the surface area of the part of the cylinder $y^2+z^2 = 9$ that lies above the rectangle with vertices $(0,0), (4,0), (0,2)$ and $(4,2)$.

4. Use the formula for surface area to verify that the surface area of the sphere of radius R is $4\pi R^2$.

5. (Stewart 15.7 #9) Evaluate the triple integral $\iiint_E y\,dV$, where E is the set of points (x,y,z) satisfying:

$$\begin{aligned} 0 \le x &\le 3 \\ 0 \le y &\le x \\ x-y \le z &\le x+y \end{aligned}$$

6. (Stewart 15.7 #13) Evaluate $\iiint_E 6xy\,dV$, where E lies under the plane $z = 1+x+y$ and above the region in the $xy-$plane bounded by the curves $y=\sqrt{x}, y=0, x=1$.

7. (Stewart 15.7 #19) Use a triple integral to find the volume of the tetrahedron enclosed by the coordinate planes, i.e. $x=0, y=0, z=0$, and the plane $2x+y+z=4$.

8. (Stewart 15.7 #37) Use symmetry to evaluate the integral $\iiint_C (4+5x^2yz^2)\,dV$ where C is the cylindrical region $x^2+y^2 \le 4, -2 \le z \le 2$.

9. Let $W \subset \mathbb{R}^3$ be the region bounded by $z = 4-y^2, y=2x, z=0$ and $x=0$. Draw a picture of W, and parameterize the triple integral

$$\iiint_W xyz\,dV$$

with three different orders of integration: $dz\,dy\,dx, dx\,dz\,dy$, and $dy\,dz\,dx$.