Math 308 O - Winter 2020
Midterm exam 1
Wednesday, January 29

Name: ______________________________

Problem 1	10	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Total	40	

- There are 4 questions on this exam. Make sure you have all four.
- **You must show your work on all problems.** The correct answer with no supporting work may result in no credit. Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.
- Give exact answers, and simplify as much as possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam. Budget your time wisely!

GOOD LUCK!

1. (10 points) Consider the vectors in $\mathbb{R}^3$ given by

$$u = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \ v = \begin{bmatrix} 5 \\ 4 \\ -5 \end{bmatrix}, \ w = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

(a) Find all solutions (x_1, x_2, x_3) to the system

$$x_1 u + x_2 v + x_3 w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

(b) Is the set $\{u, v, w\}$ linearly dependent or independent? Explain.

2. (10 points)

(a) Give a matrix A such that the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(v) = Av$ satisfies

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}-1\\3\end{bmatrix}$$

(b) Suppose $u, v, w \in \mathbb{R}^4$ are vectors, and $x_1 = 3, x_2 = -6, x_3 = 1$ is a solution to the vector equation

$$x_1 u + x_2 v + x_3 w = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}.$$

Find another solution to this equation, other than $x_1 = x_2 = x_3 = 0$, or explain why there are no other solutions.

3. (15 points) Circle **True** or **False** for each of the statements below. No justification is needed.

(a) **True** **False** Let $v, w \in \mathbb{R}^3$ be any linearly independent vectors. Then the set $\{0, v, w\}$ is linearly independent (where 0 represents the vector of all 0's).

(b) **True** **False** Any four vectors in $\mathbb{R}^2$ are always linearly dependent.

(c) **True** **False** A system of linear equations always has 0, 1, or infinitely many solutions.

(d) **True** **False** Suppose $u, v, w \in \mathbb{R}^3$ are any vectors. Then the vector $u - v + 3w$ is in $\text{span}\{u, v, w\}$.

(e) **True** **False** There exist vectors $u, v, w \in \mathbb{R}^3$ such that $\{u, v\}$ is linearly dependent, and $\{u, v, w\}$ is linearly independent.

4. (10 points) Find all values of a such that span$\{u, v, w\}$ = span$\{u, v\}$, where

$$u = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}, \quad w = \begin{bmatrix} 2 \\ a \\ 5 \end{bmatrix}.$$

For those values of a, what is the span$\{u, v, w\}$: a point, a line, a plane, or all of $\mathbb{R}^3$?