Independent Study – Learning Music Structure by Laplacian Formula

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Abstract. There are many approaches to analyzing music structure by features extracted from dimension of time series. With contruction of similarity matrix, repeated pattern can be captured which is the building block for large-scale structure. This is the work based on the Laplacian Matrix, which is essential start point of spectral clustering. We introduce variables that are trainable to reduce the cost of Laplacian Matrix from true lable, and run this method on wide variable of music recordings. Finally, we demonstrate using these trained variable for performing proper music segmentation.

1. Laplacian formula.

Give the definition of normalized Laplacian formula as followed:

$$L := I - D^{\frac{1}{2}} W D^{\frac{1}{2}} \tag{1.1}$$

D is degree matrix defined as the diagnal matrix with degrees $d_1, d_2, \dots, d_n$, which d_i is defined as followed:

$$d_i = \sum_{j \neq i}^{n} w_{ij} \tag{1.2}$$

After multiplication and the result of equation 1.1 can be rewrite as:

$$L := I - D^{\frac{1}{2}} W D^{\frac{1}{2}} = \begin{pmatrix} 1 & \frac{-w_{12}}{\sqrt{d_1 d_2}} & \cdots & \frac{-w_{1n}}{\sqrt{d_1 d_n}} \\ \frac{-w_{21}}{\sqrt{d_2 d_1}} & 1 & \cdots & \frac{-w_{2n}}{\sqrt{d_2 d_n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-w_{n1}}{\sqrt{d_n d_1}} & \frac{-w_{n2}}{\sqrt{d_n d_2}} & \cdots & 1 \end{pmatrix} = \begin{cases} \frac{-w_{i,j}}{\sqrt{d_i d_j}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases} \tag{1.3}$$

for w_{ij}, where $i = j$:

$$\frac{\partial L}{\partial w_{i,j}} = \begin{cases} 0 & \text{, if } i = j \\ \frac{-1}{\sqrt{d_i d_j}} + \frac{w_{i,j}(d_i + d_j)}{2(d_i d_i)^{\frac{3}{2}}} & \text{, for position } (i,j), (j,i) \\ \frac{w_{l,k}}{2\sqrt{d_k} d_l^{\frac{3}{2}}} & \text{, for all position } (l,k), (k,l) \text{, where } k \neq i \& j \text{ and } l = i \| j \\ 0 & \text{for any other position} \end{cases} \tag{1.4}$$

REFERENCES

[1] A Tutorial on Spectral Clustering

2. Appendix.

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2.1. Differential of Laplacian Matrix. If we would like to take derivative of Laplacian w.r.t variable $w_{i,j}$ in the symmetric matrix W. Basically, except for $L_{i,j}$ the components of $L_{i,k}$, $L_{k,i}$, $L_{k,j}$, $L_{j,k}$ will need to consider.
For position $L_{i,j}$:

$$\begin{aligned} L_{i,j} = L_{j,i} &= \frac{-w_{i,j}}{\sqrt{d_i d_j}} \\ \frac{\partial L_{i,j}}{\partial w_{i,j}} = \frac{\partial L_{j,i}}{\partial w_{i,j}} &= \frac{-1}{\sqrt{d_i d_j}} + \frac{w_{i,j}}{2(d_i d_i)^{\frac{3}{2}}}\left(\frac{\partial d_i}{\partial w_{i,j}} d_j + d_i \frac{\partial d_j}{\partial w_{i,j}}\right) \\ &= \frac{-1}{\sqrt{d_i d_j}} + \frac{w_{i,j}(d_i + d_j)}{2(d_i d_i)^{\frac{3}{2}}} \end{aligned} \tag{2.1}$$

For position $L_{l,k}$, $L_{k,l}$, where $k \neq i \& j$ and $l = i \| j$:

$$\begin{aligned} L_{k,l} = L_{l,k} &= \frac{-w_{k,l}}{\sqrt{d_k d_l}} \\ \frac{\partial L_{k,l}}{\partial w_{i,j}} = \frac{\partial L_{k,l}}{\partial w_{i,j}} &= \frac{w_{k,l}}{2\sqrt{d_k} d_l^{\frac{3}{2}}} \end{aligned} \tag{2.2}$$