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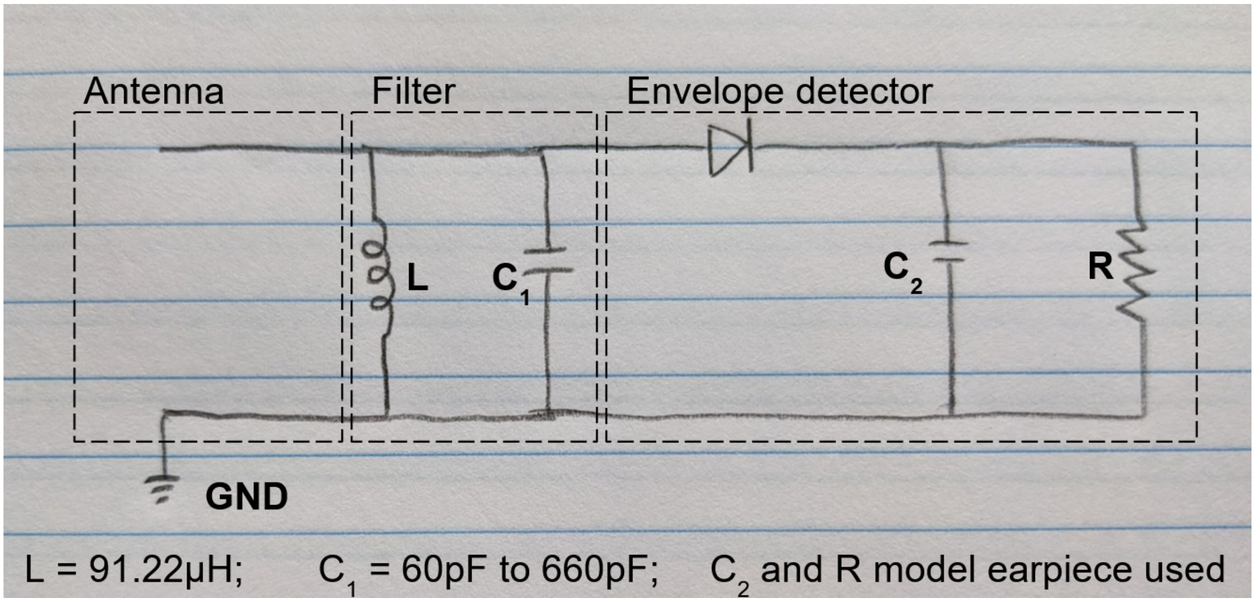
Lab 4: Designing and building an AM radio receiver

The AM radio receiver was optimized to receive a station broadcasting at a frequency of 1.4 MHz, or 1400 kHz. This is WHMP, a station providing news and talk programming.

Figure 1. AM radio receiver circuit diagram



The resonant frequency of an LC circuit is the frequency for which the “gain” of the circuit (the ratio of the output signal to the input signal) is at its highest. In other words, it’s the frequency for which the amplitude of the circuit’s output is greatest. In designing the receiver circuit, the resonant frequency should equal the broadcast frequency of the desired radio signal. This will create the strongest possible signal to go into the envelope detector. In this case, the target resonant frequency is 1400 kHz, the broadcast frequency of a local radio station.

The resonant frequency of the bandpass filter is the frequency for which the reactance of the inductor is equal to the reactance of the capacitor. From this relation and known equations for the reactance of the inductor Z_L and the reactance of the capacitor Z_C, an equation for resonant frequency in terms of inductance and capacitance can be derived.

Figure 2. Derivation of the relation between resonant frequency, inductance, and capacitance

$$Z_L + Z_C = 0$$
$$j\omega L + 1/j\omega C = 0$$
$$j\omega L = -\frac{1}{j\omega C}$$
$$-j^2\omega^2 LC = 1 \longrightarrow 1 = \omega^2 LC$$
$$\omega^2 = 1/LC \longrightarrow \omega = \frac{1}{\sqrt{LC}}$$
$$f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

There are many inductances that could produce the desired resonant frequency in combination with the given capacitance range. However, the optimal inductance is one that will produce the desired resonant frequency centered in a range of possible frequencies. This will help ensure that the receiver can be correctly tuned to the resonant frequency when it is actually constructed with non-ideal components. Python code was written to determine this optimal inductance.

Figure 3. Python code defining functions that can be used to find an optimal inductance.

```python
# import necessary libraries
import matplotlib.pyplot as plt
import numpy as np
```

```python
def calc_L(c_val, freq_val):
    '''
    Calculate an inductance from a capacitance and a resonant frequency.
    '''
    l_denom = 4 * (np.pi**2) * (freq_val**2) * c_val
    l_val = 1 / l_denom
    return l_val
```

```python
def calc_C(l_val, freq_val):
    '''
    Calculate a capacitance from an inductance and a resonant frequency.
    '''
    c_denom = 4 * (np.pi**2) * (freq_val**2) * l_val
    c_val = 1 / c_denom
    return c_val
```

```python
def calc_f(c_val, l_val):
    '''
    Calculate a resonant frequency from a capacitance and an inductance
    '''
    f_val = (1 / (2*np.pi)) * (1 / np.sqrt(c_val * l_val))
    return f_val
```

```python
def calc_frange_midpoint(l_val, c_upper, c_lower):
    '''
    Calculate the midpoint of a range of frequencies determined by a range of capacitance values.
    '''
    f_lower = calc_f(c_upper, l_val)
    f_upper = calc_f(c_lower, l_val)
    midpoint = f_lower + 0.5*(f_upper - f_lower)
    return midpoint
```

```python
def optimize(f_val, c_upper, c_lower, samples):
    '''
    Determine an inductance value that will produce the desired resonant frequency as close as possible
    to the midpoint of a range of frequencies determined by a range of capacitance values.
    '''
    # calculate range of inductances that will contain resonant frequency
    l_min = calc_L(c_upper, f_val)
    l_max = calc_L(c_lower, f_val)

    # create an array of possible inductance values
    # increasing `samples` increases the resolution
    l_values = np.linspace(l_min, l_max, samples)

    n = len(l_values)

    mid_values = np.zeros(n) # store midpoints of frequency ranges
    dist_values = np.zeros(n) # store differences between midpoints and resonant frequency

    # for each possible inductance value,
    # find the midpoint of the associated frequency range a
    # and the difference between midpoint and resonant frequency
    for i in range(n):
        current_l = l_values[i]
        mid_values[i] = calc_frange_midpoint(current_l, c_upper, c_lower)
        dist_values[i] = np.abs(mid_values[i] - f_val)

    # find the index of the inductance whose midpoint frequency is closest to the resonant frequency
    l_index = np.argmin(dist_values)

    # inductance to produce resonant frequency centered in frequency range
    optimal_l = l_values[l_index]
    return optimal_l
```

After defining all the functions, I used the known range of capacitance values to calculate an optimal inductance value for the desired resonant frequency, and checked that the resulting inductance produced a range of frequencies approximately centered around the desired resonant frequency.

Figure 4. Python code calculating the optimal inductance and resulting frequency range.

```python
# constants
f = 1.4e6
c_max = 660e-12
c_min = 60e-12
H_to_mH = 1e6
```

```python
optimal_l = optimize(f, c_max, c_min, 100000)
optimal_l_mH = optimal_l * H_to_mH
print('Optimal L =', np.round(optimal_l_mH,2), 'microH')
```

```
Optimal L = 91.22 microH
```

```python
f_min = calc_f(c_max, optimal_l)
f_max = calc_f(c_min, optimal_l)
print('minimum frequency =', f_min, 'Hz')
print('desired frequency =', f, 'Hz')
print('maximum frequency =', f_max, 'Hz')
```

```
minimum frequency = 648653.25724257 Hz
desired frequency = 1400000.0 Hz
maximum frequency = 2151339.473315486 Hz
```

The calculated, optimal inductance to produce a resonant frequency of 1400kHz is about 91μH. With capacitance varying from 60pF to 660pF, the frequencies the receiver can pick up will range from about 650kHz to about 2,200kHz. The next step is to calculate the number of inductor coils required to create an inductor with this inductance. The inductance can be written as a function of the number of coils N, the radius of the inductor A, and the turns per inch of the wire used *l*.

Figure 5. Equations used to calculate the number of inductor coils.

$$L_\mu = \frac{(NA)^2}{9A + 10l} \qquad N = 30l$$

This relation can be rewritten as a quadratic equation, where the roots of the equation are the number of coils needed to achieve the desired inductance with the given radius and wire specifications. Python code can be written to solve for those roots.

Figure 6. Python code defining functions that can be used to find the number of inductor coils.

```python
def calc_turns(l_val, a_val, tpi):
    '''
    Calculate number of coils from an inductance, radius, and turns-per-inch associated with wire used.
    '''
    # solve for roots of equation - print as a check
    poly = [a_val**2, l_val*(-10/tpi), -9*a_val*l_val]
    turns = np.roots(poly)
    print('possible number of turns:', turns, '\n')

    # extract valid roots
    if len(turns) == 0:
        # if there are no roots, print error and return nothing
        print("Uh oh! Problem calculating turns - no roots!")
        return
    else:
        # check for positive roots
        pos_index = np.where(turns >= 0)
        if len(pos_index) == 0:
            # if there are no positive roots, print error and return nothing
            print("Uh oh! Problem calculating turns - no positive roots!")
            return
        elif len(pos_index) > 1:
            # if there are multiple positive roots, return them all
            print("Heads up! Multiple roots!")
            return turns
        else:
            # if there is only one positive root, return it
            return turns[pos_index]

    return turns
```

```python
def calc_l_fromturns(n_val, a_val, tpi):
    '''
    Calculate inductance from number of coils, radius, and turns-per-inch of wire used
    '''
    l_num = (n_val**2)*(a_val**2)
    l_denom = (10*(1/tpi)*n_val) + (9*a_val)
    l_val = l_num / l_denom
    return l_val
```

After defining all the functions, I used the known radius of the inductor, turns per inch of the wire used, and previously calculated optimal inductance to calculate the number of inductor coils required. I checked that the inductor as built with the calculated number of coils would have an inductance similar to the calculated optimal inductance.

Figure 7. Python code calculating the required number of inductor coils.

```
# constants
A = 1
turns_per_inch = 30
pF_to_F = 1e-12
```

```
# calculate number of turns needed to produce desired inductance
N = calc_turns(optimal_l_mH, A, turns_per_inch)
N_int = int(N[0]+0.5) # account for truncation in int conversion
print('Inductor should be wound with', N_int, 'turns')

# calculate the actual inductance from the determined number of coils
actual_l_mH = calc_l_fromturns(N_int, A, turns_per_inch)
actual_l = actual_l_mH / H_to_mH
print('to produce an inductance of', actual_l_mH, 'microH')

# check that it is approximately equal to optimal inductance
percent_diff = 100*np.abs(optimal_l - actual_l) / actual_l
print('This differs from optimal inductance by', np.round(percent_diff,2), '%')
```

```
possible number of turns: [ 47.63822715 -17.23288698]

Inductor should be wound with 48 turns
to produce an inductance of 92.16 microH
This differs from optimal inductance by 1.02 %
```

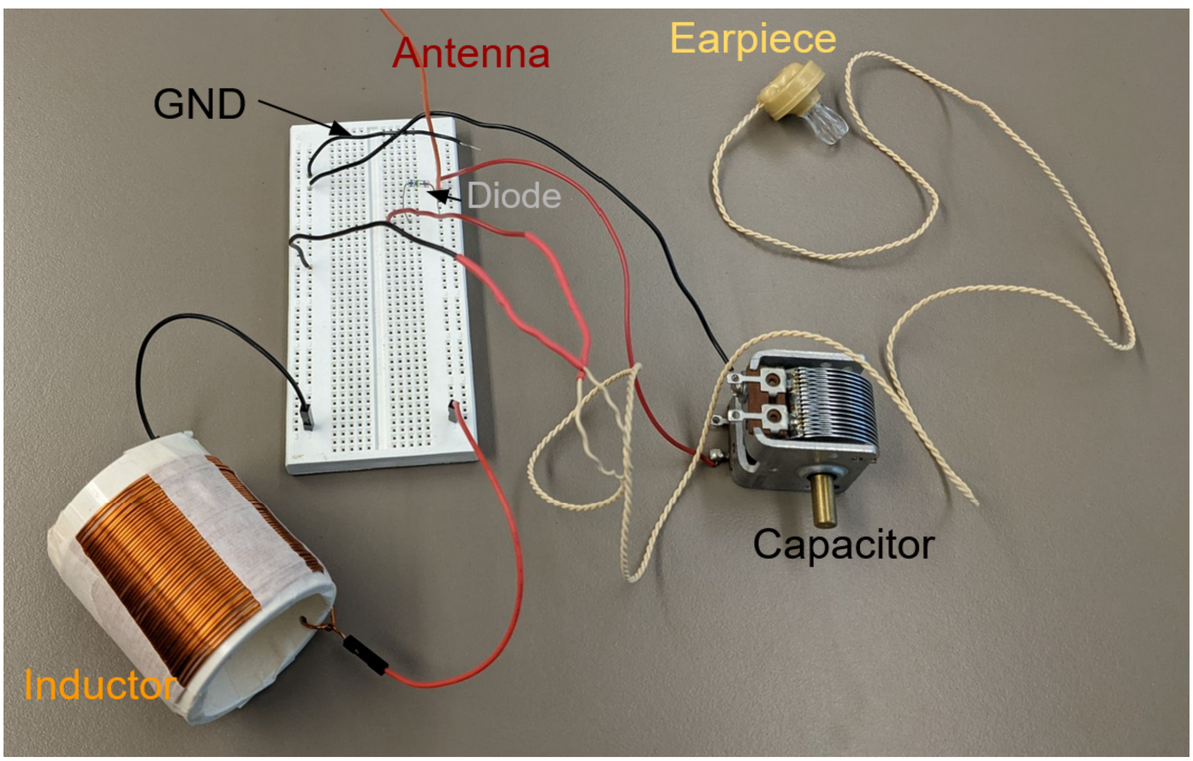
The calculated number of inductor coils to achieve optimal inductance is 48 coils. Constructing an inductor with the specified radius, wire, and number of coils will produce an actual inductance within approximately one percent of the desired optimal inductance.

Figure 8. Inductor built for use in the receiver circuit.

In the receiver circuit diagram shown in Figure 1, the antenna picks up any signals that are present in the air. These electromagnetic signals provide power to the rest of the circuit. The next part of the circuit consists of an inductor and a capacitor in parallel. The capacitor behaves as a low-pass filter, removing frequencies that are above a certain threshold by significantly reducing the amplitude of their components. The inductor behaves as a high-pass filter, removing frequencies that are below a certain threshold by significantly reducing the amplitude of their components. In combination, the inductor and capacitor function as a bandpass filter. This filter determines the range of frequencies that the receiver can detect. Ideally, the resonant frequency of the filter should match the broadcast frequency of the target input signal. This will result in the largest possible amplitude signal coming from the filter to the final part of the receiver circuit, the envelope detector.

The envelope detector demodulates the received and filtered signal to reproduce the original audio signal. It consists of a diode in series with a capacitor and resistor in parallel. In constructing the receiver circuit, the capacitor and resistor are electrically equivalent to the earpiece used. The envelope detector works by utilizing the properties of the diode. When the diode receives enough current to become forward biased, it behaves like a short circuit, so the signal that leaves the envelope detector is equal to the signal that enters the envelope detector. When the diode does not receive enough current, it becomes reverse biased and behaves like an open circuit, so the output signal decays approaching zero until the diode turns on again. If the period of the input signal is smaller than the envelope detector's time constant, then the output signal will be a decent approximation of the envelope, or the original signal modulated and broadcast by the radio station. This output is what is audible through the receiver earpiece.

Figure 9. Built receiver circuit.



The receiver did work. It picked up intermittent static and very faint voices. Due to wind, background noise from passing trucks, and low volume in general, it was difficult to distinguish actual speech. However, the receiver did pick up the station WHMP, 1400 AM. It did not receive more stations because the signal strength for other broadcast frequencies would have been much too weak--the signal amplitude of the station at resonant frequency is at its maximum, and yet was still incredibly weak and difficult to hear. Even if other stations were broadcasting within the range of frequencies calculated previously, their signals would not be strong enough to be noticed. It's also possible that there was additional resistance introduced to the circuit. Our inductor coil was connected to the circuit via somewhat long wires rather than being directly plugged into the breadboard. This introduces a resistance that was not in the original circuit, and that would have to be accounted for in the calculation of the resonant frequency. So the actual resonant frequency of the built circuit may not have matched the desired resonant frequency designed to pick up the strongest possible signal from the radio station WHMP. This could be another reason why our signal sounded weak.

Describe how AM radio works to someone without a background in engineering.

AM radio stands for Amplitude Modulation. All audio signals, such as speech or music broadcast by a radio station, take the form of a wave. Different parts of the wave have different frequencies - i.e., it takes them different amounts of time to repeat themselves. This is how complex sounds are formed--they're composed of unique frequency combinations, so every soundwave looks different. However, radio stations broadcast at a single frequency unique to each station. So how are they able to transmit tunes and conversations? This is where "amplitude modulation" comes in.

The radio station takes a soundwave with a single frequency and a constant amplitude, or signal strength, and combines it with the unique soundwave that forms music or speech. The resulting signal has an amplitude that varies with different frequencies. So the signal put out by the radio station only has one frequency, but the strength of this signal changes over time, and those changes in signal strength match the frequencies of the audio signal. Visually, it looks like the soundwave is wrapped around the broadcast frequency wave.

In order to play music, a radio receiver needs to separate the audio signal (the soundwave) from the broadcast wave. First, a receiver detects the combined signal as well as any other signals from other sources. Next, it needs to be tuned in to a particular frequency that matches the broadcast frequency of the radio station. This is what turning the dials on your car radio accomplishes! That allows the radio receiver to isolate the signal from a specific radio station. It then detects the changes in the signal strength it receives from that station. Remember, these changes in signal strength correspond to the changes in frequency associated with the soundwave being broadcast. The receiver constructs a new signal based on the detected changes in signal strength, which recreates the audio signal. And just like that, music! Or news, sports, or anything else you listen to on the radio.