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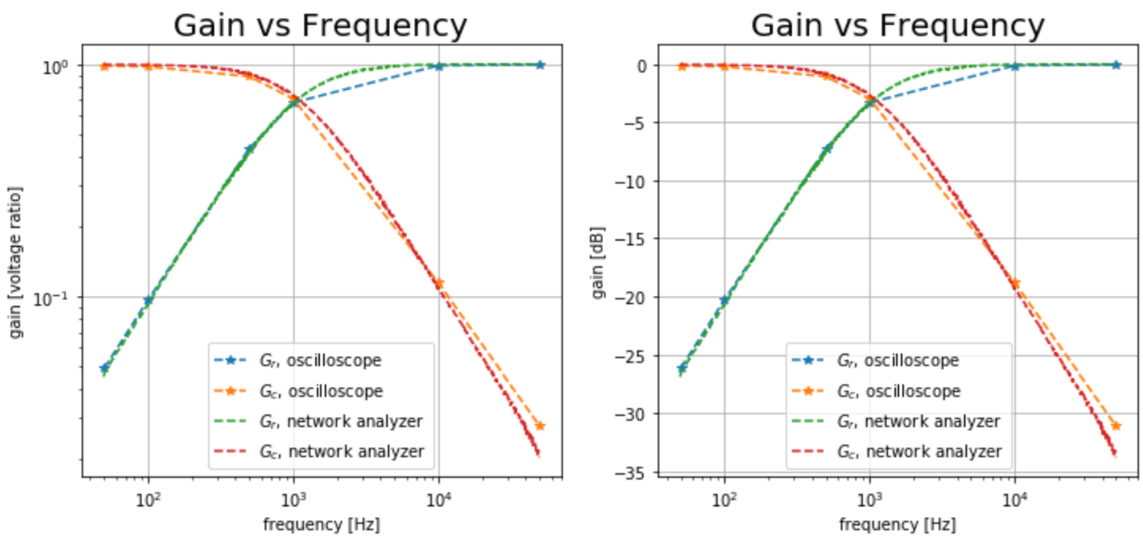
Lab 7

Figure 1. A comparison of gain vs frequency measured with the oscilloscope and the network analyzer, shown with gain in dB (right) and as a voltage ratio (left).



The measurements with the oscilloscope produced gain as a voltage ratio. The equation 'gain in dB = $20\log_{10}$(gain as voltage ratio)' was used to convert those gain measurements to dB for comparison with measurements taken with the network analyzer. The measurements with the network analyzer produced gain in dB. The equation 'gain as a voltage ratio = $10^{(\text{gain in dB} / 20)}$' was used to convert those gain measurements into voltage ratios for comparison with measurements taken with the oscilloscope.

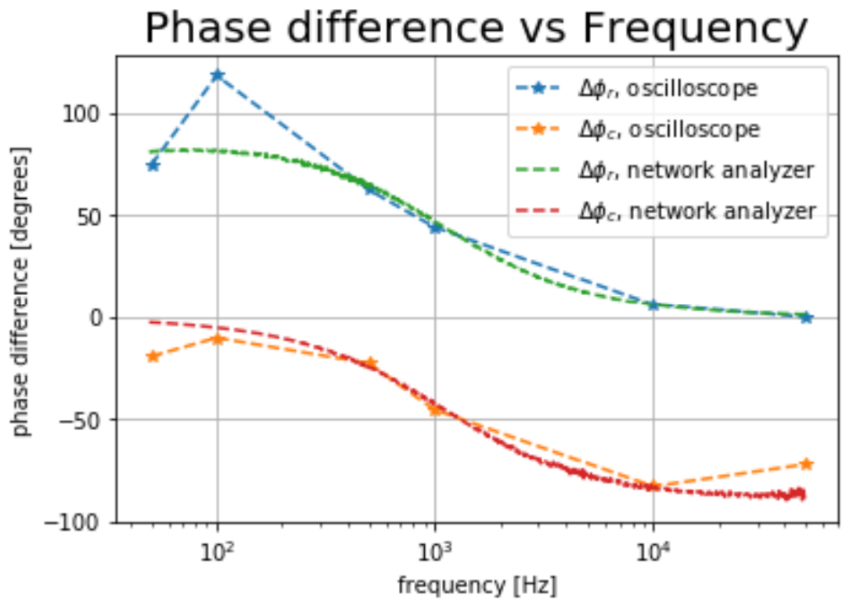
Figure 2. Python code used to create gain vs frequency plots.

```
# plotting gain vs frequency
plt.figure(figsize=(12,5))

plt.subplot(1,2,1)
plt.title('Gain vs Frequency',fontsize=20)
plt.ylabel('gain [voltage ratio]')
plt.xlabel('frequency [Hz]')
plt.loglog(freq, gain_vr, '*--', label='$G_r$, oscilloscope')
plt.loglog(freq, gain_vc, '*--', label='$G_c$, oscilloscope')
plt.loglog(res_freq, 10**(res_gain/20), '--', label='$G_r$, network analyzer')
plt.loglog(cap_freq, 10**(cap_gain/20), '--', label='$G_c$, network analyzer')
plt.legend()
plt.grid()

plt.subplot(1,2,2)
plt.title('Gain vs Frequency',fontsize=20)
plt.ylabel('gain [dB]')
plt.xlabel('frequency [Hz]')
plt.semilogx(freq, 20*np.log10(gain_vr), '*--', label='$G_r$, oscilloscope')
plt.semilogx(freq, 20*np.log10(gain_vc), '*--', label='$G_c$, oscilloscope')
plt.semilogx(res_freq, res_gain, '--', label='$G_r$, network analyzer')
plt.semilogx(cap_freq, cap_gain, '--', label='$G_c$, network analyzer')
plt.legend()
plt.grid()
```

Figure 3. A comparison of gain vs frequency measured with the oscilloscope and the network analyzer, shown in degrees.



R = 4674 Ohms ± 1%
C = 30.74 nF ± 1%
$2\pi f_c = \omega = 1 / RC$

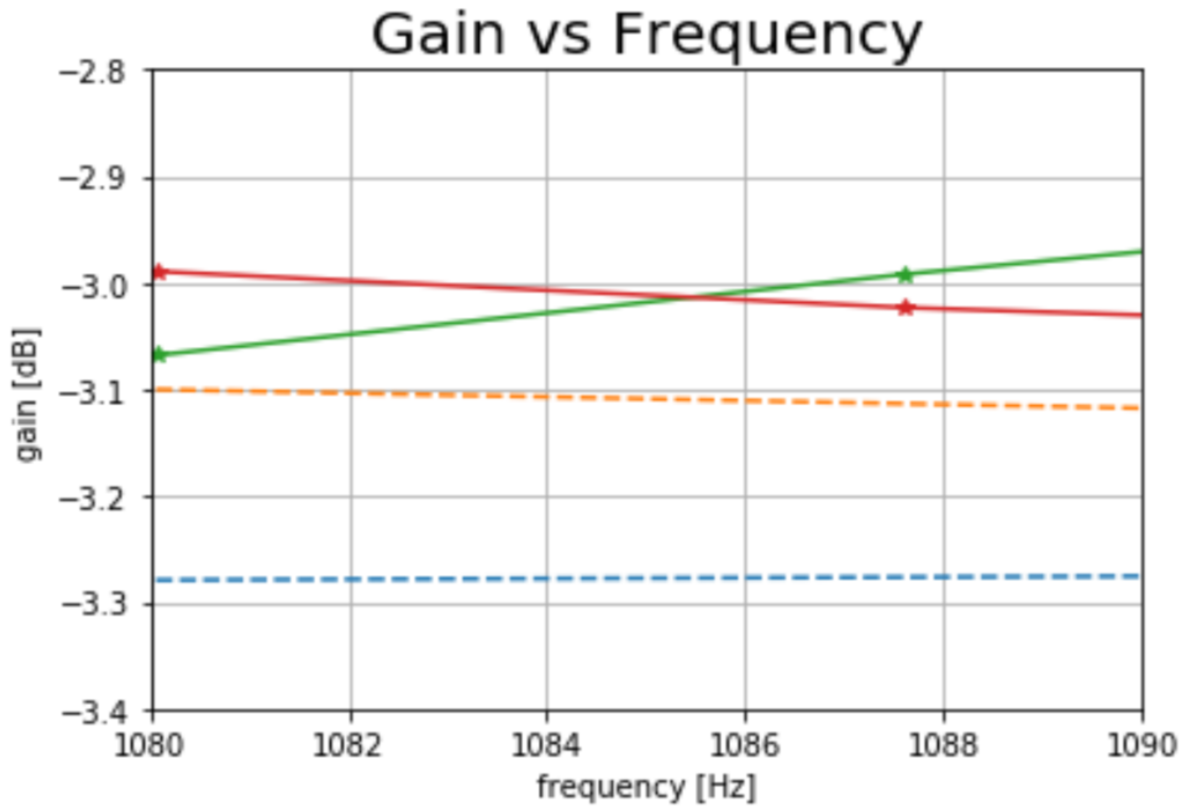
$f_c = 1 / 2\pi RC \pm \mathrm{sqrt}(1^2 + 1^2)$ = **1108 Hz ± 1.4%**

So by calculation, **f_c is between 1092 Hz and 1123 Hz.**

Examining the graphs of gain vs frequency and phase shift vs frequency can also give an estimate of the critical frequency. I adjusted the x-axis and y-axis limits on my python plots to zoom in on certain regions of the graphs. For the gain vs frequency graph, I looked at the region where the gain was between -2.8dB and -3.4dB. For the phase difference vs frequency graph, I looked at the region where the phase difference was between -60° and +60°. In determining critical frequency, I used the network analyzer data (plotted as solid lines) rather than the oscilloscope data (plotted as dotted lines) because it had far more samples.

Critical frequency is when the gain is -3dB, for both the capacitor and the resistor. It is also when the phase difference between the output and the input is 45° (or -45°).

Figure 4. Gain vs frequency plotted near the critical frequency.



By inspection, there are a few frequencies on this graph that could be considered the critical frequency. The gain of the capacitor (shown in red) is -3dB from about 1082 Hz to 1084 Hz. The gain of the resistor (shown in green) is -3dB from about 1086 Hz to 1088 Hz. The gain of the resistor is equal to the gain of the capacitor between 1085 Hz and 1086 Hz.

So based on visual inspection, f_c = 1083 Hz ± 1 Hz, 1087 Hz ± 1 Hz, or 1085.5 Hz ± 0.5 Hz.

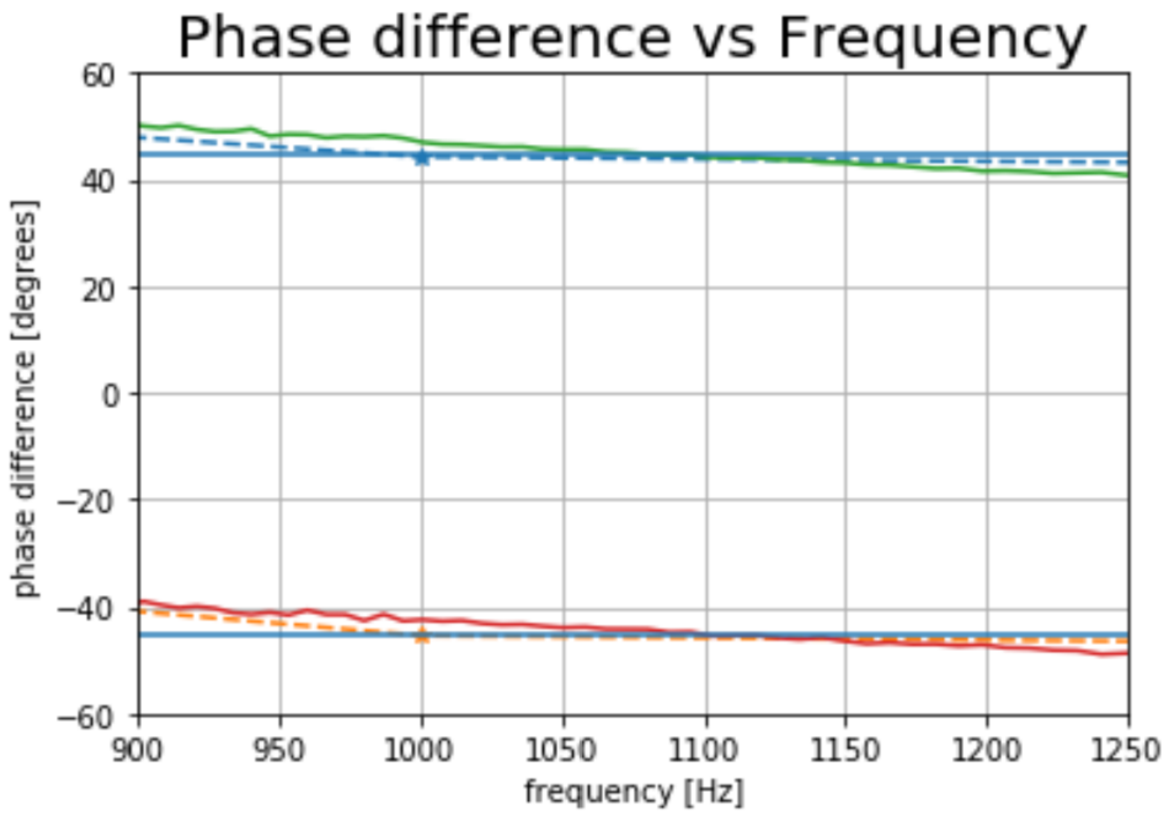
However, those uncertainties are far too small. As percentages, they are less than 0.1% of the measured values. Visual inspection is an all right method for determining the "best" values of f_c, but I would assume uncertainty of at least 1%. So based on this graph, better values of f_c would be 1083 Hz ± 1%, 1087 Hz ± 1%, or 1085 Hz ± 1%.

Taking the average of those possible f_c values:
f_c = (1083 + 1087 + 1085) / 3 Hz ± sqrt($1^2 + 1^2 + 1^2$)
f_c = 1085 Hz ± 1.7%

So based on the gain vs frequency graph, **f_c is between 1066 Hz and 1103 Hz.**

There is overlap between this range of frequencies and the calculated range of frequencies (1092 Hz to 1123 Hz), so yes, I would say that f_c is where I would expect it to be.

Figure 5. Phase difference vs frequency plotted near the critical frequency.



In this figure, I plotted two solid horizontal blue lines to indicate where the phase difference is 45° and -45°. By inspection, there are a few frequencies on this graph that could be considered the critical frequency. The phase difference of the resistor (shown in green) is 45° between 1050 Hz and 1150 Hz. The phase difference of the capacitor (shown in red) is -45° between 1100 Hz and 1150 Hz.

So based on visual inspection, f_c = 1100 Hz ± 50 Hz or 1125 Hz ± 25 Hz. I think those uncertainties (4.5% and 2.2%, respectively) are reasonable.

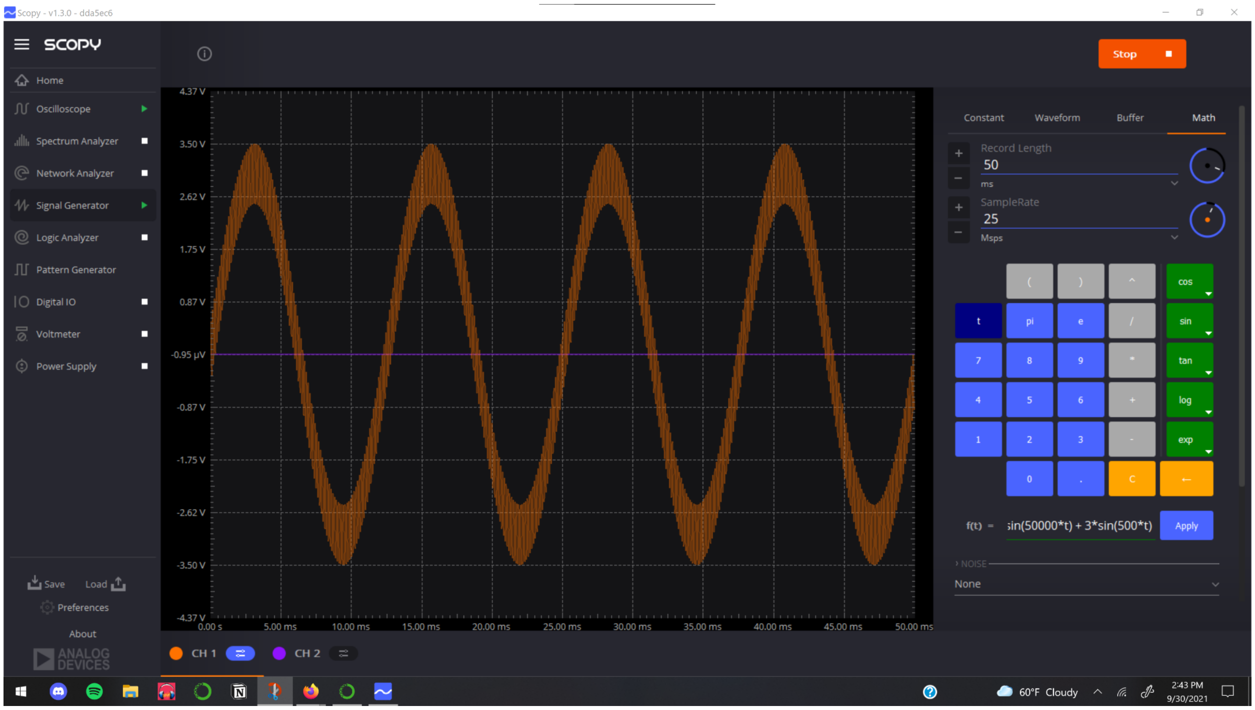
Taking the average of those possible f_c values:
f_c = (1100 + 1125) / 2 Hz ± sqrt($4.5^2 + 2.2^2$)
f_c = 1112.5 Hz ± 5%

So based on the phase difference vs frequency graph, **f_c is between 1056 Hz and 1168 Hz.**

There is some overlap between this range of frequencies and the calculated range of frequencies (1092 Hz to 1123 Hz), so I would again say that f_c is where I would expect it to be.

The fact that there is also overlap between this range of frequencies and the range of frequencies determined from the gain vs frequency graph (1066 Hz to 1103 Hz) also suggests that this is a reasonable range of values for f_c.

Figure 6. The waveform used as a source voltage for the RC circuit. V_s = 0.5sin(50000t) + 3sin(500t).



The frequency values were chosen because they are respectively much larger and much smaller than the critical frequency, so I expected to see a noticeable response from the circuit.

I calculated the expected output voltage across the capacitor from this input using the data for gain vs frequency collected from the network analyzer.

Low-pass filter when V_c = output
V_s = 0.5sin(50000t) + 3sin(500t)
V_c = G_{50kHz}0.5sin(50000t) + G_{500Hz}3sin(500t)

G_{50kHz} = -33.7429 dB = 0.02055 ratio = 2%
G_{500Hz} = -0.87214 dB = 0.90446757 ratio = 90%

V_c = 0.01sin(50000t) + 2.713sin(500t)

Based on this result, I would expect a sinusoid whose low-frequency component is of a similar amplitude to the low-frequency component of V_s, but whose high-frequency component is of a much smaller amplitude than the high-frequency component of V_s.

Figure 7. A screenshot of the Scopy oscilloscope showing V_s and V_c.

The figure above shows the expected result. In orange, the input voltage is a sinusoid with a high-frequency component at a lower amplitude and a low-frequency component at a higher amplitude. In purple, the output voltage across the capacitor is a sinusoid with the same low-frequency and high-frequency components, but the low-frequency component is at a similar amplitude as in the input signal, while the high-frequency component is at a significantly lower amplitude than in the input signal. This shows that the RC circuit with the voltage across the capacitor as the output voltage acts as a low-pass filter, because most of the voltage at a lower frequency was able to pass through, while most of the voltage at a higher frequency was not.

I repeated this process using the same source voltage V_s, but with V_r as the output. I calculated the expected output voltage across the resistor from this input using the data for gain vs frequency collected from the network analyzer.

High-pass filter when V_R = output
$V_s = V_s = 0.5\sin(50000t) + 3\sin(500t)$
$V_R = G_{50kHz}0.5\sin(50000t) + G_{500Hz}3\sin(500t)$

G_{50kHz} = 0.023134 dB = 1.00 = 100%
G_{500Hz} = -7.5539 dB = 0.419 = 42%

$V_R = 0.5\sin(50000t) + 1.25(500t)$

Based on this result, I would expect a sinusoid whose high-frequency component is of a similar amplitude to the high-frequency component of V_s, but whose low-frequency component is at a much smaller amplitude than that of V_s.

Figure 8. A screenshot of the Scopy oscilloscope showing V_s and V_r.

The figure above shows the expected result. In orange, the input voltage is a sinusoid with a high-frequency component at a lower amplitude and a low-frequency component at a higher amplitude. In purple, the output voltage across the resistor is a sinusoid with the same low-frequency and high-frequency components, but the high-frequency component is at a similar amplitude as in the input signal, while the low-frequency component is at a significantly lower amplitude than in the input signal. This shows that the RC circuit with the voltage across the resistor as the output voltage acts as a high-pass filter, because most of the voltage at a higher frequency was able to pass through, while most of the voltage at a lower frequency was not.

In the following figures, V_C is represented by the orange sinusoid, and is plotted along the x-axis of the x-y plot. V_R is represented by the purple sinusoid, and is plotted along the y-axis of the x-y plot.

Figure 9. V_R vs V_C at 1kHz.

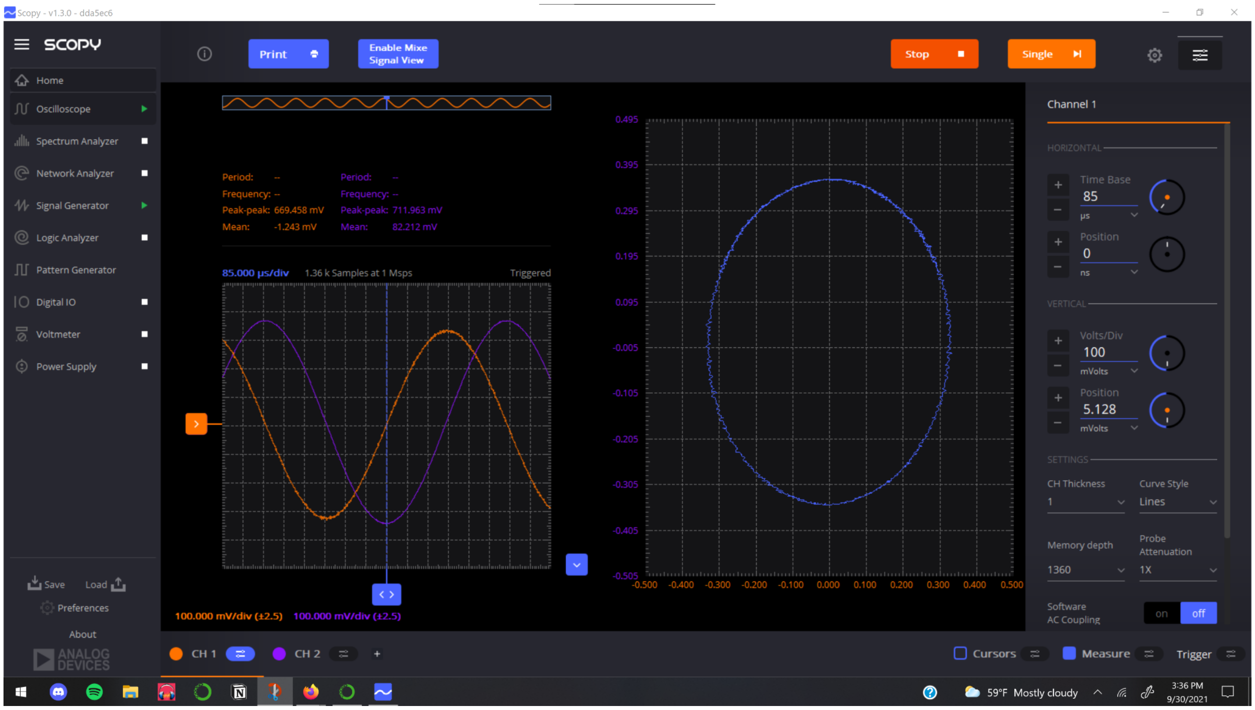


Figure 10. V_R vs V_C at 10kHz.

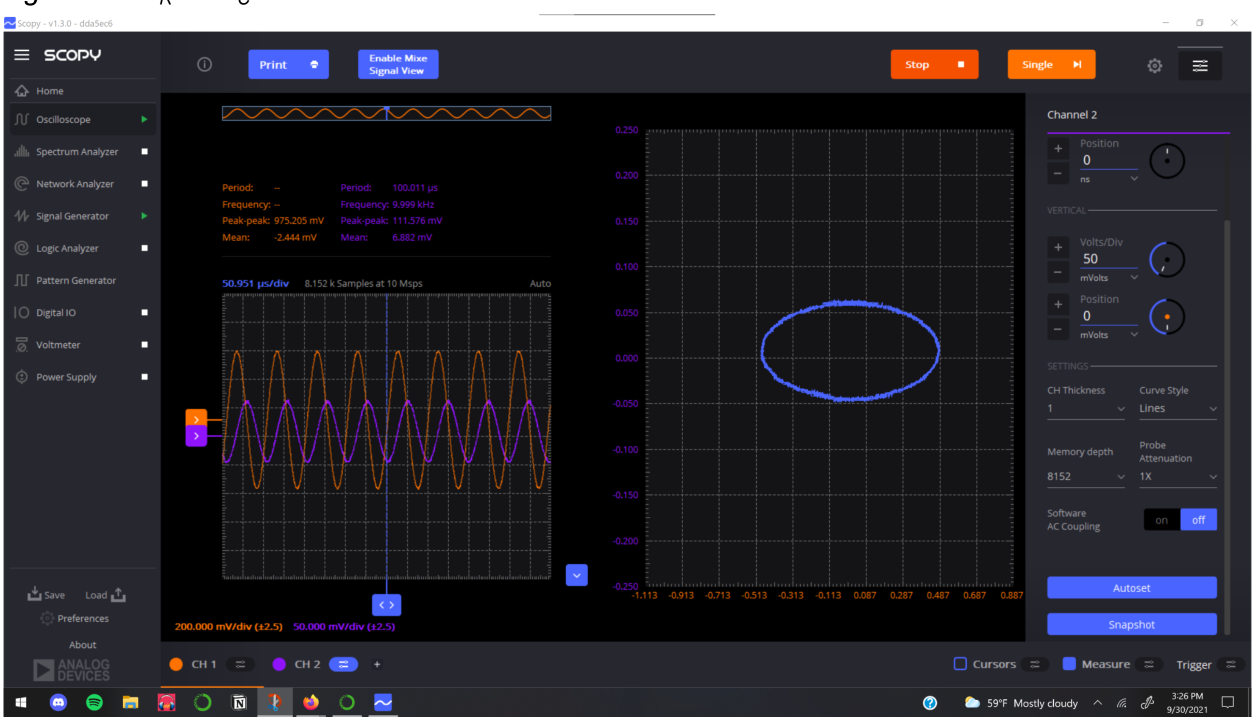
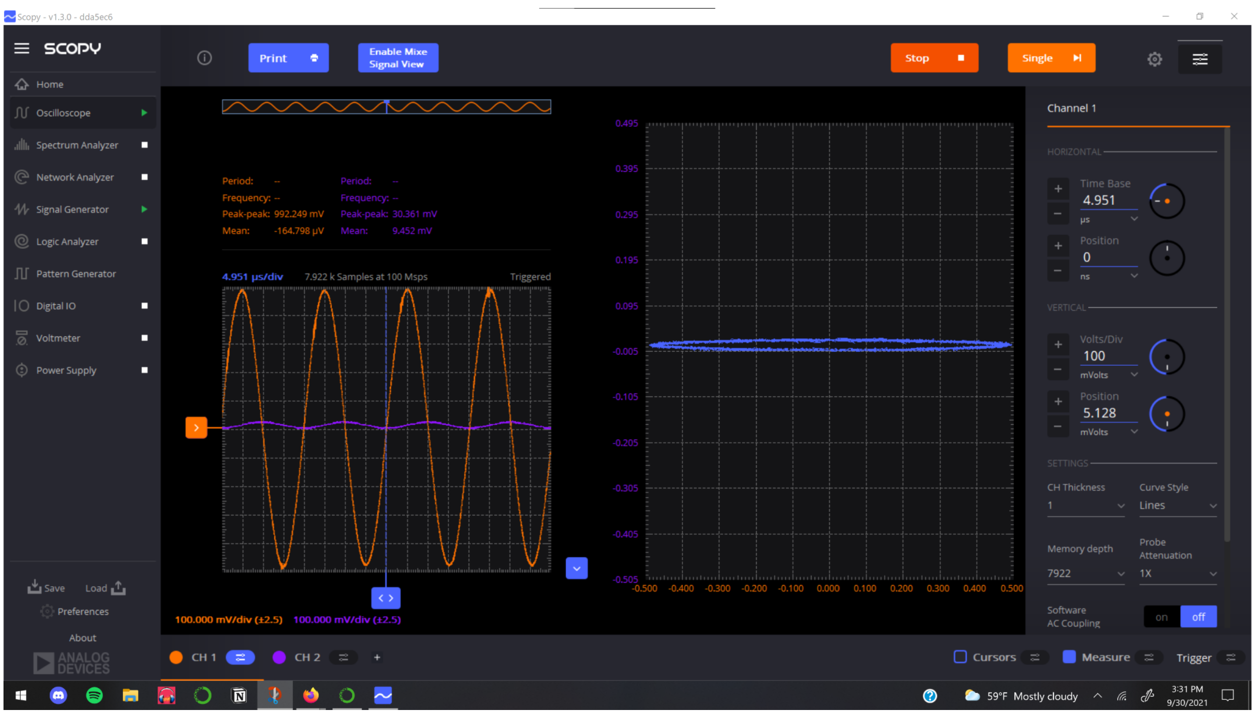


Figure 11. V_R vs V_C at 50 Hz.

Figure 12. V_R vs V_C at 50 kHz.



The plots of V_R vs V_C have the shape that they do because the voltage functions are sinusoids, and because the voltages plotted against each other have the same frequency. The amplitudes of each sinusoid influence whether the shape is longer or shorter in the x or y axis.