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Lab 8

I used the provided square wave spreadsheet as well as gain vs frequency and phase vs frequency data gathered during Lab 7 to calculate the terms of a Fourier series $V_c(t)$ modelling the output voltage across a capacitor with an input of a square wave $V_s(t)$ with amplitude 1.5V (3V peak-to-peak) and frequency 400Hz.

The spreadsheet columns utilize Excel formulas to end up with a final result. Specifically:

- Column A = N = term number in Fourier series
- Column B = fundamental frequency * N = frequency
- Column C = (2 * peak-to-peak voltage) / (N * pi) = Fourier coefficient
- Column D = measured gain in dB for corresponding frequency
- Column E = 10 ^ (gain in dB / 20) = gain ratio
- Column F = Fourier coefficient * gain ratio = amplitude at output
- Column G = measured phase in degrees for corresponding frequency
- Column H = phase in degrees * pi / 180 = phase in radians

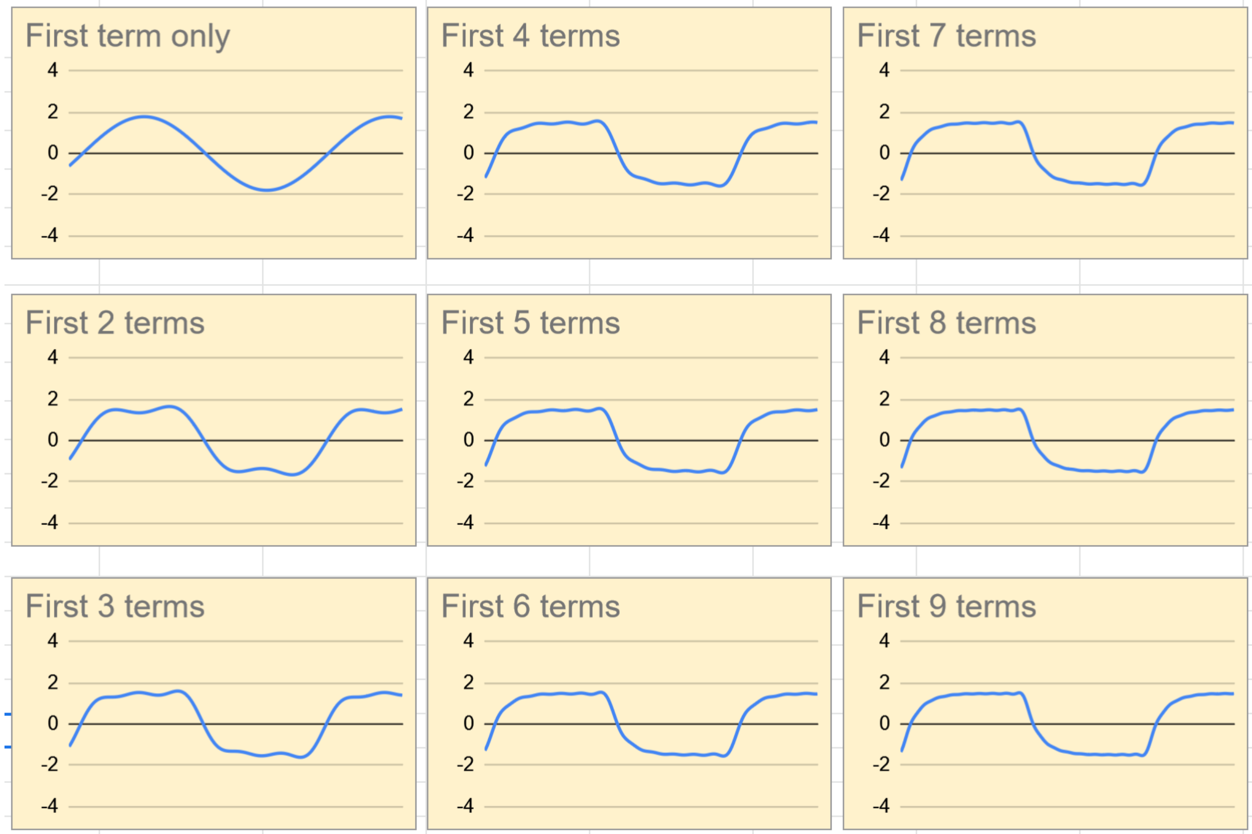
Each term in the Fourier series = column F * sin(2 * pi * column B * t + column H).

Figure 1. Spreadsheet used to determine terms of Fourier series.

	A	B	C	D	E	F	G	H
1								
2–3		f0 fundamental (Hz)	peak to peak amplitude (V)					
4		400	3					
5	N in Fourier Series	frequency	fourier coefficient	gain (measured, dB)	Gain ratio	amplitude at output	phase (degrees)	Phase (radians)
6	0	(dc offset)						
7	1	400	1.9099	-0.58062	0.9353	1.7864	-19.505	-0.3404
8	3	1200	0.6366	-3.45807	0.6716	0.4275	-47.1	-0.8221
9	5	2000	0.3820	-6.29845	0.4843	0.1850	-60.5917	-1.0575
10	7	2800	0.2728	-8.76408	0.3646	0.0995	-69.2341	-1.2084
11	9	3600	0.2122	-10.8313	0.2874	0.0610	-72.3517	-1.2628
12	11	4400	0.1736	-12.4983	0.2372	0.0412	-76.1299	-1.3287
13	13	5200	0.1469	-13.7981	0.2042	0.0300	-78.9822	-1.3785
14	15	6000	0.1273	-15.049	0.1768	0.0225	-79.9976	-1.3962
15	17	6800	0.1123	-15.8297	0.1616	0.0182	-81.0183	-1.4140

$V_c(t)$ = 1.78636572645863*sin(2*pi*400*t+-0.340426470601494)
+0.427539872086486*sin(2*pi*1200*t+-0.822050077689329)
+0.18497322705395*sin(2*pi*2000*t+-1.05752466438065)
+0.0994716536080772*sin(2*pi*2800*t+-1.20836299965501)
+0.0609809054119758*sin(2*pi*3600*t+-1.26277538441518)
+0.0411806969598718*sin(2*pi*4400*t+-1.32871741421403)
+0.0300021947199273*sin(2*pi*5200*t+-1.37849944046867)
+0.0225143865710087*sin(2*pi*6000*t+-1.39622151369342)
+0.0181579767764162*sin(2*pi*6800*t+-1.41403608936852)

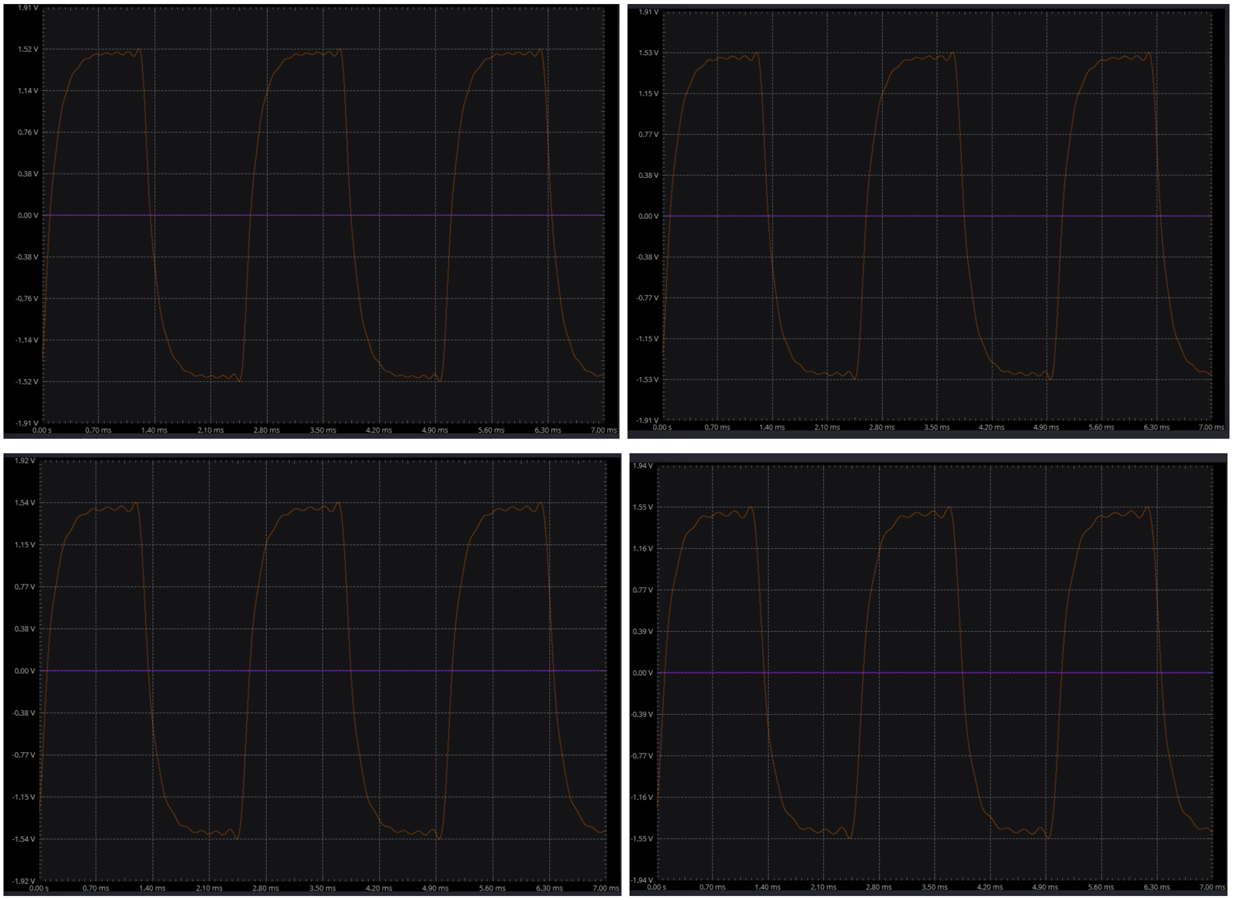
Figure 2. Plots of the resulting Fourier series with varying numbers of terms included.



Based on the above plots, about 6 terms are necessary. 4-5 terms would also be reasonable, but the amplitude of the higher frequency “bumps” in what should be the flat parts of the wave is higher. Of course, more terms results in better quality, but there is little noticeable difference between 6 terms, 7 terms, 8 terms, and 9 terms.

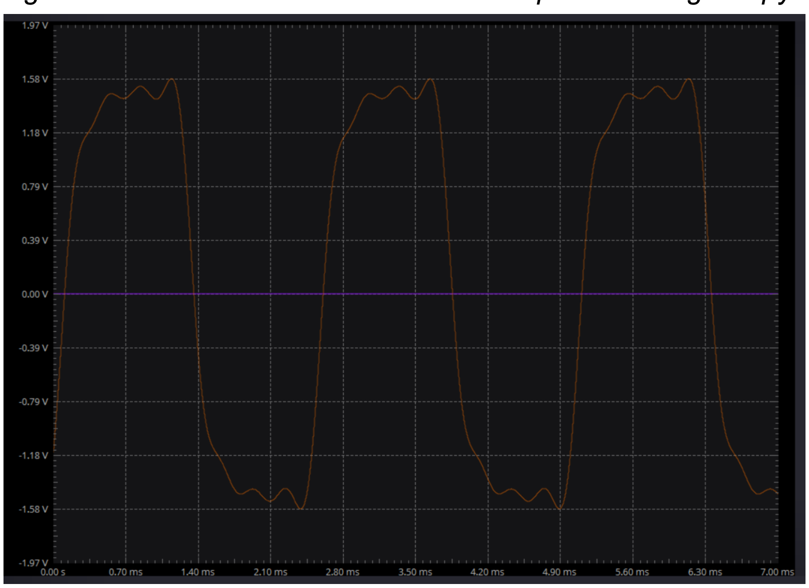
Plotting the Fourier series with different numbers of terms using the math function of Scopy’s wave generator also supports this conclusion.

Figure 3. The Fourier series plotted using Scopy's wave generator.
Terms included from top left clockwise to bottom left: 9 terms, 8 terms, 7 terms, 6 terms



Plots with 6 terms, 7 terms, 8 terms, and 9 terms are all of similar quality.

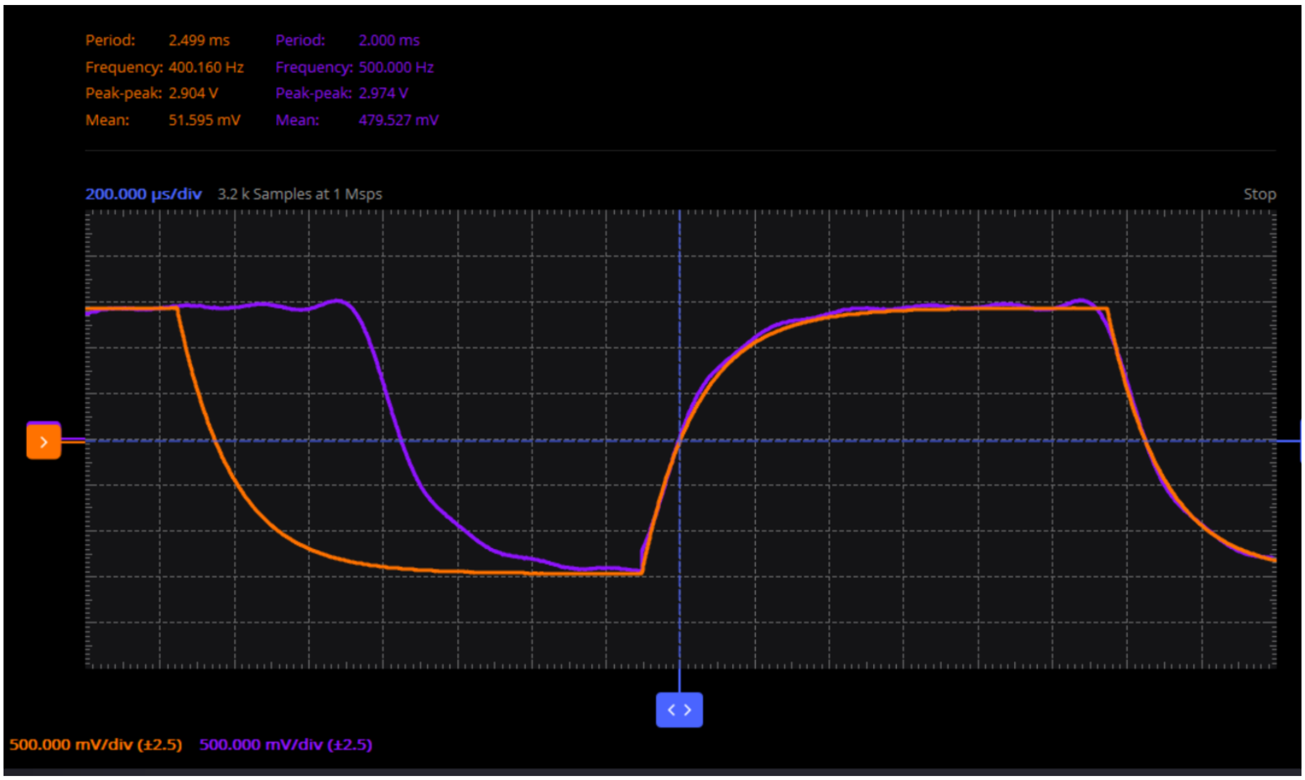
Figure 4. Five terms of the Fourier series plotted using Scopy's wave generator.



A plot with 5 terms is noticeably worse quality than those with at least 6 terms.

To compare the actual response of the capacitor to the input square wave with the Fourier series, wave generator channel 1 was used to supply the square wave $V_s(t)$ and connected to the circuit. Wave generator channel 2 was used to supply the Fourier series and was separate from the main circuit. Scope channel 1 was used to measure the voltage across the capacitor (shown in orange in the following figures). Scope channel 2 was used to measure the voltage supplied according to the Fourier series (shown in purple in the following figures).

Figure 5. Closeup comparison of Fourier series and actual response. Nine terms included.



My ADALM2000 unit seemed to have trouble measuring or possibly supplying the correct frequencies, resulting in waves that were slightly out of phase with each other. Both should have a frequency of 400 Hz. However, in the figure above, the frequency of the voltage across the capacitor was 400 Hz while the frequency of the voltage supplied according to the Fourier series was 500 Hz. The spreadsheet calculations were correct, and the waves were input correctly in the Wave Generator section of Scopy. It is likely an equipment issue.

Other than the frequency issue, the Fourier series does a good job of approximating the actual capacitor response. It is less accurate at the “corner” where voltage drops - the actual response is a lot sharper than the Fourier series. Additionally, there are higher-frequency “bumps” present along the flat top that are not present in the actual capacitor response. However, with higher numbers of terms in the Fourier series, the amplitude of those bumps decreases.

When the frequencies line up, the rise and fall of the Fourier series matches fairly closely with the actual capacitor response, as does the width of each wave.

Figure 6. Comparison of Fourier series and actual response. Nine terms included. 500mV/div

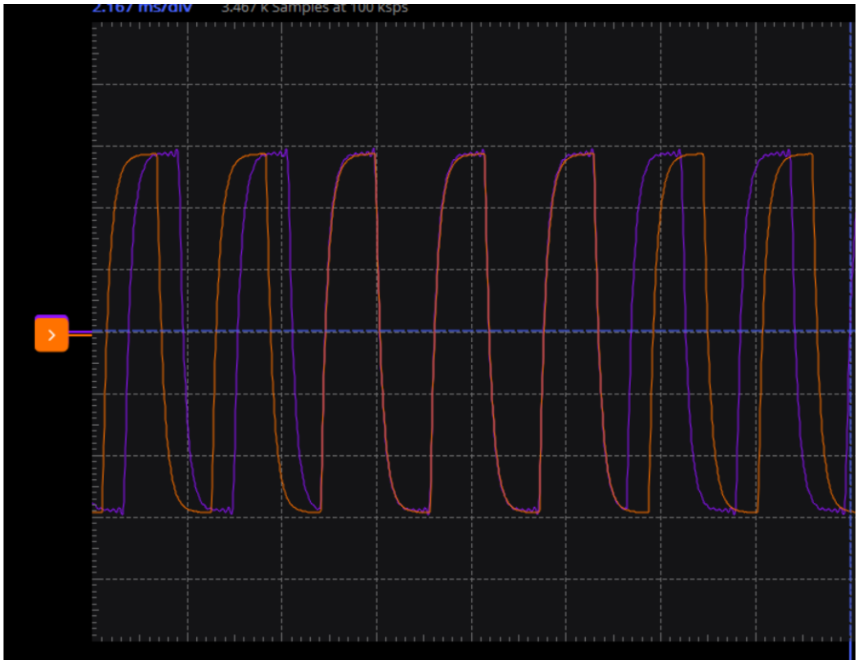
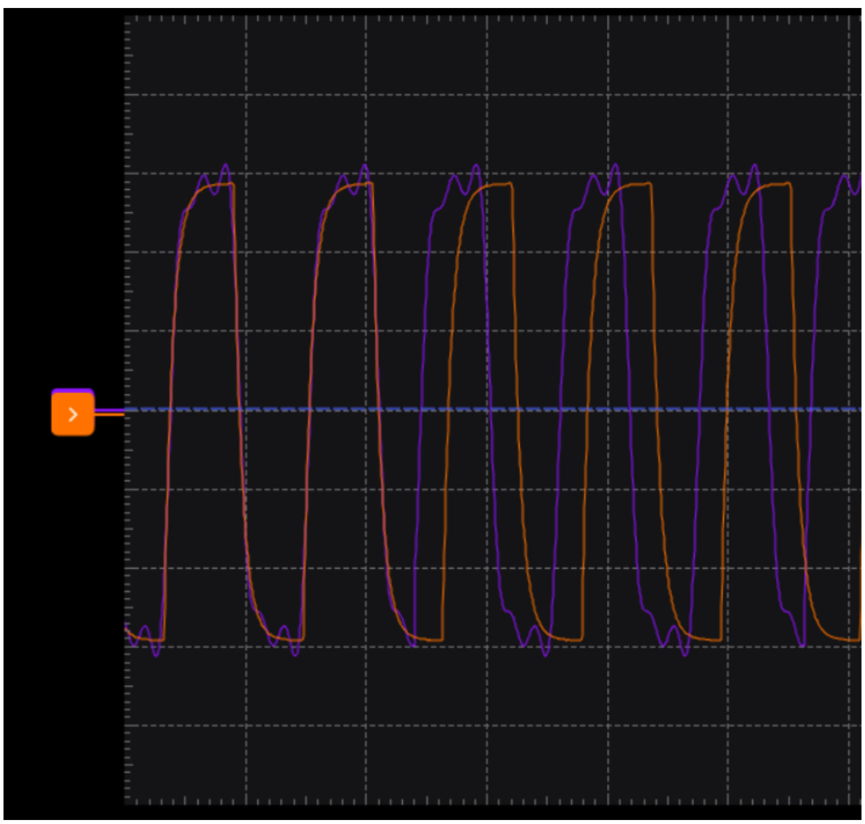


Figure 7. Comparison of Fourier series and actual response. Three terms included. 500mV/div



As the number of terms decreases, the “bumps” present in the Fourier series that are not present in the actual capacitor response become more noticeable.