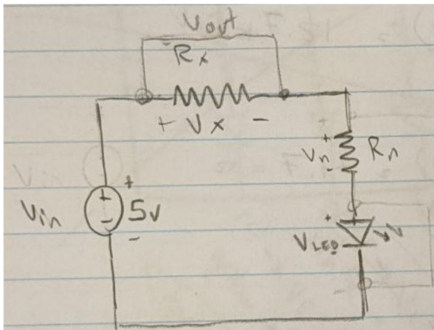
Julia Frothingham

Lab 4 - building, refining and reporting your results

The circuit is designed so that the brightness of an LED will change in response to changes in temperature. It is also designed to produce a calibration curve showing the relationship between voltage across a thermistor and temperature.

In order to achieve this, the circuit utilizes a voltage divider composed of the thermistor with a variable resistance R_x and a ⅛W resistor with a constant resistance R_n.

Figure 1: Circuit diagram, transfer function, and calculations to determine optimal R_n value



R_x max $= 23\ k\Omega$

R_x min $= 2\ k\Omega$

$V_{LED} \approx 1.8\ V$

So $V_{in} = 3.2\ V$

Transfer Function:

$$V_{out} = \frac{V_{in} R_x}{R_x + R_n}$$

$$V_{out,max} = \frac{V_{in}(R_{x,max})}{R_{x,max} + R_n} \qquad V_{out,min} = \frac{V_{in}(R_{x,min})}{R_{x,min} + R_n}$$

$$V_{out,max} - V_{out,min} = \frac{V_{in}(R_{x,max})}{R_{x,max} + R_n} - \frac{V_{in}(R_{x,min})}{R_{x,min} + R_n}$$

$$\text{difference} = \frac{V_{in} R_{xmax}(R_{xmin} + R_n) - V_{in} R_{xmin}(R_{xmax} + R_n)}{(R_{xmax} + R_n)(R_{xmin} + R_n)}$$

$$\text{difference} = \frac{\cancel{V_{in} R_{xmax} R_{xmin}} + V_{in} R_{xmax} R_n - \cancel{V_{in} R_{xmin} R_{xmax}} - V_{in} R_{xmin} R_n}{R_{xmax} R_{xmin} + R_{xmax} R_n + R_n R_{xmin} + R_n^2}$$

$$\text{difference} = \frac{V_{in} R_n (R_{xmax} - R_{xmin})}{R_{xmax} R_{xmin} + R_{xmax} R_n + R_n R_{xmin} + R_n^2}$$

$V_{in} = 3.2V$

$R_{xmax} = 23000\ \Omega$

$R_{xmin} = 2000\ \Omega$

$R_{xmax} - R_{xmin} = 21000$

$R_{xmax} R_{xmin} = 46{,}000000$

$R_n = 6782\ \Omega = \boxed{6.8\ k\Omega}$

The final value of R_n was chosen as 6.8 kΩ because it allowed for the greatest possible range of measurements of the voltage across the thermistor. In order to find this optimal R_n, value, the transfer function and measurements taken of the thermistor's minimum and maximum resistances were used to write an expression for the total range of voltages that could be measured as a function of R_n. Plotting this function showed that the function maximum, the point where the slope of the function is zero, was somewhere between 4kΩ and 8kΩ. Wolfram Alpha was used to produce a more exact value for the R_n value corresponding to the function maximum. It was found that R_n at the function maximum was approximately 6.8kΩ, which matched what would be expected based on the graph. A 6.8kΩ resistor was used in the completed circuit because it was available. However, the graph shows that if it was not available, resistors with similar values would work almost as well.

In the circuit design submitted previously, R_n was 15kΩ. This value was chosen somewhat arbitrarily and changed to avoid limiting the possible range of measurable voltages.

Figure 2. Plot of the difference between $V_{out,max}$ and $V_{out,min}$ versus resistance R_n

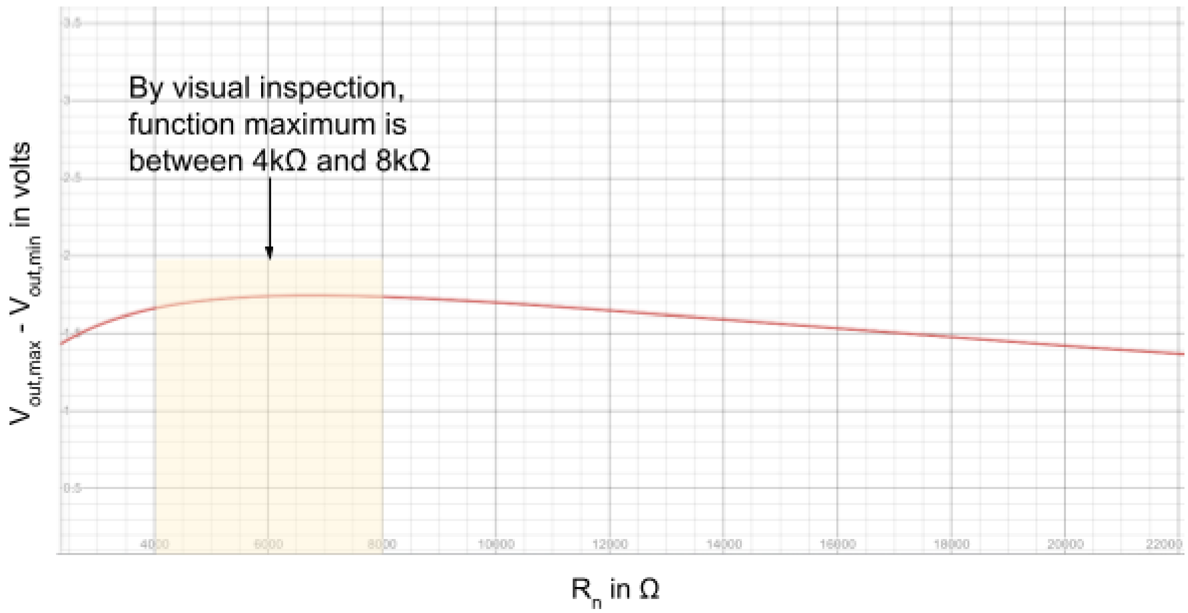


Figure 3. Wolfram Alpha calculations solving for the desired R_n value.

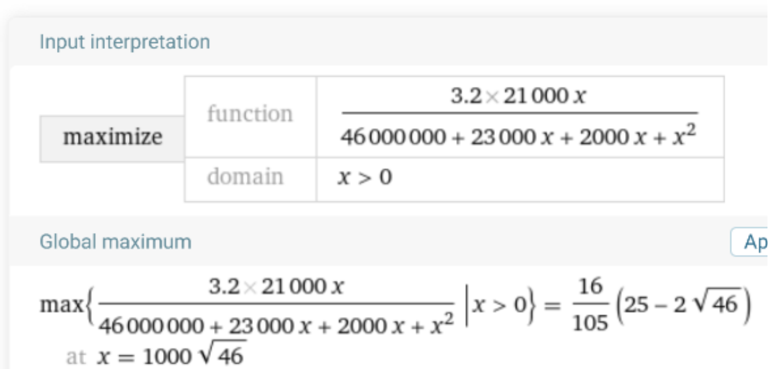
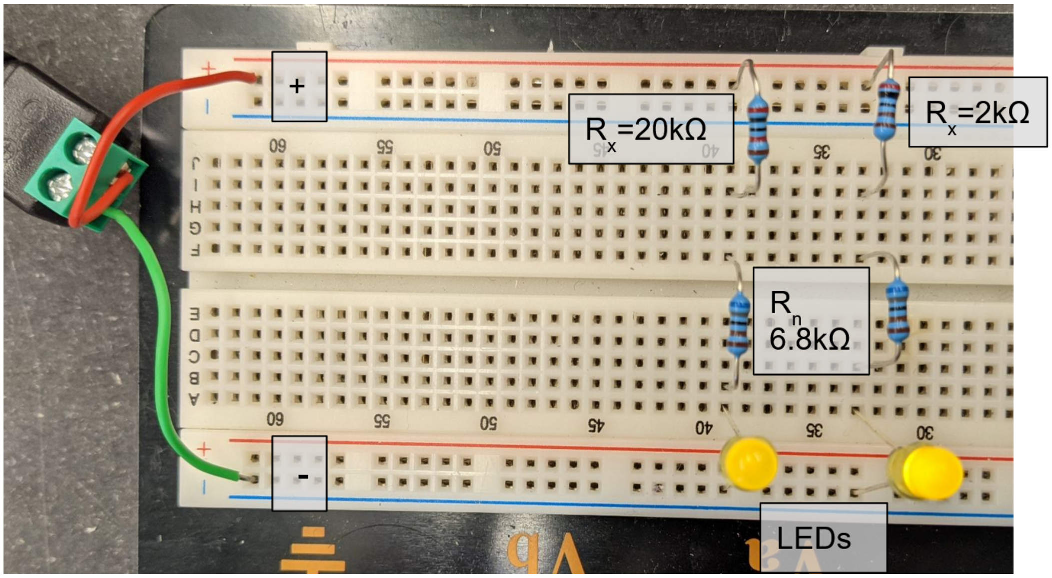


Figure 4. Circuit built to test calculated value of R_n.



As an additional check, the circuit was built with two branches in parallel, each with an LED, a 6.8kΩ resistor, and a resistor whose value approximated either $R_{x,min}$ or $R_{x,max}$ in series. In this test circuit, each LED lit up, and there was a noticeable difference between the brightness of the LED corresponding to a cold measurement and the brightness of the LED corresponding to a hot measurement. The resistance of the thermistor increases as the temperature decreases, so a dimmer LED indicates a colder temperature and a brighter LED indicates a hotter temperature.

In the actual circuit, the LED brightness is nice, but not useful for quantitative measurements. So a DMM was attached in parallel with the thermistor to measure V_{out}. In order to vary the temperature of the thermistor and take readings of the voltage at a range of temperatures, the thermistor was submerged in an insulated water bath whose temperature was varied between measurements by adding near-boiling water or ice cubes. To protect the thermistor, it was held in the cut-off finger of a latex glove and the glove was placed in the water bath through a hole in the bath's lid such that no water came in contact with the thermistor.

It is assumed that once the voltage readings displayed on the DMM reached a steady value, the temperature of the thermistor had converged with the temperature of the water bath. At that point, the thermistor and protective glove were removed from the water bath and an infrared thermometer was used to measure the water temperature.

As shown in the test circuit, the LED does produce a different brightness based on the temperature and the corresponding R_x value of the thermistor. However, in practice, these changes in brightness were quite gradual and subtle, making them difficult to notice.

Figure 5. Experimental setup to measure voltage across thermistor with varying temperature.

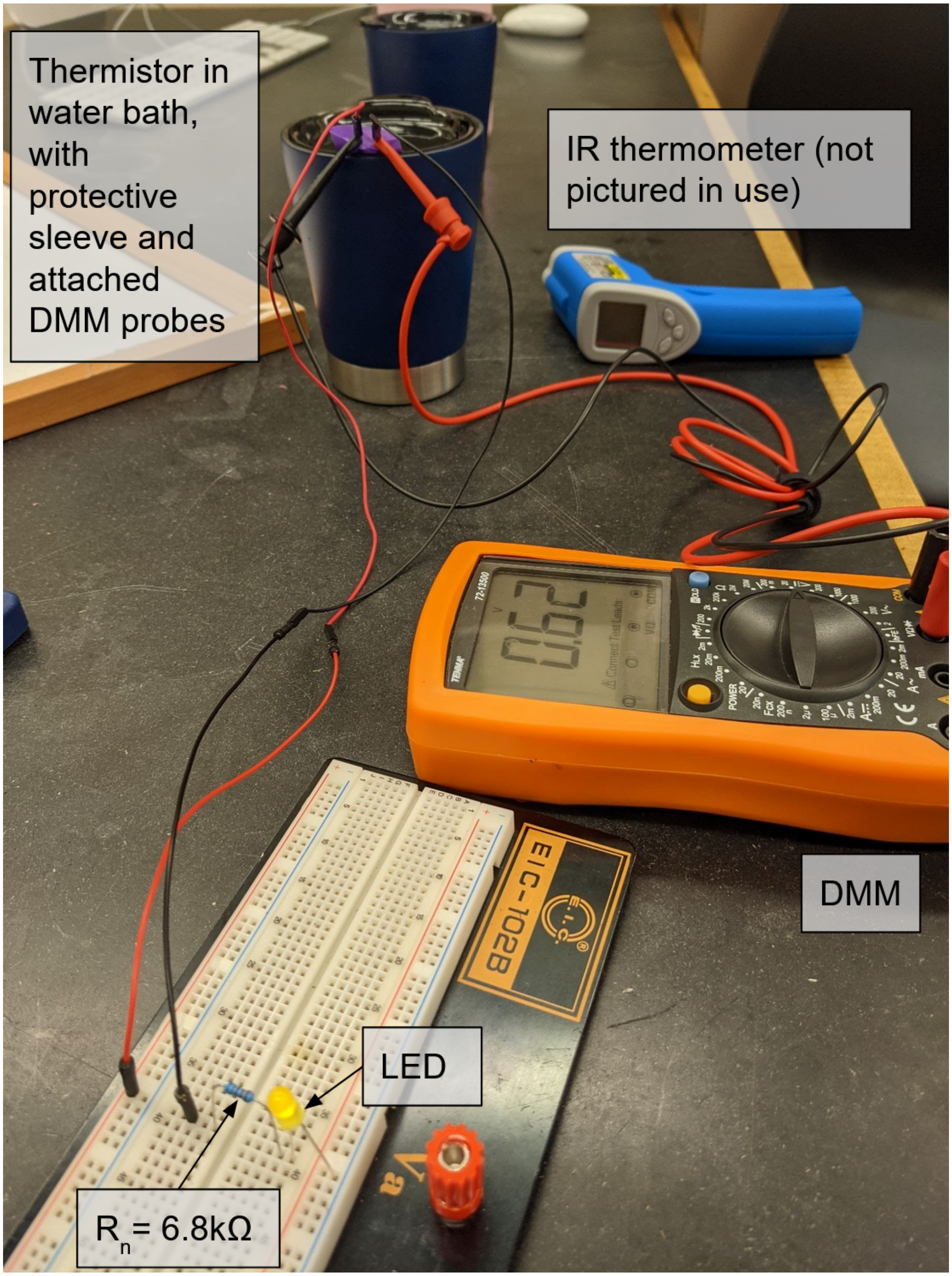


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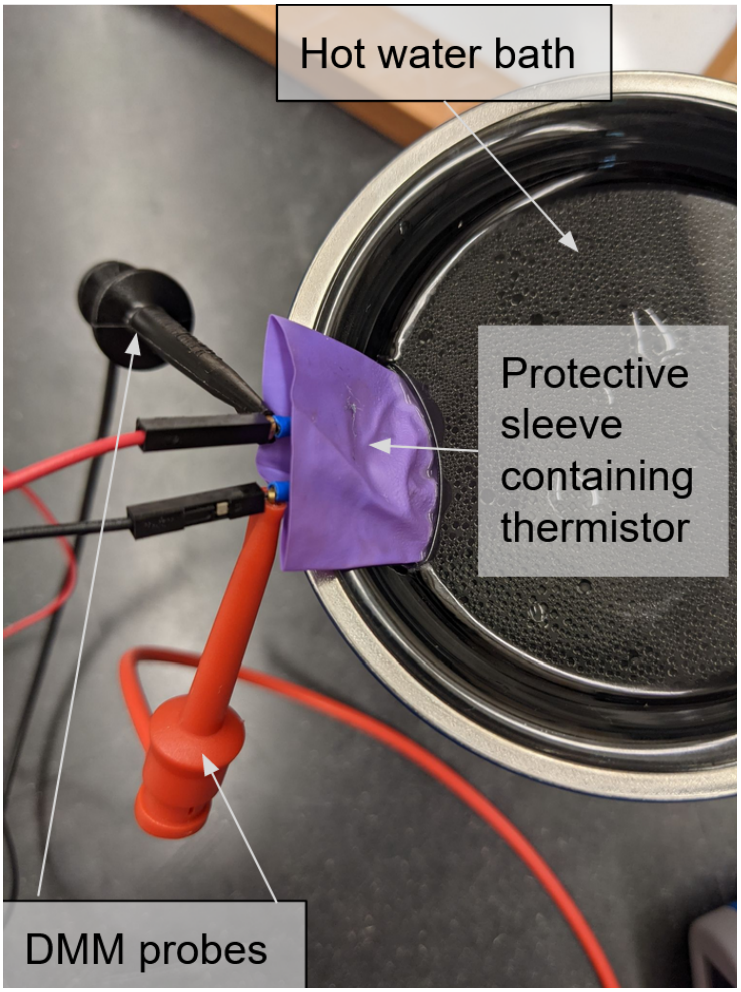
Figure 6. Close up of thermistor submerged in water bath.



After collecting data for the voltage across the thermistor at a variety of different temperatures, Python code was used to plot these voltages versus temperature and fit a function to describe their relationship. Inspection of the plotted data showed what appeared to be a linear trend, so linear regression was used.

Uncertainties were estimated to be approximately 0.1℃ for temperature measurements and 0.02V for voltage measurements. These estimates are based on measurements displayed by the DMM and the IR thermometer. The thermometer displayed temperatures up to three significant figures (with one figure to the right of the decimal point), and the third figure varied by plus or minus 0.1℃ while the first two figures remained steady. The DMM displayed voltages up to three significant figures (with two figures to the right of the decimal point), and the third figure varied by plus or minus 0.2V while the first two figures remained steady.

Figure 7. Python code used to calculate and plot linear regression with collected data and errors

```python
import matplotlib.pyplot as plt
import numpy as np
from scipy import stats
```

```python
# collected data

# both in degrees Celsius
temp = np.array([21.8,2.3,49.9,24.3,14.1,7.6,78.9,34.9,40.6,63.3,69.5,33.7,25.6,15.2])
d_temp = np.zeros(14)+0.1

# both in volts
volts = np.array([2.09,2.57,1.30,2.00,2.28,2.36,0.60,1.68,1.53,0.99,0.84,1.73,1.97,2.23])
d_volts = np.zeros(14)+0.02
```

```python
# calculate linear regression
slope, intercept, r_value, p_value, std_err = stats.linregress(temp, volts)
line = (slope*temp)+intercept
print('voltage =',round(slope,3), '* temperature +', round(intercept,3))
print('r value:', round(r_value,3))
print('p value:', '%.2e'%p_value)
print('standard error:', round(std_err,3))
```

```
voltage = -0.026 * temperature + 2.61
r value: -0.999
p value: 2.87e-17
standard error: 0.0
```

```python
# plot data with errorbars
plt.figure(figsize=(5,5))
plt.errorbar(temp, volts, xerr=d_temp,yerr=d_volts,
             fmt='b.', ecolor='r', capsize=4, label='collected data')
plt.errorbar(temp, line, fmt='-', lw=0.5, label='linear regression')
plt.title('Thermistor Voltage-Temperature Curve')
plt.xlabel('temperature in degrees Celsius')
plt.ylabel('voltage across thermistor in volts')
plt.legend()
```

Figure 8. Data for voltage and temperature plotted with linear regression and errorbars

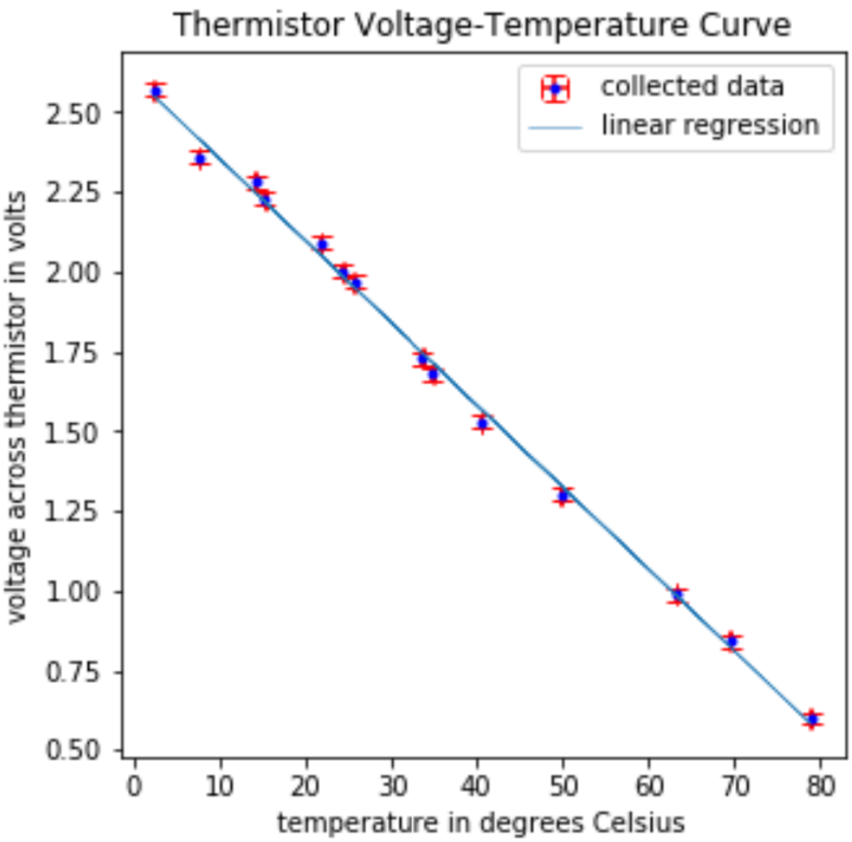


Figure 9. Calculations for the uncertainties in the relationship between V_{out} and temperature.

$V_{out} = -0.026T + 2.61$ → uncertainty on voltages is $\pm 0.02\,V$

$\dfrac{\Delta \text{Voltage}}{\Delta \text{temperature}}$ uncertainty on voltages is $\pm 0.02\,V$; uncertainty on temperatures is $\pm 0.1\,°C$

uncertainty in numerator $= \sqrt{(0.02V)^2 + (0.02V)^2} = 0.028V = 0.03V$

uncertainty in denominator $= \sqrt{(0.1°C)^2 + (0.1°C)^2} = 0.14°C$

total uncertainty $= 0.028V / 0.14°C = 0.2\,V/°C$

$$\boxed{V_{out} = -(0.026 \pm 0.2\,V/°C)T + 2.61 \pm 0.02\,V}$$