TRAJECTORY PLANNING OF
APPROACHING NON-COOPERATIVE TARGETS
BASED ON GAUSS PSEUDOSPECTRAL METHOD

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To capture a non-cooperative spacecraft whose status and information are unavailable has become a hot issue worldwide recently. This paper proposes a new trajectory planning method to solve the optimal control problem of non-cooperative target capture mission with obstacles in the final approaching phase. A continuous model based on the kinetic mechanism of spacecraft is constructed at first. Attitude of chaser craft and relative position are set as state variables. The path of avoiding obstacles is considered as constraint. Combined with terminal constraints and performance index, the parametric optimization model of algebraic constraints is established. Gauss pseudospectral method is deployed to discretize the continuous model. The problem is solved by sequential quadratic programming. Calculation and simulation module is developed to prove the feasibility of the method.

INTRODUCTION

Approaching a non-cooperative spacecraft with finite thrust is one of the key technologies in trajectory planning problem,[1,2] such as the Orbital Express made by USA,[3] ETS-VII made by Japan,[4] and TECSAS project made by Germany,[5] which verified the technology of orbital target approaching and capture. China also commenced the research of the technology.[6,7]

The target spacecraft is usually non-cooperative, whose orbital and attitude information cannot be obtained by others. The trajectory planning data can only be collected by relative movement parameters of the chasing craft.[2] So the kinetic equation of trajectory planning problem is different from that of cooperative target.[8] Besides, there may exist other uncontrollable objects surrounding the spacecraft, which increase the difficulty of the problem. Method to avoid the obstacles should be considered during the approaching process.[9] The trajectory planning when approaching a non-cooperative target is always a issue.

So far, the methods of approaching a non-cooperative target can be divided into two categories.[10,11] The former is a design without designing trajectory.[12,13] The control law of relative motion is designed directly according to the position of target. The latter is on the contrary, which contains two steps. The desired trajectory is calculated at first; then the control law is designed to follow the expected track.[14,15] When designing without trajectory, the control law is complicated since the coupling of orbit and attitude of the spacecraft, the problem of time-fuel optimal, and avoiding

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obstacles should be taken into account simultaneously.[16] When designing with trajectory planning, the trajectory planning and the control law is decoupled. Thus the complexity is reduced and the reliability is increased. Gauss pseudospectral method is one of the latter methods. It is a direct transcription method for discretizing a continuous optimal control problem into a nonlinear program.[17,18] It is more effective when dealing with a problem with initial and terminal constraints, by its high solving accuracy and convergence speed.[14] Huntington[19] used the method to study the problem of optimal reconfiguration of spacecraft formations. Duan[20] solved the optimal control problem of spacecraft fly-around releasing using continuous thrust based on the method.

In this paper, the author aims at the problem of terminal trajectory planning to approach non-cooperative targets at short range. Considering the non-cooperative features of target satellite and the problem of avoiding obstacles in a short distance, this paper proposes a model of the planned track. Gauss pseudospectral method is used to discretize the continuous model, which transforms the optimal control problem into a series of algebraic constraint parameter optimization problems. These problems are solved by sequential quadratic programming method. This article developed a trajectory planning calculation module, and proved the feasibility of the method by simulations.

1. PROBLEM DESCRIPTION

1.1. State Variables of Trajectory Planning

The trajectory planning model of this article contains 13 state variables including relative position, relative velocity, angular velocity of the chasing spacecraft, and quaternions, as shown in Eq. (1).

$$\boldsymbol{x}(t) = [x, y, z, \dot{x}, \dot{y}, \dot{z}, \omega_x^C, \omega_y^C, \omega_z^C, q_0^C, q_1^C, q_2^C, q_3^C]^T \tag{1}$$

The first three variables indicate the relative position between the chasing satellite and the target. Variables from 4 to 6 represent the relative velocity. ω denotes the attitude angular velocity. q denotes the quaternion. C denotes the chasing satellite. The control quantity that denote the control forces and moments are as following:

$$\boldsymbol{u}(t) = [f_x^C, f_y^C, f_z^C, M_x^C, My^C, M_z^C]^T \tag{2}$$

where f denotes the control force and M denotes the control moment.

In orbital coordinate system of the target spacecraft, the relative dynamic Clohessy-Wiltshire(C-W) equations between the chasing satellite and the target spacecraft is as following:

$$\begin{cases} \ddot{x} - 2n\dot{z} = f_x^C/m \\ \ddot{y} - n^2 y = f_y^C/m \\ \ddot{z} + 2n\dot{x} - 3n^2 z = f_z^C/m \end{cases} \tag{3}$$

where n denotes the orbit angular velocity and m denotes the mass of chasing satellite.

The attitude dynamics equation is as following:

$$I^C \frac{d\omega^C}{dt} + [\omega^C]^X I^C \omega^C = M^C \tag{4}$$

1.2. Track and Terminal Constraints

1.2.1. Constraint of Tracks

Assuming that the CM(center of mass) position of obstacle in orbital coordinate system of the target spacecraft is (x^B, y^B, z^B). In order to avoid the obstacle, the distance between the CM position of chasing satellite and the CM position of obstacle should be greater than the sum of chaser and obstacle's equivalent radius.

$$\sqrt{(x^C - x^B)^2 + (y^C - y^B)^2 + (z^C - z^B)^2} \geq R^C + R^B \tag{5}$$

R^C denotes the equivalent radius of the chasing satellite, while R^B denotes that of the obstacle. If the size of the chasing satellite is $[a, b, c]$, the equivalent radius of it can be calculated as Eq. (6):

$$R^C = \sqrt{a^2 + b^2 + c^2} \tag{6}$$

Also, R^B has the similar definition.

In the meantime, when approaching the non-cooperative target, the distance between the CM position of chasing satellite (x^C, y^C, z^C) and the CM position of target $(0, 0, 0)$ should be greater than the sum of R^C and R^T.

1.2.2. Terminal Constraint

When the chasing satellite approaches the non-cooperative target, assume the relative location when hovering is (x^F, y^F, z^F), thus the terminal position constraint is:

$$\sqrt{(x^C - x^F)^2 + (y^C - y^F)^2 + (z^C - z^F)^2} \leq \eta \tag{7}$$

where η denotes the tolerance of position constraint.

Considering there will be a hovering for a short period of time, the terminal velocity constraint is:

$$\sqrt{(\dot{x}^C)^2 + (\dot{y}^C)^2 + (\dot{z}^C)^2} \leq \zeta \tag{8}$$

where ζ denotes the tolerance of velocity constraint.

At the end of the approaching process, a certain direction of the chasing satellite should point to the target spacecraft in order to capture it conveniently. Assuming the attitude angular velocity and quaternion of the chasing satellite is ω^F and q^F at the end of the approaching period. Then:

$$|\omega^C - \omega^F| \leq \xi, |q^C - q^F| \leq \xi \tag{9}$$

where ξ denotes the tolerance of attitude.

1.3. Performance Index of Trajectory Planning

The trajectory planning model take two performance indexes into account. One is the fuel consumption J_1:

$$J_1 = \int_{t_0}^{t_f} f(U) \tag{10}$$

where $f(U)$ is the function of fuel consumption. t_0 is the starting time of approaching, and t_f is the ending time.

The other index is the orbit approaching time J_2:

$$J_2 = t_f - t_0 \tag{11}$$

The weighted coefficient of time λ is introduced. Considering both fuel consumption and approaching time, the performance index can be designed as:

$$J = J_1 + \lambda J_2 = \int_{t_0}^{t_f} [f(U) + \lambda] \tag{12}$$

where the changing of λ can alter the balancing of approaching time. Regard the minimum index as goal of optimizing. The greater λ becomes, the more emphasis on approaching time. If $\lambda = 0$, it will be changed into a fuel optimal problem.

By solving the continuous trajectory planning model, the expected orbit and attitude information of chasing satellite at any time is obtained. It provides a referential input for trajectory tracking control.

2. GAUSS PSEUDOSPECTRAL METHOD

The principle of solving a continuous optimal control problem using gauss pseudospectral method is to discretize the controls and states on a series of Legendre-Gauss(LG) points, and to form a Lagrange's polynomial interpolation to approach the controls and states using LG points as nodes. The optimal control problem can be transformed into a series of parametric optimization problems of algebraic constraints.

The time domain of optimal control problem is $t \in [t_0, t_f]$, and that of gauss pseudospectral method is $\tau \in [-1, 1]$. Therefore, Eq. (13) is used to change the interval.

$$\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0} \tag{13}$$

Thus, the optimal control problem with variable $\tau \in [-1, 1]$ can be described as:

$$J = \Phi(\boldsymbol{x}(-1), \boldsymbol{x}(1), t_0, t_f) + \frac{t_0 - t_f}{2} \int_{-1}^{1} g(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), t_0, t_f) d\tau \tag{14}$$

The constraints and boundary conditions are:

$$\dot{\boldsymbol{x}}(\tau) = \frac{t_0 - t_f}{2} f(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), t_0, t_f) \tag{15}$$

$$\Phi(\boldsymbol{x}(-1), \boldsymbol{x}(1), t_0, t_f) = 0 \tag{16}$$

$$C(\boldsymbol{x}(-1), \boldsymbol{x}(1), t_0, t_f) \leq 0 \tag{17}$$

In the time domain $\tau \in [-1, 1]$, approximation equation of gauss numerical integration can be expressed as:

$$\sum_{k=0}^{n} w_k f(c) \approx \int_{-1}^{1} f(\tau) d\tau \tag{18}$$

where τ_k is integration knots and w_k is integration coefficients. Integration knots τ_k is the root of the n-order Legendre polynomial $P_n(\tau) = 0$.

$$P_n(\tau) = \frac{1}{2^n n!} \frac{d^n}{d\tau^n} [(\tau^2 - 1)^n] \tag{19}$$

The integration knots are always in the range of $(-1, 1)$, which have the characteristics that sparse in the middle and dense on the edge. Gaussian numerical integral formula using n integration knots has the algebraic accuracy of $2n - 1$ order. The value of integration coefficients w_k doesn't depend on the specific form of $f(\tau)$. It is only associated with the value of integration knots τ_k. It can be expressed as:

$$w_k = \int_{-1}^{1} l_k(\tau) d\tau = \int_{-1}^{1} \prod_{i=1 i \neq k}^{n} \frac{\tau - \tau_i}{\tau_k - \tau_i} d\tau = \frac{2}{(1 - \tau_k^2)[\dot{P}_n(\tau_k)]^2}, (k = 1, 2, ..., n) \tag{20}$$

where $\dot{P}_n$ is the derivative of the n-order Legendre polynomial function.

The continuous trajectory planning model of non-cooperative targets can be discretized by approximation formula of gauss numerical integration Eq. (18), forming a series of parametric optimization problems of algebraic constraint.

3. TRAJECTORY PLANNING

3.1. Establishment of the Discrete Model using Gauss Pseudospectral Method

3.1.1. Discretization of States and Controls

According to Eq. (18), in the time interval $\tau \in [-1, 1]$, the states and controls can be approximated using Lagrange global interpolation polynomial. The state $\boldsymbol{x}(t)$ can be approximated using $K + 1$ Lagrange global interpolation polynomial as:

$$\boldsymbol{x}(t) \approx \boldsymbol{X}(\tau) = \sum_{i=1}^{K} X_i(\tau_i) l_i(\tau) \tag{21}$$

The interpolation node τ_i is determined by Eq. (19). $\tau = -1$. The control $\boldsymbol{x}(t)$ can be approximated using K Lagrange global interpolation polynomial as:

$$\boldsymbol{u}(t) \approx \boldsymbol{U}(\tau) = \sum_{i=1}^{K} U_i(\tau_i) \tilde{l}_i(\tau) \tag{22}$$

The interpolation node τ_i is determined by Eq. (19).

3.1.2. Discretization of the Differential Kinetic Equation

Take the derivative of the state variable expression in Eq. (21), and the differential kinetic equation can be transformed into an algebraic equation.

$$\dot{\boldsymbol{x}}(t) \approx \dot{\boldsymbol{X}}(\tau) = \sum_{i=1}^{K} X_i(\tau_i) D_k i(\tau_k), (k = 1, 2, ..., K) \tag{23}$$

where the differential matrix $D_{ki} \in R^{k \times (k-1)}$ is calculated as:

$$D_{ki} = \begin{cases} \dfrac{(1+r_k)\dot{P}_k(\tau_k) + P_k(\tau_k)}{(\tau_k - \tau_{ki})[(1+\tau_k)\dot{P}_k(\tau_i) + P_k(\tau_i)]}, i \neq k \\ \dfrac{(1+\tau_k)\ddot{P}_k(\tau_k) + 2\dot{P}_k(\tau_k)}{2[(1+\tau_i)\dot{P}_k(\tau_i) + P_k(\tau_i)]}, i = k \end{cases} \tag{24}$$

The differential kinetic equation is transformed as:

$$\sum_{i=0}^{K} X_i(\tau_i) D_{ki}(\tau_k) = \frac{t_0 - t_f}{2} f(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), \tau, t_0, t_f) \tag{25}$$

It can be inferred from Eq. (25) that the differential kinetic equation is only calculated on integration knots.

3.1.3. Discretization of Track and Terminal Constraint

1) Constraint of Tracks

The state of $\tau_a(a \in [1, K])$ at any time can be expressed as:

$$\boldsymbol{x}(\tau_a) = \boldsymbol{x}(\tau_0) + \int_{-1}^{\tau_a} f(\boldsymbol{x}(\tau), \boldsymbol{u}(\tau), t_0, t_a) d\tau \tag{26}$$

Expand the integral function above using approximation formula of gauss integration:

$$\boldsymbol{X}_a = \boldsymbol{X}_0 + \frac{t_0 - t_a}{2} \sum_{i=1}^{K} w_i f(\boldsymbol{X}_k, \boldsymbol{U}_k, \tau_k, t_0, t_a) \tag{27}$$

Substitute Eq. (27) into Eq. (5), the constraint of track without integral term can be obtained.

2) Terminal Constraint

The state of $\tau_f = 1$ at terminal time can be expressed using approximation formula of gauss integration as:

$$\boldsymbol{X}_f = \boldsymbol{X}_0 + \frac{t_0 - t_f}{2} \sum_{i=1}^{K} w_i f(\boldsymbol{X}_k, \boldsymbol{U}_k, \tau_k, t_0, t_f) \tag{28}$$

$\boldsymbol{X}_f$ in Eq. (28) automatically satisfies the differential kinetic equation. By substituting Eq. (28) into Eq. (9), the terminal constraint without integral term can be obtained.

3.1.4. Discretization of Performance Index

Substitute gauss integration for the integration term in performance index function in Eq. (12):

$$J = 2\lambda + \frac{t_0 - t_f}{2} \sum_{i=1}^{K} w_i g(\boldsymbol{X}_i, \boldsymbol{U}_i, t_0, t_f) \tag{29}$$

3.1.5. Discretization of Continuous Optimal control Problem

The integral term of the performance index function in Eq. (14) can be approximately expressed as:

$$J = \Phi(\boldsymbol{x}(-1), \boldsymbol{x}(1), t_0, t_f) + \frac{t_0 - t_f}{2} \sum_{i=1}^{K} w_i g(\boldsymbol{X}_i, \boldsymbol{U}_i, t_0, t_f) \tag{30}$$

subject to the dynamic constraints:

$$\boldsymbol{R}_f = \sum_{t=0}^{K} \boldsymbol{X}_i \boldsymbol{D}_{ki}(\tau_k) - \frac{t_0 - t_f}{2} f(\boldsymbol{X}_k), \boldsymbol{U}_k), \tau_k, t_0, t_f) = 0 (k = 1, ..., K) \tag{31}$$

The terminal state constraints:

$$\boldsymbol{R}_f = \boldsymbol{X}_f - \boldsymbol{X}_0 - \frac{t_0 - t_f}{2} \sum_{i=1}^{K} w_i f(\boldsymbol{X}_k, \boldsymbol{U}_k, \tau_k, t_0, t_f) = 0 \tag{32}$$

The boundary conditions:

$$\Phi(\boldsymbol{X}_0, \boldsymbol{X}_f, t_0, t_f) = 0 \tag{33}$$

And the path constraints:

$$C(\boldsymbol{X}_k, \boldsymbol{X}_k, \tau_k, t_0, t_f) \leq 0 \tag{34}$$

The discrete nonlinear programming problem after transformation can be described as: under the conditions of dynamic constraints(Eq. (31)), terminal state constraints(Eq. (32)), boundary conditions(Eq. (33)), and the path constraints(Eq. (34)), solve the control variables $(U_1, U_2, ..., U_K)$ and the discrete state variables $(X_1, X_2, ..., X_K)$, to minimize the performance index((Eq. (30))). Then the problem can be solved by sequential quadratic programming (SQP) methods.

3.1.6. Trajectory Planning Procedure

Using the method above, the trajectory planning calculation program can be established on Matlab. The program consists of 4 modules including parameters setting module, Gauss Pseudospectral discrete state module, sequential quadratic programming solution module, and visualization of results module. The relationship between modules is shown in Figure 1. Users are allowed to set up the planning parameters, track constraint, and terminal constraint parameters in the parameters setting module.
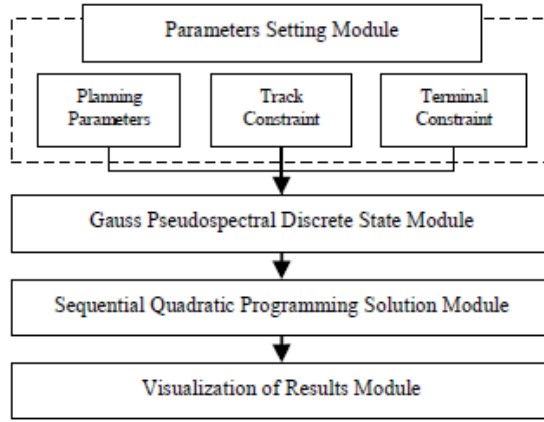
Figure 1. Principle Diagram of the Trajectory Planning Program

3.2. Simulation and Analysis

Assuming that the non-cooperative target is a defunct satellite in high earth orbit whose attitude is out of control. The satellite is rotating slowly about an random axis (maximum moment of inertia). An obstacle is located close to the defunct satellite.

Initial kinetic parameters of the chaser and the target spacecraft are as shown in Table 1. The parameters are in body coordinate system of the chaser.

Table 1. Initial kinetic parameters

Kinetic Parameters	Chaser	Target
Mass(kg)	100.00	946.00
Center-of-mass Position(m)	(0.00,0.00,0.00)	(1.07,0.01,0.01)
$J_x(kg \cdot m^2)$	100.00	218.54
$J_y(kg \cdot m^2)$	100.00	309.82
$J_z(kg \cdot m^2)$	100.00	345.14
$J_{xy}(kg \cdot m^2)$	0.00	-2.09
$J_{yz}(kg \cdot m^2)$	0.00	-12.00
$J_{xz}(kg \cdot m^2)$	0.00	-12.00
Roll(deg)	0.00	0.00
Pitch(deg)	0.00	0.00
Yaw(deg)	0.00	0.00
ω_x(deg/s)	0.00	0.11
ω_y(deg/s)	0.00	0.33
ω_z(deg/s)	0.00	1.09

The orbital elements of the chaser and the target spacecraft are as shown in Table 2.

The expected distance between chaser and target spacecraft is $L = 3$ m. The expected relative velocity is 0. Positional tolerance is $\eta' = 0.01$ m. Speed tolerance is $\zeta' = 0.01$ m/s.

In orbital coordinate system of the target spacecraft, the initial position of the chasing satellite is $(-10, 0, 0)$, and the velocity is $(0, 0.2, -0.05)$. The hovering position is $(-3, 0, 0)$. The initial Euler

Table 2. Orbital elements of chaser and target spacecraft

Orbital Elements	Chaser	Target
Semi-major axis(m)	42164003.0	42164000.0
Eccentricity	1.48×10^{-6}	0.0
Inclination Angle (deg)	1.30×10^{-4}	0.0
RAAN(deg)	8.95×10^{-1}	0.0
Argument of Perigee (deg)	87.58	0.0
True Anomaly (deg)	-88.47	0.0
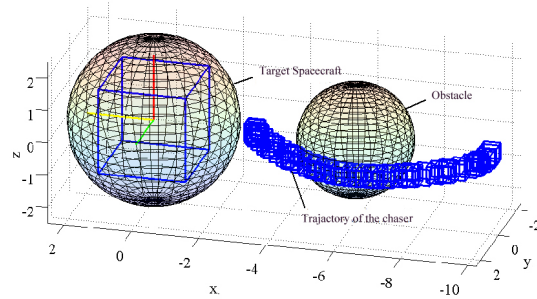
Figure 2. Trajectory of Chaser in Body Coordinate System of Target Satellite

angle is $(0, 0, 0)$, and the angular velocity is $(0.11, 0.33, 1.09)$. The expected value of the angle and angular velocity when hovering are both 0. $R^C = 2.5$. The position of the obstacle is $(-6, 0, 0)$. $R^B = 2$. A matlab simulation program is carried out to make the planning. The track of the chaser in body coordinate system and orbit coordinate system of the target satellite are shown in Figure 2 and Figure 3.

In Figure 2 and Figure 3, the big sphere with a cube frame is the target spacecraft. The small sphere is the obstacle. The curve chain is the trajectory of the chasing satellite. The entire process of the chaser bypassing the obstacle and approaching the target could be seen. The program result of the position and velocity of the chasing satellite in orbital coordinate system of the target spacecraft is shown in Figure 4.

It can be inferred from Figure 4 that the chasing satellite approached the target in direction X, which is in accord with expectation. In order to avoid the obstacle, the chasing satellite flew away from the target at first, then became closer, in direction Y and Z. The entire process lasted nearly 31.44 seconds. The program result of Euler angle and angular velocity of the chasing satellite in orbital coordinate system of the target spacecraft is shown in Figure 5.

A 3D simulation program is also carried out to demonstrate the approaching course, as shown is Figure 6. The bigger cube satellite with solar panals is the target spacecraft. The sphere connected is the obstacle. The smaller cube denotes the chaser which flew from the right side to capture the target.
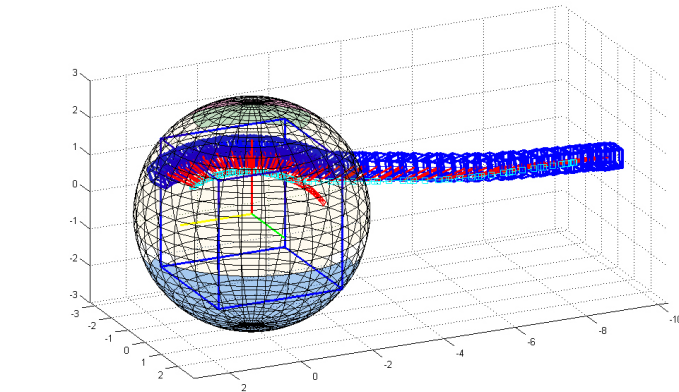
Figure 3. Trajectory of Chaser in Orbit Coordinate System of Target Satellite
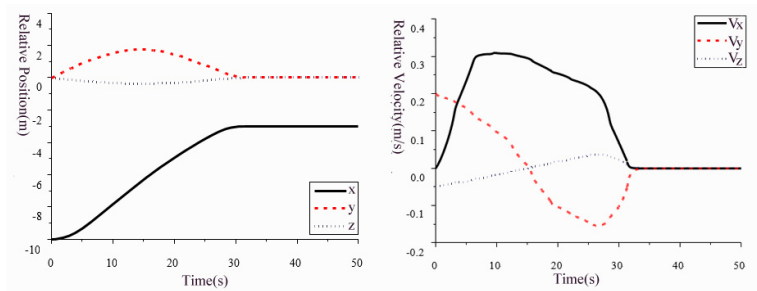

Figure 4. Position and Velocity of the Chasing Spacecraft

It can be inferred from the result that the program result is in accordance with expectation, and the approaching trajectory is correct. It verified the feasibility of planning trajectory of approaching non-cooperative targets using Gauss Pseudospectral method.

4. CONCLUSION

In this article, a trajectory planning model of approaching non-cooperative targets is established. The model is discretized by Gauss pseudospectral method, and the problem is translated into a parameter optimization problem with algebraic constraints, which can be solved by sequential quadratic programming. A calculating program of trajectory planning is developed. Aiming at
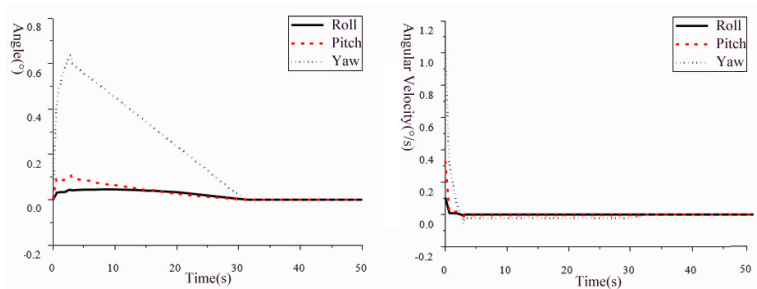

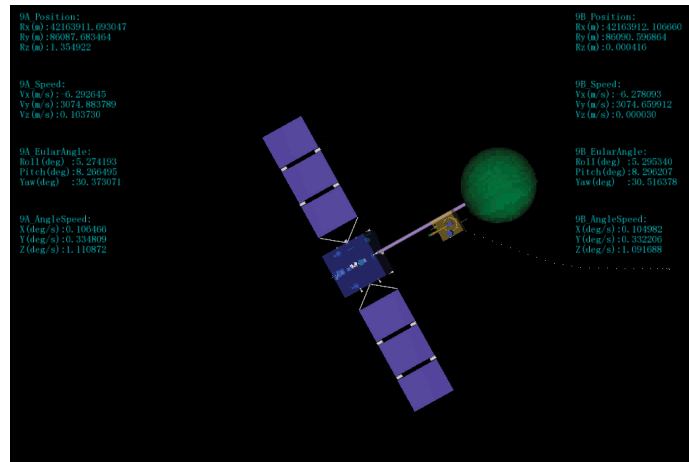**Figure 5. Euler Attitude and Angular Velocity of the Chaser**

1536

Figure 6. 3D Simulation of the Approaching Process

a typical working condition, the approaching trajectory is designed and the expected trajectory is given. The program result is in accordance with expectation, which proved the correctness of planning algorithm.

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