NE 523 Prof. Azmy
Spring 2026 January 26, 2026

Homework Set # 1
The Boltzmann Equation
Due February 2, 2026

1. **Two-species Boltzmann Equation** (adapted from Huang 3.4) **20 *Points***
Consider a mixture of two gases whose molecules have masses m & M respectively and which are subjected to external forces $\mathbf{F}$ & $\mathbf{Q}$, respectively. Denote the respective distribution functions by f & g. Assuming that only binary collisions between molecules are important, derive the Boltzmann equation for the system.

2. **Invariance of Phase-space Volume Element** **20 *Points***
Consider a dilute gas for which the external force field $\mathbf{F}$ is independent of the particle velocity $\mathbf{v}$. Show that, to first order in δt, the translation of the phase-space element $d^3\mathbf{r}d^3\mathbf{v}$ at time t to $d^3\mathbf{r}'d^3\mathbf{v}'$ at time $t+\delta t$ leaves its volume invariant, namely $d^3\mathbf{r}'d^3\mathbf{v}' = d^3\mathbf{r}d^3\mathbf{v}$. You may only neglect quadratic terms in δt, but not terms in $dx\ \delta t$ or $dv_x\ \delta t$, *etc*. Note that this will cause the $dx\ dv_x$ element to change shape during δt, but it should retain its volume.

3. **Moments of the Maxwell-Boltzmann Equilibrium Distribution** **20 *Points/Part***
Consider the Maxwell-Boltzmann equilibrium distribution for a dilute, single-species gas whose particles are of mass m:
$$f_0(\mathbf{v}) = n\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m\left|\mathbf{v}-\mathbf{v}_0\right|^2}{2kT}\right]$$
where n is the number of particles/cc, T is the gas temperature, k is Boltzmann's constant, and $\mathbf{v}_0$ is the bulk velocity of the gas. Derive an expression for:
    a. The zeroth moment: $\int d^3v\, f_0(\mathbf{v})$
    b. The first moment: $\int d^3v\, \mathbf{v}\, f_0(\mathbf{v})$
    c. The second moment: $\int d^3v \left(\frac{1}{2}mv^2\right) f_0(\mathbf{v})$

    [Take the limits of integration to be $-\infty \le v_x, v_y, v_z \le \infty$ in Cartesian coordinates and $0 \le v \le \infty, \mathbf{\Omega}$ covers a full unit sphere in spherical coordinates of velocity space]