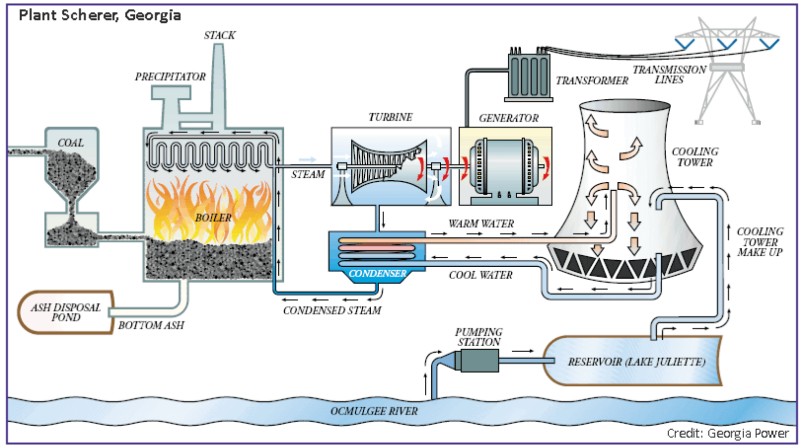
NE 511: Multi-Physics of Nuclear Reactors

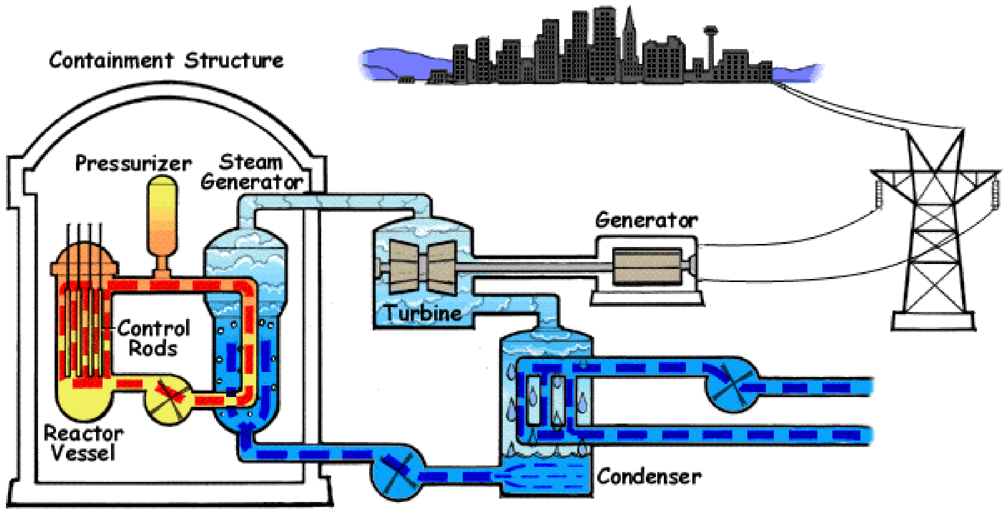
UNIT 2: Short-Time Multi-Physics Phenomena in Reactor Core

Lecture 3 : PROMPT AND DELAYED NEUTRONS
DYNAMIC EQUATION & SIMPLIFIED NEUTRON CYCLE
TOTAL DELAYED NEUTRON YIELDS & YIELDS OF DELAYED NEUTRON GROUPS

Nuclear Power Plants — How They Produce Electricity?

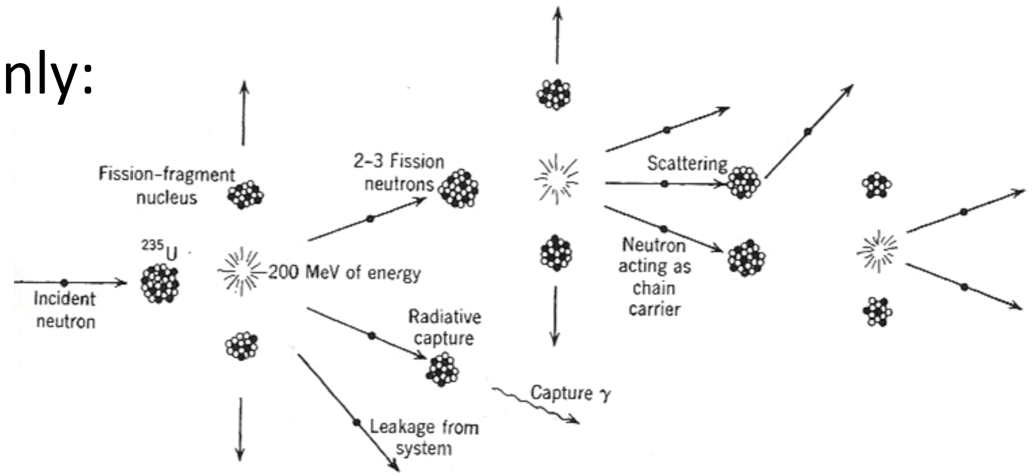
- A nuclear power plant works on the same basic principle as a conventional thermal plant
  - The difference is the heat source:
    - *Fossil plants burn fuel*
    - *Nuclear plants use fission in a reactor core*
- Major Components:
  - Reactor core — contains fuel and produces heat
  - Control rods — absorb neutrons to regulate power
  - Moderator — slows neutrons (water or graphite)
  - Coolant — transfers heat from core to steam system
  - Containment — reinforced structure preventing release
- Types:
  - LWR (Light Water Reactor), LMFR, HTGR, HWR, MSR, etc.
  - Large, small modular reactors (SMRs), and microreactors





Reactor Physics — What Happens in the Core?

- Nuclear energy comes from the fission of heavy nuclei, mainly:
  - U-235
  - Pu-239 (bred from U-238 in some reactors)
- Chain Reaction Basics
  - A neutron hits a fissile nucleus
  - Nucleus splits → releases energy + 2–3 neutrons
  - Some neutrons cause more fissions → sustained reaction
- The system is governed by the neutron balance
- **Key Physics Concepts**
  - Moderation — slows neutrons to thermal energies
  - Delayed neutrons — enable controllable power changes
  - Reactivity feedback
  - Negative temperature / Doppler feedback is stabilizing
  - Burnup — energy extracted per unit fuel
  - Neutron poisons (e.g., xenon-135) affect reactor control



$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right) $$

Criticality and multiplication factor – *k*

$$ k = \frac{\textit{Number of neutrons in one generation}}{\textit{Number of neutrons in preceding generation}} $$

$$ k \begin{cases} < 1 & - \textit{subcritical} \\ = 1 & - \textit{critical} \\ > 1 & - \textit{supercritical} \end{cases} $$

Neutron balance in nuclear reactors

$$ \frac{dN}{dt} = \begin{bmatrix} Production \\ rate \end{bmatrix} - \begin{bmatrix} Loss \\ rate \end{bmatrix} $$

Prompt and Delayed Neutrons

The Basic Concepts

Which is the essential phenomenon in nuclear reactors?

The self-sustaining process of neutron induced fission

What are the main differences between fast and thermal nuclear reactors?

1. *Neutron energy at which fission reactions are caused*
2. *Isotopes to undergo a fission reaction*

Can we use the same basic dynamic principles to describe nuclear fission reactors?

Yes, but there is a very important difference – the time scale of neutron reproduction (due to different fraction of delayed neutrons)

The basic concepts of time-dependent reactor are:

1. *Reactivity*
2. *Average Neutron Generation Time and Lifetime*
3. *Delayed Neutrons*

REACTIVITY

Reactivity is a measure of the departure from criticality, or relative departure of the neutron multiplication factor from unity: $\rho = \frac{k-1}{k}$

- *Integral property of the reactor*
- *Can be measured (via k), but usually deduced from observation of dynamics behavior*
- *Depends on:*
    1. *reactor size;*
    2. *amounts and densities of various materials;*
    3. *cross-sections for fission, scattering, absorption*

NEUTRON GENERATION TIME

Neutron generation time is the mean time of neutron reproduction in a multiplying medium (average time between two birth events in successive generations):

$$\Lambda = \frac{1}{\bar{v}\,\nu\Sigma_f}$$

$$\frac{1}{\bar{v}}\left[\frac{s}{cm}\right] \qquad \frac{1}{\Sigma_f}[cm] \qquad = \qquad \Delta t_f[s]$$

mean free path for fission (the average distance traveled from birth to fission)

average time between birth and fission it may cause

- *Integral property of the reactor*
- *Fast reactors: $10^{-7} \div 10^{-8}$ sec*
- *Thermal reactor: $10^{-3} \div 10^{-4}$ sec*
- *Depends on:*
    1. *Number of scattering collisions before leakage or absorption*

NEUTRON LIFETIME

Neutron lifetime is the time between birth and death (absorption or leakage):

$$\ell = \frac{1}{\bar{v}} \frac{1}{\Sigma_a + DB^2}$$

For infinite systems: $\ell_\infty = \frac{1}{\bar{v}} \frac{1}{\Sigma_a}$

Assuming only prompt neutrons:

$$\ell_p = \Lambda \quad - \textit{critical system/reactor}$$
$$\ell_p < \Lambda \quad - \textit{subcritical system/reactor}$$
$$\ell_p > \Lambda \quad - \textit{supercritical system/reactor}$$

PROMPT AND DELAYED NEUTRONS

Prompt Neutrons

Nearly all the neutrons produced as a result of a fission process are emitted "promptly," i.e., without noticeable delay.

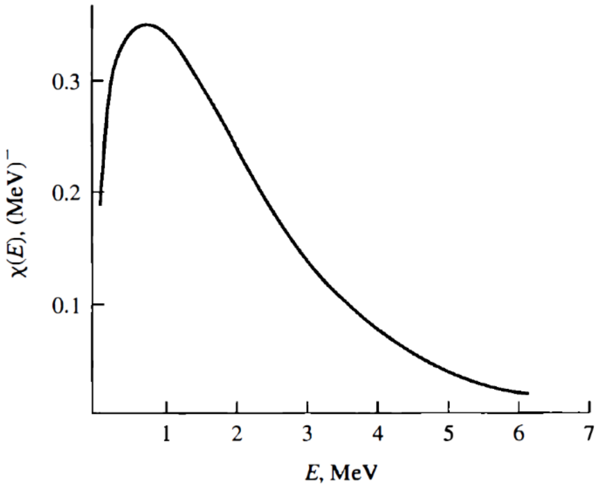
- *The prompt neutrons are emitted by the "direct" fission products immediately after the fission process, since the excitation energy of the fission product nuclei is generally much larger than the neutron binding energy E_{bn}.*

- *The typical decay time of such excited states may be 10^{-15} sec or less, which is completely negligible.*

- Prompt neutron spectrum:

$$\chi(E) = 0.453e^{-1.036E}\sinh\sqrt{2.29E}$$

- The function $\chi(E)$ is normalized
- The average energy of the prompt neutrons is 1.98 MeV
- The most probable energy of the prompt neutrons is 0.73 MeV



PROMPT AND DELAYED NEUTRONS

Delayed Neutrons

After a fission event, the reaction products consist of (1) two radioactive nuclei; (2) several prompt neutrons; (3) several gamma rays. None of the resultant fission product nuclei can directly emit an additional neutron.

- *Some of the fission product nuclei may decay into daughter nuclei for which the excitation energy is larger than the neutron binding energy.*
- *Such nuclei may then immediately emit a neutron, which has been delayed by the comparatively long time it took such a nucleus to undergo a beta decay.*
- *Less than 1% of the neutron production in fission.*

PROMPT AND DELAYED NEUTRONS

Delayed Neutrons

Delayed neutron can be emitted if the maximum electron energy in the beta decay is larger than the neutron binding energy.

Practically, the entire delay comes from the beta decay and only a negligibly small amount comes from the actual neutron emission.

"Emitter" *is the daughter nuclei that produce delayed neutrons.*

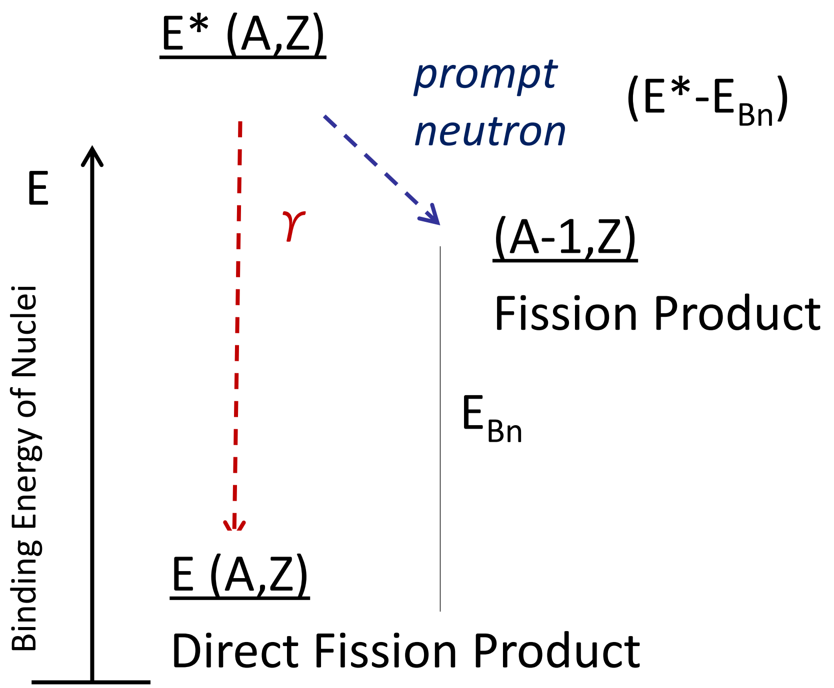
"Precursor" *is the parent of the daughter nuclei that produce delayed neutrons.*

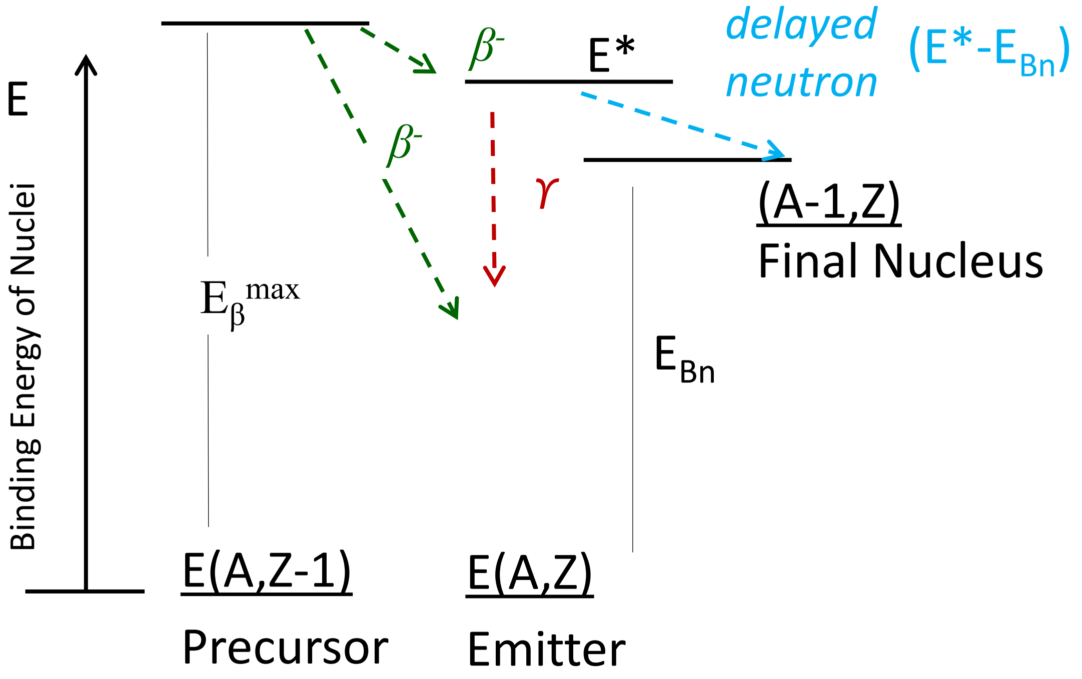
The average total number of neutrons, ν, which is composed of prompt and delayed neutrons, is expressed in terms of the relevant yields (neutrons per fission):

$$\nu = \nu_p + \nu_d$$

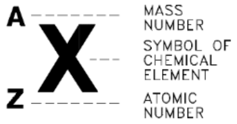
The production of delayed neutrons is often described by the delayed neutron fraction: $\beta = \frac{\nu_d}{\nu}$

PROMPT AND DELAYED NEUTRONS





- Atomic number, Z – total number of protons in a nucleus
- Total electrical charge of a nucleus is +Ze
- Number of neutrons in a nucleus, N
- Atomic mass number (or nucleon number) = total number of nucleons in a nucleus or A=(Z+N)



PROMPT AND DELAYED NEUTRONS

Delayed Neutrons

The physical process of production of delayed neutrons is basically the same as the production of prompt neutrons – what is the difference?

→ ***Delayed neutrons are emitted on the average with considerably smaller energies than prompt neutrons.***

Why?

The excitation energy and the neutron binding energy in delayed neutron emitter nuclei is normally much smaller than in the highly excited "direct" fission products.

Static reactor problems - the prompt and the delayed fission neutrons always appear together as the total number of fission neutrons.

Delayed neutrons - less 1% of the fission neutrons produced after time delays of about a second to 80 seconds.

→ *Delayed neutrons may play a dominant role in many kinetics phenomena.*

PROMPT AND DELAYED NEUTRONS

Delayed Neutrons

In $^{235}U/^{238}U$-fueled thermal reactors at high burnup, as well as in Pu-fueled fast reactors, several isotopes contribute comparably or significantly to the production of delayed neutrons.

→ *Delayed neutron data of several fissionable isotopes must be considered.*

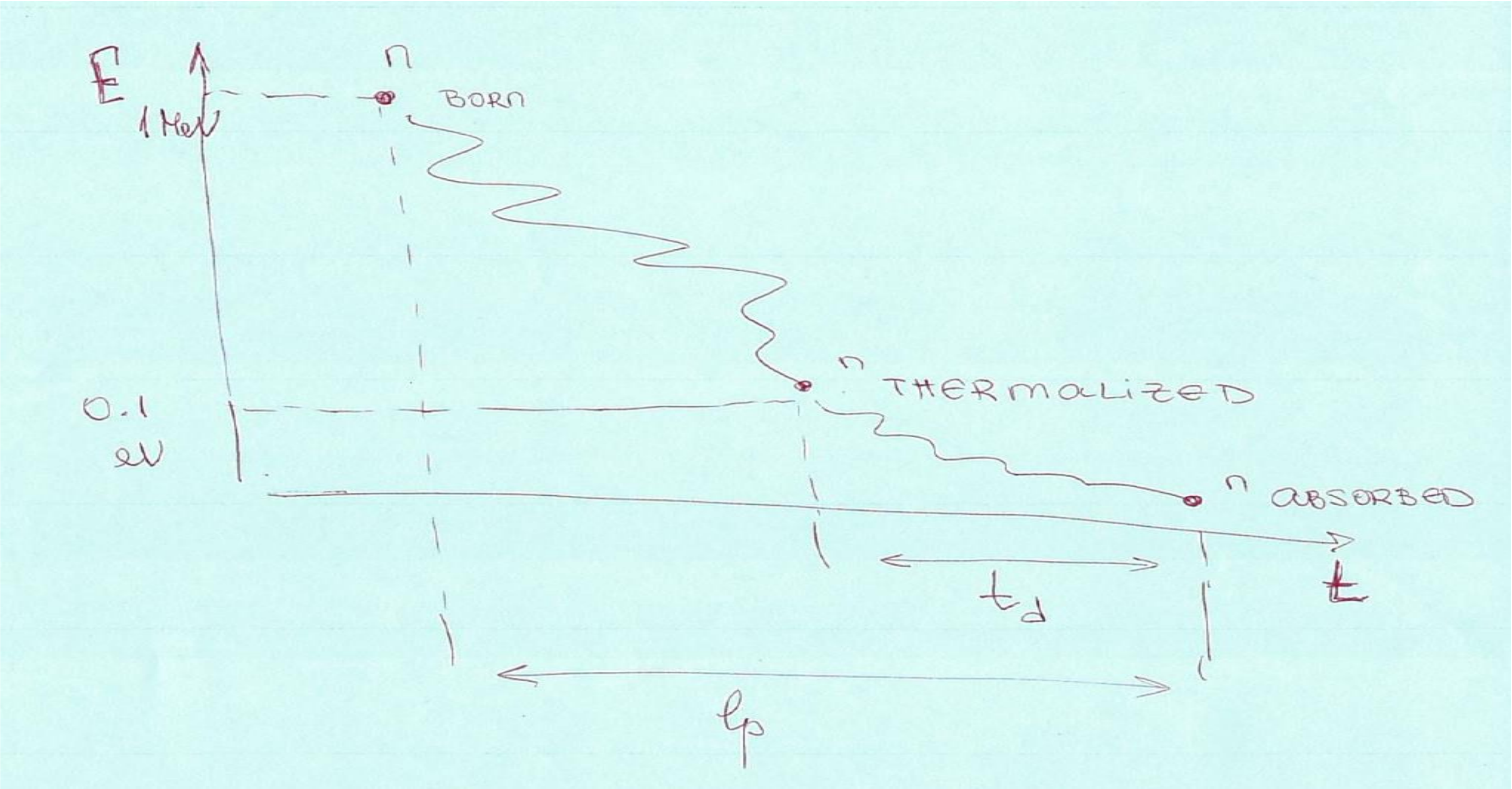
A further complication appears in fast reactors where fissions are induced by neutrons in a wide energy range.

→ *Therefore, the isotope as well as the energy dependencies of the delayed neutron production need to be considered.*

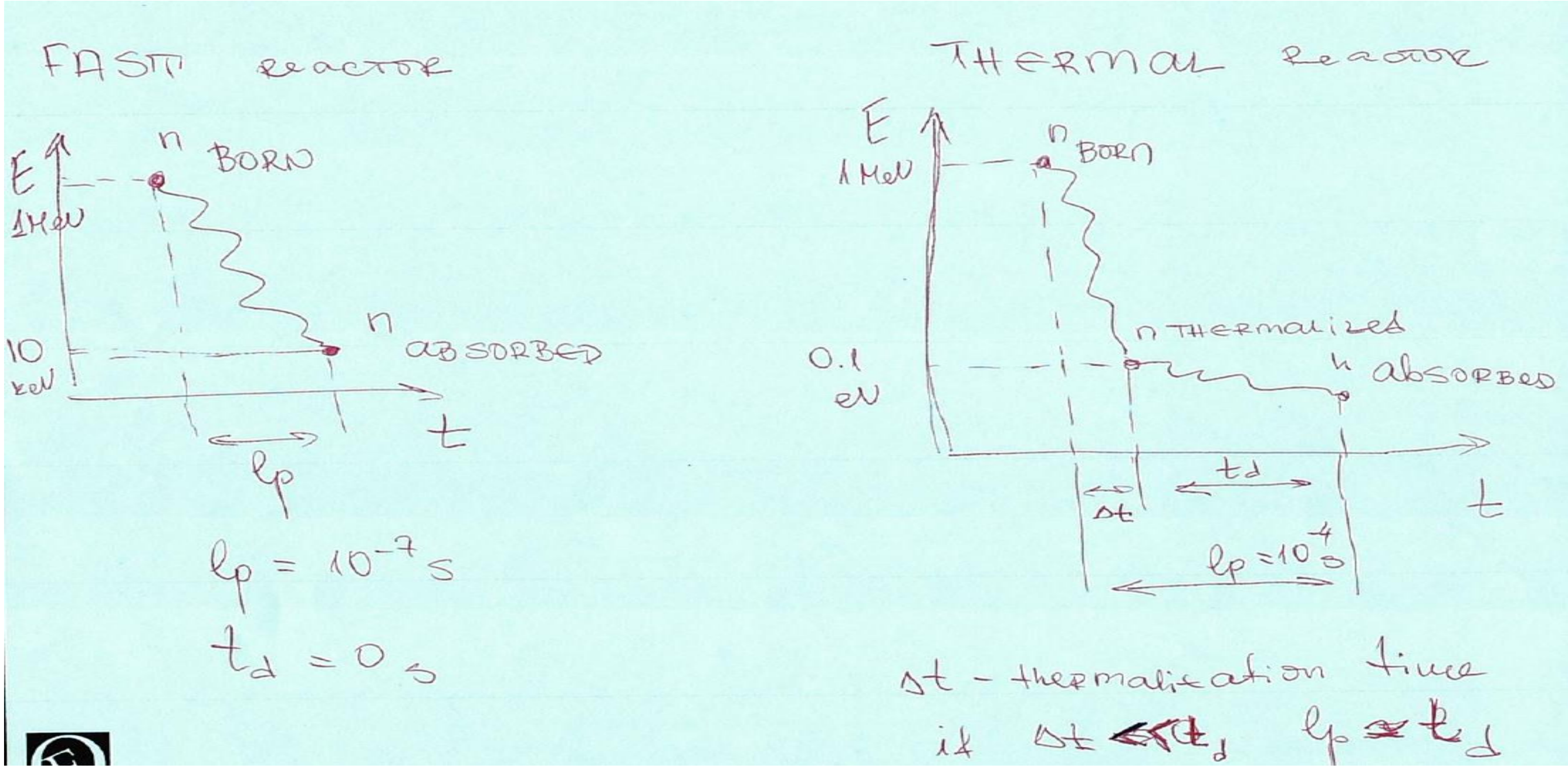
Prompt neutron Lifetime and Mean Diffusion Time

Prompt Neutron Lifetime l_p – *the average time between the emission of a prompt neutron and its death (absorption or leakage)*

Mean Diffusion Time t_d – *the average lifetime of a thermal neutron in an infinite system*



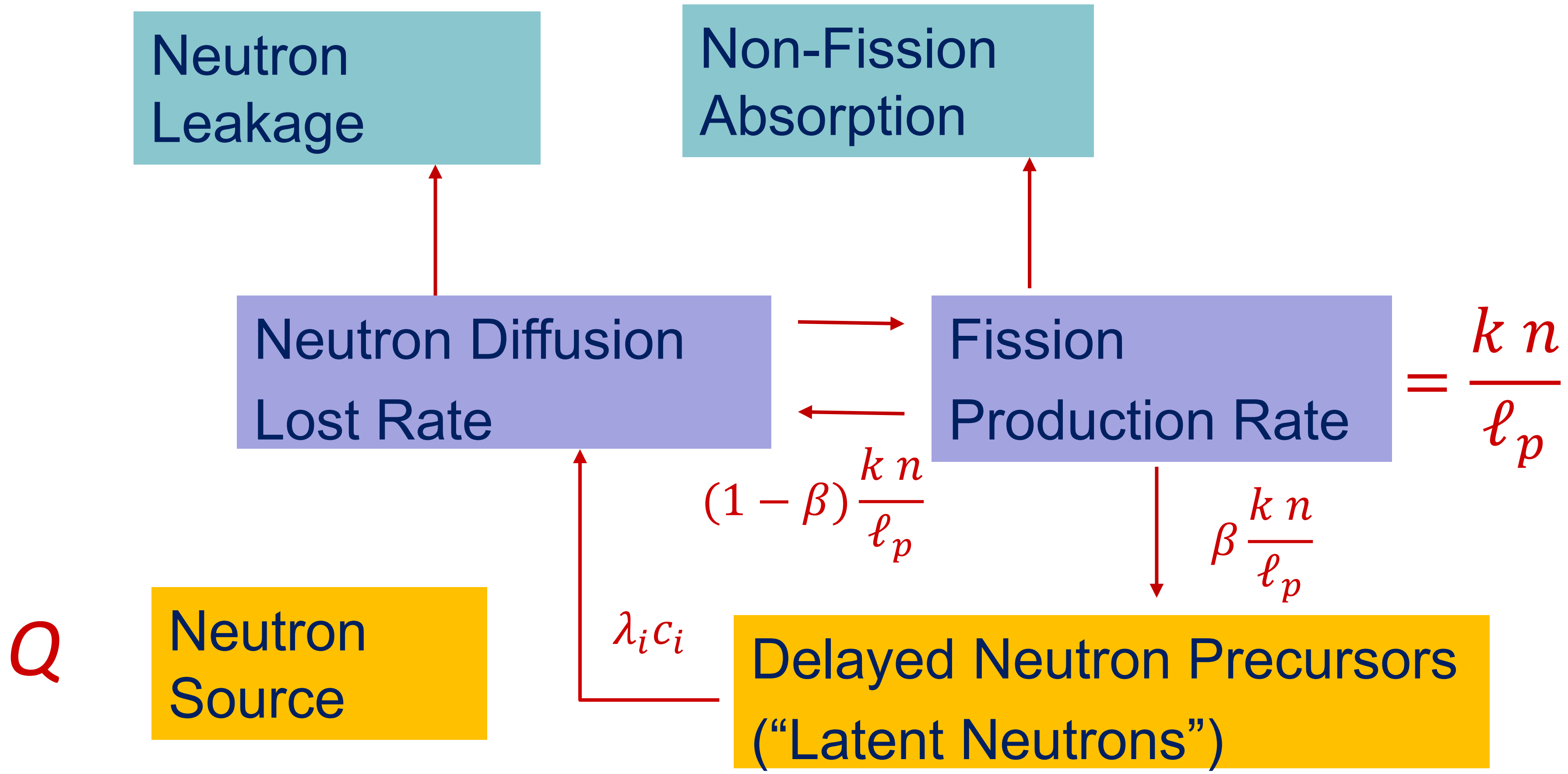
PROMPT NEUTRON LIFETIME AND MEAN DIFFUSION TIME



Dynamic Equation & Simplified Neutron Cycle

REACTOR KINETICS - SIMPLIFIED NEUTRON CYCLE

n(t) – number of neutrons in the system at time t
ℓ_p *– neutron lifetime (characteristic time)*
k – total number of neutrons (prompt and delayed) produced per neutron lost
$\beta = \sum_i \beta_i$ *– total delayed neutron fraction*
c_i *– number of precursors of i^th type*
λ_i *– decay constant of i^th precursor*
Q – external neutron source (#/s)



Reactor Kinetics - Simplified Neutron Cycle

The neutron balance equation is then:

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + \nu\Sigma_f\phi + Q$$

$$D\nabla^2\phi - \Sigma_a\phi = -n/\ell_p$$

$$\nu = \nu_p + \nu_d$$

$$\beta = \frac{\nu_d}{\nu}$$

$$\frac{\partial n}{\partial t} = -\frac{n}{\ell_p} + (1-\beta)\nu\Sigma_f\phi + \sum_{i=1}^{6}\lambda_i c_i + Q$$

$$\sum_{i=1}^{6}\lambda_i c_i = \beta k \frac{n}{\ell_p}$$

$$\frac{\partial n}{\partial t} = -\frac{n}{\ell_p} + (1-\beta)k\frac{n}{\ell_p} + \sum_{i=1}^{6}\lambda_i c_i + Q$$

$$\frac{\partial n}{\partial t} = \frac{k-1-\beta}{\ell_p}n + \sum_{i=1}^{6}\lambda_i c_i + Q$$

$$\frac{\partial c_i}{\partial t} = \frac{k\,\beta_i}{\ell_p}n - \lambda_i c_i$$

$$\rho = \frac{k-1}{k}$$

$$\Lambda = \frac{\ell_p}{k}$$

$$\frac{\partial n}{\partial t} = \frac{\rho-\beta}{\Lambda}n + \sum_{i=1}^{6}\lambda_i c_i + Q$$

$$\frac{\partial c_i}{\partial t} = \frac{\beta}{\Lambda}n - \lambda_i c_i$$

Total Delayed Neutron Yields & Yields of Delayed Neutron Groups

TOTAL DELAYED NEUTRON YIELDS

To recall:

The average total number of neutrons: $\nu = \nu_p + \nu_d$

The production of delayed neutrons is often described by the delayed neutron fraction: $\beta^{ph} = \frac{\nu_d}{\nu_d + \nu_p}$

Theoretical arguments and measurements show that ν_d is practically independent of the energy of the fission inducing neutron, i.e.,

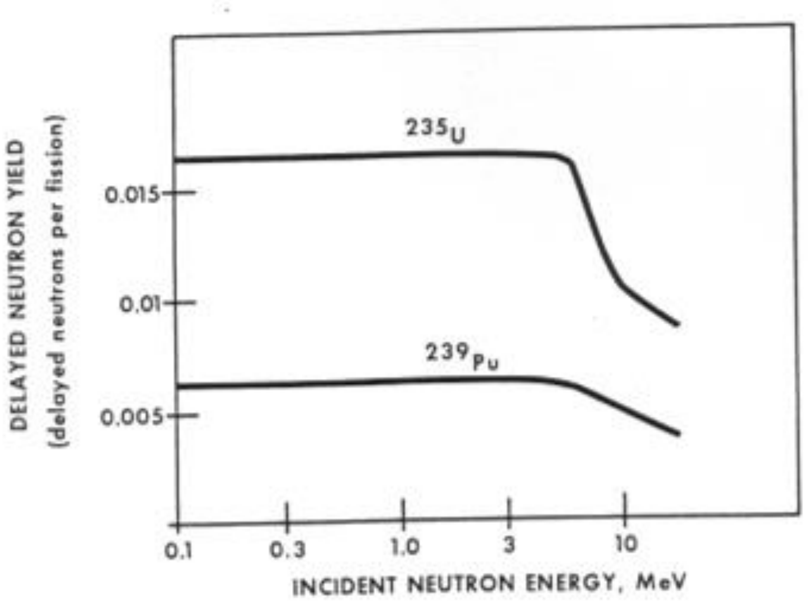
$$\nu_d(E) \approx \text{constant for } 0 \leq E \leq 4\ MeV$$

For comparison, the prompt yield, ν_p , shows a significant energy variation in this range (an increase of ~20%).

Therefore, by using $\beta^{ph} = \frac{\nu_d}{\nu_d + \nu_p}$ an artificial energy dependence would be introduced into the description of delayed neutrons.

TOTAL DELAYED NEUTRON YIELDS

Energy dependence of the delayed neutron yields of ^{235}U and ^{239}Pu:



The total delayed neutron yields are significantly different for different isotopes, but there seem to be two regularities:

Fission Nuclide	ν_d
^{233}U	0.0070 ±0.0004
^{235}U	0.0165 ±0.0005
^{238}U	0.0412 ±0.0017
^{239}Pu	0.0063 ±0.0003
^{240}Pu	0.0088 ±0.0006
^{241}Pu	0.0154 ±0.0015
^{242}Pu	0.0160 ±0.0050

1. *The total delayed neutron yield increases with increasing atomic weight for a given element.*

2. *The total delayed neutron yield decreases with increasing number of protons.*

Yields of Delayed Neutron Groups

Some facts:

‼ There are only about 40 out of approximately 500 different fission products that have the special property required for being a delayed neutron emitter.

‼ All 40 precursors have different lifetimes and therefore the corresponding neutrons will appear at different delay times.

→ *The consequence of the different precursor lifetimes is that the corresponding delayed neutrons will have a different effect on the time dependence of the neutron flux.*

Yields of Delayed Neutron Groups

The most direct way to consider the effect of the differences in the precursor lifetimes is to take all of them into account individually.

‼ This approach has three serious drawbacks:

1. *The lifetimes and abundances of many precursors are not known accurately enough.*

2. *Even if the lifetimes and abundances of all precursors were known accurately, their inclusion into the theoretical formulation of the kinetics problem would lead to an impractically lengthy set of differential equations.*

3. *Several precursors are themselves products of beta decays.*

Yields of Delayed Neutron Groups

Experimental knowledge on delayed neutron production was historically available in the form of average source curves, $S_d(t)$.

These source curves are obtained by exposing a sample of fissionable material to a very short neutron pulse, which instantaneously produces through fission a large number of precursors.

The decay of these precursors results in the source of delayed neutrons $S_d(t)$.

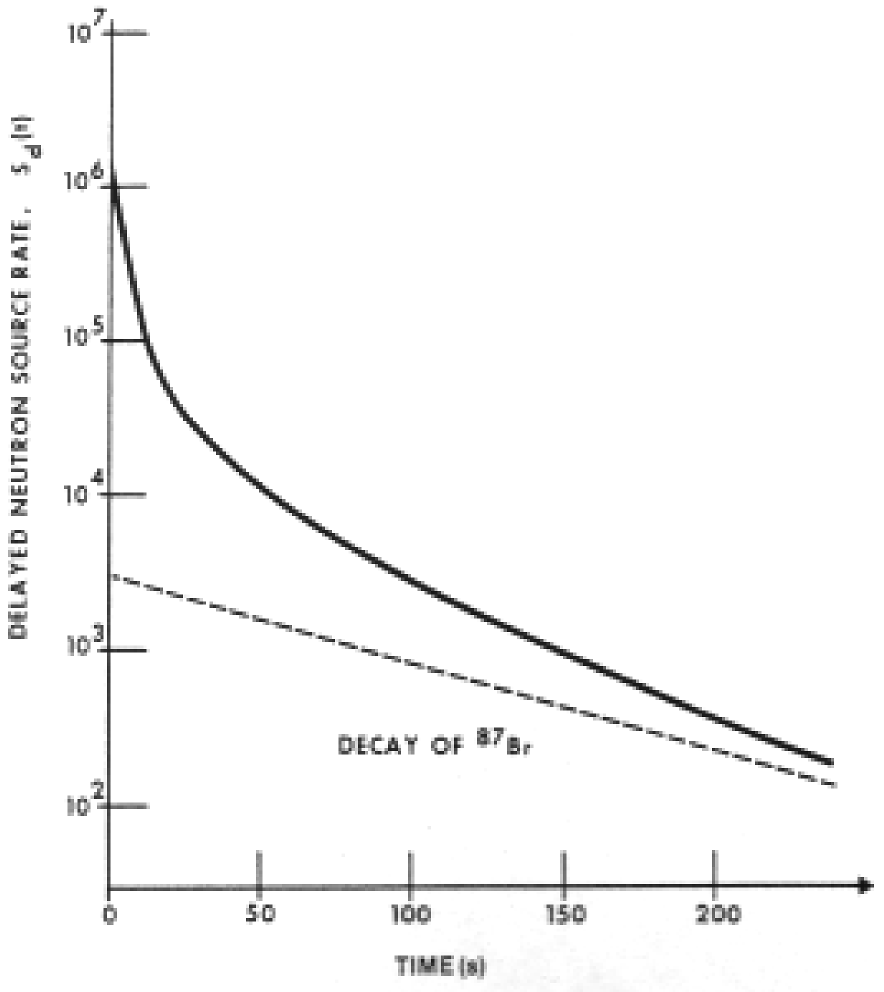
If n_f is the number of fissions that occurred in the sample during the flux pulse, then:

$$v_d n_f = total\ number\ of\ precursors.$$

The term $S_d(t)$ describes the decay rate of these precursors and thus the production rate of delayed neutrons.

YIELDS OF DELAYED NEUTRON GROUPS

Example of a source curve:



The decay rate of the single isotope ^{87}Br (Bromine) with its mean decay time of ~80 s.

The experimental points in the actual experiment are average values of 80 measured decay curves.

Physically, the decay curve $S_d(t)$ is a superposition of contributions from all precursors including those that are themselves beta decay products.

YIELDS OF DELAYED NEUTRON GROUPS

This complicated superposition $S_d(t)$ can be fairly accurately represented by just six exponential functions without including a buildup term, representing production of precursors through beta decay:

$$S_d(t) = n_f \sum_{k=1}^{6} \nu_{dk}\lambda_{dk}\exp(-\lambda_k t)$$

This equation represents a delayed neutron source that results from the decay of six "average" groups of precursors, all of them produced at t = 0.

The "delayed group" yields, ν_{dki}, and the average "delayed group" decay constants, λ_{dki}, are obtained for each isotope i by a least-squares fit of the right side of this equation to an experimentally determined left side:

$$S_{di}(t) = n_{fi} \sum_{k=1}^{6} \nu_{dki}\lambda_{dki}\exp(-\lambda_{ki} t)$$

This 6- delay group structure is in general use in reactor kinetics.

Yields of Delayed Neutron Groups

Some remarks:

1. In a thermal reactor (LWR) ^{235}U is the dominant fuel:

   → *In modeling, it is often assumed that all of the delayed neutrons are produced from fission products of ^{235}U.*

2. In high-burnup LWR fuel and in a fast breeder reactor (FBR) there are several nuclides that contribute to the delayed neutron source:

   The ^{238}U contribution in an FBR is comparable to the ^{239}Pu contribution because the ^{238}U delayed neutron yield is ~7 times larger than that for ^{239}Pu; thereby largely compensating for its lower fission rate.

3. Other contributions such as those of ^{240}Pu and ^{241}Pu may also be of practical significance.

4. The contribution of ^{242}Pu is generally small.

YIELDS OF DELAYED NEUTRON GROUPS

Some remarks:

If more than one isotope is of importance, the simplicity of the six-delay group representation is lost

→ *the total delayed neutron source has to be found as a sum of all isotopic contributions:*

$$S_{di}(t) = n_{fi} \sum_{k=1}^{6} \nu_{dki} \lambda_{dki} \exp(-\lambda_{ki} t)$$

$$\downarrow$$

$$S_{di}(t) = \sum_{i} n_{fi} \sum_{k=1}^{6} \nu_{dki} \lambda_{dki} \exp(-\lambda_{ki} t)$$

YIELDS OF DELAYED NEUTRON GROUPS

Some remarks:

The isotope dependence of the decay constants is not very pronounced, because most of them differ only within their statistical errors → *the use of a single set of isotope-independent decay constants can be expected to fit the experimental delayed neutron sources with only an insignificant loss of accuracy:*

$$S_{di}(t) = n_{fi} \sum_{k=1}^{6} \nu_{dki} \lambda_{dki} \exp(-\lambda_{ki} t)$$

↓

$$S_{di}(t) = \sum_{i} n_{fi} \sum_{k=1}^{6} \nu_{dki} \lambda_{k} \exp(-\lambda_{k} t)$$

Yields of Delayed Neutron Groups

Some remarks:

!!! The advantages of having a single set of decay constants are that:

1. Precursor concentrations and delayed neutron sources can be readily summed up for all isotopes.
2. Macroscopic cross sections can be introduced in the same way as that for prompt fission neutrons.

Next Class

Emission Spectra of Delayed Neutrons

Evaluation of Delayed Neutron Data

Theoretical Background for Calculation of Kinetics Data