12: Support Vector Machines (SVMs)

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Support Vector Machine (SVM) - Optimization objective

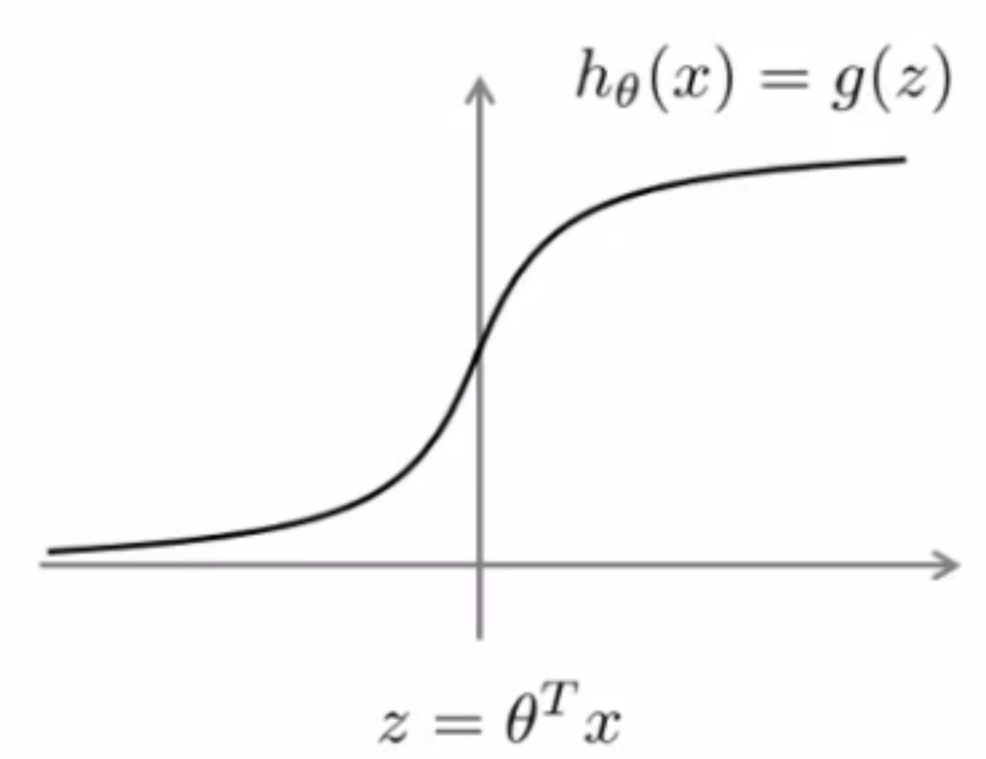
- So far, we've seen a range of different algorithms
    - With supervised learning algorithms - performance is pretty similar
        - What matters more often is;
            - The amount of training data
            - Skill of applying algorithms
- One final supervised learning algorithm that is widely used - **support vector machine (SVM)**
    - Compared to both logistic regression and neural networks, a SVM sometimes gives a cleaner way of learning non-linear functions
    - Later in the course we'll do a survey of different supervised learning algorithms

An alternative view of logistic regression

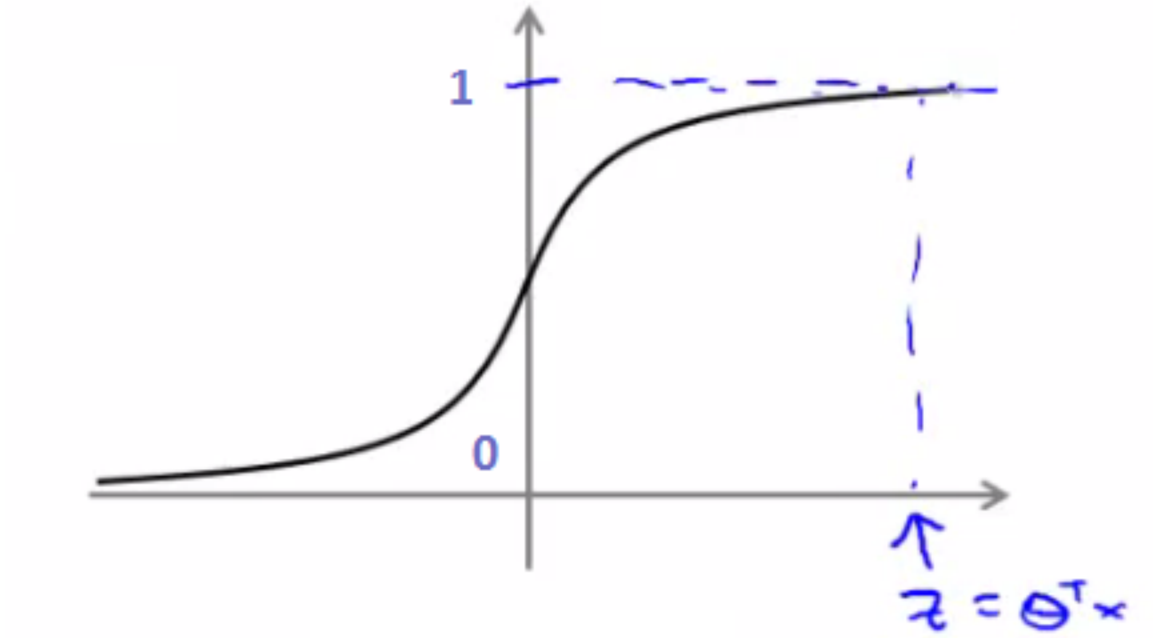
- Start with logistic regression, see how we can modify it to get the SVM
    - As before, the logistic regression hypothesis is as follows

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

    - And the sigmoid activation function looks like this



    - In order to explain the math, we use z as defined above
- What do we want logistic regression to do?
    - We have an example where y = 1
        - Then we hope $h_\theta(x)$ is close to 1
        - With $h_\theta(x)$ close to 1, $(\theta^T x)$ must be **much larger** than 0



    - Similarly, when y = 0

        - Then we hope $h_\theta(x)$ is close to 0
        - With $h_\theta(x)$ close to 0, (θ^T x) must be **much less** than 0
    - This is our classic view of logistic regression
        - Let's consider another way of thinking about the problem
- Alternative view of logistic regression
    - If you look at cost function, each example contributes a term like the one below to the overall cost function

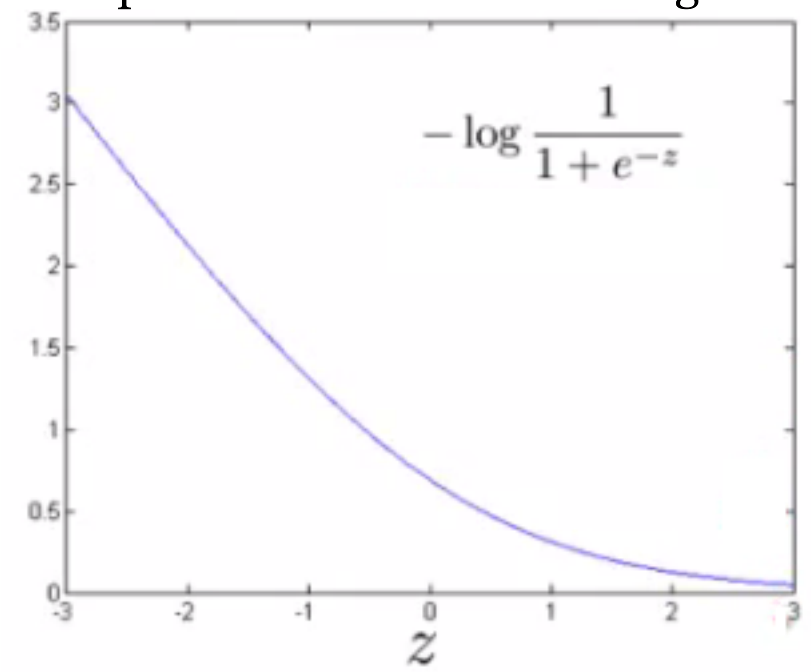
$$-(y \log h_\theta(x) + (1-y)\log(1-h_\theta(x)))$$

        - For the overall cost function, we sum over all the training examples using the above function, and have a 1/m term
- If you then plug in the hypothesis definition ($h_\theta(x)$), you get an expanded cost function equation;

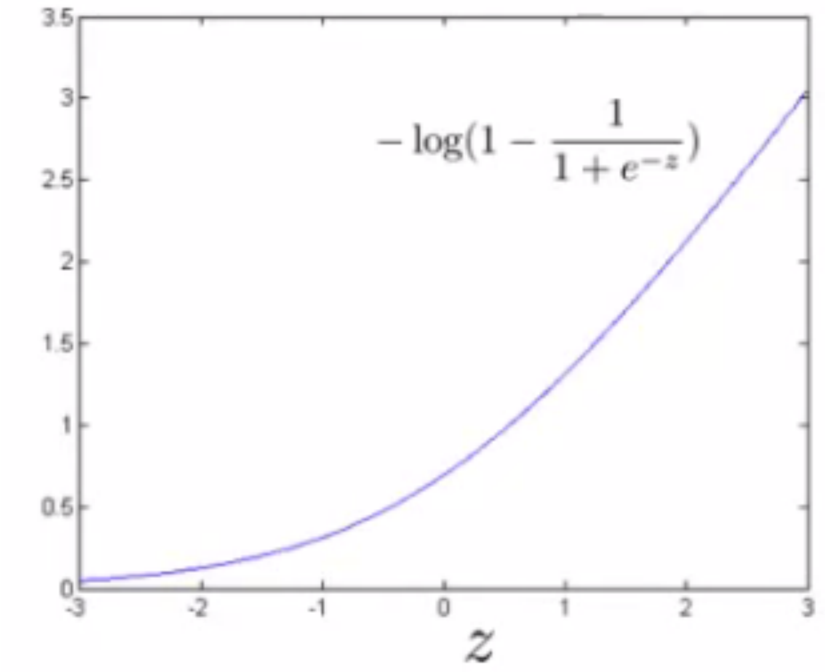
$$= -y \log \frac{1}{1+e^{-\theta^T x}} - (1-y)\log(1 - \frac{1}{1+e^{-\theta^T x}})$$

    - So each training example contributes that term to the cost function for logistic regression

- If y = 1 then only the first term in the objective matters
    - If we plot the functions vs. z we get the following graph



        - This plot shows the cost contribution of an example when y = 1 given z
            - So if z is big, the cost is low - this is good!
            - But if z is 0 or negative the cost contribution is high
            - This is why, when logistic regression sees a positive example, it tries to set θ^T x to be a very large term
- If y = 0 then only the second term matters
    - We can again plot it and get a similar graph

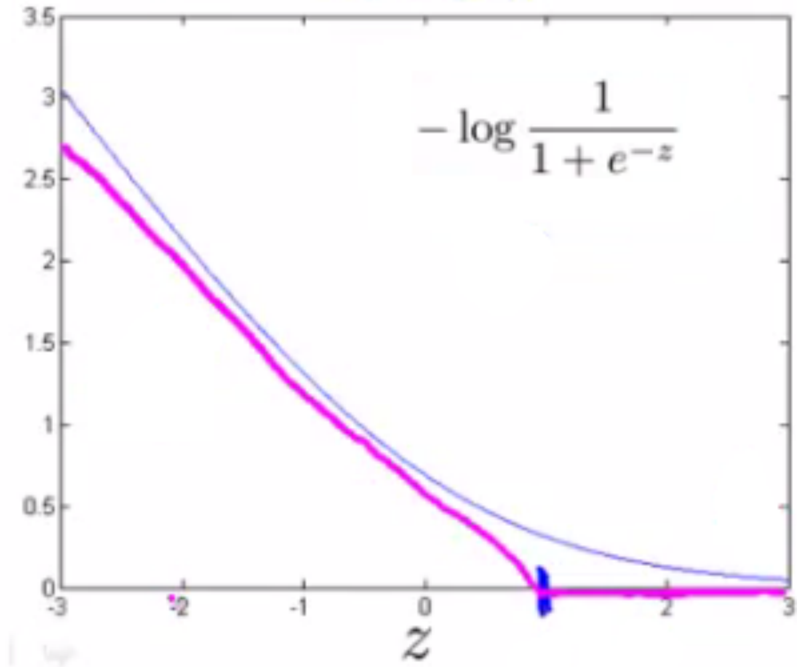


        - Same deal, if z is small then the cost is low
            - But if s is large then the cost is massive

SVM cost functions from logistic regression cost functions

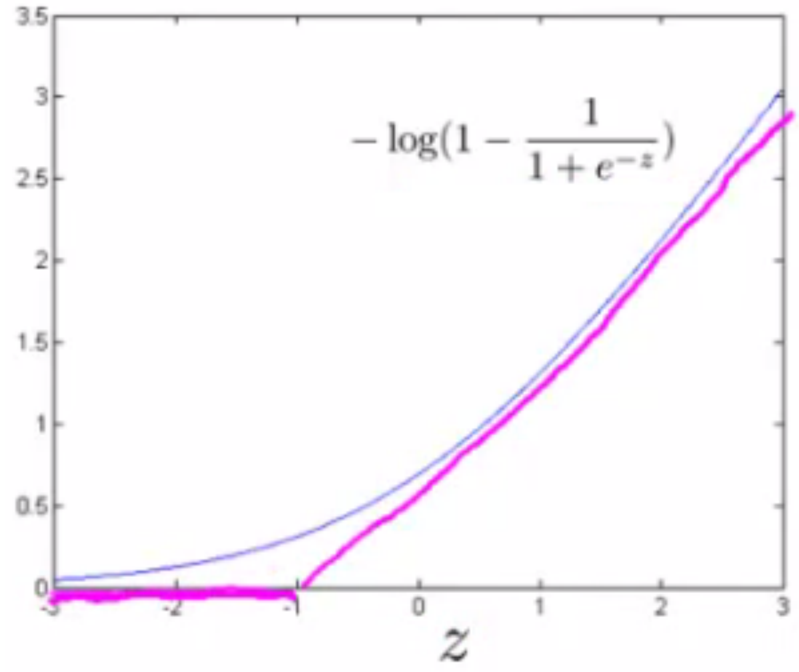
- To build a SVM we must redefine our cost functions

    - When y = 1
        - Take the y = 1 function and create a new cost function
        - Instead of a curved line create two straight lines (magenta) which acts as an approximation to the logistic regression y = 1 function



            - Take point (1) on the z axis
                - Flat from 1 onwards
                - Grows when we reach 1 or a lower number
            - This means we have two straight lines
                - Flat when cost is 0
                - Straight growing line after 1
        - So this is the new y=1 cost function
            - Gives the SVM a computational advantage and an easier optimization problem
            - We call this function **cost$_1$(z)**

- Similarly
    - When y = 0
        - Do the equivalent with the y=0 function plot



        - We call this function **cost$_0$(z)**
- So here we define the two cost function terms for our SVM graphically
    - How do we implement this?

The complete SVM cost function

- As a comparison/reminder we have logistic regression below

$$\min_\theta \frac{1}{m} \left[\sum_{i=1}^{m} y^{(i)} \left(-\log h_\theta(x^{(i)}) \right) + (1 - y^{(i)}) \left((-\log(1 - h_\theta(x^{(i)})) \right) \right] + \frac{\lambda}{2m} \sum_{j=1}^{n} \theta_j^2$$

    - If this looks unfamiliar its because we previously had the - sign outside the expression
- For the SVM we take our two logistic regression y=1 and y=0 terms described previously and replace with
    - cost$_1$(θ^T x)
    - cost$_0$(θ^T x)
- So we get

$$\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} cost_1(\theta^T x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T x^{(i)}) \right] + \frac{\lambda}{2m} \sum_{i=1}^{n} \theta_j^2$$

SVM notation is slightly different

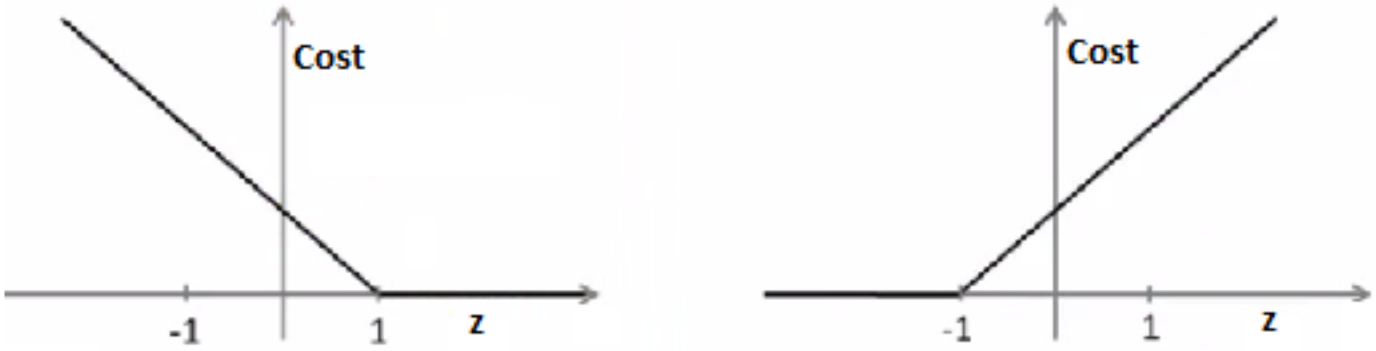
- In convention with SVM notation we rename a few things here
- 1) Get rid of the 1/m terms
    - This is just a slightly different convention
    - By removing 1/m we should get the same optimal values for
        - 1/m is a constant, so should get same optimization
        - e.g. say you have a minimization problem which minimizes to u = 5
            - If your cost function * by a constant, you still generates the minimal value
            - That minimal value is different, but that's irrelevant
- 2) For logistic regression we had two terms;
    - Training data set term (i.e. that we sum over m) = **A**
    - Regularization term (i.e. that we sum over n) = **B**
        - So we could describe it as A + λB
        - Need some way to deal with the trade-off between regularization and data set terms
        - Set different values for λ to parametrize this trade-off
    - Instead of parameterization this as A + λB
        - For SVMs the convention is to use a different parameter called C
        - So do CA + B
        - If C were equal to 1/λ then the two functions (CA + B and A + λB) would give the same value
- So, our overall equation is

$$\min_{\theta} C \sum_{i=1}^{m} \left[y^{(i)} cost_1(\theta^T x^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T x^{(i)}) \right] + \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$

- Unlike logistic, $h_\theta(x)$ doesn't give us a probability, but instead we get a direct prediction of 1 or 0
    - So if θ^T x is equal to or greater than 0 --> $h_\theta(x) = 1$
    - Else --> $h_\theta(x) = 0$

Large margin intuition

- Sometimes people refer to SVM as **large margin classifiers**
    - We'll consider what that means and what an SVM hypothesis looks like
    - The SVM cost function is as above, and we've drawn out the cost terms below

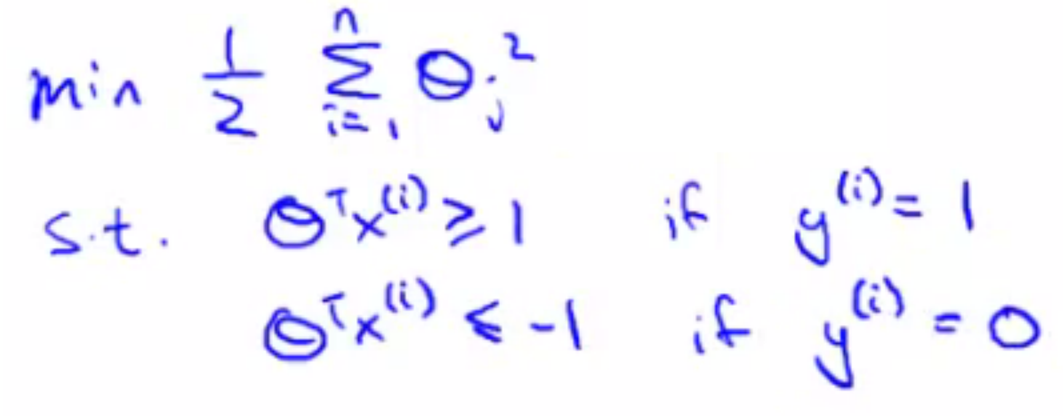


If $y = 1$, we want $\theta^T x \geq 1$ (not just ≥ 0)

If $y = 0$, we want $\theta^T x \leq -1$ (not just < 0)

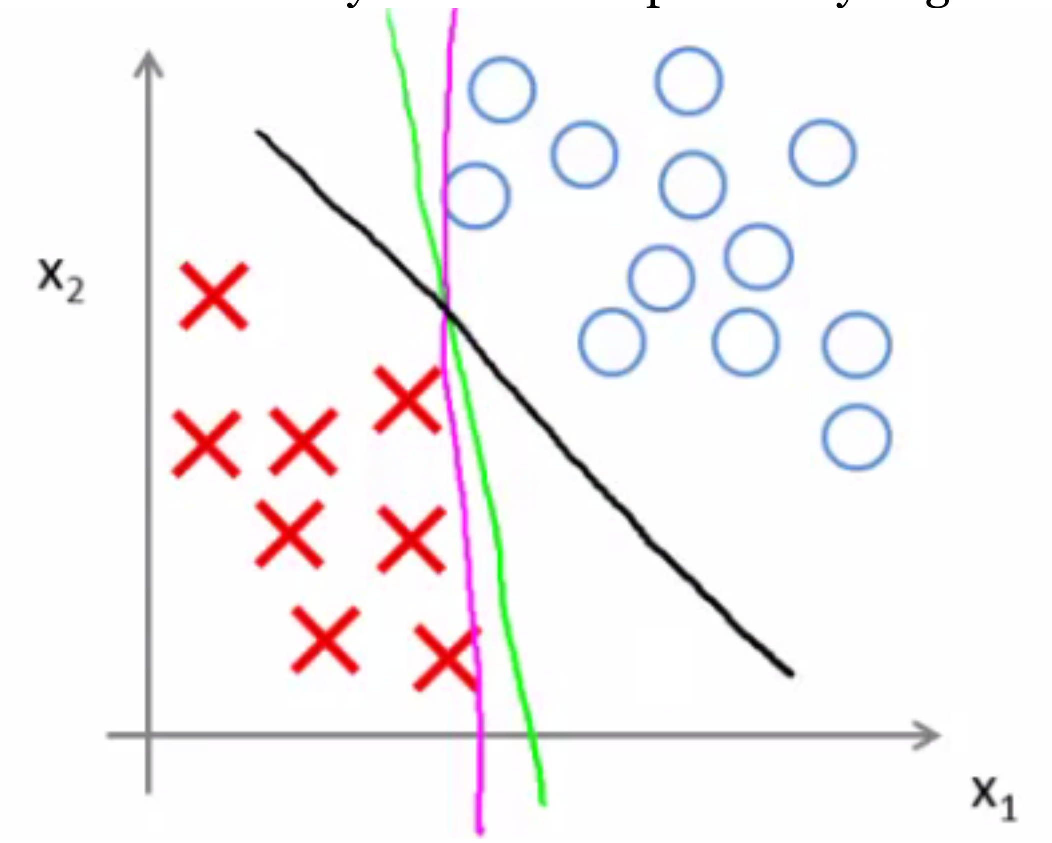
-
    - Left is cost_1 and right is cost_0
    - What does it take to make terms small
        - If y =1
            - $\text{cost}_1(z) = 0$ only when z >= 1
        - If y = 0
            - $\text{cost}_0(z) = 0$ only when z <= -1
    - Interesting property of SVM

        - If you have a positive example, you only really *need* z to be greater or equal to 0
            - If this is the case then you predict 1
        - SVM wants a bit more than that - doesn't want to *just* get it right, but have the value be quite a bit bigger than zero
            - Throws in an extra safety margin factor
- Logistic regression does something similar
- What are the consequences of this?
    - Consider a case where we set C to be huge
        - C = 100,000
        - So considering we're minimizing CA + B
            - If C is huge we're going to pick an A value so that A is equal to zero
            - What is the optimization problem here - how do we make A = 0?
        - Making A = 0
            - If y = 1
                - Then to make our "A" term 0 need to find a value of θ so (θ^T x) is greater than or equal to 1
            - Similarly, if y = 0
                - Then we want to make "A" = 0 then we need to find a value of θ so (θ^T x) is equal to or less than -1
        - So - if we think of our optimization problem a way to ensure that this first "A" term is equal to 0, we re-factor our optimization problem into just minimizing the "B" (regularization) term, because
            - When A = 0 --> A*C = 0
        - So we're minimizing B, under the constraints shown below

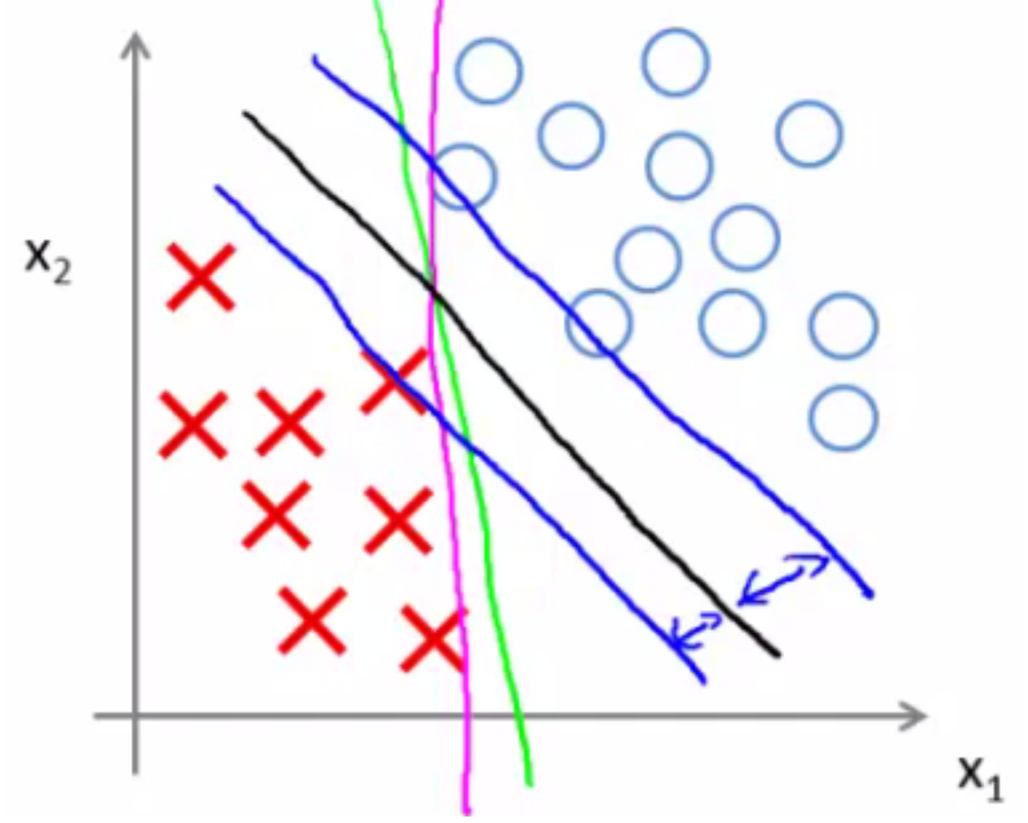


$$\min \frac{1}{2} \sum_{i=1}^{n} \theta_j^2$$
$$\text{s.t.} \quad \theta^T x^{(i)} \geq 1 \quad \text{if } y^{(i)} = 1$$
$$\theta^T x^{(i)} \leq -1 \quad \text{if } y^{(i)} = 0$$

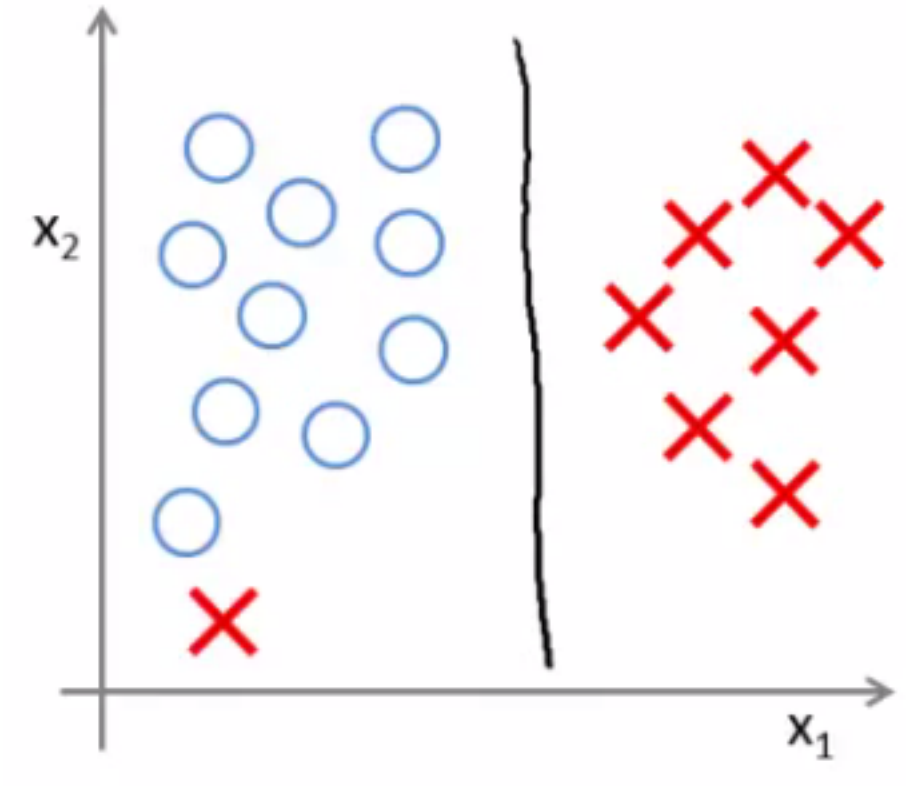
    - Turns out when you solve this problem you get interesting decision boundaries



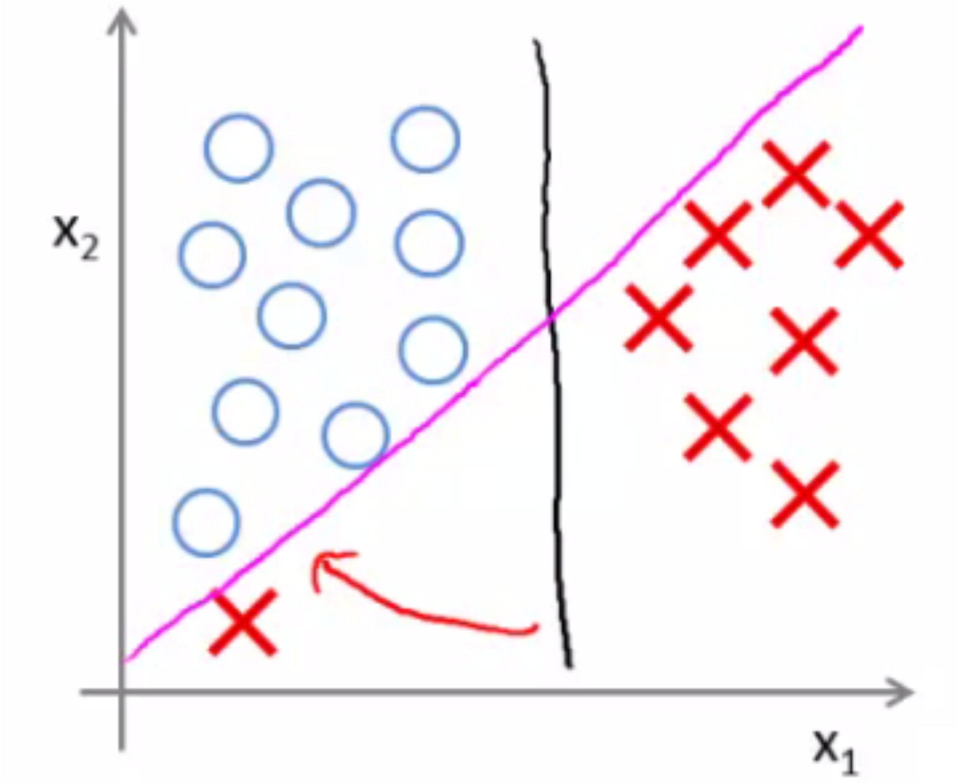
    - The green and magenta lines are functional decision boundaries which could be chosen by logistic regression
        - But they probably don't generalize too well
    - The black line, by contrast is the the chosen by the SVM because of this safety net imposed by the optimization graph
        - More robust separator
    - Mathematically, that black line has a larger minimum distance (margin) from any of the training examples



  - By separating with the largest margin you incorporate robustness into your decision making process
- We looked at this at when C is very large
  - SVM is more sophisticated than the large margin might look
    - If you were just using large margin then SVM would be very sensitive to outliers



    - You would risk making a ridiculous hugely impact your classification boundary
      - A single example might not represent a good reason to change an algorithm
      - If C is very large then we *do* use this quite naive maximize the margin approach



      - So we'd change the black to the magenta
    - But if C is reasonably small, or a not too large, then you stick with the black decision boundary
  - What about non-linearly separable data?
    - Then SVM still does the right thing if you use a normal size C
    - So the idea of SVM being a large margin classifier is only really relevant when you have no outliers and you can easily linearly separable data
  - Means we ignore a few outliers

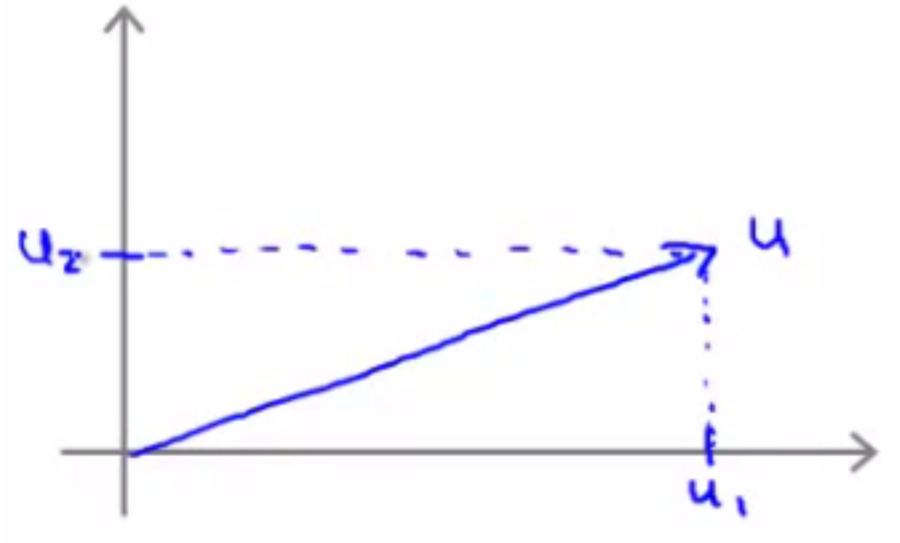
Large margin classification mathematics (optional)

Vector inner products

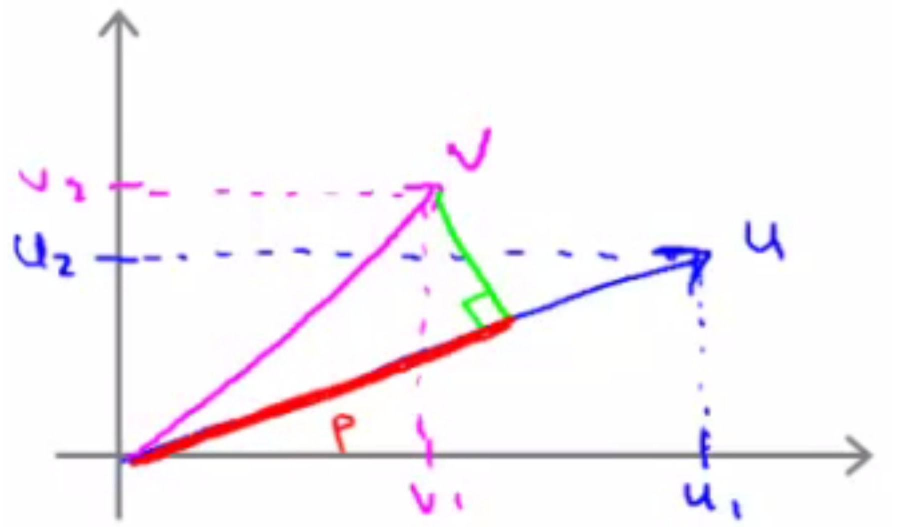
- Have two (2D) vectors u and v - what is the inner product ($u^T v$)?

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

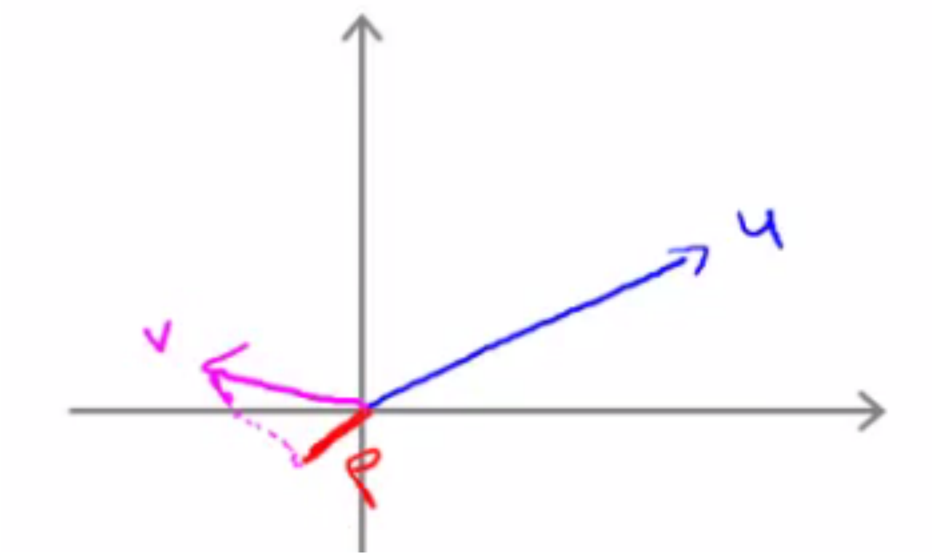
  - Plot u on graph
    - i.e u_1 vs. u_2



  - One property which is good to have is the **norm** of a vector
    - Written as ||u||
      - This is the euclidean length of vector u
    - So ||u|| = SQRT($u_1^2 + u_2^2$) = real number
      - i.e. length of the arrow above
      - Can show via Pythagoras
  - For the inner product, take v and orthogonally project down onto u
    - First we can plot v on the same axis in the same way (v_1 vs v_1)
    - Measure the length/magnitude of the projection

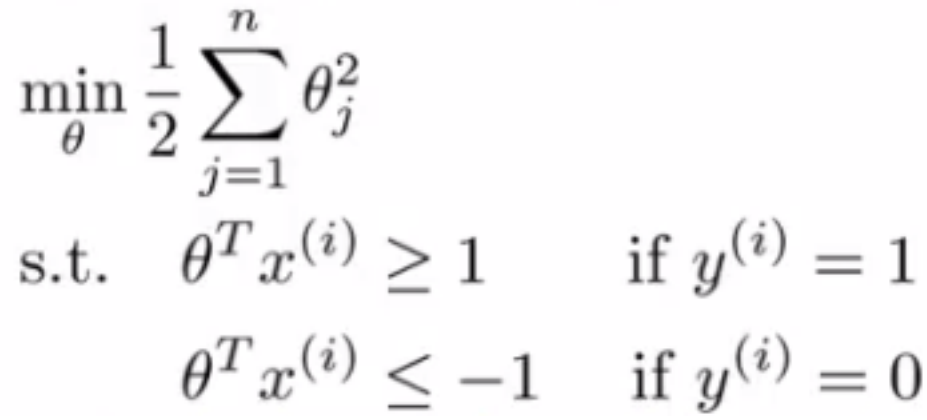


    - So here, the green line is the projection
      - p = length along u to the intersection
      - p is the magnitude of the projection of vector v onto vector u
  - Possible to show that
    - $u^T v$ = p * ||u||
      - So this is one way to compute the inner product
    - $u^T v = u_1v_1 + u_2v_2$
    - So therefore
      - **p * ||u|| = u_1v_1+ u_2v_2**
      - This is an important rule in linear algebra
    - We can reverse this too
      - So we could do
        - $v^T u = v_1u_1$+ v$_2u_2$
        - Which would obviously give you the same number
  - p can be negative if the angle between them is 90 degrees or more



  - So here p is negative
- Use the vector inner product theory to try and understand SVMs a little better

SVM decision boundary

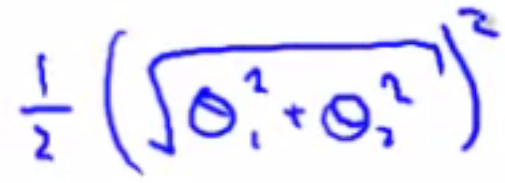


$$\min_{\theta} \frac{1}{2}\sum_{j=1}^{n}\theta_j^2$$
$$\text{s.t.}\quad \theta^T x^{(i)} \geq 1 \quad \text{if } y^{(i)} = 1$$
$$\theta^T x^{(i)} \leq -1 \quad \text{if } y^{(i)} = 0$$

- For the following explanation - two simplification
  - Set θ_0= 0 (i.e. ignore intercept terms)
  - Set n = 2 - (x_1, x_2)
    - i.e. each example has only 2 features
- Given we only have two parameters we can simplify our function to

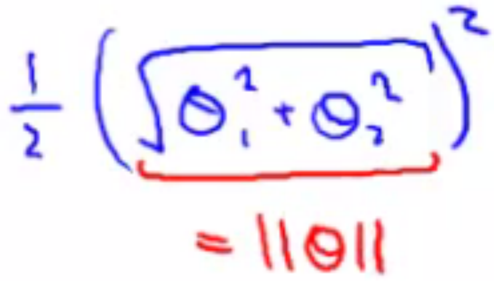
$$\frac{1}{2}(\theta_1^2 + \theta_2^2)$$

- And, can be re-written as



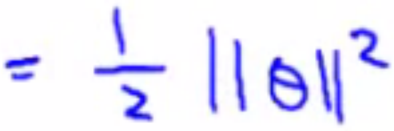
$$\frac{1}{2}\left(\sqrt{\theta_1^2 + \theta_2^2}\right)^2$$

  - Should give same thing
- We may notice that



$$\frac{1}{2}\left(\underbrace{\sqrt{\theta_1^2 + \theta_2^2}}_{= \|\theta\|}\right)^2$$

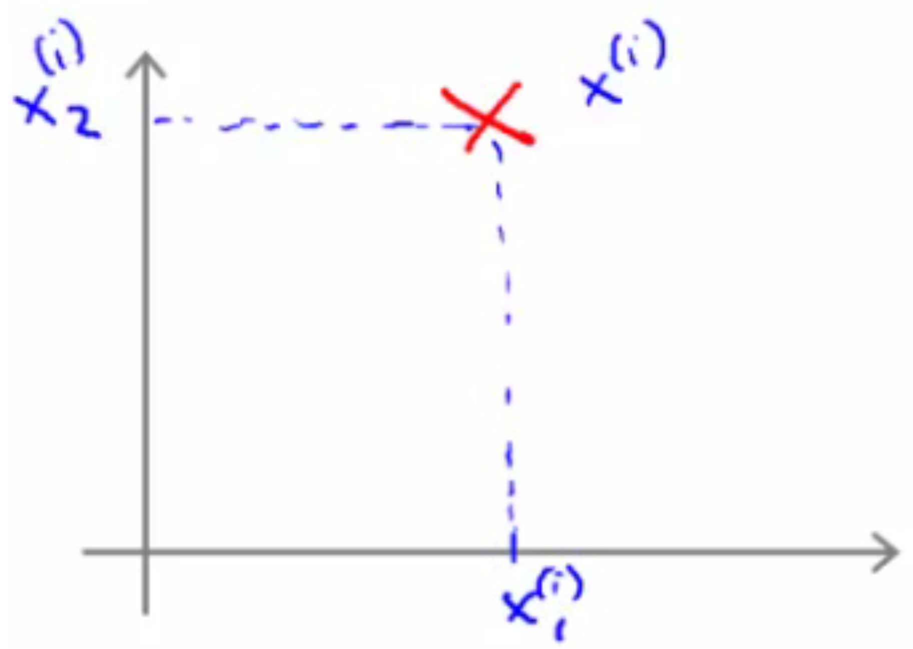
  - The term in red is the norm of θ
    - If we take θ as a 2x1 vector
    - If we assume θ_0 = 0 its still true
- So, finally, this means our optimization function can be re-defined as



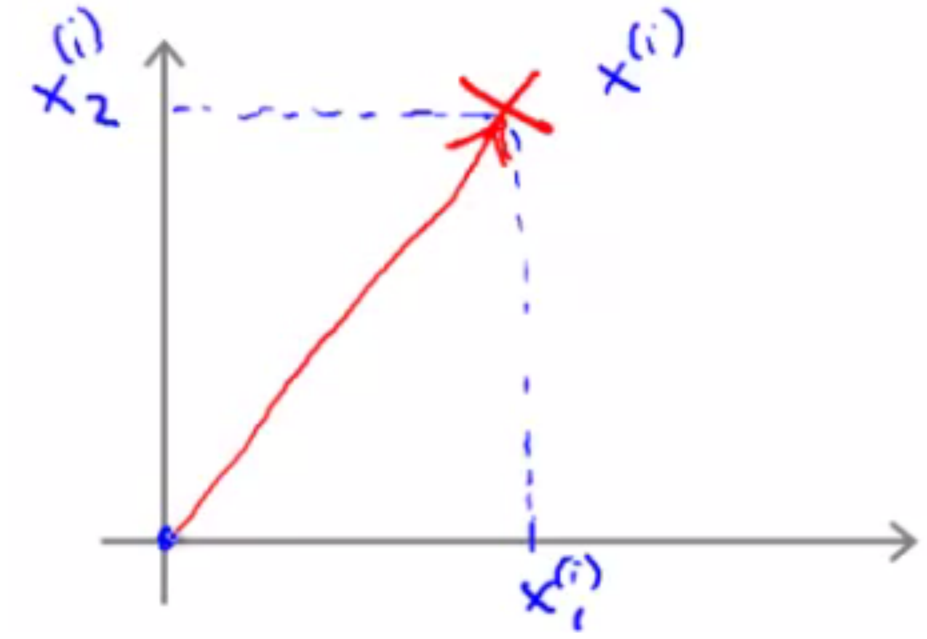
$$= \frac{1}{2}\|\theta\|^2$$

- So the SVM is minimizing the squared norm

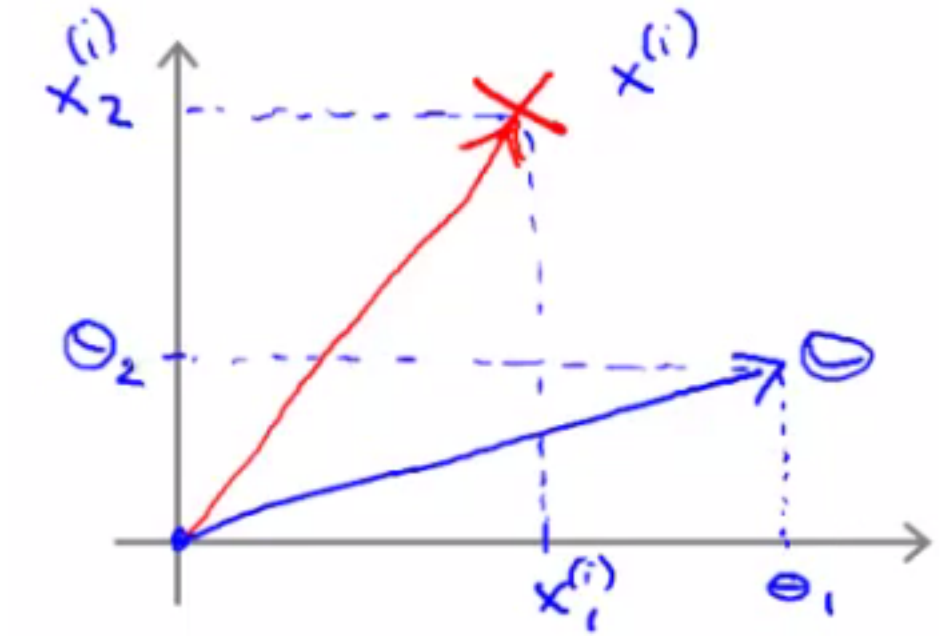
- Given this, what are the (θ^T x) parameters doing?
  - Given θ and given example x what is this equal to
    - We can look at this in a comparable manner to how we just looked at u and v
  - Say we have a single positive training example (red cross below)



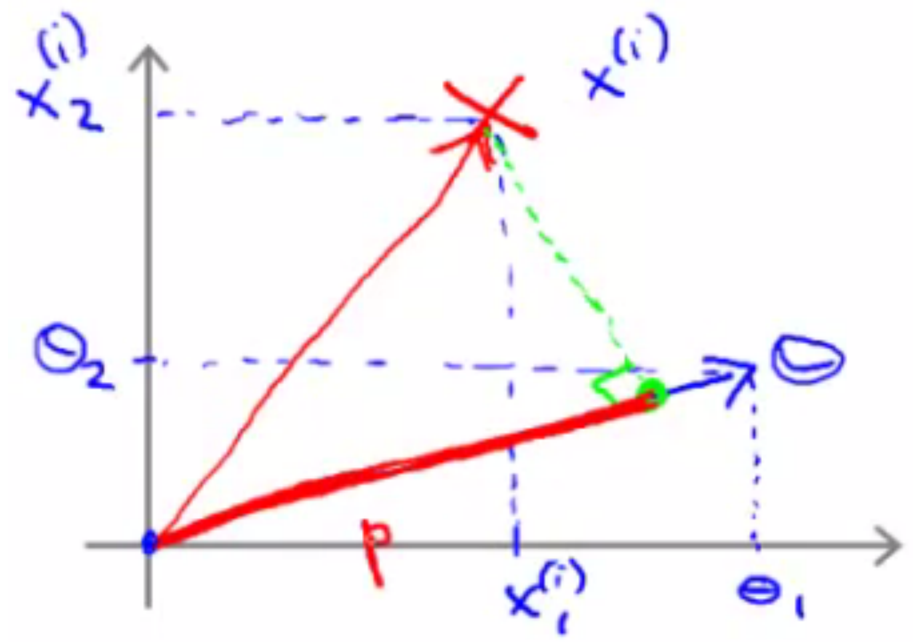
- Although we haven't been thinking about examples as vectors it can be described as such



- Now, say we have our parameter vector θ and we plot that on the same axis



- The next question is what is the inner product of these two vectors



  - p, is in fact p^i, because it's the length of p for example i
    - Given our previous discussion we know
      $(\theta^T x^i) = p^i * ||\theta||$
      $= \theta_1 x^i_1 + \theta_2 x^i_2$
    - So these are both equally valid ways of computing $\theta^T x^i$

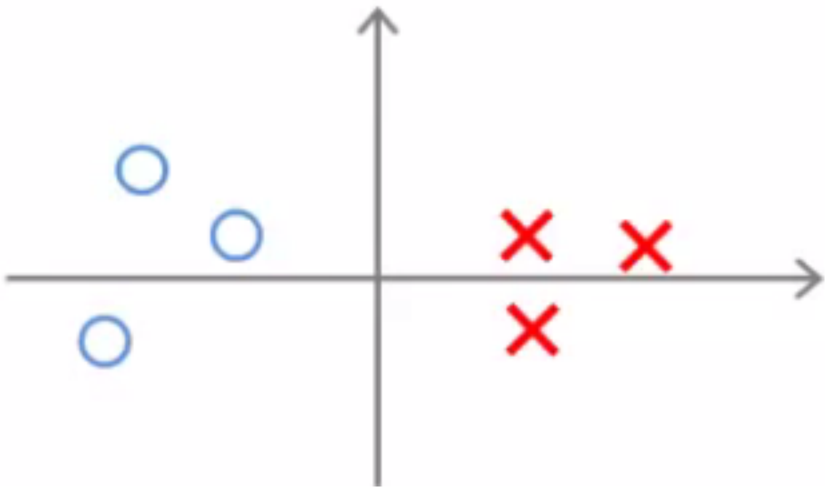
- What does this mean?
  - The constraints we defined earlier
    - $(\theta^T x) >= 1$ if $y = 1$

    - (θ^T x) <= -1 if y = 0
  - Can be replaced/substituted with the constraints
    - p^i * ||θ|| >= 1 if y = 1
    - p^i * ||θ|| <= -1 if y = 0
  - Writing that into our optimization objective

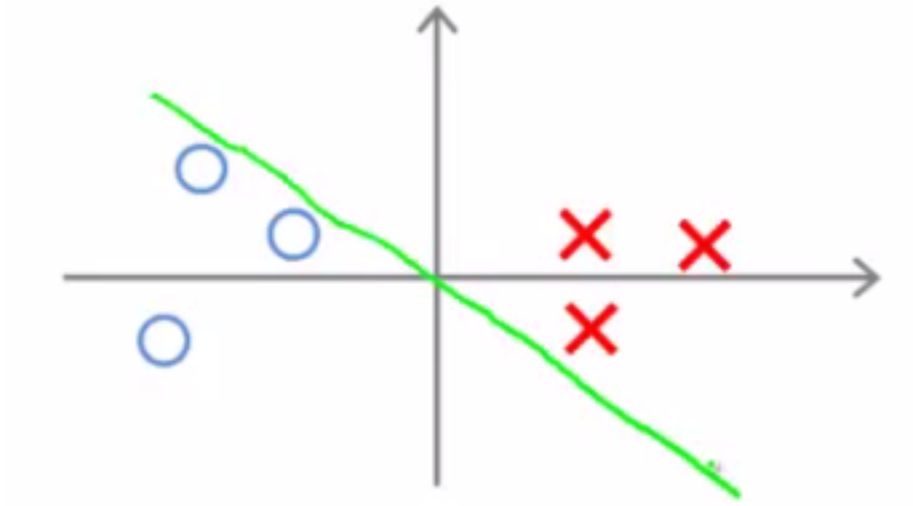
$$\min_\theta \frac{1}{2}\sum_{j=1}^{n}\theta_j^2 = \frac{1}{2}\left\|\theta\right\|^2$$

$$\begin{aligned} \text{s.t.} \quad & p^{(i)} \cdot \|\theta\| \geq 1 \quad \text{if } y^{(i)} = 1 \\ & p^{(i)} \cdot \|\theta\| \leq -1 \;\; \text{if } y^{(i)} = 1 \end{aligned}$$

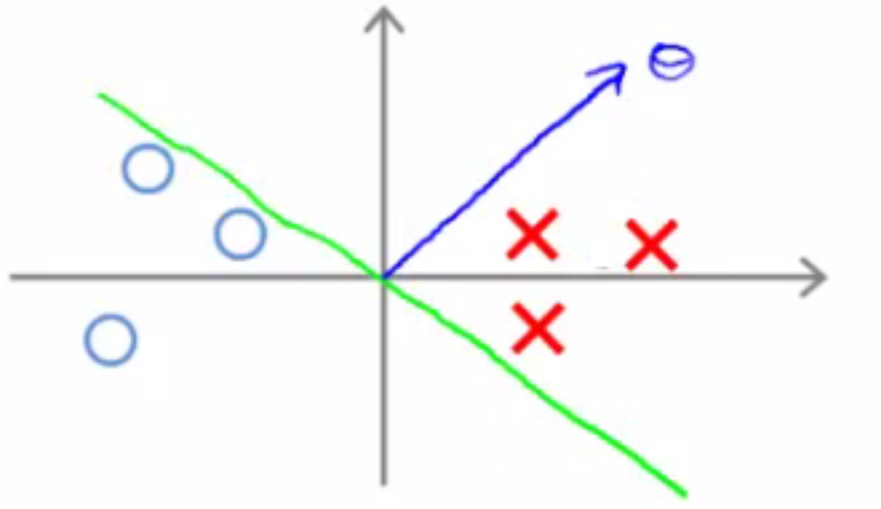
- So, given we've redefined these functions let us now consider the training example below



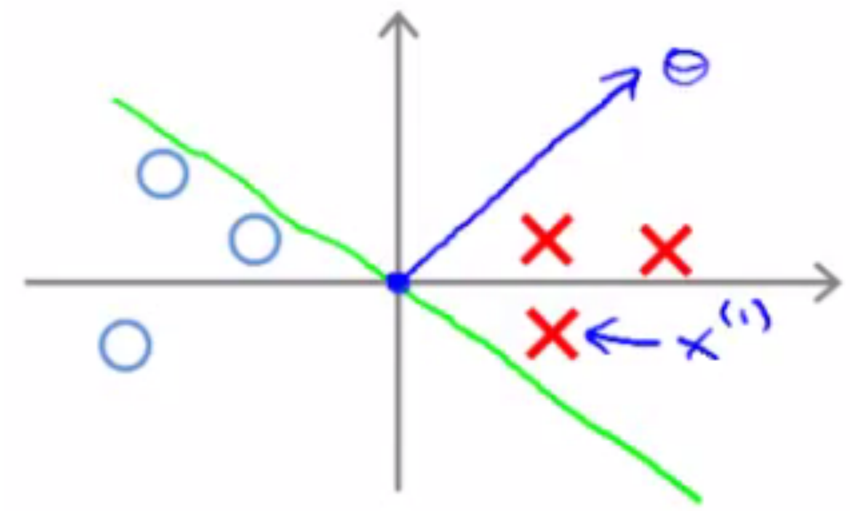
  - Given this data, what boundary will the SVM choose? Note that we're still assuming θ_0 = 0, which means the boundary has to pass through the origin (0,0)
    - Green line - small margins



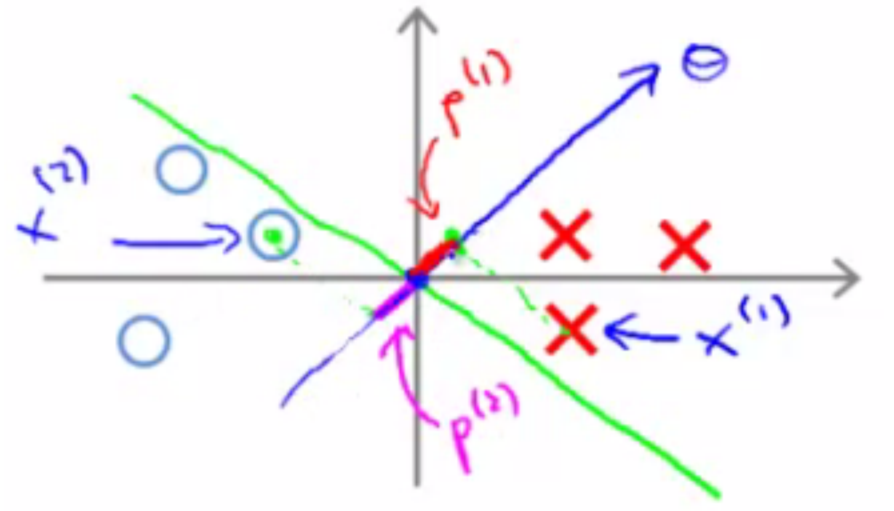
      - SVM would not chose this line
        - Decision boundary comes very close to examples
        - Lets discuss *why* the SVM would **not** chose this decision boundary
  - Looking at this line
    - We can show that θ is at 90 degrees to the decision boundary



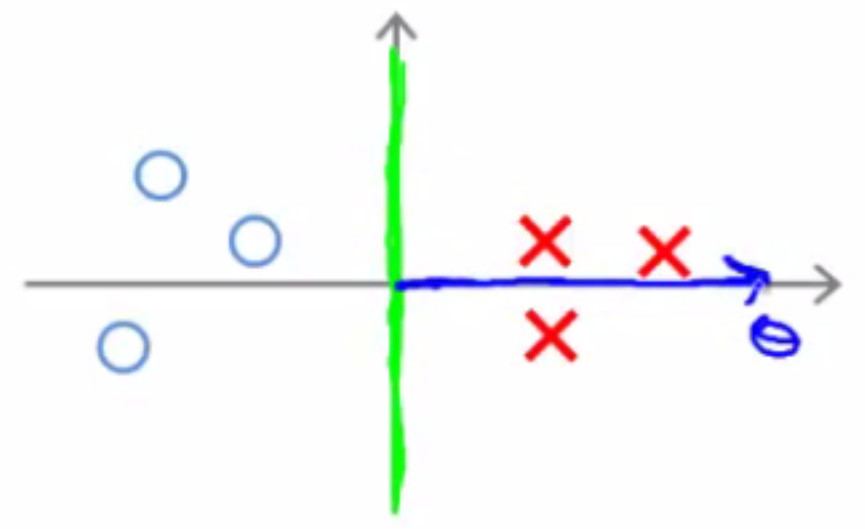
      - **θ is always at 90 degrees to the decision boundary** (can show with linear algebra, although we're not going to!)
- So now lets look at what this implies for the optimization objective
  - Look at first example (x^1)



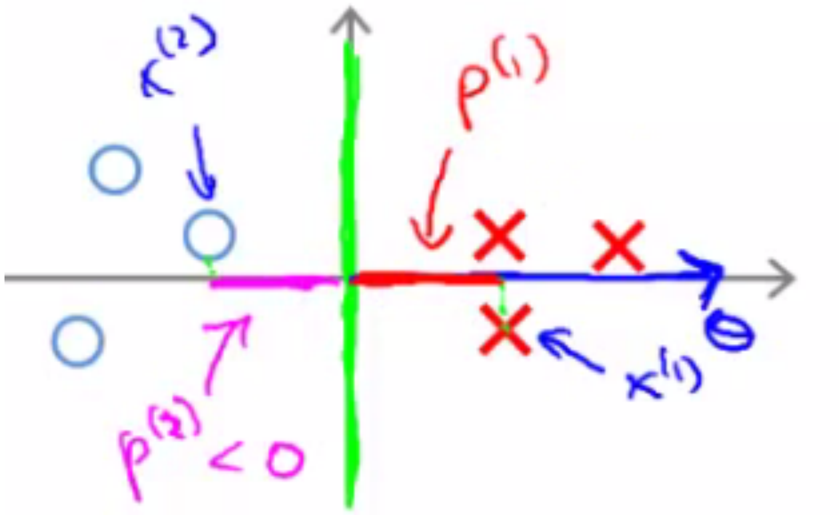
   - Project a line from x^1 on to to the θ vector (so it hits at 90 degrees)
      - The distance between the intersection and the origin is (**p^1**)
   - Similarly, look at second example (x^2)
      - Project a line from x^2 into to the θ vector
      - This is the magenta line, which will be **negative** (**p^2**)
   - If we overview these two lines below we see a graphical representation of what's going on;



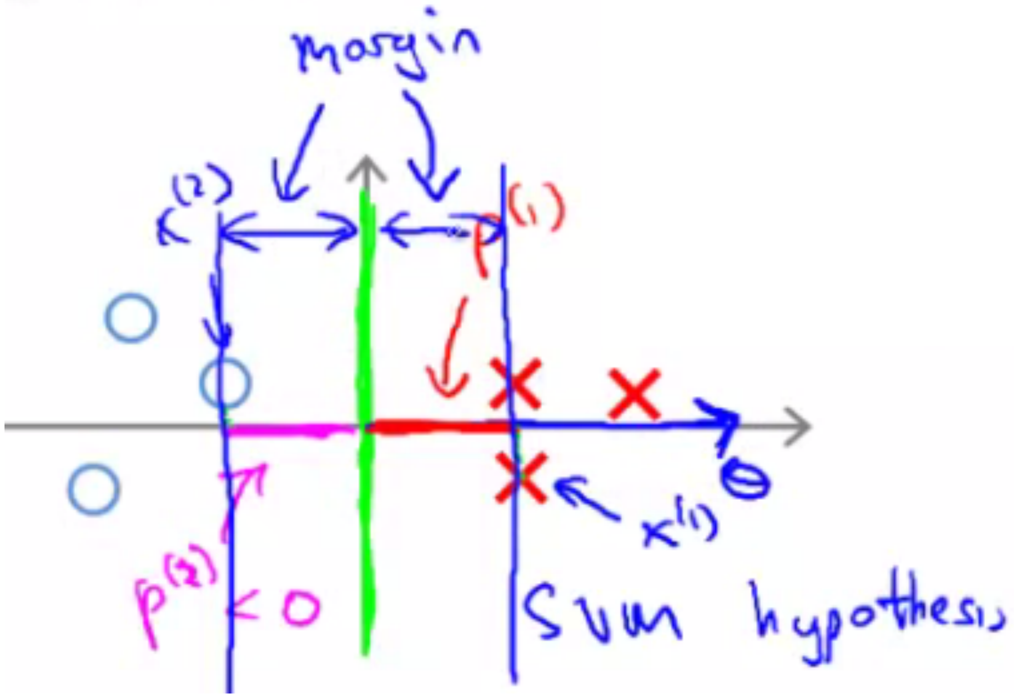
   - We find that both these p values are going to be pretty small
   - If we look back at our optimization objective
      - We know we need p^1 * ||θ|| to be bigger than or equal to 1 for positive examples
         - If p is small
            - Means that ||θ|| must be pretty large
      - Similarly, for negative examples we need p^2 * ||θ|| to be smaller than or equal to -1
         - We saw in this example p^2 is a small negative number
            - So ||θ|| must be a large number
   - Why is this a problem?
      - The optimization objective is trying to find a set of parameters where the norm of theta is small
         - So this doesn't seem like a good direction for the parameter vector (because as p values get smaller ||θ|| must get larger to compensate)
            - So we should make p values larger which allows ||θ|| to become smaller
- So lets chose a different boundary



   - Now if you look at the projection of the examples to θ we find that p^1 becomes large and ||θ|| can become small
   - So with some values drawn in



- This means that by choosing this second decision boundary we can make ||θ|| smaller
    - Which is why the SVM choses this hypothesis as better
    - This is how we generate the large margin effect

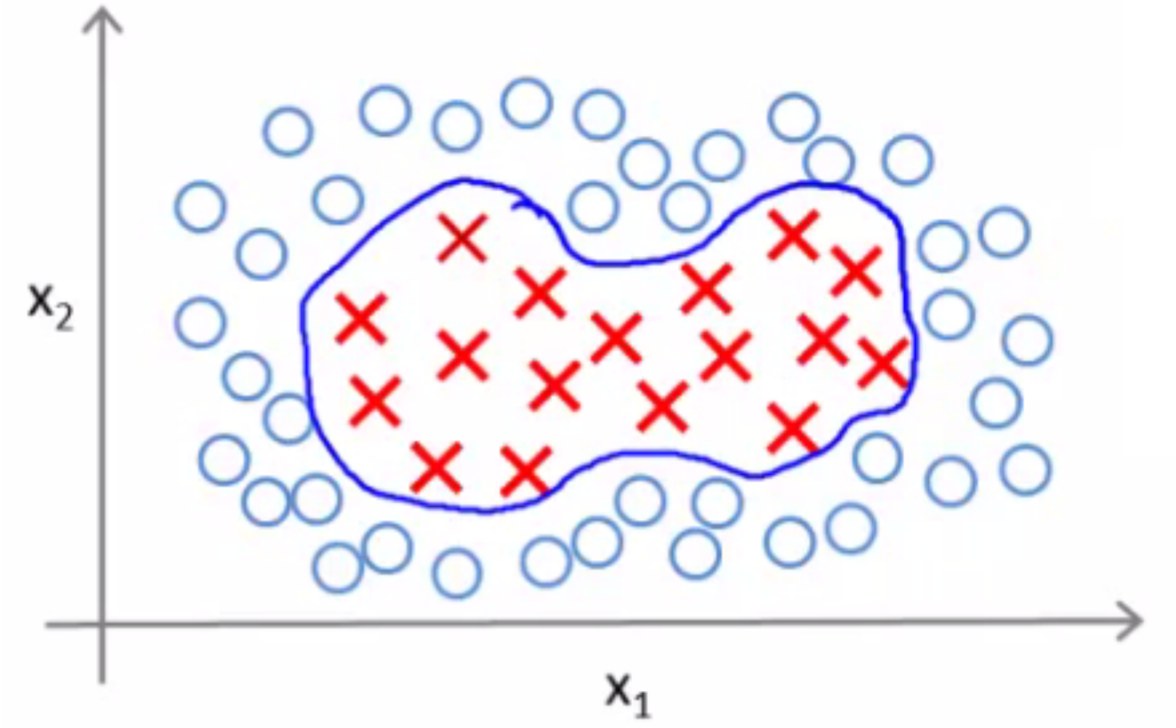


- The magnitude of this margin is a function of the p values
    - So by maximizing these p values we minimize ||θ||

- Finally, we did this derivation assuming $\theta_0 = 0$,
    - If this is the case we're entertaining only decision boundaries which pass through (0,0)
    - If you allow θ_0 to be other values then this simply means you can have decision boundaries which cross through the x and y values at points other than (0,0)
    - Can show with basically same logic that this works, and even when θ_0 is non-zero when you have optimization objective described above (when C is very large) that the SVM is looking for a large margin separator between the classes

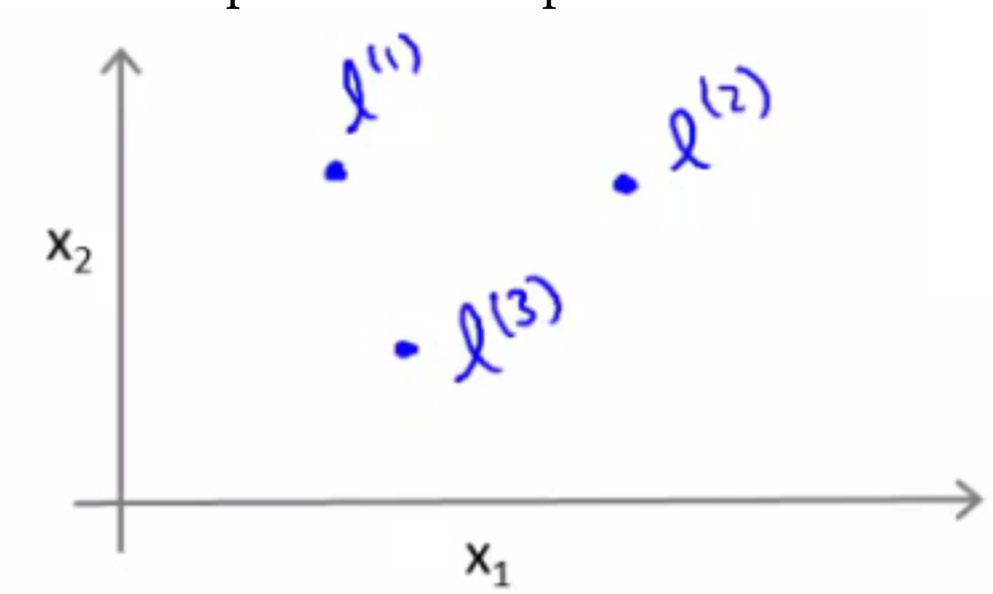
Kernels - 1: Adapting SVM to non-linear classifiers

- What are kernels and how do we use them
    - We have a training set
    - We want to find a non-linear boundary



- Come up with a complex set of polynomial features to fit the data
    - Have $h_\theta(x)$ which
        - Returns 1 if the combined weighted sum of vectors (weighted by the parameter vector) is less than or equal to 0

            - Else return 0
        - Another way of writing this (new notation) is
            - That a hypothesis computes a decision boundary by taking the sum of the parameter vector multiplied by a **new feature vector f**, which simply contains the various high order x terms
            - e.g.
                - $h_\theta(x) = \theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3$
                - Where
                    - $f_1 = x_1$
                    - $f_2 = x_1 x_2$
                    - $f_3 = ...$
                    - i.e. not specific values, but each of the terms from your complex polynomial function
        - Is there a better choice of feature f than the high order polynomials?
            - As we saw with computer imaging, high order polynomials become computationally expensive
- New features
    - Define three features in this example (ignore x_0)
    - Have a graph of x_1 vs. x_2 (don't plot the values, just define the space)
    - Pick three points in that space



    - These points l^1, l^2, and l^3, were chosen manually and are called **landmarks**
        - Given x, define f1 as the similarity between (x, l^1)
            - = exp(- (|| x - l^1 ||^2) / 2σ^2)
            - = $\exp\left(-\frac{\|x-l^{(1)}\|^2}{2\sigma^2}\right)$
            - **|| x - l^1 ||** is the euclidean distance between the point x and the landmark l^1 squared
                - Disussed more later
            - If we remember our statistics, we know that
                - σ is the **standard deviation**
                - σ^2 is commonly called the **variance**
        - Remember, that as discussed

$$\|x-l^{(1)}\|^2 = \sum_{j=1}^{n}(x_j - l_j^{(1)})^2$$

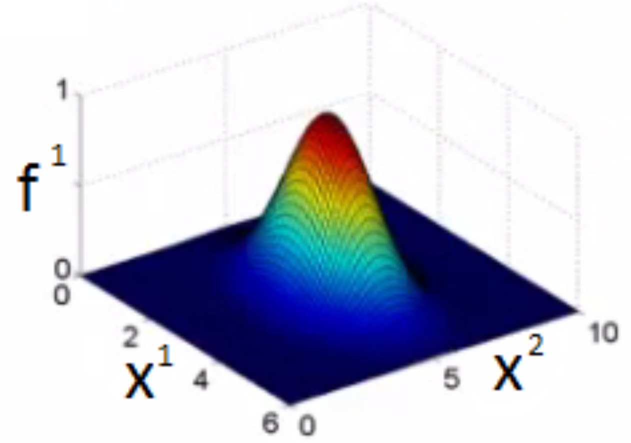
    - So, f2 is defined as
        - f2 = similarity(x, l^1) = exp(- (|| x - l^2 ||^2) / 2σ^2)
    - And similarly
        - f3 = similarity(x, l^2) = exp(- (|| x - l^1 ||^2) / 2σ^2)
    - This similarity function is called a **kernel**
        - This function is a **Gaussian Kernel**
    - So, instead of writing similarity between x and l we might write
        - f1 = k(x, l^1)

Diving deeper into the kernel

- So lets see what these kernels do and why the functions defined make sense
  - Say x is close to a landmark
    - Then the squared distance will be ~0
      - So

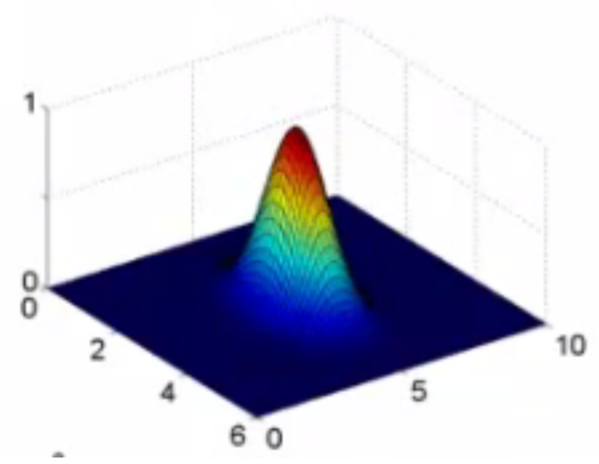
$$f_1 \approx \exp\left(-\frac{0^2}{2\sigma^2}\right)$$

        - Which is basically e^{-0}
          - Which is close to 1
    - Say x is far from a landmark
      - Then the squared distance is big
        - Gives $e^{\text{-large number}}$
          - Which is close to zero
    - Each landmark defines a new features
- If we plot f1 vs the kernel function we get a plot like this
  - Notice that when x = [3,5] then f1 = 1
  - As x moves away from [3,5] then the feature takes on values close to zero
  - So this measures how close x is to this landmark

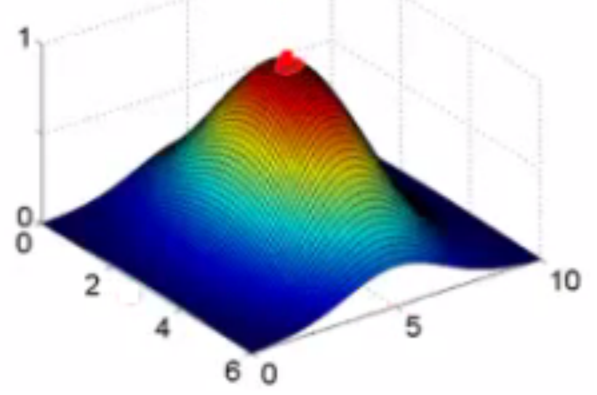


What does σ do?

- **σ^2** is a parameter of the Gaussian kernel
  - Defines the steepness of the rise around the landmark
- Above example $\sigma^2 = 1$
- Below $\sigma^2 = 0.5$

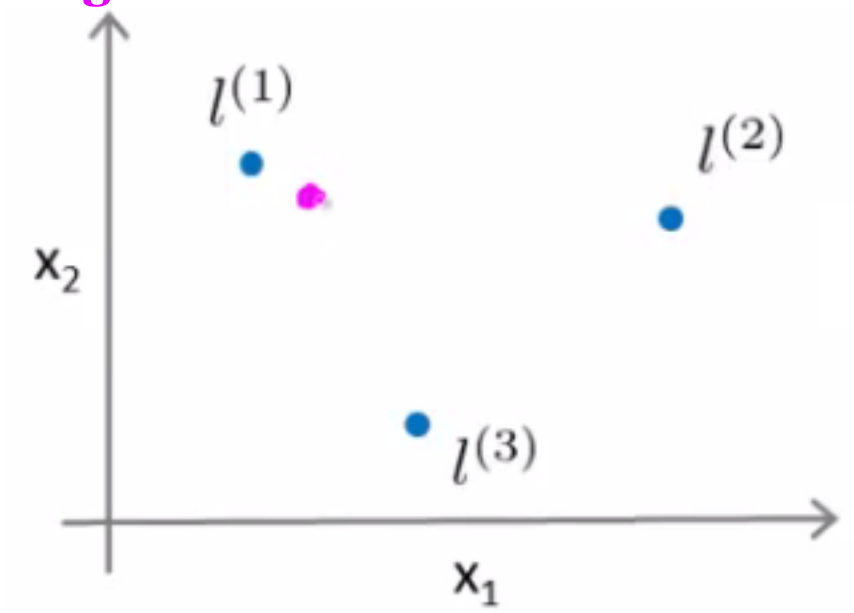


  - We see here that as you move away from 3,5 the feature f1 falls to zero much more rapidly
- The inverse can be seen if $\sigma^2 = 3$

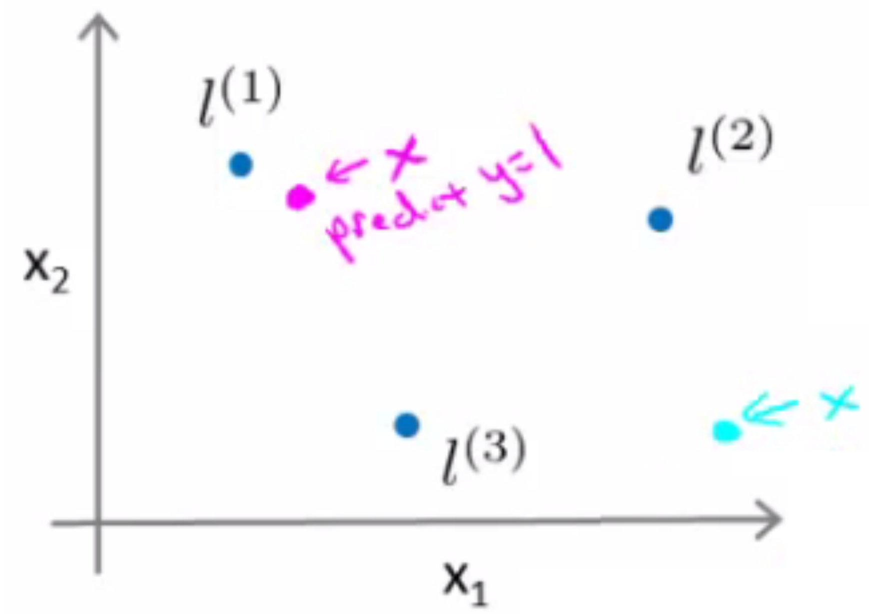


- Given this definition, what kinds of hypotheses can we learn?
  - With training examples x we predict "1" when
  - $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 >= 0$

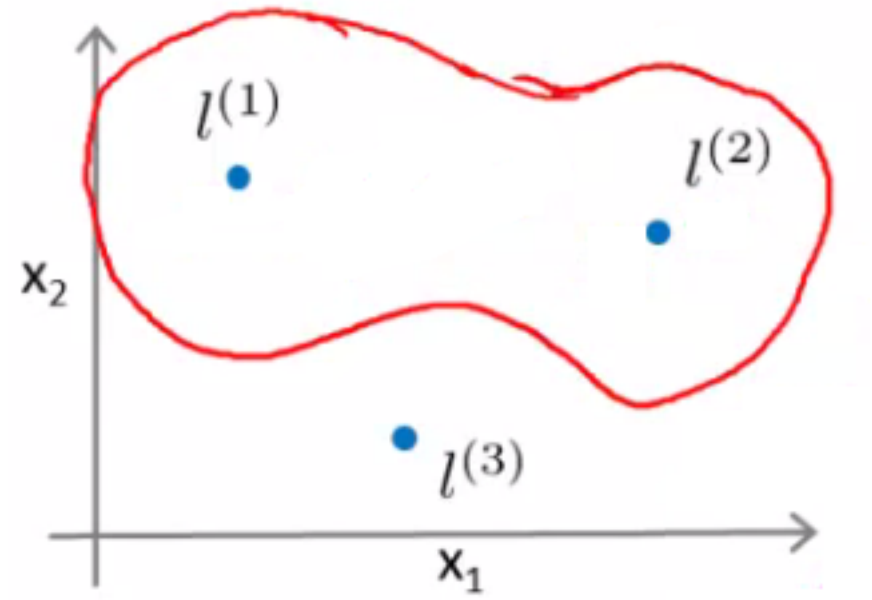
        - For our example, lets say we've already run an algorithm and got the
            - $\theta_0 = -0.5$
            - $\theta_1 = 1$
            - $\theta_2 = 1$
            - $\theta_3 = 0$
        - Given our placement of three examples, what happens if we evaluate an example at the **magenta dot** below?



        - Looking at our formula, we know f1 will be close to 1, but f2 and f3 will be close to 0
            - So if we look at the formula we have
                - $\theta_0 + \theta_1 f_1 + \theta_2 f_2 + \theta_3 f_3 >= 0$
                - $-0.5 + 1 + 0 + 0 = 0.5$
                    - 0.5 is greater than 1
        - If we had **another point** far away from all three



            - This equates to -0.5
                - So we predict 0
    - Considering our parameter, for points near l^1 and l^2 you predict 1, but for points near l^3 you predict 0
    - Which means we create a non-linear decision boundary that goes a lil' something like this;



        - Inside we predict y = 1
        - Outside we predict y = 0
- So this show how we can create a non-linear boundary with landmarks and the kernel function in the support vector machine
    - But
        - How do we get/chose the landmarks
        - What other kernels can we use (other than the Gaussian kernel)

Kernels II

- Filling in missing detail and practical implications regarding kernels
- Spoke about picking landmarks manually, defining the kernel, and building a hypothesis function
  - Where do we get the landmarks from?
  - For complex problems we probably want lots of them

Choosing the landmarks

- Take the training data
- For each example place a landmark at exactly the same location
- So end up with m landmarks
  - One landmark per location per training example
  - Means our features measure how close to a training set example something is
- Given a new example, compute all the f values
  - Gives you a feature vector f (f_0 to f_m)
    - f_0 = 1 always
- A more detailed look at generating the f vector
  - If we had a training example - features we compute would be using (x^i, y^i)
    - So we just cycle through each landmark, calculating how close to that landmark actually x^i is
      - f_1^i, = k(x^i, l^1)
      - f_2^i, = k(x^i, l^2)
      - ...
      - f_m^i, = k(x^i, l^m)
    - Somewhere in the list we compare x to itself... (i.e. when we're at f_i^i)
      - So because we're using the Gaussian Kernel this evalues to 1
    - Take these m features (f_1, f_2 ... f_m) group them into an [m +1 x 1] dimensional vector called f
      - f^i is the f feature vector for the ith example
      - And add a 0th term = 1
- Given these kernels, how do we use a support vector machine

SVM hypothesis prediction with kernels

- Predict y = 1 if (θ^T f) >= 0
  - Because θ = [m+1 x 1]
  - And f = [m +1 x 1]
- So, this is how you make a prediction assuming you already have θ
  - How do you get θ?

SVM training with kernels

- Use the SVM learning algorithm

$$\min_{\theta} C \sum_{i=1}^{m} y^{(i)} cost_1(\theta^T f^{(i)}) + (1 - y^{(i)}) cost_0(\theta^T f^{(i)}) + \frac{1}{2} \sum_{j=1}^{n} \theta_j^2$$

  - Now, we minimize using f as the feature vector instead of x
  - By solving this minimization problem you get the parameters for your SVM
- In this setup, m = n
  - Because number of features is the number of training data examples we have
- One final mathematic detail (not crucial to understand)
  - If we ignore θ_0 then the following is true

$$\sum_{j=1}^{n} \theta_j^2 = \theta^T\theta$$

- What many implementations do is
  $\theta^T M \theta$
  - Where the matrix M depends on the kernel you use
  - Gives a slightly different minimization - means we determine a rescaled version of θ
  - Allows more efficient computation, and scale to much bigger training sets
  - If you have a training set with 10 000 values, means you get 10 000 features
    - Solving for all these parameters can become expensive
    - So by adding this in we avoid a for loop and use a matrix multiplication algorithm instead

- You can apply kernels to other algorithms
  - But they tend to be very computationally expensive
  - But the SVM is far more efficient - so more practical
- Lots of good off the shelf software to minimize this function

- **SVM parameters (C)**
  - Bias and variance trade off
  - Must chose C
    - C plays a role similar to 1/LAMBDA (where LAMBDA is the regularization parameter)
  - Large C gives a hypothesis of **low bias high variance** --> overfitting
  - Small C gives a hypothesis of **high bias low variance** --> underfitting
- **SVM parameters (σ^2)**
  - Parameter for calculating f values
    - Large σ^2 - f features vary more smoothly - higher bias, lower variance
    - Small σ^2 - f features vary abruptly - low bias, high variance

SVM - implementation and use

- So far spoken about SVM in a very abstract manner
- What do you need to do this
  - Use SVM software packages (e.g. liblinear, libsvm) to solve parameters θ
  - Need to specify
    - Choice of parameter C
    - Choice of kernel

Choosing a kernel

- We've looked at the **Gaussian kernel**
  - Need to define σ (σ^2)
    - Discussed σ^2
  - When would you chose a Gaussian?
    - If n is small and/or m is large
      - e.g. 2D training set that's large
  - If you're using a Gaussian kernel then you may need to implement the kernel function
    - e.g. a function
      fi = kernel(x1,x2)
      - Returns a real number
    - Some SVM packages will expect you to define kernel
    - Although, some SVM implementations include the Gaussian and a few others
      - Gaussian is probably most popular kernel
  - NB - make sure you perform **feature scaling** before using a Gaussian kernel
    - If you don't features with a large value will dominate the f value
- Could use no kernel - **linear kernel**
  - Predict y = 1 if (θ^T x) >= 0
    - So no f vector

        - Get a standard linear classifier
    - Why do this?
        - If n is large and m is small then
            - Lots of features, few examples
            - Not enough data - risk overfitting in a high dimensional feature-space
- Other choice of kernel
    - Linear and Gaussian are most common
    - Not all similarity functions you develop are valid kernels
        - Must satisfy **Merecer's Theorem**
        - SVM use numerical optimization tricks
            - Mean certain optimizations can be made, but they must follow the theorem
    - **Polynomial Kernel**
        - We measure the similarity of x and l by doing one of
            - $(x^T l)^2$
            - $(x^T l)^3$
            - $(x^T l+1)^3$
        - General form is
            - $(x^T l+Con)^D$
        - If they're similar then the inner product tends to be large
        - Not used that often
        - Two parameters
            - Degree of polynomial (D)
            - Number you add to l (Con)
        - Usually performs worse than the Gaussian kernel
        - Used when x and l are both non-negative
    - **String kernel**
        - Used if input is text strings
        - Use for text classification
    - **Chi-squared kernel**
    - **Histogram intersection kernel**

Multi-class classification for SVM

- Many packages have built in multi-class classification packages
- Otherwise use one-vs all method
- Not a big issue

Logistic regression vs. SVM

- When should you use SVM and when is logistic regression more applicable
- If n (features) is large vs. m (training set)
    - e.g. text classification problem
        - Feature vector dimension is 10 000
        - Training set is 10 - 1000
        - Then use logistic regression or SVM with a linear kernel
- If n is small and m is intermediate
    - n = 1 - 1000
    - m = 10 - 10 000
    - Gaussian kernel is good
- If n is small and m is large
    - n = 1 - 1000
    - m = 50 000+
        - SVM will be slow to run with Gaussian kernel
    - In that case
        - Manually create or add more features
        - Use logistic regression of SVM with a linear kernel
- Logistic regression and SVM with a linear kernel are pretty similar
    - Do similar things

    - Get similar performance
- A lot of SVM's power is using diferent kernels to learn complex non-linear functions
- For all these regimes a well designed NN should work
    - But, for some of these problems a NN might be slower - SVM well implemented would be faster
- SVM has a convex optimization problem - so you get a global minimum
- It's not always clear how to chose an algorithm
    - Often more important to get enough data
    - Designing new features
    - Debugging the algorithm
- SVM is widely perceived a very powerful learning algorithm