/tmp/test.tex

```
1 Let $n$ be a natural number.
2 Then $n + \frac12$ is a rational number.
```

Commands: enter h (help) to see all available commands

[h]elp
[q]uit
[0] natural number (Q21199): ambiguous mathematical term used either for non-negative or for strictly positive integers, depending on usage
[1] non-negative integer (Q28920052): integer greater than or equal to zero; natural number explicitly including zero
[2] positive integer (Q28920044): integer greater than zero; natural number explicitly excluding zero
[s]kip once

>>>

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```
1 Let $n$ be a \wdalign{Q28920052}{natural number}.
2 Then $n + \frac12$ is a rational number.
```

Commands: enter h (help) to see all available commands

[h]elp
[q]uit
[0] rational number (Q1244890): quotient of two integers
[s]kip once

>>>

/tmp/1201.6656.html

```
  51 <body>
  52 <div class="ltx_page_main">
  53 <div class="ltx_page_content">
  54 <article class="ltx_document ltx_authors_1line">
  55 <h1 class="ltx_title ltx_title_document">Every odd number greater than <math id="id1.m
1.1" class="ltx_Math" alttext="1" display="inline"><semantics id="id1.m1.1b"><mn id="id1.m1
.1.1" xref="id1.m1.1.1.cmml">1</mn><annotation-xml encoding="MathML-Content" id="id1.m1.1c"
><cn type="integer" id="id1.m1.1.1.cmml" xref="id1.m1.1.1">1</cn></annotation-xml><annotati
on encoding="application/x-tex" id="id1.m1.1d">1</annotation></semantics></math> is the sum
 of at most five primes</h1>
  56 <div class="ltx_authors">
  57 <span class="ltx_creator ltx_role_author">
  58 <span class="ltx_personname">Terence Tao
  59 </span><span class="ltx_author_notes">

Commands: enter h (help) to see all available commands
  [h]elp
  [q]uit
  [0] Q199 (Q199): natural number one
  [s]kip once
>>>
```

/tmp/1201.6656.html

Every odd number greater than 1 is the sum of at most five primes

Terence Tao UCLA Department of Mathematics, Los Angeles, CA 90095-1596.
tao@math.ucla.edu

Commands: enter h (help) to see all available commands
[h]elp
[q]uit
[0] Q199 (Q199): natural number one
[s]kip once
>>>

/tmp/1201.6656.html

tao@math.ucla.edu

Abstract.

We prove that every odd number N greater than 1 can be expressed as the sum of at most five primes, improving the result of Ramaré that every even natural number can be expressed as the sum of at most six primes. We follow the circle method of Hardy-Littlewood and Vinogradov, together with Vaughan's identity; our additional techniques, which may be of interest for other Goldbach-type

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Abstract.

We prove that every odd number N greater than 1 can be expressed as the sum of at most five primes, improving the result of Ramaré that every even natural number can be expressed as the sum of at most six primes. We follow the circle method of Hardy-Littlewood and Vinogradov, together with Vaughan's identity; our additional techniques, which may be of interest for other Goldbach-type

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[q]uit
[0] Q199 (Q199): natural number one
[s]kip once
>>>

/tmp/1201.6656.html

method of Hardy-Littlewood and Vinogradov, together with Vaughan's identity, our additional techniques, which may be of interest for other Goldbach-type problems, include the use of smoothed exponential sums and optimisation of the Vaughan identity parameters to save or reduce some logarithmic losses, the use of multiple scales following some ideas of Bourgain, and the use of Montgomery's uncertainty principle and the large sieve to improve the L^2 estimates on major arcs. Our argument relies on some previous numerical work, namely the verification of Richstein of the even Goldbach conjecture up to 4×10^{14}, and the verification of van de Lune and (independently) of Wedeniwski of the Riemann hypothesis up to height 3.29×10^9

Commands: enter h (help) to see all available commands
[h]elp
[q]uit
[0] Q200 (Q200): natural number
[s]kip once
>>>

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1. Introduction

Commands: enter h (help) to see all available commands
[h]elp
[q]uit
[0] Cartesian product (Q173740): set of the ordered pairs such that the first element of the pair is in the first element of the product and the second element of the pair is in the second element of the product
[1] cross product (Q178192): mathematical operation on two vectors giving a vector as result
[2] multiplication (Q40276): mathematical operation
[s]kip once
>>>