

What does it mean? A framework for prototyping Montague-style semantics

One approach to study the meaning of natural language is to translate utterances to logical expressions, which have well-defined semantics and allow for rigorous inference. Richard Montague, a well-known pioneer of this approach, described the translation of a particular subset of English into logic in great detail. Over the years, there have been many similar publications that propose a novel way to translate sentences into a logic (often first-order logic, but more obscure logics are also common). Sometimes, the approach was extended with pragmatic analysis based on logical inference.

We claim that research (and education) in this direction could benefit from more prototyping to test and demonstrate new ideas.

In this talk, I will present GLIF, a declarative framework for prototyping the translation of natural language to logics. GLIF combines existing, specialized frameworks that solve part of the problem: the Grammatical Framework (development of natural language grammars), MMT (logic development) and ELPI (inference). These frameworks can be connected seamlessly because of their compatible underlying logical frameworks. We successfully use GLIF in a lecture on symbolic natural language semantics.

What does it mean?
A framework for prototyping Montague-style semantics

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Seminar: Computing Meaning

Hildesheim

July 21, 2022

- PhD student in the KWARC group (Erlangen) *Knowledge representation*
- Supervisor: Michael Kohlhase
- My interest: Mathematical language, precise semantics extraction
- Background for this talk:
 - We teach a lecture in *logic-based natural language semantics*
 - Wanted more hands-on experience
 - \rightsquigarrow new framework: GLIF *Grammar, Logic, Inference*

Natural Language Semantics (Symbolic)

For this talk:

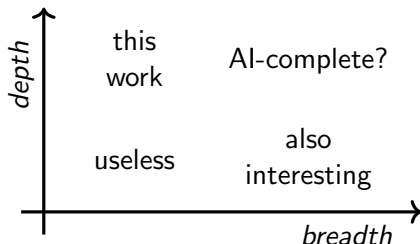
Translating natural language into a formal semantic representation (logic).

Example:

"Every student paints and is quiet." $\rightsquigarrow \forall x.s(x) \Rightarrow (p(x) \wedge q(x))$

Rule-based (no ML):

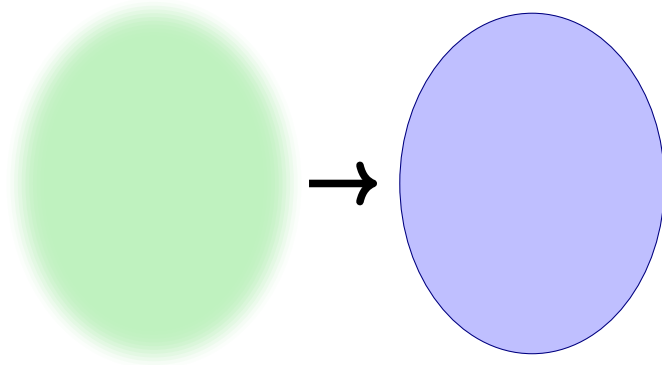
Parsing \rightsquigarrow semantics construction \rightsquigarrow inference.



Method of Fragments

Natural Language

Logic



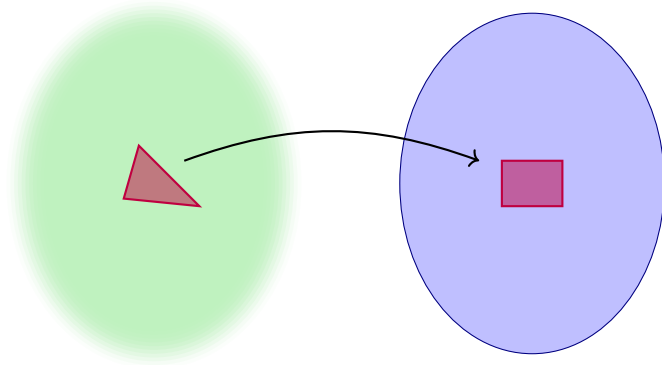
How do we get from messy language to formal logic?

Montague [Mon70]: Look at a “nice” subset and map into logic.

Method of Fragments

Natural Language

Logic



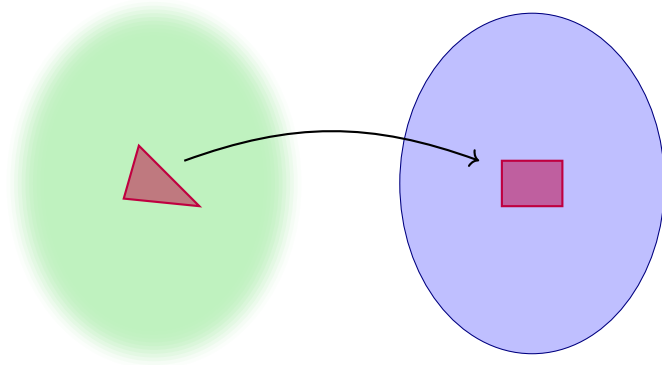
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Method of Fragments

Natural Language

Logic



"Ahmed paints and Berta is quiet."

"Ahmed doesn't paint."

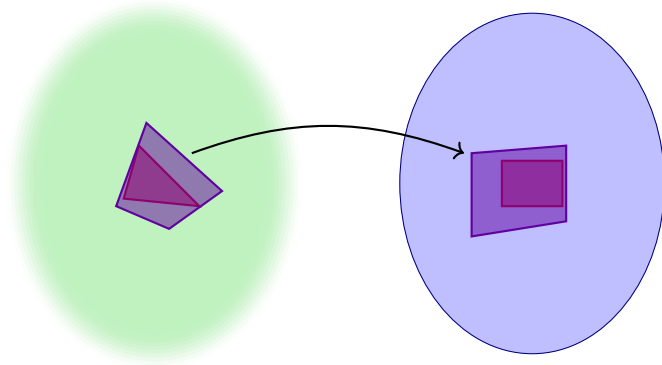
$p(a) \wedge q(b)$

$\neg p(a)$

Method of Fragments

Natural Language

Logic



“Every student paints and is quiet.”

“Nobody paints.”

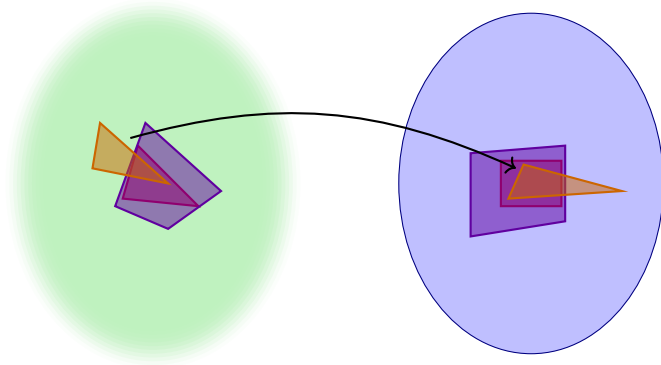
$\forall x.s(x) \Rightarrow (p(x) \wedge q(x))$

$\neg \exists x.p(x)$

Method of Fragments

Natural Language

Logic



“Ahmed isn’t allowed to paint.”

“Ahmed and Berta must paint.”

$\neg \Diamond p(a)$

$(\Box p(a)) \wedge \Box p(b)$

Method of Fragments

Hand-waving is problematic:

“Ahmed paints. He is quiet.” $\overset{?}{\rightsquigarrow} p(a) \wedge q(a)$

Montague: Specify

- grammar,
- target logic,
- semantics construction.

fixes NL subset

maps parse trees to logic

Example from [Mon74]

- | |
|--|
| <p>T11. If $\phi, \psi \in P_I$ and ϕ, ψ translate into ϕ', ψ' respectively, then ϕ and ψ translates into $[\phi \wedge \psi]$, ϕ or ψ translates into $[\phi \vee \psi]$.</p> <p>T12. If $\gamma, \delta \in P_{IV}$ and γ, δ translate into γ', δ' respectively, then γ and δ translates into $\hat{x}[\gamma'(x) \wedge \delta'(x)]$, γ or δ translates into $\hat{x}[\gamma'(x) \vee \delta'(x)]$.</p> <p>T13. If $\alpha, \beta \in P_T$ and α, β translate into α', β' respectively, then α or β translates into $\hat{P}[\alpha'(P) \vee \beta'(P)]$.</p> |
|--|

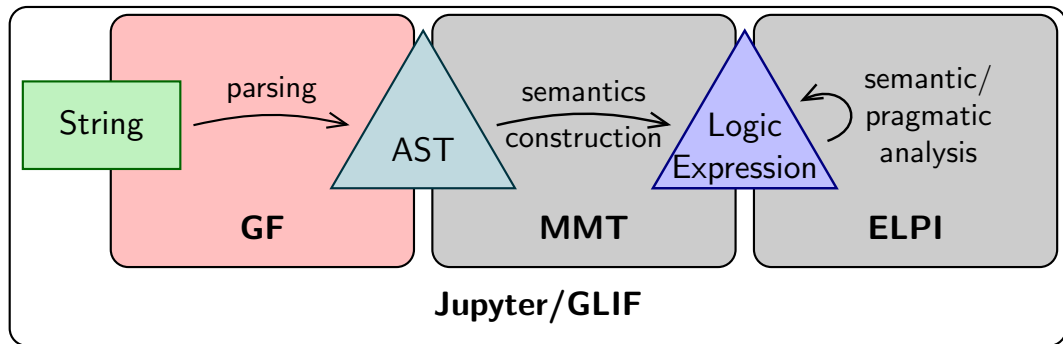
Claim: That doesn't scale well \rightsquigarrow **We need prototyping!**

NLU Prototyping

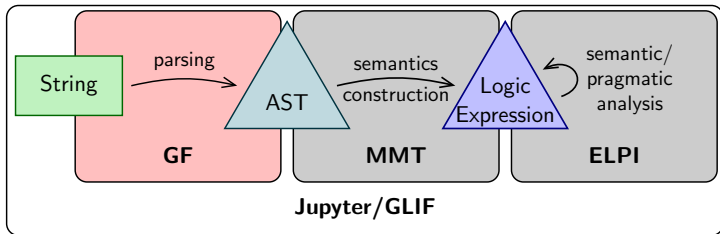
```
> translate "Every student paints and is quiet."  
 $\forall x.s(x) \Rightarrow (p(x) \wedge q(x))$ 
```

- Traditionally done in Prolog/Haskell
 - requires a lot of work
- A dedicated framework might be better
 - only partial solutions exist
- Can we combine existing partial solutions?
 - ↪ GLIF

Components of GLIF: GF



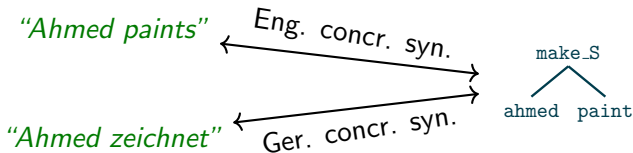
Components of GLIF: GF



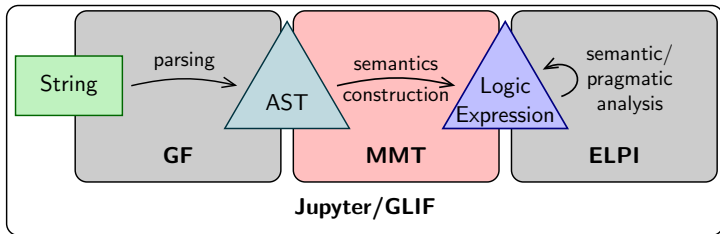
Components of GLIF: Grammatical Framework [GF]

- Specialized for developing natural language grammars
- Separates abstract and concrete syntax

```
make_S : NP -> VP -> S;                                abstract  
make_S np vp = np.s ++ vp.s!np.n;                        concrete
```
- Abstract syntax based on LF
- Comes with large library ≥ 36 *languages*



Components of GLIF: MMT



Components of GLIF: MMT

- Modular logic development and knowledge repr.
- Not specialized in one logical framework *we use LF*
- We will use MMT to:
 - ① represent abstract syntax
 - ② specify target logic and discourse domain theory
 - ③ specify semantics construction

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 - ① **represent abstract syntax**
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GF

```
cat
  NP; VP; S;
fun
  make_S :
    NP -> VP -> S;
```



MMT

```
NP : type
VP : type
S  : type
make_S :
  NP → VP → S
```

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Logic Syntax

```
o : type //propositions
¬ : o → o
∧ : o → o → o
∨ : o → o → o

ι : type //individuals
∀ : (ι → o) → o
∃ : (ι → o) → o
```

Discourse Domain

```
paint : ι → o
quiet : ι → o
ahmed : ι
berta : ι
```

idea: $\forall f$ or $\forall \lambda x.f(x)$
instead of $\forall x.f(x)$

Components of GLIF: MMT

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 - ③ **specify semantics construction**

Semantics Construction

map symbols in abstract syntax to terms in logic/domain theory

Simple setting

S	\mapsto o
NP	\mapsto ι
VP	\mapsto ι \rightarrow o
make_S	\mapsto $\lambda n. \lambda v. v$ n
ahmed	\mapsto ahmed

More advanced

NP	\mapsto (ι \rightarrow o) \rightarrow o
sentence	\mapsto $\lambda n. \lambda v. n$ v
everyone	\mapsto $\lambda p. \forall \lambda x. p$ x
berta	\mapsto $\lambda p. p$ berta

Example: Parsing + Semantics Construction

“Ahmed and Berta paint”

↓ parsing

make_S (andNP ahmed berta) paint

↓ semantics construction

$(\lambda n. \lambda v. n \ v) \ ((\lambda a. \lambda b. \lambda p. a \ p \ \wedge \ b \ p) \ (\lambda p. p \ \text{ahmed}) \ (\lambda p. p \ \text{berta})) \ \text{paint}$

↓ β -reduction

$\text{paint} \ \text{ahmed} \ \wedge \ \text{paint} \ \text{berta}$

Example: Student Project [Int]

parse "John has not always run" | **construct**

$\neg H \text{ (run john)}$

parse "John has to have been allowed to always run" | **construct**

$\Box P \Diamond (H \text{ (run john)} \wedge G \text{ (run john)})$

parse "John probably will never run" | **construct**

$\text{Prob } G \neg(\text{run john})$

parse "it has to be possible that John runs" | **construct**

$\Box \Diamond (\text{run john})$

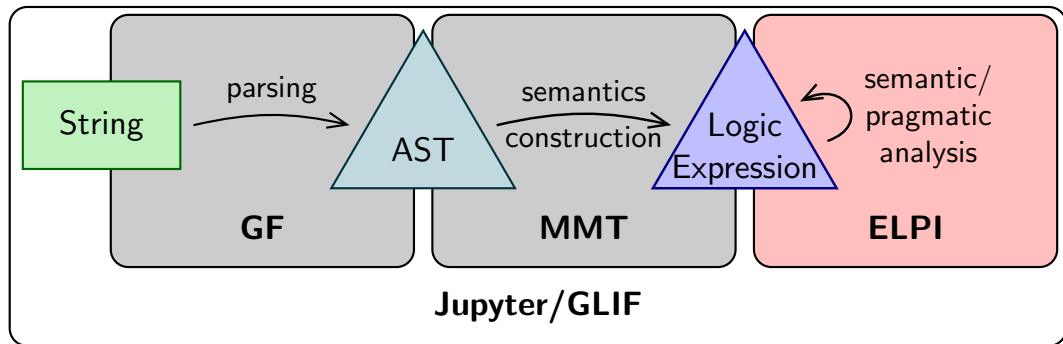
parse "Mary saw that John would kill the dog" | **construct**

$P \text{ } \textcircled{\text{mary}} F \text{ (kill john dog)}$

parse "Mary runs and John sees it" | **construct**

$(\text{run mary}) \wedge \text{ } \textcircled{\text{john}} (\text{run mary})$

Components of GLIF: ELPI



Components of GLIF: ELPI

- Implementation and extension of λ Prolog
- MMT can generate logic signatures
- Generic inference/reasoning step after semantics construction
- Goal: Use it for semantic/pragmatic analysis

\approx *Prolog + HOAS*

MMT

```
o : type //propositions
¬ : o → o
∧ : o → o → o
∨ : o → o → o
```

```
ι : type //individuals
∀ : (ι → o) → o
∃ : (ι → o) → o
```

ELPI

```
kind o type.
not : o -> o.
and : o -> o -> o.
or  : o -> o -> o.
```

```
kind i type.
type forall (i -> o) -> o.
type exists (i -> o) -> o.
```

“The trophy doesn’t fit in the brown suitcase because it’s too big.” [LDM12]

Semantic/Pragmatic Analysis

“The trophy doesn’t fit in the brown suitcase because it’s too big.” [LDM12]
“The trophy doesn’t fit in the brown suitcase because it’s too small.” [LDM12]

Semantic/Pragmatic Analysis

“The trophy doesn’t fit in the brown suitcase because it’s too big.” [LDM12]
“The trophy doesn’t fit in the brown suitcase because it’s too small.” [LDM12]

“The ball has a radius of 2m.”
“The ball has a mass of 2m.”

“We saw her duck.”

~> semantics construction creates preliminary semantic representation(s) that get refined by the semantic/pragmatic analysis

Example: Discard wrong readings in controlled natural language

"the ball has a mass of 5kg" \rightarrow AST \longrightarrow `mass(theball, quant(5, kilo gram))`

Example: Discard wrong readings in controlled natural language

"the ball has a mass of 5kg" \rightarrow AST \longrightarrow $\text{mass}(\text{theball}, \text{quant}(5, \text{kilo gram}))$

"a kinetic energy of 12mN" \nearrow AST₁ \longrightarrow $\lambda x. E_{\text{kin}}(x, \text{quant}(2, \text{milli Newton}))$
"a kinetic energy of 12mN" \searrow AST₂ \longrightarrow $\lambda x. E_{\text{kin}}(x, \text{quant}(2, \text{meter} \cdot \text{Newton}))$

Example: Discard wrong readings in controlled natural language

"the ball has a mass of 5kg" \rightarrow AST \longrightarrow `mass(theball, quant(5, kilo gram))`

"a kinetic energy of 12mN" \rightarrow AST₁ \longrightarrow ~~`$\lambda x.E_{kin}(x, quant(2, milli\ Newton))$`~~

"a kinetic energy of 12mN" \rightarrow AST₂ \longrightarrow `$\lambda x.E_{kin}(x, quant(2, meter \cdot Newton))$`

```
In [20]: 1 parse "the ball has a mass of 5 k g and a kinetic energy of 12 m N" |
          2 construct
```

(mass theball (quant 5 kilo gram)) \wedge (ekin theball (quant 12 milli Newton))
(mass theball (quant 5 kilo gram)) \wedge (ekin theball (quant 12 meter·Newton))

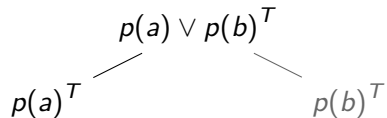
```
In [21]: 1 parse "the ball has a mass of 5 k g and a kinetic energy of 12 m N" |
          2 construct | filter -predicate=filter pred
```

(mass theball (quant 5 kilo gram)) ^ (ekin theball (quant 12 meter·Newton))

Example: Tableaux Machine [KK03]

- Can use tableaux for model generation
- Tableau machine: pick “best” branch as model and continue there with next sentence
like a human?

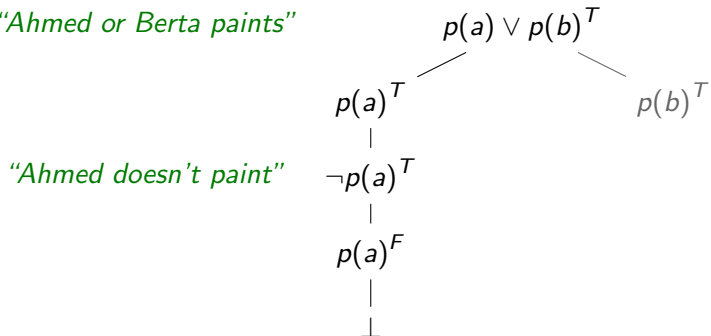
“Ahmed or Berta paints”



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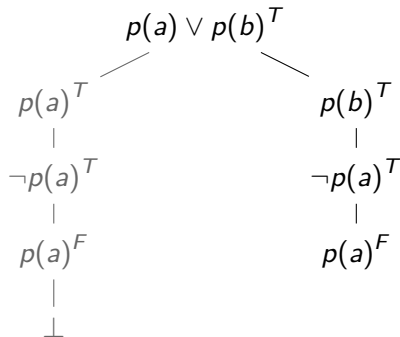
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Example: Tableaux Machine

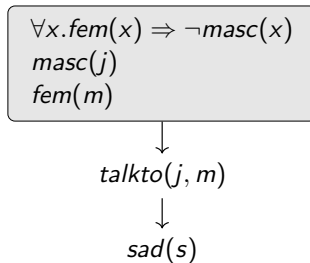
Background Knowledge

"John talks to Mary."

talkto(j, m)

"Sasha is sad."

sad(s)



Example: Tableaux Machine

Background Knowledge

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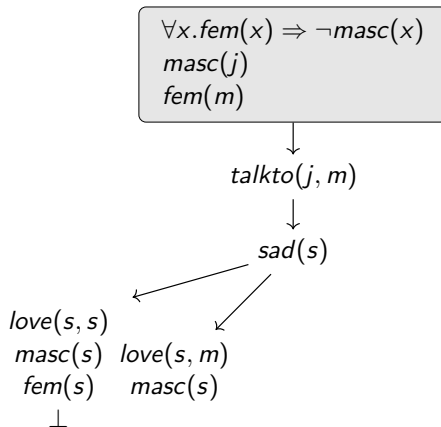
"Sasha is sad."

$sad(s)$

"He loves her."

$\exists X.masc(X) \wedge$

$\exists Y.fem(Y) \wedge love(X, Y)$



Example: Tableaux Machine

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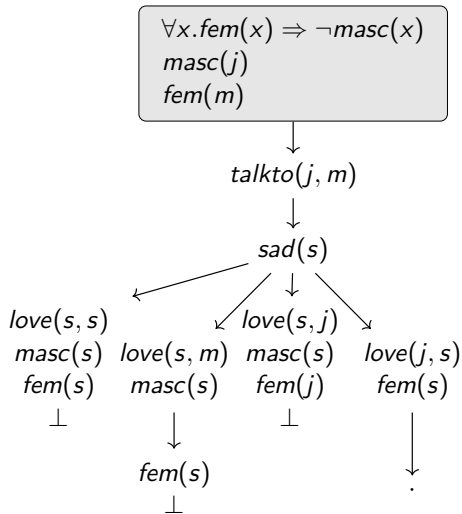
"He loves her."

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"Sasha is a woman."

$fem(s)$



Example: Tableaux Machine

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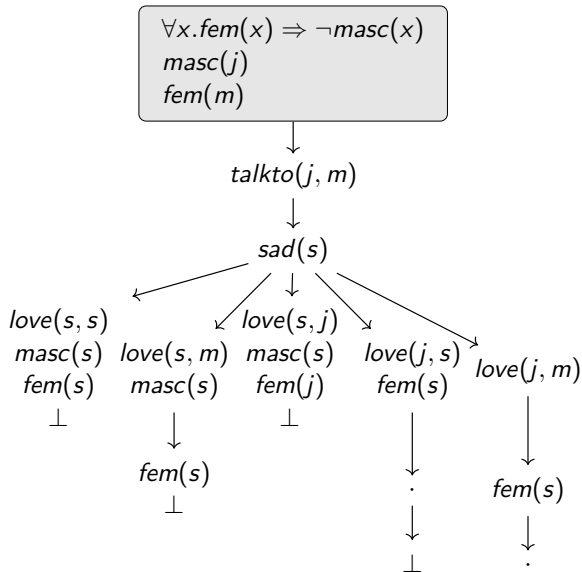
$\exists X.masc(X) \wedge$
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"Sasha is a woman."

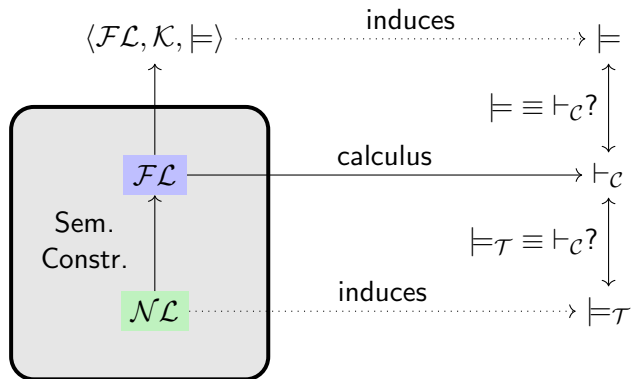
$fem(s)$

"John doesn't love Sasha."

$\neg love(j, s)$



Making it a Science



- 1 Test: Does "*Ahmed and Berta paint.*" $\models_{\mathcal{T}}$ "*Berta paints.*"?
- 2 Model prediction: Yes, because $p(a) \wedge p(b) \vdash_c p(b)$.
- 3 Correct result: Ask people.

Example: Epistemic Q&A

John knows that Mary or Eve knows that Ping has a dog. (S_1)

Mary doesn't know if Ping has a dog. (S_2)

Does Eve know if Ping has a dog? (Q)

$$S_1 = \Box_{\text{john}}(\Box_{\text{mary}}hd(\text{ping}) \vee \Box_{\text{eve}}hd(\text{ping}))$$

$$S_2 = \neg(\Box_{\text{mary}}hd(\text{ping}) \vee \Box_{\text{mary}}\neg hd(\text{ping}))$$

$$Q = \Box_{\text{eve}}hd(\text{ping}) \vee \Box_{\text{eve}}\neg hd(\text{ping})$$

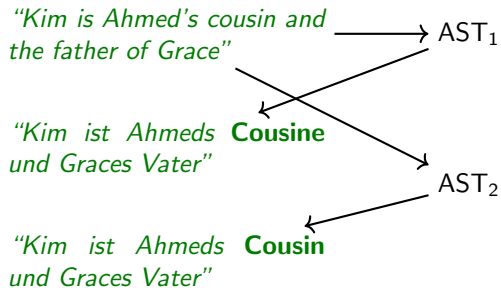
$$S_1, S_2 \vdash_{S5_n} Q \quad \rightsquigarrow \quad \text{yes}$$

$$S_1, S_2 \vdash_{S5_n} \neg Q \quad \rightsquigarrow \quad \text{no}$$

$$\text{else} \quad \rightsquigarrow \quad \text{maybe}$$

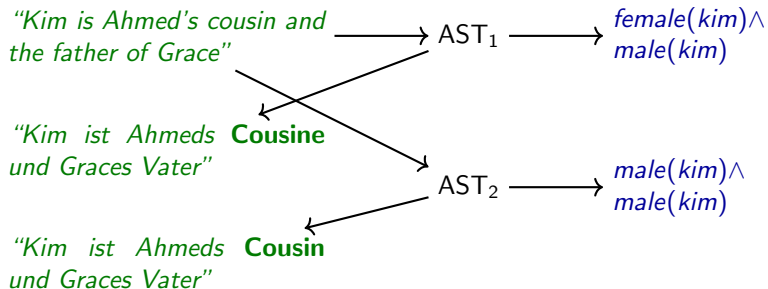
Example: Translation

- Two German words for “*cousin*”, depending on the gender
- Two entries in abstract syntax: `cousin_female` and `cousin_male`
- Use inference to discard ASTs



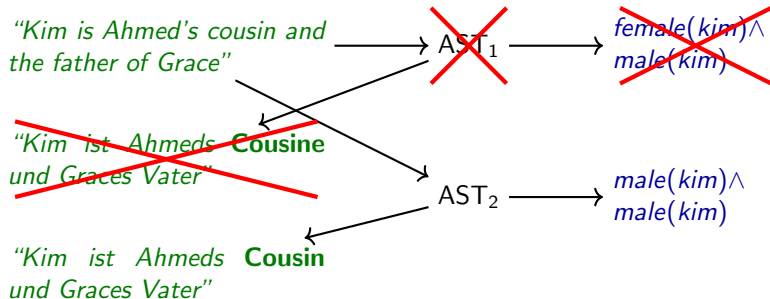
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Example: Input Language for SageMath

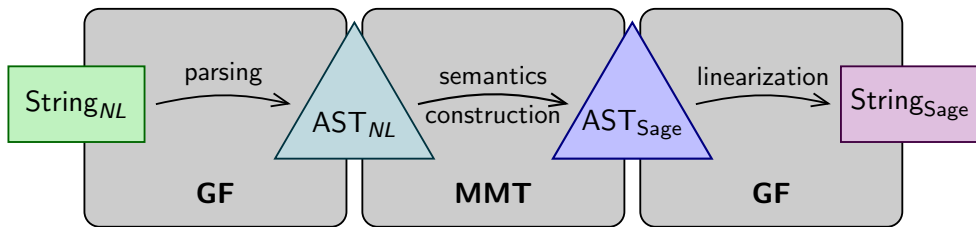
- Can we make a natural input language for SageMath?

WolframAlpha-like

```
sage: g = AlternatingGroup(5)
sage: g.cardinality()
60
```

“Let G be the alternating group on 5 symbols. What is the cardinality of G ?”

Example: Input Language for SageMath



Example: Input Language for SageMath

> Let G be the alternating group on 5 symbols.

```
# G = AlternatingGroup(5)
```

> Let $|H|$ be a notation for the cardinality of H .

```
# def bars(H): return H.cardinality()
```

> What is $|G|$?

```
# print(bars(G))
```

```
60
```

> Let A_N be a notation for the alternating group on N symbols.

```
# def A(N): return AlternatingGroup(N)
```

> What are the cardinalities of A_4 and A_5 ?

```
# print(A(4).cardinality()); print(A(5).cardinality())
```

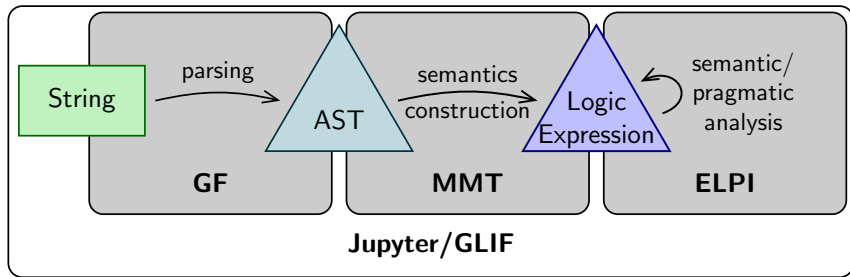
```
12
```

```
60
```

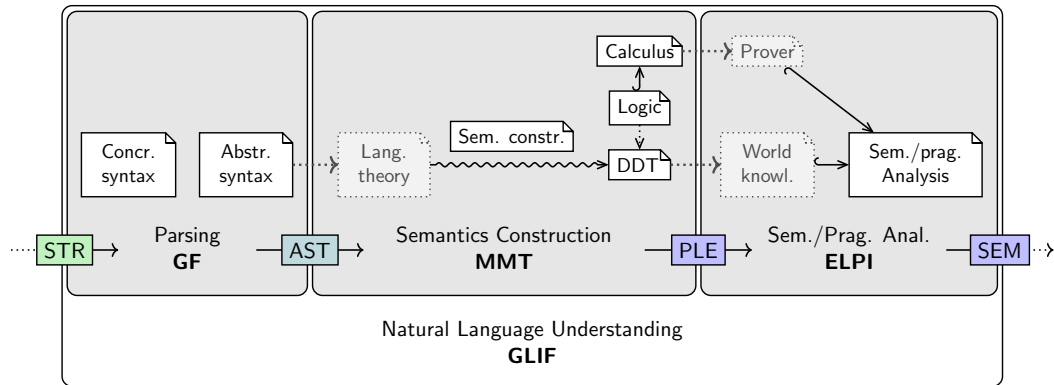
Summary

- $GLIF = GF + MMT + ELPI$
- Prototyping natural language semantics
- We use it for teaching

(symbolic)



Pipeline Specification



Natural Deduction in MMT/LF

$$\frac{A \wedge B}{A} \wedge E$$
$$\frac{A \vee B \quad \begin{array}{c} [A]^1 \\ \vdots \\ C \end{array} \quad \begin{array}{c} [B]^1 \\ \vdots \\ C \end{array}}{C} \vee E^1$$

// $\vdash X$ is type of proofs for X (judgments as types)
 $\vdash : o \rightarrow type$

$\wedge E1 : \prod_{A:o} \prod_{B:o} \vdash A \wedge B \rightarrow \vdash A$

$\vee E : \prod_{A:o} \prod_{B:o} \prod_{C:o} \vdash A \vee B \rightarrow (\vdash A \rightarrow \vdash C) \rightarrow (\vdash B \rightarrow \vdash C) \rightarrow \vdash C$

Generating Provers in ELPI

LF rule $\wedge E1 : \Pi_{A:o} \Pi_{B:o} \vdash A \wedge B \rightarrow \vdash A$

ELPI equivalent

direct: `pi A \ pi B \ ded (and A B) => ded A.`

syn. sugar: `ded A :- ded (and A B).`

Generating Provers in ELPI

LF rule $\wedge E1 : \Pi_{A:o} \Pi_{B:o} \vdash A \wedge B \rightarrow \vdash A$

ELPI equivalent

direct: `pi A \ pi B \ ded (and A B) => ded A.`

syn. sugar: `ded A :- ded (and A B).`

Example: Or-Elimination

LF: $\vee E : \Pi_{A:o} \Pi_{B:o} \Pi_{C:o} \vdash A \vee B \rightarrow (\vdash A \rightarrow \vdash C) \rightarrow (\vdash B \rightarrow \vdash C) \rightarrow \vdash C$

ELPI: `ded C :- ded (or A B), ded A => ded C, ded B => ded C.`

Example: Forall-Introduction

LF: $\forall I : \Pi_{P:l \rightarrow o} (\Pi_{x:l} \vdash P \ x) \rightarrow \vdash \forall P$

ELPI: `ded (forall P) :- pi x \ ded (P x).`

Controlling the Proof Search

- Problem: Search diverges *searching harder than checking*
- Solution: Control search with helper predicates:
inspired by ProofCert project by Miller et al.
 - Intuition: Decide whether to apply rule
 - Do not affect correctness
 - Extra argument tracks aspects of proof state

Before: `ded A :- ded (and A B) .`

Now: `ded X A :- help/andEl X A B X1, ded X1 (and A B) .`

Helper Predicates

Name	Predicate	Argument
Iter. deepening	checks depth	remaining depth
Proof term	generates term	proof term
Product	calls other predicates	arguments for other predicates
Backchaining	Prolog's backchaining (\approx forward reasoning from axioms via \Rightarrow/\forall elimination rules)	pattern of formula to be proven (e.g. a conjunction)

Example helper: Iterative deepening

```
help/andEl (idcert  $N$ ) _ _ (idcert  $N1$ ) :-  $N > 0$ ,  $N1$  is  $N - 1$ .
```

Tableau Provers

$$\frac{A \wedge B^F}{A^F \mid B^F} \wedge^F \qquad \frac{A \wedge B^F \quad \begin{array}{c} [A^F] \\ \vdots \\ \perp \end{array} \quad \begin{array}{c} [B^F] \\ \vdots \\ \perp \end{array}}{\perp} \wedge^F$$


LF: $\wedge^F : \Pi_{A:o} \Pi_{B:o} A \wedge B^F \rightarrow (A^F \rightarrow \perp) \rightarrow (B^F \rightarrow \perp) \rightarrow \perp$


ELPI: `closed X :- help/andF X A B X1 X2 X3, f X1 (and A B),
f/hyp A => closed X2, f/hyp B => closed X3.`


With iterative deepening we get a working prover!


→ Other helpers result in more efficient provers

References I

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<https://github.com/us77ipis/glif-intensional-logic> (visited on 07/19/2022).

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References II



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