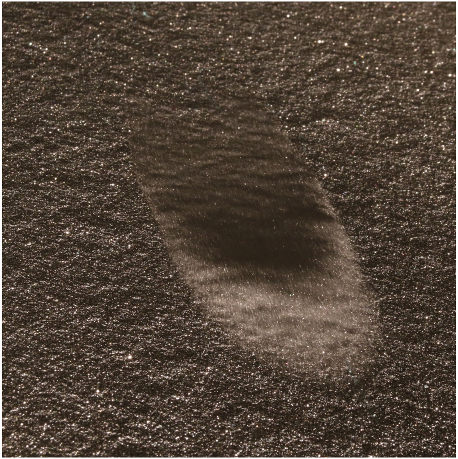
Granular avalanches:

release of a finite mass on a rough inclined plane



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Experiments: Dr. Sylvain Viroulet, PI: Prof. Nico Gray (University of Manchester)

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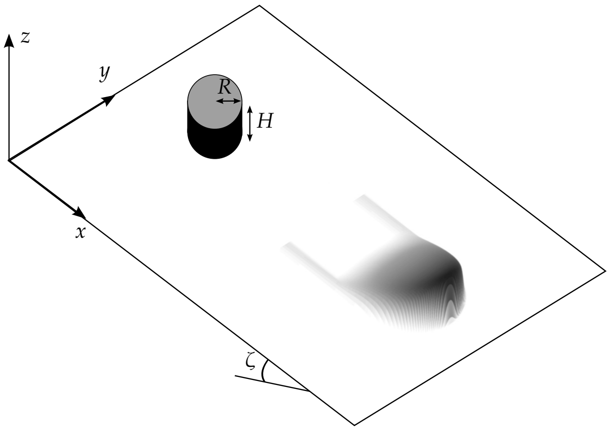
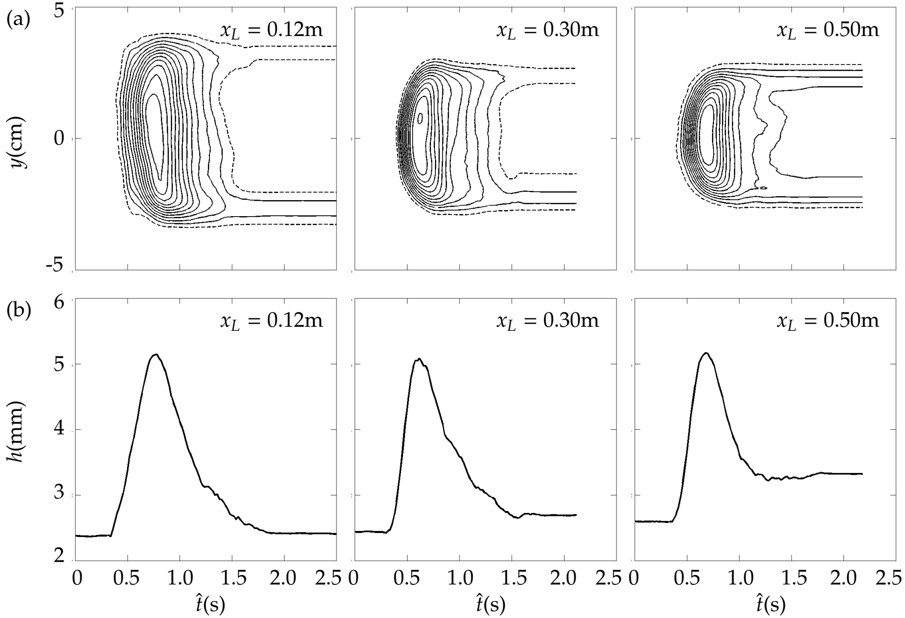
Experimental Setup

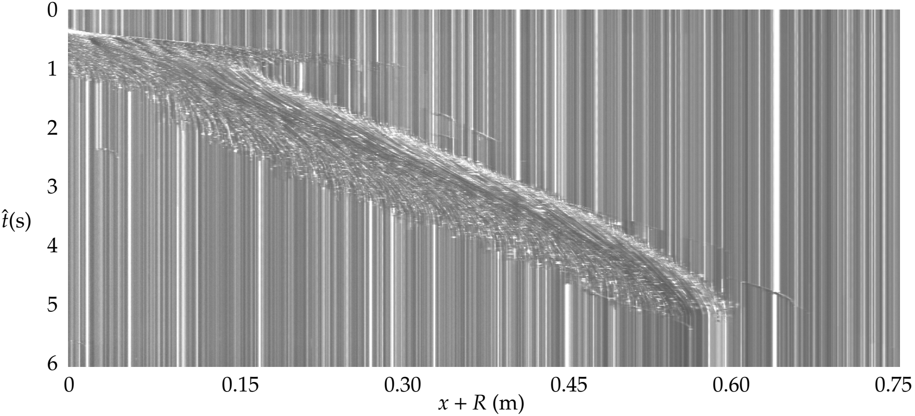


- Release of cylindricial mass ($R = 1.4$cm, $H = 1.8$cm) of $280 - 350\mu$m diameter carborundum particles on a static erodible layer of the same material
- Rough bed of attached $750 - 1000\mu$m diameter spherical glass beads

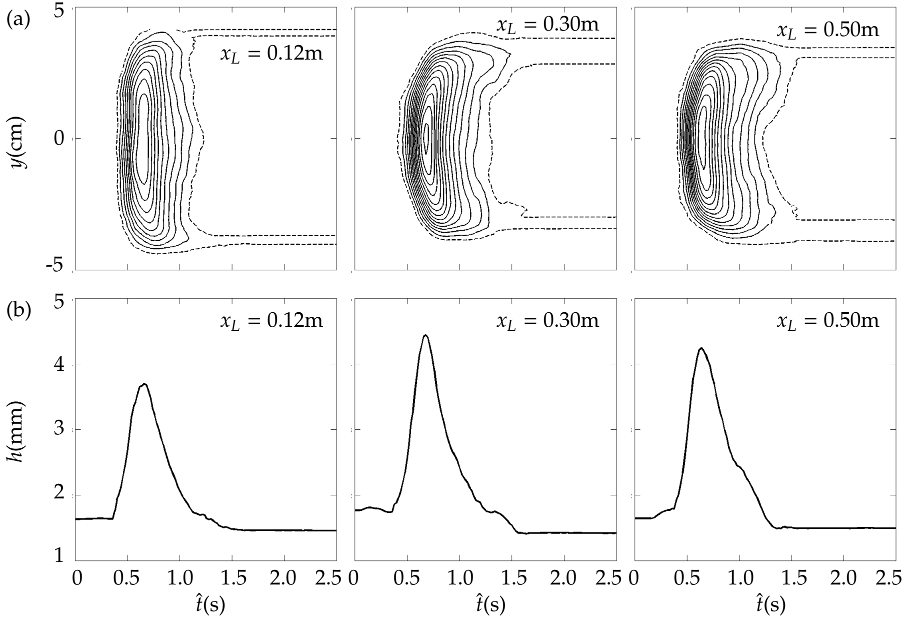
Experimental Observations for $\zeta = 34.0°$



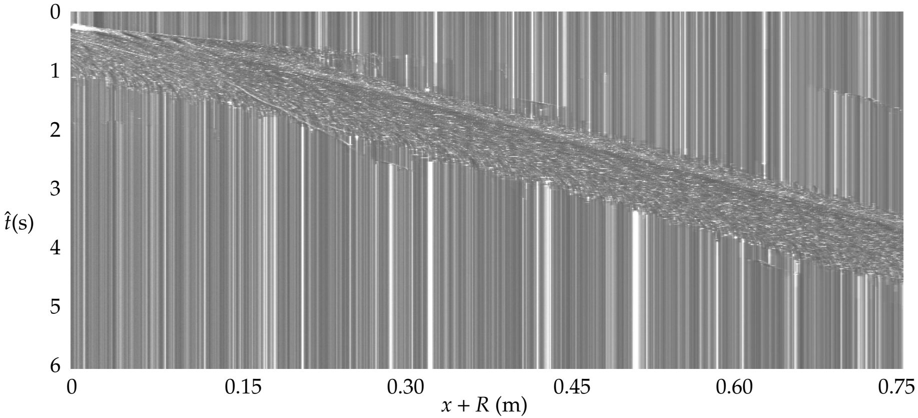
Experimental Observations for $\zeta = 34.0°$



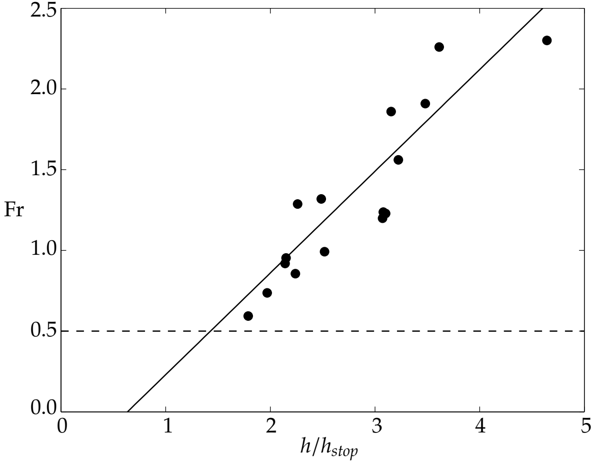
Experimental Observations for $\zeta = 35.2°$



Experimental Observations for $\zeta = 35.2°$

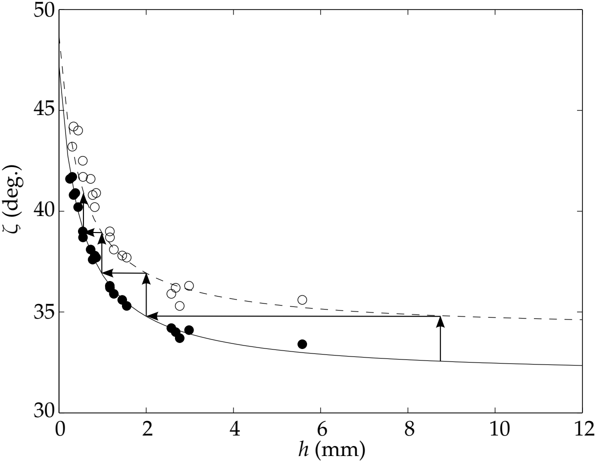


Basal friction law



- Steady uniform flow rule: $\mathrm{Fr} = \dfrac{\bar{u}}{\sqrt{hg\cos\zeta}} = \beta \dfrac{h}{h_{\mathrm{stop}}(\zeta)} - \Gamma$
- Here $\beta = 0.63$, $\Gamma = 0.40$ and $\alpha = 0.5$ (unsteady cut-off)
- Spherical glass beads: $\beta = 0.136$, $\Gamma = 0$
  Sand: $\beta = 0.65$, $\Gamma = 0.77$ (Pouliquen & Forterre 2002)

Basal friction law



- Critical slope angle functions:

$$\tan \zeta_{stop,start} = \tan \zeta_{1,3} + \frac{\tan \zeta_2 - \tan \zeta_1}{1 + h/\mathscr{L}}$$

- $\zeta_1 = 31.7^\circ$, $\zeta_2 = 47.2^\circ$, $\zeta_3 = 34.0^\circ$, $\mathscr{L} = 0.39$mm

Basal friction law

- $\mu(h, \mathrm{Fr}) =$

$$\begin{cases} \mu_1 + \dfrac{\mu_2 - \mu_1}{1 + h\beta/\mathscr{L}(\mathrm{Fr} + \Gamma)} & \mathrm{Fr} \geq \alpha \\ \left(\dfrac{\mathrm{Fr}}{\alpha}\right)^{\kappa} \left(\mu_1 + \dfrac{\mu_2 - \mu_1}{1 + h\beta/\mathscr{L}(\alpha + \Gamma)} - \mu_3 - \dfrac{\mu_2 - \mu_1}{1 + h/\mathscr{L}}\right) + \mu_3 + \dfrac{\mu_2 - \mu_1}{1 + h/\mathscr{L}} & 0 < \mathrm{Fr} \leq \alpha \\ \mu_3 + \dfrac{\mu_2 - \mu_1}{1 + h/\mathscr{L}} & \mathrm{Fr} = 0 \end{cases}$$

  where $\mu_i = \tan \zeta_i \quad (i = 1 \ldots 3)$

- Deposit layer thickness $h = h_{stop}$ when $\mathrm{Fr} = \beta - \Gamma \geq \alpha \Rightarrow$

$$h_{stop}(\zeta) = \mathscr{L}\left(\frac{\tan \zeta_2 - \tan \zeta_1}{\tan \zeta - \tan \zeta_1} - 1\right)$$

- $h_{\mathrm{stop}}(34.0°) = 2.778\mathrm{mm}$, $h_{\mathrm{stop}}(35.2°) = 1.663\mathrm{mm}$

2D Governing Equations

- Mass and momentum balance:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) = 0,$$

$$\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}\left(h\bar{u}^2\right) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) + \frac{\partial}{\partial x}\left(\frac{1}{2}h^2 g\cos\zeta\right) = hgS_x + \frac{\partial}{\partial x}\left(\nu h^{3/2} D_x\right),$$

$$\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}\left(h\bar{v}^2\right) + \frac{\partial}{\partial y}\left(\frac{1}{2}h^2 g\cos\zeta\right) = hgS_y + \frac{\partial}{\partial y}\left(\nu h^{3/2} D_y\right),$$

- Source terms:

$$S^x = \sin\zeta - \mu\frac{\bar{u}}{|\bar{\boldsymbol{u}}|}\cos\zeta,$$

$$S^y = \qquad - \mu\frac{\bar{v}}{|\bar{\boldsymbol{u}}|}\cos\zeta,$$

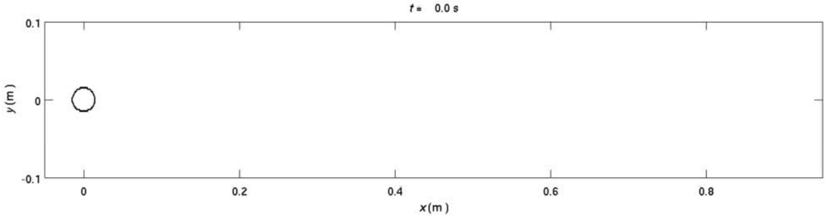
- Diffusive terms (see Gray & Edwards 2014)

$$D_x = \frac{\partial \bar{u}}{\partial x} + \frac{1}{2}\left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y}\right),$$

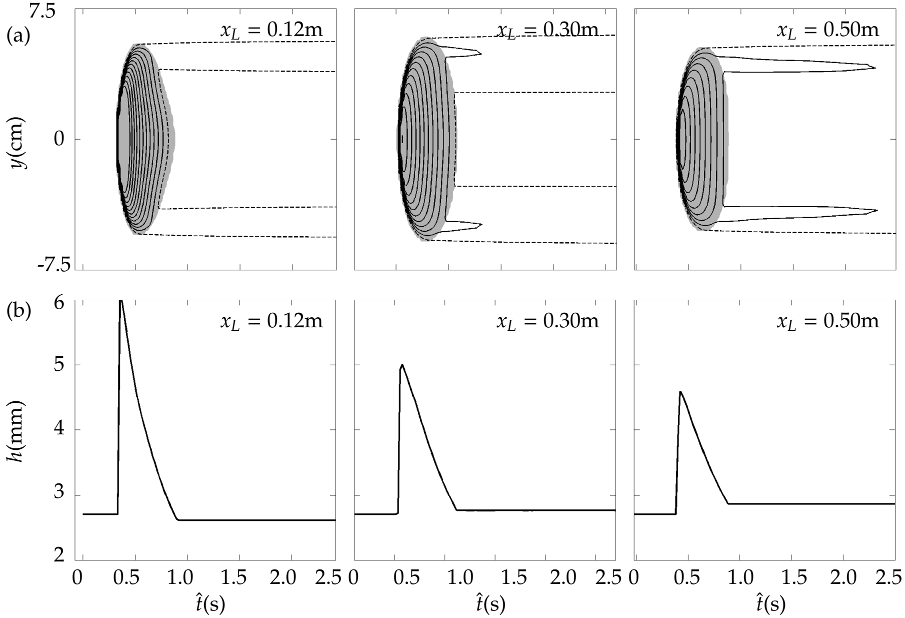
$$D_y = \frac{1}{2}\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right) + \frac{\partial \bar{v}}{\partial y}.$$

Numerical simulations for $\zeta = 34.0°$

- ICs: cylindrical cap $R = 1.4$cm, $H = 1.8$cm on a stationary layer $h_0 = 2.7$mm $< h_{stop}$
- BCs: stationary layer $h_0 = 2.7$mm, $\bar{u} = \bar{v} = 0$ inflow on LHS, free outflow on RHS

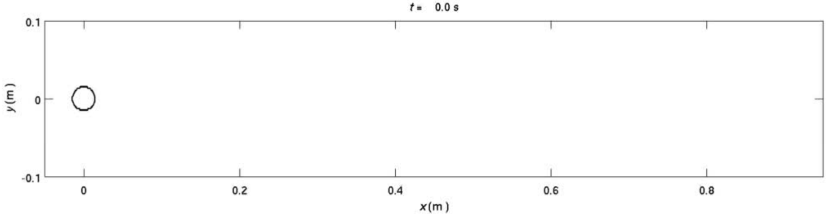


NUMERICAL SIMULATION FOR $\zeta = 34.0°$

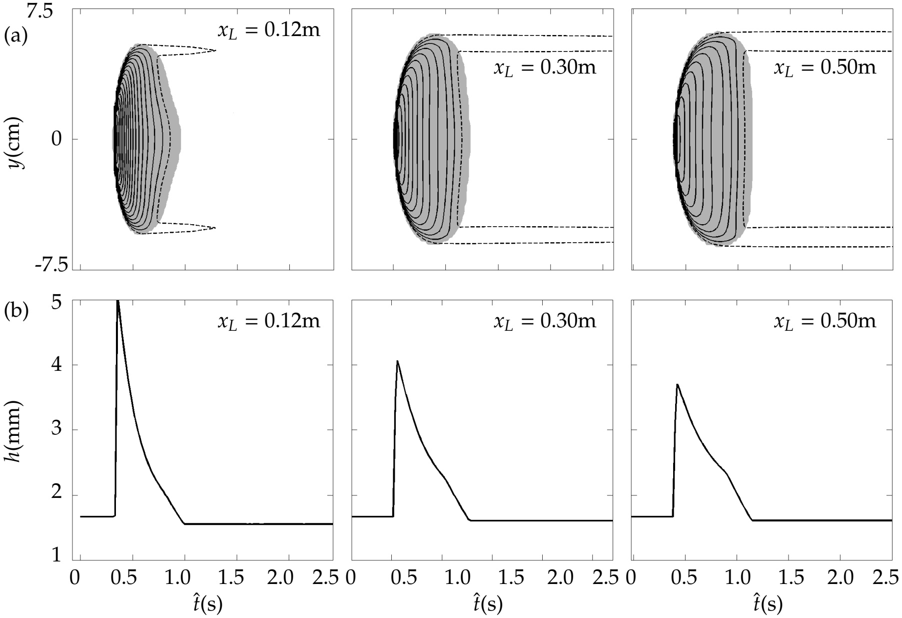


Numerical simulations for $\zeta = 35.2°$

- ICs: cylindrical cap $R = 1.4$cm, $H = 1.8$cm on a stationary layer $h_0 = 1.66$mm $< h_{stop}$
- BCs: stationary layer $h_0 = 1.66$mm, $\bar{u} = \bar{v} = 0$ inflow on LHS, free outflow on RHS



Numerical simulation for $\zeta = 35.2°$



Travelling-frame Equations

- Transform to coordinates (ξ, y, t) moving at wavespeed u_w by $\xi = x - u_w t$
- Mass and momentum balance:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial \xi}\left(h(\bar{u} - u_w)\right) + \frac{\partial}{\partial y}\left(h\bar{v}\right) = 0,$$

$$\frac{\partial}{\partial t}\left(h\bar{u}\right) + \frac{\partial}{\partial \xi}\left(h\bar{u}(\bar{u} - u_w)\right) + \frac{\partial}{\partial y}\left(h\bar{u}\bar{v}\right) + \frac{\partial}{\partial \xi}\left(\frac{1}{2}h^2 g\cos\zeta\right) = hgS^{\xi} + D^{\xi}(\xi, y),$$

$$\frac{\partial}{\partial t}\left(h\bar{v}\right) + \frac{\partial}{\partial \xi}\left(h\bar{v}(\bar{u} - u_w)\right) + \frac{\partial}{\partial y}\left(h\bar{v}^2\right) + \frac{\partial}{\partial y}\left(\frac{1}{2}h^2 g\cos\zeta\right) = hgS^{y} + D^{y}(\xi, y),$$

- Source terms:

$$S^{\xi} = \sin\zeta - \mu\frac{\bar{u}}{|\bar{\boldsymbol{u}}|}\cos\zeta,$$

$$S^{y} = \qquad - \mu\frac{\bar{v}}{|\bar{\boldsymbol{u}}|}\cos\zeta,$$

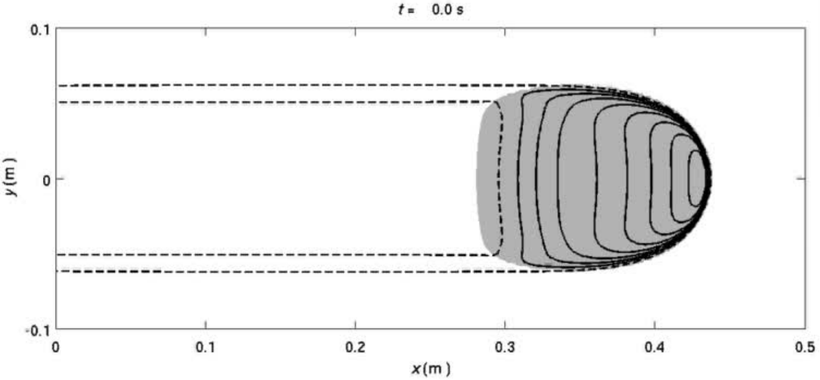
- Diffusive terms

$$D^{\xi}(\xi, y) = \frac{\partial}{\partial \xi}\left(\nu h^{3/2}\frac{\partial \bar{u}}{\partial \xi}\right) + \frac{\partial}{\partial y}\left(\frac{1}{2}\nu h^{3/2}\left(\frac{\partial \bar{v}}{\partial \xi} + \frac{\partial \bar{u}}{\partial y}\right)\right),$$

$$D^{y}(\xi, y) = \frac{\partial}{\partial \xi}\left(\frac{1}{2}\nu h^{3/2}\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial \xi}\right)\right) + \frac{\partial}{\partial y}\left(\nu h^{3/2}\frac{\partial \bar{v}}{\partial y}\right).$$

Travelling-frame Simulations

- $u_w = 0.21\text{ms}^{-1}$
- ICs: profile from $\zeta = 35.2°$ simulation
- BCs: stationary layer $h_0 = 1.66\text{mm}$, $\bar{u} = \bar{v} = 0$ inflow on RHS, free outflow on LHS



Thank you for listening