

# Trace of “SubMatrix” algorithm<sup>1</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

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<sup>1</sup>Note: Two elements by cache line

## Trace of “SubMatrix” algorithm<sup>2</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ \hline 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ \hline 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ \hline 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

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<sup>2</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>3</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ \mathbf{5} & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ \hline 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} \mathbf{a} & \mathbf{b} & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ \mathbf{5a + 6e + 7i + 8m} & \mathbf{5b + 6f + 7j + 8n} & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ \hline 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>3</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>4</sup>

$$\begin{pmatrix} 1 & 2 & | & 3 & 4 \\ 5 & \mathbf{6} & | & 7 & 8 \\ 9 & 10 & | & 11 & 12 \\ 13 & 14 & | & 15 & 16 \end{pmatrix} \times \begin{pmatrix} a & b & | & c & d \\ \mathbf{e} & \mathbf{f} & | & g & h \\ i & j & | & k & l \\ m & n & | & o & p \end{pmatrix} =$$

$$\begin{pmatrix} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & | & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + \mathbf{6e} + 7i + 8m & 5b + \mathbf{6f} + 7j + 8n & | & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & | & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & | & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{pmatrix}$$

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<sup>4</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>5</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ \hline 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ \hline 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ \hline 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ \hline 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

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<sup>5</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>6</sup>

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix} \times \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} =$$

$$\begin{pmatrix} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{pmatrix}$$

<sup>6</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>7</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

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<sup>7</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>8</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ \hline i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>8</sup>Note: Two elements by cache line



# Trace of “SubMatrix” algorithm<sup>9</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

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<sup>9</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>10</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>10</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>11</sup>

$$\begin{pmatrix} 1 & 2 & | & 3 & 4 \\ \mathbf{5} & 6 & | & 7 & 8 \\ \hline 9 & 10 & | & 11 & 12 \\ \hline 13 & 14 & | & 15 & 16 \end{pmatrix} \times \begin{pmatrix} a & b & | & \mathbf{c} & \mathbf{d} \\ \hline e & f & | & g & h \\ \hline i & j & | & k & l \\ \hline m & n & | & o & p \end{pmatrix} =$$

$$\begin{pmatrix} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & | & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ \hline 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & | & \mathbf{5c} + 6g + 7k + 8o & \mathbf{5d} + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & | & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ \hline 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & | & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{pmatrix}$$

<sup>11</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>12</sup>

$$\begin{pmatrix} 1 & 2 & | & 3 & 4 \\ 5 & \mathbf{6} & | & 7 & 8 \\ 9 & 10 & | & 11 & 12 \\ 13 & 14 & | & 15 & 16 \end{pmatrix} \times \begin{pmatrix} a & b & | & c & d \\ e & f & | & \mathbf{g} & \mathbf{h} \\ i & j & | & k & l \\ m & n & | & o & p \end{pmatrix} =$$

$$\begin{pmatrix} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & | & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & | & 5c + \mathbf{6g} + 7k + 8o & 5d + \mathbf{6h} + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & | & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & | & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{pmatrix}$$

<sup>12</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>13</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ \hline 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ \hline 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ \hline 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ \hline 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>13</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>14</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>14</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>15</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>15</sup>Note: Two elements by cache line

# Trace of “SubMatrix” algorithm<sup>16</sup>

$$\left( \begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right) \times \left( \begin{array}{cc|cc} a & b & c & d \\ e & f & g & h \\ \hline i & j & k & l \\ m & n & o & p \end{array} \right) =$$

$$\left( \begin{array}{cc|cc} 1a + 2e + 3i + 4m & 1b + 2f + 3j + 4n & 1c + 2g + 3k + 4o & 1d + 2h + 3l + 4p \\ 5a + 6e + 7i + 8m & 5b + 6f + 7j + 8n & 5c + 6g + 7k + 8o & 5d + 6h + 7l + 8p \\ \hline 9a + 10e + 11i + 12m & 9b + 10f + 11j + 12n & 9c + 10g + 11k + 12o & 9d + 10h + 11l + 12p \\ 13a + 14e + 15i + 16m & 13b + 14f + 15j + 16n & 13c + 14g + 15k + 16o & 13d + 14h + 15l + 16p \end{array} \right)$$

<sup>16</sup>Note: Two elements by cache line



## Trace of “SubMatrix” algorithm

and so on ...