

# Stat 610 Homework 7

Thursday, November 10, 11:59pm

## Assignment

This assignment is (slightly) adapted from Lange, “Numerical Analysis for Statisticians.”

Consider the data from The London Times during the years 1910-1912 given in the table below.

Deaths $i$	Frequency $n_i$
0	162
1	267
2	271
3	185
4	111
5	61
6	27
7	8
8	3
9	1

The column labeled “Deaths  $i$ ” refers to the number of deaths to women 80 years and older reported by day. The column labeled “Frequency  $n_i$ ” refers to the number of days with  $i$  deaths. A Poisson distribution gives a poor fit to these data, possibly because of different patterns of deaths in winter and summer. A mixture of two Poissons provides a much better fit. Under the Poisson admixture model, the likelihood of the observed data is

$$\prod_{i=0}^9 \left[ \alpha e^{-\mu_1} \frac{\mu_1^i}{i!} + (1 - \alpha) e^{-\mu_2} \frac{\mu_2^i}{i!} \right]^{n_i}$$

where  $\alpha$  is the admixture parameter and  $\mu_1$  and  $\mu_2$  are the means of the two Poisson distributions.

Implement an EM algorithm for this model. Let  $\theta = (\alpha, \mu_1, \mu_2)^T$  and

$$z_i(\theta) = \frac{\alpha e^{-\mu_1} \mu_1^i}{\alpha e^{-\mu_1} \mu_1^i + (1 - \alpha) e^{-\mu_2} \mu_2^i}$$

be the posterior probability that a day with  $i$  deaths belongs to Poisson population 1.

Check that the EM algorithm is given by

$$\begin{aligned} \alpha_{m+1} &= \frac{\sum_i n_i z_i(\theta_m)}{\sum_i n_i} \\ \mu_{m+1,1} &= \frac{\sum_i n_i i z_i(\theta_m)}{\sum_i n_i z_i(\theta_m)} \\ \mu_{m+1,2} &= \frac{\sum_i n_i i [1 - z_i(\theta_m)]}{\sum_i n_i [1 - z_i(\theta_m)]} \end{aligned}$$

From the initial estimates  $\alpha_0 = .3$ ,  $\mu_{0,1} = 1$ , and  $\mu_{0,2} = 2.5$ , compute via the EM algorithm the maximum likelihood estimates. Note how slowly the EM algorithm converges in this example.

## Submission parameters

Submit two files:

- A pdf writeup containing an explanation of the update formulas and the parameter estimates after each iteration of the algorithm.
- A file containing the code you used.