The Quite Reasonable Effectiveness of Formalization in Programming Language Design

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- What is reasonable effectiveness?

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- What is reasonable effectiveness?
- Reasonable effectiveness as a measure of utility

The Experience of Formalization

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- "Premature *abstraction* is the root of all evil."

Premature Abstraction

```
(* We define our total category of graphs
   which is fibered over the base category *)
Section TotalCategoryG.
  Context `{Countable K}.
  Context `{FMap SH}.
  Context {elts : forall A, Elements A (SH A)}.

  Definition G_obj :=
  { G : graph | @GraphWF K _ _ SH _ G }.

  Definition G_hom (A B : G_obj) :=
    { f : B_hom (interface (`A)) (interface (`B)) &
      forall n, ElemOf n dom (node (`A)) ->
        exists m, ElemOf m dom (node (`B)) /\
  ...
```

The Root of All Evil

Abstraction as a luxury

- Premature optimization
- Moves away from the problem

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Abstraction as a necessity

- Occam's Razor
- Minimally sufficient definitions

Case Study: Abstraction as a Necessity

The EPIC language

- Confluent, nondeterministic lambda calculus variant
- Parallel by default
- Tree-like denotation

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```
Inductive term :=
| lam : lets -> term
with lets :=
| def (t:term) (l:lets)
| app (f:id) (x:id) (l:lets)
| tpl (xs:list id) (l:lets)
| prj (n:nat) (x:id) (l:lets)
| cut (x:id) (l:lets)
| ret (y:id).
```

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Define `id := nat.` (De Bruijn indeces).

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```
Definition wf_var (G : nat) (x : id) : bool := x <? G.
```

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Unfortunately...

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Unfortunately... most of Category Theory

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- Programming languages allow us to define an *effective procedure* for a computer.
- Formalization is the *effective procedure* of mathematics.

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