CS 2850 – Networks HW 4

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1. (a) The largest strongly connected component of this graph is the set $\{5, 6, 7, 8, 9, 10\}$.

   (b) If we were to add a link from node 14 to node 2, the largest strongly connected component would become the set

   $$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14\}$$

   which is the largest possible set of strongly connected components from a single-link addition. Without loss of generality with respect to node 14 or 2, note that linking 14 to 1 is not the solution because 4 would be inaccessible. Similarly, linking 14 to 13 would do the same.

   (c) From the original set that we mentioned in part (a), notice that each of those nodes depend on the link between 7 and 8 to be strongly connected. If we were to remove that link, there would be no strongly connected component with a larger size than one. Thus, deleting the link between 7 and 8 is the solution.

2. After one iteration of the Basic PageRank Update Rule, we get the following table of values:

Node	Value
A	4/17
B	3/17
C	3/17
D	2/17
E	4/17
F	1/17
Sum	17/17

   Although they do add to 1, the probabilities are not the same as the original values. Therefore, these numbers do not form an equilibrium.

3. (a) The following table shows the value of each node in terms of x:

Node	Value in Terms of x
A	x
B	$x/2$
C	$x/2$
D	$x/4$
E	$x/2$
F	$x/4$
G	$5x/8$
H	$3x/8$

(b) We can solve for x by summing these values and setting them equal to 1. We get the following equation:

$$x + \frac{x}{2} + \frac{x}{2} + \frac{x}{4} + \frac{x}{2} + \frac{x}{4} + \frac{5x}{8} + \frac{3x}{8} = 1$$

which yields $x = \frac{1}{4}$. Thus, our values become

Node	Value in Terms of x
A	$\frac{1}{4}$
B	$\frac{1}{8}$
C	$\frac{1}{8}$
D	$\frac{1}{16}$
E	$\frac{1}{8}$
F	$\frac{1}{16}$
G	$\frac{5}{16}$
H	$\frac{3}{16}$
Sum	1

4. (a) After $k = 2$ steps of the computation, we get the following table of values:

Node	Auth	Normalized Auth	Hub	Normalized Hub
A	11	0.61	0	0.00
B	7	0.39	0	0.00
C	0	0.00	11	0.23
D	0	0.00	11	0.23
E	0	0.00	18	0.38
F	0	0.00	7	0.15

(b) We would expect option 2 to yield a higher hub score for node Y, which would in turn yield a higher authority score for node X After $k = 2$ steps of the computation with each addition, we get the following table of values

Node	Normalized Auth Opt 1	Normalized Auth Opt 2
A	0.58	0.47
B	0.37	0.37
X	0.05	0.16

which are aligned with our expectations.

(c) My strategy is to connect Z to A, C, X, Y, connect Y to B, X, Z and connect X to Y, X which yields the following table after $k = 2$ rounds:

Node	Auth	Normalized Auth	Hub	Normalized Hub
A	25	0.26	0	0.00
B	17	0.18	0	0.00
C	10	0.10	25	0.09
D	0	0.00	25	0.09
E	0	0.00	42	0.16
F	0	0.0	17	0.06
X	22	0.23	37	0.14
Y	15	0.16	46	0.17
Z	7	0.07	27	0.27

From this table, it is clear that A has the highest authority, and X has the second-highest authority.