CS 2850 – Networks HW 8

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1. (a) One iteration after the new behavior is introduced, we will see node 3 adopt A because it only has three neighbors. On the next iteration, nodes 9 and 4 will also adopt A because $1/3, 2/5 > 0.3$ of their neighbors respectively have adopted A. This will continue until every node on the left hand side of the graph has adopted A. At this point, adoption will cease because $1/4 < 0.3$ of the neighbors of node 6 have adopted A.

   (b) The sets are constituted by the two sides of the graph:

   $$S = \{1, 3, 4, 5, 9, 10\} \text{ and } T = \{2, 6, 7, 8, 11\}.$$

   (c) Under the current adoption rule, this is not possible. The only way to get from node 6 to node 5 (or vice versa) is to have an adoption rule such that a node will adopt A if at least $1/4$ of its neighbors have adopted A. Thus, changing the starting node will not impact the ability for A to be adopted between nodes $5, 6$.

2. (a) We would observe this pattern with a value of $q = 1/4$. In the first iteration, we would see nodes 2 and 4 switch to A, and then in the next iteration, we would see nodes $3, 5, 7, 8$ switch to A. Finally, in the last iteration, we would see node 6 switch to A.

   (b) Firstly, we see that node 2 has 3 neighbors and node 4 has 4 neighbors. Thus, choosing $q = 1/3$ would lead to node 2 adopting A in the first iteration, and node 4 not adopting A until node 3 has adopted A. The rest of the pattern follows from running the rest of the iterations.

   (c) Suppose for contradiction that q exists and node 2 adopts A on the first iteration. For the desired pattern to be observed, it must be the case that $q < 1/2$ for node 5 to adopt A on the second iteration, and $q > 2/5$ for node 3 not to adopt A. However, if $q > 2/5$, then node 2 will not adopt A on the first iteration–therein lies the contradiction. Therefore, there is no value of q that exists which yields the desired pattern.

3. From the question, it must be the case that u should choose A if $x > q$ and should choose B if $x < q$ (we ignore the case that $x = q$). Then we

can answer this question by treating x as the probability that u chooses A and solving for x using the payoff matrix:

$$5x - 2(1-x) > 1x - 3(1-x)$$
$$\implies x > \frac{1}{5}.$$

Thus, u should choose A if $x > \frac{1}{5}$ and should choose B if $x < \frac{1}{5}$. Therefore, $q = \frac{1}{5}$.

4. (a) We can find a value for q by looking at the ratio of friends in each region. More specifically, in order for the new feature to not spread from X, to Y, it must be the case that q is greater than the number of friends that users in Y have in X divided by the number of friends that users in X have in X, or rather, $q > \frac{1}{6}$. Moreover, for the new feature to spread from Y to X, it must be the case that q is at most the number of friends that users in X have in Y divided by the total number of friends that users in X have in both regions, or rather, $q \leq \frac{4}{16} = \frac{1}{4}$. Thus, the desired value of q exists and is $\frac{1}{6} < q < \frac{1}{4}$.

   (b) The desired pattern could be observed by taking the inverse of the situation that we described in part (a). This yields a value of $\frac{1}{4} < q \leq \frac{1}{6}$. However, this is not possible.