CS 2850 – Networks

jfw225

August 2022

1. Let $\psi(\mathcal{P}, \mathcal{R}, \mathcal{C})$ be the payoff for player $\mathcal{P} \in \{A, B\}$ for row $\mathcal{R} \in \{U, D\}$ and column $\mathcal{C} \in \{L, R\}$. Additionally, let $\mathbb{E}[\psi(\mathcal{P}, \mathcal{R}, \mathcal{C})]$ be the expected payoff for player $\mathcal{P}$ for row $\mathcal{R}$ and column $\mathcal{C}$.

   (a) The dominant strategy for Player A is always to choose row D because they will always have a better payoff than if they picked row U, regardless of the column chosen by player B. More rigidly, $\psi(A, D, \mathcal{C}) > \psi(A, U, \mathcal{C})$ for all $\mathcal{C} \in \{L, R\}$.

   Likewise, the dominant strategy for Player B is always to choose row R because $\psi(B, \mathcal{R}, R) > \psi(B, \mathcal{R}, L)$ for all $\mathcal{R} \in \{U, D\}$.

   Therefore, the Nash equilibrium is (D, R).

   (b) Unlike the in part (a), Player A does not have a dominant strategy. That is, Player A cannot guarantee a better payoff by simply always picking some row.

   However, Player B does have a dominant strategy, which is to always pick column R. This is because $\psi(B, \mathcal{R}, R) > \psi(B, \mathcal{R}, L)$ for all $\mathcal{R} \in \{U, D\}$.

   Given that Player B will always choose column R, Player A is better off choosing row U because $\psi(A, U, R) > \psi(A, D, R)$. Since there is no change in strategy that will result in a better payoff for Player A, (U, R) is the Nash equilibrium.

   (c) First, observe that there is no pure strategy that is a part of the Nash equilibrium for this game. That is, there is no strategy that will result in a better payoff for both players. Thus, we must consider mixed strategies. Let p be the probability that Player A chooses row U and q be the probability that Player B chooses column L. Then we can write the expected payoffs for each player in terms of p, q:

$$
\begin{aligned}
\underline{\textbf{Player A:}} \quad \mathbb{E}\left[\psi\left(A, U, \mathcal{C}\right)\right] &= q \cdot \psi\left(A, U, L\right) + \left(1-q\right) \cdot \psi\left(A, U, R\right) \\
&= q + \left(1-q\right) \cdot 0 = q; \\
\mathbb{E}\left[\psi\left(A, D, \mathcal{C}\right)\right] &= q \cdot \psi\left(A, D, L\right) + \left(1-q\right) \cdot \psi\left(A, D, R\right) \\
&= q \cdot 0 + \left(1-q\right) \cdot 1 = 1-q; \\
\underline{\textbf{Player B:}} \quad \mathbb{E}\left[\psi\left(B, \mathcal{R}, L\right)\right] &= p \cdot \psi\left(B, U, L\right) + \left(1-p\right) \cdot \psi\left(B, D, L\right) \\
&= p \cdot 1 + \left(1-p\right) \cdot 2 = 1-p; \\
\mathbb{E}\left[\psi\left(B, \mathcal{R}, R\right)\right] &= p \cdot \psi\left(B, U, R\right) + \left(1-p\right) \cdot \psi\left(B, D, R\right) \\
&= p \cdot 2 + \left(1-p\right) \cdot 1 = p+1.
\end{aligned}
$$

From section 6.7 of the textbook, we know that $\mathbb{E}\left[\psi\left(A, U, \mathcal{C}\right)\right] = \mathbb{E}\left[\psi\left(A, D, \mathcal{C}\right)\right]$ and $\mathbb{E}\left[\psi\left(B, \mathcal{R}, L\right)\right] = \mathbb{E}\left[\psi\left(B, \mathcal{R}, R\right)\right]$. If this were not the case, we would have a contradiction because we established that there are no pure strategies. Thus, we can solve for p, q:

$$
\begin{aligned}
\mathbb{E}\left[\psi\left(A, U, \mathcal{C}\right)\right] &= \mathbb{E}\left[\psi\left(A, D, \mathcal{C}\right)\right] \\
dostuffhere & \\
\mathbb{E}\left[\psi\left(B, \mathcal{R}, L\right)\right] &= \mathbb{E}\left[\psi\left(B, \mathcal{R}, R\right)\right]
\end{aligned}
$$