CS 2850 – Networks HW 7

jfw225

November 2022

1. (a) Solving the equation yields equilibrium points of 0, 0.25, and 0.75.
   (b) 0.25 is the only unstable equilibrium while the others are all stable.
   (c) The product will become more popular over time because $z = 0.3$ is between 0.25 and 0.75, and thus, it will grow in popularity over time.
   (d) Our answer would not change it the value changed to $z = 0.5$ because the function is still increasing between 0.25 and 0.75.

2. (a) For $z = 1/4$, we have

   $$\begin{aligned} r(1/4)\,f(1/4) &= (1 - 1/4) \cdot \min(1, 1) \\ &= 3/4. \end{aligned}$$

   Since $3/4 > 7/16$, the fraction of interested population will be increasing. Therefore, $z = 1/4$ is not an equilibrium.

   (b) When $z < 1/4$, we have

   $$\begin{aligned} 7/16 = r(z)\,f(z) &= (1 - z) \cdot \min(1, 4z) \\ 7/16 &= = (1 - z) \cdot 4z \\ 7/16 &= = 4z - 4z^2. \end{aligned}$$

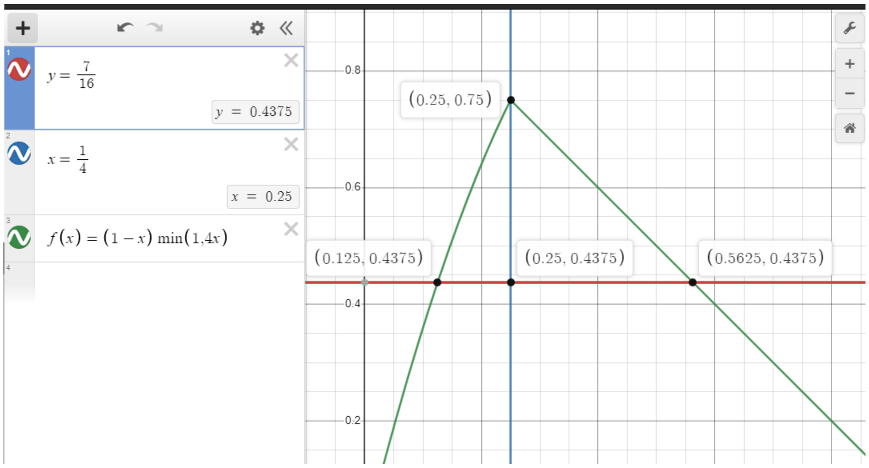
   Solving this equation yields $z = 0, 1/8$ as the equilibrium solutions.

   (c) When $z > 1/4$, we have

   $$\begin{aligned} 7/16 = r(z)\,f(z) &= (1 - z) \cdot \min(1, 4z) \\ 7/16 &= = (1 - z) \cdot 1 \\ 7/16 &= = 1 - z. \end{aligned}$$

   Solving this equation yields $z = 9/16$ as an equilibrium solution.

(d) We will observe that $z = 1/4 > 1/8$ will grow to $z = 9/16$ as shown in the following graph:



3. Let n be the number of articles that it takes to receive k views. Then it is theorized that $n = \frac{c}{k^{1.5}}$ for some constant c.

(a) Let $n = 200$ and $k = 25$. Then we have

$$n = \frac{c}{k^{1.5}} \implies 200 = \frac{c}{25^{1.5}} \implies c = 200 \cdot 25^{1.5} = 25,000.$$

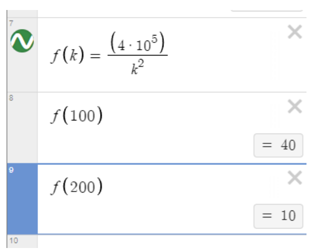
(b) Let $n = 10$ and $k = \max(100, 25) = 100$. Then we have

$$n = \frac{c}{k^{1.5}} \implies 10 = \frac{c}{100^{1.5}} \implies c = 10 \cdot 100^{1.5} = 10,000.$$

(c) The question is equivalent to asking if there is some function f whose parent is the power law such that $f(100) = 40$ and $f(200) = 10$. Using this information, we create the following two equations:

$$
\begin{aligned}
40 &= \frac{b}{100^a} \\
10 &= \frac{b}{200^a} \\
\implies 10 &= \frac{40 \cdot 100^a}{200^a} \\
\frac{1}{4} &= \left(\frac{1}{2}\right)^a \\
\implies a &= 2 \\
\implies b &= 40 \cdot 100^2 = 400,000.
\end{aligned}
$$

Checking the answer, we have



4. (a) Observe that it is impossible for any graph that is produced by the RICH-GET-RICHER algorithm to contain a node who does not have exactly one outbound edge (other than node 1 because it is the oldest). Graph (a) could not have been produced from the algorithm because there are nodes that have more than one outbound edge. Graph (c) has a node other than node 1 with zero outbound edges. Thus, the only graphs that could have been produced by the algorithm are (b) and (d).

   (b) A value of $p = 1/2$ means that a node is equally as likely to connect to some node i or copy the decision of node i. In graph (d), none of the nodes made the former choice–each node chose node i. Thus, it is more likely that (b) was produced by the algorithm.