CS 2850 – Networks HW 3

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1. Let b_i be the bid made by the i-th bidder, and let b_j be the bid made by the j-th bidder where $i, j \in \{2, 3\}$ and $i \neq j$. Suppose that bidder i bids at least as much as bidder j (i.e. $b_i \geq b_j$). In order to win, we must bid $b_1 > b_i$ for the item. Since this is a second-price sealed-bid auction, we will pay b_i for the item if we win which yields a payoff of $v_1 - b_i$. If instead we lose the auction, then our payoff is zero. From this, we consider two cases:

   - <u>**Case 1: $\mathbf{b_i > v_1}$**</u> If $b_i > v_1$ is the second highest bet and we won the auction, then it must be the case that we bid more than v_1. In this case, we are guaranteed to have a payoff of $v_1 - b_i \leq 0$. Notice that winning when $b_i > v_1$ is at best as valuable as not playing at all. Thus, we should never bid more than v_1 when $b_i > v_1$.
   - <u>**Case 2: $\mathbf{b_i \leq v_1}$**</u> In this case, we know that both b_i and b_j are at most v_i. We will still pay b_i for the item, but we are guaranteed to have a payoff of at least zero. More specifically, our payoff is $v_1 - b_i \geq 0$. Notice that when $b_i \leq v_1$, a bid of $b_1 = v_i$ is guaranteed to yield the exact same payoff as a bid of $b_1 > v_i$.

   Thus, bidding above $v_1 = 30$ never increases the payoff and only adds additional risk. Therefore, our friend is incorrect and we should never bid more than 30.

2. Let $\psi_i\left(R, v_i, b_i, b_j\right)$ be the payoff for bidder i when placing a bid of b_i for an object valued at v_i while facing off in a two-buyer auction against some other bidder j who places a bid of b_j. In addition, the auction is a second-price auction with a reserve price R. Let us further define this payoff function as

$$\psi_i\left(R, v_i, b_i, b_j\right) = \begin{cases} 0 & \text{if } b_i \leq b_j \text{ or } b_i \leq R, \\ v_i - \max\{b_j, R\} & \text{if } b_i > b_i \text{ and } b_i > R. \end{cases}.$$

   (a) Let us refer to the opposing buyer as bidder j and ourselves as bidder i. From the question, we know that $R = 10$ and our value is $v_i = 15$. Our goal is to bid such that our expected payoff is maximized, or rather, pick a value b_i such that $\mathbb{E}\left[\psi_i\left(R, v_i, b_i, b_j\right)\right]_{b_j}$ is maximized, where the expected payoff is given by

$$\mathbb{E}\left[\psi_i\left(R, v_i, b_i, b_j\right)\right]_{b_j} = \sum_{b \in \{5,10,15\}} \mathbb{P}\left(b_j = b\right) \cdot \psi_i\left(R = 10, v_i = 15, b_i, b_j = b\right).$$

Computing each value of b_i gives us

$$\begin{aligned}
\mathbf{b_i = 5:} \quad & \mathbb{E}\left[\psi_i\left(R, v_i, b_i = 5, b_j\right)\right]_{b_j} &&= \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot 0 \\
& &&= 0, \\
\mathbf{b_i = 10:} \quad & \mathbb{E}\left[\psi_i\left(R, v_i, b_i = 10, b_j\right)\right]_{b_j} &&= \frac{1}{3}(15 - 10) + \frac{1}{3}(15 - 10) + \frac{1}{3} \cdot 0 \\
& &&= \frac{5}{3}, \\
\mathbf{b_i = 15:} \quad & \mathbb{E}\left[\psi_i\left(R, v_i, b_i = 15, b_j\right)\right]_{b_j} &&= \frac{1}{3}(15 - 15) + \frac{1}{3}(15 - 10) + \frac{1}{3} \cdot 0 \\
& &&= \frac{5}{3}.
\end{aligned}$$

Since choosing $b_i = 15$ has a higher win probability than any other value and also yields an expected at least as good as the next choice, we should bid $b_i = 15$.

(b) something similar