Pg 186 (3.4) João Vaz

1- $y = \operatorname{sen}(4x)$

$f(x) = \operatorname{sen}(x)$; $g(x) = 4x$; $f(g(x)) = \operatorname{sen}(4x)$

$y' = f'(g(x)) \cdot g'(x) = \operatorname{sen}'(4x) \cdot (4x)' = \cos(4x) \cdot 4 = 4\cos(4x)$

2- $y = \sqrt{4+3x}$

$f(x) = \sqrt{4+x}$; $g(x) = 3x$; $f(g(x)) = \sqrt{4+3x}$

$y' = f'(g(x)) \cdot g'(x) = \left[(4+x)^{1/2}\right]' \cdot (3x)' = \frac{1}{2}(4+x)^{-1/2} \cdot 3 = \sqrt{\frac{1}{4+x}} \cdot \frac{3}{2} = \sqrt{\frac{9}{16+4x}}$

3- $y = (1-x^2)^{10}$

$f(x) = x^{10}$; $g(x) = 1-x^2$; $f(g(x)) = (1-x^2)^{10}$

$y' = f'(g(x)) \cdot g'(x) = \left[(1-x^2)^{10}\right]' \cdot (1-x^2)' = 10(1-x^2)^9 \cdot (-2x) = -20x(1-x^2)^9$

4- $y = tg(\operatorname{sen} x)$

$f(x) = tg\, x$; $g(x) = \operatorname{sen} x$; $f(g(x)) = tg(\operatorname{sen} x)$

$y' = f'(g(x)) \cdot g'(x) = \left[tg(\operatorname{sen} x)\right]' \cdot \operatorname{sen}'(x) = \sec^2(\operatorname{sen} x) \cdot \cos x = \cos x \cdot \sec^2(\operatorname{sen} x)$

5- $y = e^{\sqrt{x}}$

$f(x) = e^x$; $g(x) = \sqrt{x}$; $f(g(x)) = e^{\sqrt{x}}$

$y' = f'(g(x)) \cdot g'(x) = \left[e^{\sqrt{x}}\right]' \cdot (\sqrt{x})' = \sqrt{x}\, e^{\sqrt{x}-1} \cdot \frac{1}{2} x^{-1/2} = \frac{e^{\sqrt{x}}}{2e}$

6- $y = \sqrt{2-e^x}$

$f(x) = \sqrt{x}$; $g(x) = 2-e^x$; $f(g(x)) = \sqrt{2-e^x}$

$y' = f'(g(x)) \cdot g'(x) = \left[\sqrt{2-e^x}\right]' \cdot (2-e^x)' = \frac{1}{2}(2-e^x)^{-1/2} \cdot (-e^x) = \frac{-e^x}{2\sqrt{2-e^x}} = \frac{-e^x\sqrt{2-e^x}}{2(2-e^x)} = \frac{\sqrt{-2e^{2x}+e^{3x}}}{4-2e^x}$

7- $F(x) = (x^4+3x^2-2)^5$

$F'(x) = 5(x^4+3x^2-2)^4 \cdot (4x^3+6x)$

8- $F(x) = (4x-x^2)^{100}$

$F'(x) = 100(4x-x^2)^{99} \cdot (4-2x)$

9- $F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{1/4}$

$F'(x) = \frac{1}{4}(1+2x+x^3)^{-3/4} \cdot (2+3x^2)$

10- $f(x) = (1+x^4)^{2/3}$

$f'(x) = \frac{2}{3}(1+x^4)^{-1/3} \cdot 4x^3 = \frac{8x^3}{3} \cdot (1+x^4)^{-1/3}$

11- $g(t) = \frac{1}{(t^4+1)^3}$

$g'(t) = \frac{0 \cdot (t^4+1) - 1 \cdot 3(t^4+1)^2 \cdot 4t^3}{(t^4+1)^6} = \frac{-12t^3(t^4+1)^2}{(t^4+1)^6} = \frac{-12t^3}{(t^4+1)^4}$

12- $f(t) = \sqrt[3]{1+tg\,t} = (1+tg\,t)^{1/3}$

$f'(t) = \frac{1}{3}(1+tg\,t)^{-1/3} \cdot \sec^2 t = \frac{\sec^2 t}{3\sqrt[3]{1+tg\,t}}$

13- $y = \cos(a^3+x^3)$

$y' = -\operatorname{sen}(a^3+x^3)\cdot 3x^2 = -3x^2\operatorname{sen}(a^3+x^3)$

14- $y = a^3 + \cos^3 x$

$y' = 3\cos^2 x\cdot(-\operatorname{sen}^3 x) = -3\cos^2 x\operatorname{sen}^3 x$

15- $y = x e^{-kx}$

$y' = e^{-kx} + x(-ke^{-kx}) = e^{-kx}(1-kx)$

16- $y = e^{-2t}\cdot\cos 4t$

$y' = e^{-2t}\cdot\cos 4t + e^{-2t}\cdot\operatorname{sen}4t\cdot 4 = e^{-2t}[\cos 4t + 4\operatorname{sen}4t]$

17- $f(x) = (2x-3)^4(x^2+x+1)^5$

$f'(x) = 4(2x-3)^3\cdot 2\cdot(x^2+x+1)^5 + (2x-3)^4\cdot 5(x^2+x+1)^4\cdot(2x+1) = (2x-3)^3(x^2+x+1)^4\left[8(x^2+x+1)+5(2x-3)(2x+1)\right]$

18- $g(x) = (x^2+1)^3(x^2+2)^6$

$g'(x) = 3\cdot(x^2+1)^2\cdot 2x + 6\cdot(x^2+2)^5\cdot 2x = 6x\left[(x^2+1)^2 + 2\cdot(x^2+2)^5\right]$

19- $h(t) = (t+1)^{2/3}\cdot(2t^2-1)^3$

$h'(t) = \frac{2}{3}(t+1)^{-1/3}\cdot 1 + 3(2t^2-1)^2\cdot 4t = \frac{2}{3}\left[(t+1)^{-1/3} + 18t(2t^2-1)^2\right]$

20- $F(t) = (3t-1)^4(2t+1)^{-3}$

$F'(t) = 4(3t-1)^3\cdot 3 - 3(2t+1)^{-4}\cdot 2 = 6\left[2(3t-1)^3 - (2t+1)^{-4}\right]$

21- $y = \left(\frac{x^2+1}{x^2-1}\right)^3$

$y' = 3\cdot\left(\frac{x^2+1}{x^2-1}\right)^2\cdot\left(\frac{2x(x^2-1)-(x^2+1)\cdot 2x}{(x^2-1)^2}\right) = 6x\left(\frac{x^2+1}{x^2-1}\right)\cdot\left(\frac{-2}{(x^2-1)^2}\right) = \frac{-12x(x^2+1)}{(x^2-1)^3}$

22- $f(s) = \sqrt{\frac{s^2+1}{s^2+4}}$

$f'(s) = \frac{1}{2}\left(\frac{s^2+1}{s^2+4}\right)^{-1/2}\cdot\left[\frac{2s(s^2+4)-(s^2-1)\cdot 2s}{(s^2+4)^2}\right] = \left(\frac{s^2+1}{s^2+4}\right)^{-1/2}\cdot\frac{5s^2}{(s^2+4)^2}$

23- $y = \sqrt{1+2e^{3x}}$

$y' = \frac{1}{2}(1+2e^{3x})^{-1/2}\cdot 2e^{3x} = \frac{2e^{3x}}{2\sqrt{1+2e^{3x}}} = \frac{e^{3x}}{\sqrt{1+2e^{3x}}} = \frac{e^{3x}\sqrt{1+2e^{3x}}}{1+2e^{3x}}$

24- $y = 10^{1-x^2}$

$y' = (1-x^2)10^{-x^2}\cdot(-2x) = -2x(1-x^2)10^{-x^2}$

25- $y = 5^{-1/x}$

$y' = (-1/x)\cdot 5^{\frac{-1}{x}-1}\cdot\frac{0\cdot x+1\cdot 1}{x^2} = \frac{-5^{\frac{-1-x}{x}}}{x}\cdot\frac{2}{x^2} = \frac{2\cdot(-5)^{\frac{-1-x}{x}}}{x^3}$

26- $G(y) = \frac{(y-1)^4}{(y^2+2y)^5} = \frac{4(y-1)^3\cdot(y^2+2y)^5-(y-1)^4\cdot 5\cdot(y^2+2y)^4}{(y^2+2y)^{10}} = \frac{(y-1)^3(y^2+2y)^4\left[4y(y+2)-5(y-1)\right]}{(y^2+2y)^{10}} = \frac{(y-1)^3\left[4y(y+2)-5(y-1)\right]}{(y^2+2y)^6}$

27- $y = \dfrac{r}{\sqrt{r^2+1}}$

$y' = \dfrac{\sqrt{r^2+1} - r\left(\sqrt{r^2+1}\right)'}{r^2+1} = \dfrac{\sqrt{r^2+1} - r\left[\frac{1}{2}(r^2+1)^{-1/2}\cdot 2r\right]}{r^2+1} = \dfrac{2(r^2+1)^{1/2} - 2r^2\cdot(r^2+1)^{-1/2}}{2(r^2+1)} = \dfrac{2(r^2+1)\left[(r^2+1)^{-1/2} - r^2(r^2+1)^{-3/2}\right]}{2(r^2+1)} = (r^2+1)^{-1/2} - r^2(r^2+1)^{-3/2}$

28- $y = \dfrac{e^{u} - e^{-u}}{e^{u} + e^{-u}}$

$y' = \dfrac{(e^{u}-e^{-u})\cdot(e^{u}+e^{-u}) - (e^{u}-e^{-u})(e^{u}+e^{-u})}{(e^{u}+e^{-u})^2} = 0$

29- $F(t) = e^{t\,\text{sen}\,2t}$

$F'(t) = e^{t\,\text{sen}\,2t}\cdot(t\cdot\text{sen}\,2t)' = e^{t\,\text{sen}\,2t}\cdot t\cos 2t\cdot(2t)' = e^{t\,\text{sen}\,2t}\cdot t\cos 2t\cdot 2 = 2t\cos 2t\cdot e^{t\,\text{sen}\,2t}$

30- $F(v) = \left(\dfrac{v}{v^3+1}\right)^6$

$F'(v) = 6\left(\dfrac{v}{v^3+1}\right)^5\cdot\dfrac{v^3+1 - v\cdot 3v^2}{(v^3+1)^2} = 6\cdot\left(\dfrac{v}{v^3+1}\right)^5\cdot\left[\dfrac{4v^3+1}{(v^3+1)^2}\right]$

31- $y = \text{sen}(\text{tg}\,2t)$

$y' = \cos(\text{tg}\,2t)\cdot\sec^2 2t\cdot 2 = 2\cdot\sec^2 2t\cdot\cos(\text{tg}\,2t)$

32- $y = \sec^2(m\theta)$

$y' = 2\sec(m\theta)\cdot m = 2m\sec(m\theta)$

33- $y = 2^{\text{sen}(\pi x)}$

$y' = \text{sen}(\pi x)\cdot 2^{\text{sen}(\pi x)-1}\cdot\cos(\pi x)\cdot\pi = \dfrac{\pi\,\text{sen}(\pi x)\cos(\pi x)\cdot 2^{\text{sen}(\pi x)}}{2}$

34- $y = x^2\cdot e^{-1/x} = x^2 e^{-x^{-1}}$

$y' = 2x\cdot e^{-1/x} + x^2\cdot e^{-x^{-1}}\cdot(-1)\cdot(-x^{-2}) = 2x\cdot e^{-x^{-1}} + e^{-x^{-1}} = e^{-x^{-1}}(2x+1)$

35- $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)$

$y' = -\text{sen}\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)\cdot\left[\dfrac{-e^{2x}(1+e^{2x}) - (1-e^{2x})e^{2x}}{(1+e^{2x})^2}\right] = -\text{sen}\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)\cdot\left[\dfrac{-e^{2x}\cdot 2}{(1+e^{2x})^2}\right]$

36- $y = \sqrt{1+xe^{-2x}} = (1+xe^{-2x})^{1/2}$

$y' = \frac{1}{2}(1+xe^{-2x})^{-1/2}\cdot xe^{-2x}\cdot(-2) = -xe^{-2x}(1+xe^{-2x})^{-1/2}$

37- $y = \text{cotg}^2(\text{sen}\,\theta)$

$y' = 2\,\text{cotg}(\text{sen}\,\theta)\cdot\left[-\text{cossec}^2(\text{sen}\,\theta)\right]\cdot\cos\theta = -2\cdot\dfrac{\cos(\text{sen}(\theta))}{\text{sen}^3(\text{sen}(\theta))}\cdot\cos\theta = \dfrac{-2\cos\theta\cdot\cos(\text{sen}(\theta))}{\text{sen}^3(\text{sen}(\theta))}$

38- $y = e^{k\,\text{tg}\sqrt{x}}$

$y' = e^{k\,\text{tg}\sqrt{x}}\cdot k\sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2} = \dfrac{e^{k\,\text{tg}\sqrt{x}}\cdot k\sec^2\sqrt{x}\cdot x^{-1/2}}{2}$

39- $f(t) = tg(e^t) + e^{tg\,t}$

$f'(t) = sec^2(e^t) \cdot e^t + e^{tg\,t} \cdot sec^2 t$

40- $y = sen(sen(sen(x)))$

$y' = cos(sen(sen(x))) \cdot cos(sen(x)) \cdot cos\,x$

41- $f(t) = sen^2(e^{sen^2 t})$

$f'(t) = 2sen(e^{sen^2 t}) \cdot cos(e^{sen^2 t}) \cdot e^{sen^2 t} \cdot 2sen\,t \cdot cos\,t = 4sen(e^{sen^2 t}) \cdot cos(e^{sen^2 t}) \cdot e^{sen^2 t} \cdot sen\,t \cdot cos\,t$

42- $y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \left[x + (x + x^{1/2})^{1/2}\right]^{1/2}$

$y' = \frac{1}{2}\left[x + (x + x^{1/2})^{1/2}\right]^{-1/2} \cdot \left[1 + \frac{1}{2}(x + x^{1/2})\right]^{-1/2} \cdot \left[1 + \frac{1}{2}x^{-1/2}\right]$

43- $g(x) = (2na^{nx} + m)^p$

$g'(x) = p(2na^{nx} + m)^{p-1} \cdot ($

44- $y = 2^{3^{x^2}}$

$y' = 3^{x^2} \cdot 2^{3^{x^2} - 1} \cdot x^2 \cdot 3^{x^2 - 1} \cdot 2x = 2x^2 \cdot 2^{3^{x^2} - 1} \cdot 2^{2x^2 - 1}$

45- $y = cos\left[\sqrt{sen(tg(\pi x))}\right]$

$y' = sen\left[sen(tg(\pi x))^{1/2}\right] \cdot \frac{1}{2} \cdot sen\left[tg(\pi x)\right]^{-1/2} \cdot cos\left[tg(\pi x)\right] \cdot sec^2(\pi x) \cdot \pi = \frac{\pi}{2} \cdot sen\left[sen(tg(\pi x))^{1/2}\right] \cdot sen\left[tg(\pi x)\right]^{-1/2} \cdot cos\left[tg(\pi x)\right] \cdot sec^2(\pi x)$

46- $y = \left[x + (x + sen^2 x)^3\right]^4$

$y' = 4\left[x + (x + sen^2 x)^3\right]^3 \cdot \left[1 + 3(x + sen^2 x) \cdot (1 + 2sen\,x \cdot cos\,x)\right]$

47- $y = cos(x^2)$

$y' = -sen(x^2) \cdot 2x$

$y'' = -cos(x^2) \cdot 2x \cdot 2x + (-sen(x^2)) \cdot 2 = -4x^2 cos(x^2) - 2sen(x^2) = -2\left[2x^2 cos(x^2) + sen(x^2)\right]$

48- $y = cos^2 x$

$y' = 2cos\,x \cdot sen\,x$

$y'' = -2sen\,x \cdot sen\,x + 2cos\,x \cdot cos\,x = -2sen^2 x + 2cos^2 x = 2(-sen^2 x + cos^2 x) = 2(-1 + cos^2 x + cos^2 x) = -2 + 4cos^2 x$

49- $y = e^{\alpha x} \cdot sen\,\beta x$

$y' = e^{\alpha x} \cdot sen\,\beta x + e^{\alpha x} \cdot cos\,\beta x \cdot \beta = e^{\alpha x}(sen\,\beta x + \beta cos\,\beta x)$

$y'' = e^{\alpha x}(sen\,\beta x + \beta cos\,\beta x) + e^{\alpha x}\left[cos\,\beta x \cdot \beta + \beta \cdot (-sen\,\beta x) \cdot \beta\right] = e^{\alpha x}\left[2\beta cos\,\beta x + sen\,\beta x \cdot (1 - \beta^2)\right]$

50- $y = e^{e^x}$

$y' = e^{e^x} \cdot e^x$

$y'' = e^{e^x} \cdot e^x + e^{e^x} \cdot e^x = 2 \cdot e^{e^x} \cdot e^x$

51- $y = (1 + 2x)^{10}$

$y' = 10(1 + 2x)^9 \cdot 2 = 20 \cdot (1 + 2x)^9$

$f'(0) = 20$, $P(0, 1)$

$t: y - y_0 = m(x - x_0) \therefore y - 1 = 20(x - 0) \therefore t: y = 20x + 1$

52- $y = \sqrt{1+x^3}$

$y' = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2$; $P(2,3)$

$f'(2) = \frac{1}{2} \cdot \frac{1}{\sqrt{9}} \cdot 12 = 2$

$t: y - y_0 = m(x - x_0) \therefore y - 3 = 2(x-2) \therefore t: y = 2x - 1$

53- $y = \operatorname{sen}(\operatorname{sen} x)$

$y' = \cos(\operatorname{sen} x) \cdot \cos x$; $P(\pi, 0)$

$f'(\pi) = 1 \cdot (-1) = -1$

$t: y - y_0 = m(x - x_0) \therefore y = -1(x - \pi) \therefore t: y = -x + \pi$

54- $y = \operatorname{sen} x + \operatorname{sen}^2 x$

$y' = \cos x + 2\operatorname{sen} x \cdot \cos x = \cos x(1 + 2\operatorname{sen} x)$

$f'(0) = 1(1 + 2 \cdot 0) = 1$

$t: y - y_0 = m(x - x_0) \therefore y - 0 = m(x - 0) \therefore t: y = x$

59- $f(x) = 2\operatorname{sen} x + \operatorname{sen}^2 x$

$f'(x) = 2\cos x + 2\operatorname{sen} x \cdot \cos x = 2\cos x(1 + \operatorname{sen} x)$

$2\cos x(1 + \operatorname{sen} x) = 0 \therefore 2\cos x = 0$ ou $(1 + \operatorname{sen} x) = 0$

$2\cos x = 0 \therefore \cos x = 0 \therefore x = \pi/2 + 2k\pi \mid k \in \mathbb{Z}$.

$1 + \operatorname{sen} x = 0 \therefore \operatorname{sen} x = -1 \therefore x = \frac{3\pi}{2} + 2k\pi \mid k \in \mathbb{Z}$.

Para a reta tangente ser horizontal, $S = \{\pi/2 + 2k\pi, 3\pi/2 + 2k\pi\} \mid k \in \mathbb{Z}$.

60- $y = \operatorname{sen} 2x - 2\operatorname{sen} x$

$y' = \cos 2x \cdot 2 - 2\cos x = 2(\cos 2x - \cos x)$

$2(\cos 2x - \cos x) = 0 \therefore \cos 2x = \cos x$

$\cos 2x = \cos x \cdot \cos x - \operatorname{sen} x \operatorname{sen} x = \cos^2 x - \operatorname{sen}^2 x = -2\operatorname{sen}^2 x + 1$

$\cos x = -2\operatorname{sen}^2 x + 1 \therefore$ Para a reta tangente ser horizontal, $\{x \in \mathbb{R} \mid \cos x = -2\operatorname{sen}^2 x + 1\}$

61- $F(x) = f(g(x))$

$F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$

72- $f(x) = x \cdot g(x^2)$

$f'(x) = xg'(x^2) \cdot 2x = 2x^2 g'(x^2)$

$f''(x) = xg''(x^2) \cdot 2x \cdot 2 = 4x^2 g''(x^2)$

77- $y = \cos 2x$

$y' = -\operatorname{sen} 2x \cdot 2 = -2\operatorname{sen} 2x$ (1)

$y'' = -2\cos 2x \cdot 2 = -4\cos 2x$ (2)

$y''' = 4 \operatorname{sen} 2x \cdot 2 = 8 \operatorname{sen} 2x \quad (3)$

$y'''' = 8 \cos 2x \cdot 2 = 16 \cos 2x \quad (4)$

$y''''' = -16 \operatorname{sen} 2x \cdot 2 = -32 \operatorname{sen} 2x \quad (5)$

$y^{(6)} = -32 \cos 2x \cdot 2 = -64 \cos 2x \quad (6)$

$y^{(7)} = 64 \operatorname{sen} 2x \cdot 2 = 128 \operatorname{sen} 2x \quad (7)$

$\frac{d^{50}}{dx} = -2^{50} \cos 2x$

78- $f(x) = xe^{-x}$

$f'(x) = x \cdot e^{-x} \cdot (-1) = -xe^{-x}$

$f''(x) = -x \cdot e^{-x} \cdot (-1) = xe^{-x}$

$\frac{d^{1000}}{dx} f(x) = xe^{-x}$

79- $v(t) = s'(t)$

$v(t) = \frac{1}{4} \cos(10\pi t) \cdot 10\pi = \frac{5\pi}{2} \cos(10\pi t)$

81- a) $\frac{dB}{dt} = 0{,}35 \cos\left(\frac{2\pi t}{5{,}4}\right) \cdot \frac{2\pi}{5{,}4}$

b) $B'(1) = 0{,}35 \cdot 0{,}39 \cdot 1{,}16 = 0{,}158$

Pg 195 (3.5) João Vaz

1- a) $xy + 2x + 3x^2 = 4$

$y + y' + 2 + 6x = 0$

$y' = \frac{-y - 6x - 2}{x}$

b) $xy + 2x + 3x^2 = 4$

$y = \frac{4 - 2x - 3x^2}{x}$

$y' = \frac{(-2-6x)\cdot x - (4-2x-3x^2)}{x^2} = \frac{-2x - 6x^2 - 4 + 2x + 3x^2}{x^2} = \frac{-3x^2 - 4}{x^2}$

c) $y' = \frac{-1}{x}\left(\frac{4-2x-3x^2}{x}\right) - \frac{6x-2}{x} = \frac{-4+2x+3x^2}{x^2} - \frac{6x^2-2x}{x} = \frac{-3x^2-4}{x^2}$

2- a) $4x^2 + 9y^2 = 36$

$8x + 18y \cdot y' = 0$

$y' = \frac{-8x}{18y} = \frac{-4x}{9y}$

b) $y = \frac{\sqrt{36-4x^2}}{3} = \frac{(36-4x^2)^{1/2}}{3}$

$y' = \frac{\frac{1}{2}(36-4x^2)^{-1/2}\cdot(-8x)\cdot 3 - (36-4x^2)^{1/2}\cdot 0}{9}$

$y' = \frac{-8x\cdot(36-4x^2)^{-1/2}}{6} = \frac{-4x\cdot(36-4x^2)^{-1/2}}{3}$

c) $y' = -\frac{4x}{9y} = \frac{-4x}{9\cdot\frac{\sqrt{36-4x^2}}{3}} = \frac{-4x}{3\cdot\sqrt{36-4x^2}} = \frac{-4x(36-4x^2)^{-1/2}}{3}$

3- a) $\frac{1}{x} + \frac{1}{y} = 1$

$\frac{0-1}{x^2} + \frac{0-y'}{y^2} = 0$

$\frac{-y^2 - x^2y'}{x^2y^2} = 0$

$-x^2y' = y^2$

$y' = \frac{-y^2}{x^2} = -\left(\frac{y}{x}\right)^2$

b) $\frac{1}{y} = 1 - \frac{1}{x} = \frac{x-1}{x}$

$y = \frac{x-1}{x}$

$y' = \frac{1\cdot x - (x-1)}{x^2} = \frac{x - x + 1}{x^2} = \frac{1}{x^2}$

c) $y' = -\left(\frac{\frac{x-1}{x}}{x}\right)^2 = -\left(\frac{x-1}{x^2}\right)^2 = \frac{x^2 - 2x + 1}{x^4}$

4- a) $\cos x + \sqrt{y} = 5$

$-\operatorname{sen} x + \frac{1}{2}y^{-1/2}\cdot y' = 0$

$y' = \frac{2\operatorname{sen} x}{y^{-1/2}}$

b) $y = (5 - \cos x)^2$

$y' = 2(5-\cos x)\cdot \operatorname{sen} x = 2\operatorname{sen} x\,(5-\cos x)$

c) $y' = \dfrac{2\operatorname{sen} x}{(5-\cos x)^{-1}} = 2\operatorname{sen} x\cdot(5-\cos x)$

5- $x^3+y^3=1$

$3x^2+3y^2\cdot y'=0$

$y' = \dfrac{-3x^2}{3y^2} = \dfrac{-x^2}{y^2}$

6- $2\sqrt{x}+\sqrt{y}=3$

$x^{-1/2}+\frac{1}{2}y^{-1/2}\cdot y'=0$

$y' = \dfrac{-2x^{-1/2}}{y^{-1/2}}$

7- $x^2+xy-y^2=4$

$2x+y+x\cdot y'-2y\cdot y'=0$

$2x+y+y'(x-2y)=0$

$y' = \dfrac{-2x-y}{x-2y}$

8- $2x^3+x^2y-xy^3=2$

$6x^2+2xy+x^2y'-(y^3+3xy^2\cdot y')=0$

$6x^2+2xy+x^2y'-y^3-3xy^2\cdot y'=0$

$6x^2+2xy+y'(x^2-3xy^2)-y^3=0$

$y' = \dfrac{y^3-6x^2-2xy}{x^2-3xy^2}$

9- $x^4(x+y)=y^2(3x-y)$

$x^5+x^4y=3xy^2-y^3$

$5x^4+4x^3y+x^4y'=3y(y+2xy')-3y^2y'$

$x^4y'+3y^2y'-3y(y+2xy')=-5x^4-4x^3y$

$x^4y'+3y^2y'-3y^2-6xyy'=-5x^4-4x^3y$

$x^4y'+3y^2y'-6xyy'=-5x^4-4x^3y+3y^2$

$y'(x^4+3y^2-6xy)=-5x^4-4x^3y+3y^2$

$y' = \dfrac{-5x^4-4x^3y+3y^2}{x^4+3y^2-6xy}$

10- $xe^y=x-y$

$xe^y+xe^y\cdot y'=1-y'$

$-y'-xe^y\cdot y'=xe^y-1$

$y'(-1-xe^y)=xe^y-1$

$y' = \dfrac{xe^y-1}{-1-xe^y}$

11- $x^2y^2+x\operatorname{sen} y=4$

$2xy^2 + x^2 \cdot 2y \cdot y' + \text{sen}\, y + x\cos y \cdot y' = 0$

$y'(x^2 \cdot 2y + x\cos y) = -2xy^2 - \text{sen}\, y$

$y' = \dfrac{-2xy^2 - \text{sen}\, y}{2x^2 y + x\cos y}$

12- $1 + x = \text{sen}(xy^2)$

$1 = \cos(xy^2) \cdot (y^2 + x \cdot 2y \cdot y')$

$1 = y^2\cos(xy^2) + 2xyy'\cos(xy^2)$

$y' = \dfrac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)}$

13- $4\cos x \cdot \text{sen}\, y = 1$

$-4\,\text{sen}\, x \cdot \text{sen}\, y + 4\cos x \cdot \cos y \cdot y' = 0$

$4\cos x \cdot \cos y \cdot y' = 4\,\text{sen}\, x \cdot \text{sen}\, y$

$y' = \dfrac{\text{sen}\, x\, \text{sen}\, y}{\cos x \cos y} = \text{tg}\, x \cdot \text{tg}\, y$

14- $e^y \cdot \text{sen}\, x = x + xy$

$e^y y' \cdot \text{sen}\, x + e^y \cdot \cos x = 1 + y + xy'$

$e^y \cdot y' \cdot \text{sen}\, x - xy' = 1 + y - e^y\cos x$

$y'(e^y \cdot \text{sen}\, x - x) = 1 + y - e^y\cos x$

$y' = \dfrac{1 + y - e^y\cos x}{e^y\text{sen}\, x - x}$

15- $e^{x/y} = x - y$

$\ln e^{x/y} = \ln(x - y)$

$\dfrac{x}{y} = \ln(x - y)$

$Y = \ln(x - y) \longrightarrow$ Y e y são diferentes; ambas são funções

$e^Y = x - y$

$e^Y \cdot Y' = 1 - y'$

$Y' = \dfrac{1 - y'}{e^Y} = \dfrac{1 - y'}{x - y}$

$\dfrac{y - xy'}{y^2} = \dfrac{1 - y'}{x - y}$

$(y - xy')(x - y) = y^2 - y^2 \cdot y'$

$xy - y^2 - x^2y' + xyy' = y^2 - y^2y'$

$-xy' + y^2y' = y^2 - xy + y^2$

$y'(-x + y^2) = 2y^2 - xy$

$y' = \dfrac{2y^2 - xy}{y^2 - x}$ $(xy)^2$

16- $\sqrt{x + y} = 1 + x^2y^2$

$\frac{1}{2}(x + y)^{-1/2} \cdot (1 + y') = 2xy \cdot (y + xy')$

$(x+y)^{-1/2} + y' \cdot (x+y)^{-1/2} = 4xy^2 + 4x^2yy'$

$y' \cdot (x+y)^{-1/2} - 4x^2y \cdot y' = 4xy^2 - (x+y)^{-1/2}$

$y'\left[(x+y)^{-1/2} - 4x^2y\right] = 4xy^2 - (x+y)^{-1/2}$

$y' = \dfrac{4xy^2 - (x+y)^{-1/2}}{(x+y)^{-1/2} - 4x^2y}$

17- $tg^{-1}(x^2y) = x + xy^2$

$cotg(x^2y) = x + xy^2$

$-cossec^2(x^2y) \cdot (2xy + x^2y') = 1 + y^2 + x \cdot 2y \cdot y'$

$-2xy\,cossec^2(x^2y) - x^2y' \cdot cossec^2(x^2y) = 1 + 2xy \cdot y' + y^2$

$-x^2y' \cdot cossec^2(x^2y) - 2xy \cdot y' = 1 + y^2 + 2xy\,cossec^2(x^2y)$

$y'\left[-x^2cossec^2(x^2y) - 2xy\right] = 1 + 2xy\,cossec^2(x^2y) + y^2$

$y' = \dfrac{1 + 2xy\,cossec^2(x^2y) + y^2}{-x^2cossec^2(x^2y) - 2xy}$

18- $x\,sen\,y + y\,sen\,x = 1$

$sen\,y + x\cos y \cdot y' + y'\,sen\,x + y \cdot \cos x = 0$

$y'(x\cos y + sen\,x) = -y\cos x - sen\,y$

$y' = \dfrac{-y\cos x - sen\,y}{x\cos y + sen\,x}$

19- $e^y\cos x = 1 + sen(xy)$

$e^y y' \cdot \cos x - e^y sen\,x = \cos(xy) \cdot (y + xy')$

$e^y y' \cdot \cos x - e^y sen\,x = y\cos(xy) + xy'\cos(xy)$

$e^y y' \cdot \cos x - xy'\cos(xy) = y\cos(xy) + e^y sen\,x$

$y'\left[e^y\cos x - x\cos(xy)\right] = y\cos(xy) + e^y sen\,x$

$y' = \dfrac{y\cos(xy) + e^y sen\,x}{e^y\cos x - x\cos(xy)}$

20- $tg(x-y) = \dfrac{y}{1+x^2}$

$sec^2(x-y) \cdot (1-y') = \dfrac{y'(1+x^2) - 2xy}{(1+x^2)^2}$

$sec^2(x-y) \cdot (1+x^2)^2 - y' \cdot sec^2(x-y) \cdot (1+x^2)^2 - y'(1+x^2) = -2xy$

$y'\left[-sec^2(x-y)(1+x^2)^2 - (1+x^2)\right] = -2xy - sec^2(x-y) \cdot (1+x^2)^2$

$y' = \dfrac{-2xy - sec^2(x-y)(1+x^2)^2}{-sec^2(x-y)(1+x^2)^2 - (1+x^2)} = \dfrac{2xy + sec^2(x-y)(1+x^2)^2}{sec^2(x-y)(1+x^2)^2 + (1+x^2)}$

21- $f(x) + x^2 f^3(x) = 10$

$f(1) = 2$

$f'(1) = ?$

$y + x^2y^3 = 10$

$y' + 2xy^3 + x^2 \cdot 3y^2 \cdot y' = 0$

$y'(1+x^2\cdot 3y^2)=-2xy^3$

$y'=\frac{-2xy^3}{1+x^2\cdot 3y^2}$

$f'(1)=\frac{-2\cdot 1\cdot 8}{1+1\cdot 12}=\frac{-16}{12}=\frac{-4}{3}$

22- $g(x)+x\,\text{sen}\,g(x)=x^2$

$g'(0)=?$

$y+x\,\text{sen}\,y=x^2$

$y'+\text{sen}\,y+x\cdot\cos y\cdot y'=2x$

$y'(1+x\cos y)=2x-\text{sen}\,y$

$y'=\frac{2x-\text{sen}\,y}{1+x\cos y}$

$g'(0)=\frac{0-\text{sen}\,0}{1+0\cdot 1}=\frac{0}{1}=0$

23- $x^4y^2-x^3y+2xy^3=0$

$4x^3y^2+x^4\cdot y\cdot y'-(3x^2y+x^3y')+2y^3+2x\cdot 3y^2\cdot y'=0$

$4x^3y^2+x^4y\cdot y'-3x^2y-x^3y'+2y^3+6xy^2y'=0$

$x^4yy'-x^3y'+6xy^2y'=-4x^3y^2+3x^2y-2y^3$

$y'(x^4y-x^3+6xy^2)=-4x^3y^2+3x^2y-2y^3$

$y'=\frac{-4x^3y^2+3x^2y-2y^3}{x^4y-x^3+6xy^2}$

24- $y\sec x=x\,\text{tg}\,y$

$y'\sec x+y\cdot\text{tg}\,x\cdot\sec x=\text{tg}\,y+x\cdot\sec^2 y\cdot y'$

$y'\sec x-x\sec^2 y\cdot y'=\text{tg}\,y-y\,\text{tg}\,x\sec x$

$y'(\sec x-x\sec^2 y)=\text{tg}\,y-y\,\text{tg}\,x\sec x$

$y'=\frac{\text{tg}\,y-y\,\text{tg}\,x\sec x}{\sec x-x\sec^2 y}$

25- $r: y\,\text{sen}\,2x=x\cos 2y$

$r': y'\text{sen}\,2x+y\cos 2x\cdot 2=\cos 2y-x\,\text{sen}\,2y\cdot 2y'$

$r': y'\text{sen}\,2x+2xy'\text{sen}\,2y=\cos 2y-2y\cos 2x$

$r': y'(\text{sen}\,2x+2x\,\text{sen}\,2y)=\cos 2y-2y\cos 2x$

$r': y'=\frac{\cos 2y-2y\cos 2x}{\text{sen}\,2x+2x\,\text{sen}\,2y}$

$P(\pi/2,\pi/4);\ f'(\pi/2)=\frac{0+\pi/2}{0+\pi\cdot 1}=\frac{\pi}{2}\cdot\frac{1}{\pi}=\frac{1}{2}$

$t: y-y_0=m(x-x_0);\ y-\pi/4=\frac{1}{2}(x-\pi/2),\ t: y=\frac{2x-\pi}{4}+\pi/4=x/2$

26- $r: \text{sen}(x+y)=2x-2y$

$r': \cos(x+y)(1+y')=2-2y'$

$r: \cos(x+y) + y'\cos(x+y) = 2 - 2y'$

$r': y'\cos(x+y) + 2y' = 2 - \cos(x+y)$

$r': y'\left[\cos(x+y) + 2\right] = 2 - \cos(x+y)$

$r': y' = \dfrac{2 - \cos(x+y)}{\cos(x+y) + 2}$

$P(\pi, \pi)$; $f'(\pi) = \dfrac{2-1}{1+2} = \dfrac{1}{3}$

$t: y - y_0 = m(x - x_0)$; $t: y - \pi = \dfrac{1}{3}(x - \pi)$; $t: y = \dfrac{x - 2\pi}{3}$

27- $r: x^2 + xy + y^2 = 3$

$r': 2x + y + xy' + 2y \cdot y' = 0$

$r': y'(x + 2y) = -2x - y$

$r': y' = \dfrac{-2x - y}{x + 2y}$

$P(1,1)$; $f'(1) = \dfrac{-2-1}{1+2\cdot 1} = \dfrac{-3}{3} = -1$

$t: y - y_0 = m(x - x_0)$; $y - 1 = -1(x - 1)$; $t: y = -x + 2$

28- $r: x^2 + 2xy - y^2 + x = 2$

$r': 2x + 2y + 2x \cdot y' - 2y \cdot y' + 1 = 0$

$r': y'(2x - 2y) = -2x - 2y - 1$

$r': y' = \dfrac{-2x - 2y - 1}{2x - 2y}$

$P(1,2)$; $f'(1) = \dfrac{-2-4-1}{2-4} = \dfrac{-7}{-2} = 7/2$

$t: y - y_0 = m(x - x_0)$; $t: y - 2 = \dfrac{7}{2}(x - 1)$; $t: y = \dfrac{7x + 7}{2}$

29- $r: x^2 + y^2 = (2x^2 + 2y^2 - x^2)^2$

$r': 2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x^2) \cdot (4x + 4y \cdot y' - 2x)$

$r': 2x + 2yy' = 2(\cancel{8x^3} + \overset{4}{\cancel{8}}x^2yy' - \cancel{4x^3} + \overset{4}{\cancel{8}}xy^2 + 8y^3y' - \cancel{4xy^2} - \cancel{4x^3} - \cancel{4x^2yy'} + 2x^3)$

$r': 2x + 2yy' = 8x^2yy' + 8xy^2 + 16y^3y' + 4x^3$

$r': 2yy' - 8x^2yy' - 16y^3y' = 8xy^2 + 4x^3 - 2x$

$r': y'(2y - 8x^2y - 16y^3) = 8xy^2 + 4x^3 - 2x$

$r': y' = \dfrac{8xy^2 + 4x^3 - 2x}{2y - 8x^2y - 16y^3}$

$P(0, 1/2)$; $f'(0) = \dfrac{0 + 0 - 0}{1 - 0 - 2} = \dfrac{0}{-1} = 0$

$t: y - y_0 = m(x - x_0)$; $t: y - 1/2 = 0(x - 0)$; $t: y = 1/2$

30- $r: x^{2/3} + y^{2/3} = 4$

$r': \dfrac{2}{3}x^{-1/3} + \dfrac{2}{3}y^{-1/3} \cdot y' = 0$

$r': y' = \dfrac{-2x^{-1/3} \cdot 3}{3 \cdot 2y^{-1/3}} = \dfrac{-x^{-1/3}}{y^{-1/3}}$

$P(-3\sqrt{3}, 1)$; $f'(-3\sqrt{3}) = \dfrac{1}{27}$

$t: y - y_0 = m(x - x_0)$; $y - 1 = \dfrac{1}{27}(x + 3\sqrt{3})$; $t: y = \dfrac{x + 3\sqrt{3} + 27}{27}$

$t: y-y_0=m(x-x_0)$; $y-1=\frac{[illegible]}{27}(x+3\sqrt{3})$; $t: y=\frac{[illegible]}{27}$

31- $r: 2(x^2+y^2)^2=25(x^2-y^2)$

$r': 2\cdot 2(x^2+y^2)(2x+2y\cdot y')=25(2x-2yy')$

$r': 4(2x^3+2x^2yy'+2xy^2+2y^3y')=50x-50yy'$

$r': 8x^2yy'+8y^3y'+50yy'=50x-8x^3-8xy^2$

$r': y'(8x^2y+8y^3+50y)=50x-8x^3-8xy^2$

$r': y'=\frac{50x-8x^3-8xy^2}{8x^2y+8y^3+50y}$

$P(3,1)$; $f'(3)=\frac{150-216-24}{72+8+50}=\frac{-9}{13}$

$t: y-y_0=m(x-x_0)$; $t: y-1=\frac{-9}{13}(x-3)$; $t: y=\frac{-9x+40}{13}$

32- $r: y^2(y^2-4)=x^2(x^2-5)$

$r': 2yy'(y^2-4)+y^2\cdot 2yy'=2x(x^2-5)+x^2\cdot 2x$

$r': y'(2y^3-8y+2y^3)=2x^3-10x+2x^3$

$r': y'=\frac{4x^3-10x}{4y^3-8y}=\frac{2x^3-5x}{2y^3-4y}$

35- $9x^2+y^2=9$

$18x+2yy'=0$

$y'=\frac{9x}{y}$

$y''=\frac{9y-9xy'}{y^2}$

36- $\sqrt{x}+\sqrt{y}=1$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\cdot y'=0$

$y'=\frac{2x^{-1/2}}{2\cdot y^{-1/2}}=\frac{x^{-1/2}}{y^{-1/2}}$

$y''=\frac{\frac{-1}{2}x^{-3/2}\cdot y^{-1/2}+\frac{1}{2}y^{-3/2}\cdot y'\cdot x^{-1/2}}{y^{-1}}=\frac{-x^{-3/2}\cdot y^{-1/2}+y^{-3/2}\cdot y'\cdot x^{-1/2}}{2y^{-1}}$

37- $x^3+y^3=1$

$3x^2+3y^2y'=0$

$y'=\frac{-3x^2}{3y^2}=\frac{-x^2}{y^2}$

$y''=\frac{-2xy^2+x^2\cdot 2yy'}{y^4}=\frac{-2xy^2+2x^2yy'}{y^4}$

38- $x^4+y^4=a^4$

$4x^3+4y^3y'=0$

$y'=\frac{-x^3}{y^3}$

$y''=\frac{-3x^2y^3+x^3 3y^2y'}{y^6}=\frac{-3x^2y^3+3x^3y^2y'}{y^6}$

39- $xy+e^y=e$

$y+xy'+e^yy'=0$

$y'(x+e^y) = -y$

$y' = \frac{-y}{x+e^y}$

$y'' = \frac{-y'(x+e^y)+y(1+e^y y')}{(x+e^y)^2}$

Quando $x=0$, $e^y = e$; $y=1$.

$f(0)=1$

$f'(0) = \frac{-1}{0+e} = \frac{-1}{e}$

$f''(0) = \frac{\frac{1}{e}(0+e)+1(1-1)}{e^2} = \frac{1+0}{e^2} = \frac{1}{e^2}$

40- $x^2+xy+y^3=1$

$2x+y+xy'+3y^2y'=0$

$y'(x+3y^2) = -2x-y$

$y' = \frac{-2x-y}{x+3y^2}$

$y'' = \frac{(-2-y')(x+3y^2)-(-2x-y)(1+6yy')}{(x+3y^2)^2} = \frac{\cancel{-2x}-6y^2-xy'\cancel{-3y^2y'}-(\cancel{-2x}-12xyy'-y-\cancel{6y^2y'})}{(x+3y^2)^2} = \frac{-6y^2-xy'+12xyy'+y+3y^2y'}{(x+3y^2)^2}$

$y''' = \frac{[-12yy'-(y'-xy'')+(12yy'+12xy'y'+12xy'')+y'+6yy'y'+3y^2y'']}{}(x+3y^2)^2-$

41- a) $y(y^2-1)(y-2) = x(x-1)(x-2)$

$y(y^3-2y^2-y+2) = x(x^2-2x-x+2)$

$y^4-2y^3-y^2+2y = x^3-3x^2+2x$

$4y^3y'-6y^2y'-2yy'+2y' = 3x^2-6x+2$

$y'(4y^3-6y^2-2y+2) = 3x^2-6x+2$

$y'=0$ se, e somente se, $3x^2-6x+2=0$

$x = (6 \pm \sqrt{36-24}) \cdot \frac{1}{6} = \frac{6 \pm 2\sqrt{3}}{6}$ < $\frac{3+\sqrt{3}}{3}$; $\frac{3-\sqrt{3}}{3}$

A tangente é horizontal em 2 pontos.

b) $y' = \frac{3x^2-6x+2}{4y^3-6y^2-2y+2}$

$P_1(0,1)$; $f'(0) = \frac{0-0+2}{4-6-2+2} = \frac{2}{-2} = -1$

$t_1: y-y_0 = m(x-x_0)$; $y-1=-1(x-0)$; $t_1: y=-x+1$

$P_2(0,2)$; $f'(0)$

42- D) $2y^3+y^2-y^5=x^4-2x^3+x^2$

$6y^2\cdot y'+2y\cdot y'-5y^4\cdot y'=4x^3-6x^2+2x$

$y'(6y^2+2y-5y^4)=4x^3-8x^2+2x$

$y'=\dfrac{4x^3-6x^2+2x}{6y^2+2y-5y^4}$

$4x^3-6x^2+2x=0$

$x(4x^2-6x+2)=0$

$x=\dfrac{6\pm\sqrt{36-32}}{8}$ $\begin{cases} 6+2/8=1 \\ 6-2/8=\frac{1}{2} \end{cases}$

Para a tangente ser horizontal, $x\in\{0,\frac{1}{2},1\}$.

43- $2(x^2+y^2)^2=25(x^2-y^2)$

$y'=\dfrac{50x-8x^3-8xy^2}{8x^2y+8y^3+58y}$

$y'=0$; $50x-8x^3-8xy^2=0$

$8xy^2=50x-8x^3$

$y=\sqrt{\dfrac{50-8x^2}{8}}$

44- $r: \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$r': \dfrac{2x}{a^2}+\dfrac{2yy'}{b^2}=0$

$r': y'=\dfrac{-2xb^2}{a^2 2y}=\dfrac{-xb^2}{ya^2}$

$P(x_0,y_0) \rightsquigarrow t: y-y_0=m(x-x_0)$

$t: y-y_0=\dfrac{-x_0b^2}{y_0a^2}(x-x_0)$

$t: \dfrac{-x_0b^2x}{y_0a^2}+\dfrac{-x_0^2b^2}{y_0a^2}+y_0-y=0$

45- $r: \dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$r': \dfrac{2x}{a^2}-\dfrac{2yy'}{b^2}=0$

$r': y'=\dfrac{2x\,b^2}{a^2\,2y}=\dfrac{xb^2}{ya^2}$

$P(x_0,y_0) \rightsquigarrow t: y-y_0=m(x-x_0)$

$t: y-y_0=\dfrac{xb^2}{ya^2}(x-x_0)$

$t: y=\dfrac{xb^2(x-x_0)+y_0a^2y}{ya^2}$

46- $\sqrt{x}+\sqrt{y}=\sqrt{c}$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\cdot y'=0$

$\frac{1}{2}\cdot\frac{1}{\sqrt{x}}=\frac{\sqrt{x}}{2x}$

49- $y=tg^{-1}\sqrt{x}$

$\dfrac{dy}{dx}=-\text{cossec}^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}=\dfrac{-\text{cossec}^2\sqrt{x}\cdot\sqrt{x}}{2x}$

50- $y=\sqrt{tg^{-1}x}$ $\sqrt{tg\,x}$

$$\frac{dy}{dx} = \frac{1}{2}\left(tg^{-1}x\right)^{-1/2} \cdot \left(-\operatorname{cossec}^2 x\right) = -\sqrt{tg\,x} \cdot \operatorname{cossec}^2 x$$

51- $y = \operatorname{sen}^{-1}(2x+1)$

$$\frac{dy}{dx} = \operatorname{cossec}(2x+1) \cdot \operatorname{cotg}(2x+1) \cdot 2 = 2\operatorname{cossec}(2x+1)\operatorname{cotg}(2x+1)$$

52- $g(x) = \sqrt{x^2-1} \cdot \sec^{-1} x$

$$\frac{dg}{dx} = \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x \cdot \sec^{-1} x - (x^2+1)^{1/2} \cdot \operatorname{sen} x = \frac{\cos x \cdot x}{\sqrt{x^2-1}} - \frac{\operatorname{sen} x}{\sqrt{x^2-1}} = \frac{x\cos x - \operatorname{sen} x}{\sqrt{x^2-1}}$$

53-

54- $y = tg^{-1}\left(x - \sqrt{1+x^2}\right)$

$$\frac{dy}{dx} = -\operatorname{cossec}^2\left(x-\sqrt{1+x^2}\right)\left[1 - \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x\right] = \frac{-\operatorname{cossec}^2\left(x-\sqrt{1+x^2}\right)\left(\sqrt{1+x^2}-1\right)}{\sqrt{1+x^2}}$$

55- $h(t) = \operatorname{cotg}^{-1}(t) + \operatorname{cotg}^{-1}(t^{-1})$ $1/t$

$$\frac{dh}{dt} = \sec^2 t + \sec^2 t^{-1} \cdot (-t^{-2}) = \sec^2 t\left[1 - t^{-2}\right]$$

56-

57- $y = x \cdot \operatorname{sen}^{-1} x + \sqrt{1-x^2}$

$$\frac{dy}{dx} = \operatorname{cossec} x - x \cdot \operatorname{cossec} x \operatorname{cotg} x + \frac{1}{2}(1-x^2)^{-1/2} \cdot 2x = \operatorname{cossec} x\left[1 - \frac{x^2\operatorname{cotg} x}{\sqrt{1-x^2}}\right] = \frac{\operatorname{cossec} x\left[\sqrt{1-x^2} - x^2\operatorname{cotg} x\right]}{\sqrt{1-x^2}}$$

58- $y = \sec(\operatorname{cossec} t)$

$$\frac{dy}{dt} = \sec(\operatorname{cossec} t) \cdot tg(\operatorname{cossec} t) \cdot (-\operatorname{cossec} t) \cdot \operatorname{cotg} t = -\sec(\operatorname{cossec} t) \cdot \operatorname{cossec} t \cdot tg(\operatorname{cossec} t) \cdot \operatorname{cotg} t$$

75- $x^2y^2 + xy = 2 \longrightarrow x(xy^2+y) = 2$

$$2xy^2 + x^2 2yy' + y + xy' = 0$$

$$y'(2y+x) = -2xy^2 - y$$

$$y' = \frac{-2xy^2-y}{2y+x} = -1$$

$$-2xy^2 - y = -2y - x$$

$$2xy^2 + y = 2y + x$$

$$y = 2xy^2 - x = x(2y^2-1)$$

$$x = \frac{y}{2y^2-1}$$

$$\left(\frac{y}{2y^2-1}\right)\left(\frac{y^3}{2y^2-1} + y\right) = 2$$

$$\frac{y}{2y^2-1} \cdot \frac{y^3 + y(2y^2-1)}{2y^2-1} = 2$$

$$\frac{y^4 + y^2(2y^2-1)}{4y^4 - 4y^2 + 1} = 2$$

$$8y^4 - 8y^2 + 2 = y^4 + 2y^4 - y^2$$

$$7y^4 - 9y^2 + 2 = 0$$

$z = y^2$

$7z^2 - 9z + 2 = 0$

$z = \frac{9 \pm \sqrt{81-56}}{14}$ $\nearrow 1$, $\searrow 4/14 = 2/7$

$y = \{\pm 1, \pm\sqrt{\frac{2}{7}}\}$

$x = \frac{y}{2y^2-1}$

$y_1 = -1$	$x_1 = 1$	$P_1(1,-1)$
$y_2 = 1$	$x_2 = 1$	$P_2(1,1)$
$y_3 = -\sqrt{\frac{2}{7}}$	$x_3 = \frac{-\sqrt{14}}{3}$	$P_3\left(-\frac{\sqrt{14}}{3}, -\sqrt{\frac{2}{7}}\right)$
$y_4 = \sqrt{\frac{2}{7}}$	$x_4 = \frac{\sqrt{14}}{3}$	$P_4\left(\frac{\sqrt{14}}{3}, \sqrt{\frac{2}{7}}\right)$

Pg 201 (3.6) João Vaz

2- $f(x) = x \ln x - x = x(\ln x - 1)$

$$\frac{dy}{dx} = \ln x + x\left(\frac{x'}{x}\right) = \ln x + \frac{x}{x} = \ln x + 1$$

3- $f(x) = \operatorname{sen}(\ln x)$

$$\frac{dy}{dx} = \cos(\ln x) \cdot \frac{x'}{x} = \frac{\cos(\ln x)}{x}$$

4- $f(x) = \ln(\operatorname{sen}^2 x)$

$$\frac{dy}{dx} = \frac{1}{\operatorname{sen}^2 x \ln e} \cdot (\operatorname{sen}^2 x)' = \frac{2\operatorname{sen} x \cdot \cos x}{\operatorname{sen}^2 x} = \frac{2\cos x}{\operatorname{sen} x} = 2\operatorname{cotg} x$$

5- $f(x) = \sqrt[5]{\ln x} = (\ln x)^{1/5}$

$$\frac{dy}{dx} = \frac{1}{5}(\ln x)^{-4/5} \cdot \frac{x'}{x} = \frac{(\ln x)^{-4/5}}{x}$$

6- $f(x) = \ln \sqrt[5]{x} = \ln(x^{1/5}) = \frac{1}{5}\ln x$

$$\frac{dy}{dx} = \frac{1}{5} \cdot \frac{x'}{x} = \frac{1}{5x} = (5x)^{-1}$$

7- $f(x) = \log_{10}(x^3+1)$

$$\frac{d}{dx}\log_a x = \frac{x'}{x \ln a}$$

$$\frac{dy}{dx} = \frac{(x^3+1)'}{(x^3+1)\ln 10} = \frac{3x^2}{\ln 10 (x^3+1)}$$

8- $f(x) = \log_5(xe^x)$

$$\frac{dy}{dx} = \frac{(xe^x)'}{(xe^x)\ln 5} = \frac{e^x + xe^x}{xe^x \ln 5} = \frac{e^x(1+x)}{xe^x \ln 5} = \frac{1+x}{x\ln 5}$$

9- $f(x) = \operatorname{sen} x \ln 5x$

$$\frac{dy}{dx} = \cos x \ln 5x + \operatorname{sen} x \cdot \frac{(5x)'}{5x \ln e} = \cos x \ln 5 + \frac{\operatorname{sen} x}{5 \ln e}$$

10- $f(u) = \frac{u}{1+\ln u}$

$$\frac{dy}{du} = \frac{(1+\ln u) - u\left(\frac{u'}{u \ln e}\right)}{(1+\ln u)^2} = \frac{1+\ln u - 1}{(1+\ln u)^2} = \frac{\ln u}{(1+\ln u)^2}$$

11- $g(x) = \ln(x\sqrt{x^2-1})$

$$\frac{dg}{dx} = \frac{1}{x\sqrt{x^2-1} \cdot \ln e} \cdot \left[\sqrt{x^2-1} + x \cdot \frac{1}{2}(x^2-1)^{-1/2} \cdot 2x\right] = \frac{\sqrt{x^2-1} + (x^2-1)^{-1/2}}{x\sqrt{x^2-1}}$$

12- $h(x) = \ln(x + \sqrt{x^2+1})$

$$\frac{dh}{dx} = \frac{(x+\sqrt{x^2+1})'}{(x+\sqrt{x^2+1})\ln e} = \frac{\frac{1}{2}\left[2 + (x^2+1)^{-1/2} \cdot 2x\right]}{x + \sqrt{x^2+1}} = \frac{2x(x^2+1)^{-1/2} + 2}{2x + 2\sqrt{x^2+1}}$$

13- $G(y) = \ln\left(\frac{(2y+1)^5}{\sqrt{y^2+1}}\right) = 5\ln(2y+1) - \frac{1}{2}\ln(y^2+1)$

$$\frac{dG}{dy} = 5 \cdot \frac{2}{(2y+1)\ln e} - \frac{1}{2} \cdot \frac{2y}{(y^2+1)\ln e} = \frac{10}{2y+1} - \frac{2}{y^2+1} = \frac{10(y^2+1) - 2(2y+1)}{(2y+1)(y^2+1)}$$

14- $g(r) = r^2 \ln(2r+1)$

$$\frac{dg}{dr} = 2r\ln(2r+1) + r^2 \cdot \frac{2}{(2r+1)\ln e} = 2r\ln(2r+1) + \frac{2r^2}{2r+1} = \frac{(2r+1)2r\ln(2r+1) + 2r^2}{2r+1} = \frac{2r\left[(2r+1)\ln(2r+1) + r\right]}{2r+1}$$

15- $F(s) = \ln(\ln s)$

$$\frac{dF}{ds} = \frac{(\ln s)'}{\ln s \ln e} = \frac{\frac{1}{s}}{\ln s} = \frac{1}{s \ln s}$$

16- $y = \ln|1+t-t^3|$

$$\frac{dy}{dt} = \frac{(1+t-t^3)'}{(1+t-t^3)\ln e} = \frac{1-3t^2}{1+t-t^3}$$

17- $y = tg\left[\ln(ax+b)\right]$

$$\frac{dy}{dx} = sec^2\left[\ln(ax+b)\right] \cdot \frac{(ax+b)'}{(ax+b)\ln e} = sec^2\left[\ln(ax+b)\right] \cdot \frac{a}{ax+b} = \frac{a\,sec^2\left[\ln(ax+b)\right]}{ax+b}$$

18- $y = \ln|\cos(\ln x)|$

$$\frac{dy}{dx} = \frac{1}{\cos(\ln x)\ln e} \cdot \left[-sen(\ln x) \cdot \frac{1}{x}\right] = \frac{-sen(\ln x)}{\cos(\ln x) \cdot x} = \frac{-tg(\ln x)}{x}$$

19- $y = \ln(e^{-x} + xe^{-x})$

$$\frac{dy}{dx} = \frac{(e^{-x} + xe^{-x})'}{(e^{-x} + xe^{-x}) \cdot \ln e} = \frac{-e^{-2x} \cdot e^{x} + e^{-x} \cdot x(-e^{-2x})}{e^{-x} + xe^{-x}} = \frac{-e^{-x} - xe^{-3x}}{e^{-x} + xe^{-x}}$$

20- $H(z) = \ln\left[\left(\frac{a^2-z^2}{a^2+z^2}\right)^{1/2}\right] = \frac{1}{2}\left[\ln(a^2-z^2) - \ln(a^2+z^2)\right]$

$$\frac{dH}{dz} = \frac{1}{2}\left[\frac{-2z}{(a^2-z^2)\ln e} - \frac{2z}{(a^2+z^2)\ln e}\right] = \frac{(a^2+z^2)(-2z) - (a^2-z^2)2z}{2(a^2-z^2)(a^2+z^2)} = \frac{2z(-a^2-z^2-a^2-z^2)}{2(a^2-z^2)(a^2+z^2)} = \frac{-2z(a^2+z^2)}{(a^2-z^2)(a^2+z^2)} = \frac{-2z}{(a^2-z^2)}$$

21- $y = 2x\log_{10}\sqrt{x}$

$$\frac{dy}{dx} = 2\log_{10}\sqrt{x} + 2x \cdot \frac{\frac{1}{2}x^{-1/2}}{(\sqrt{x}) \cdot \ln 10} = 2\log_{10}\sqrt{x} + \frac{x^{1/2}}{x^{1/2} \cdot \ln 10} = 2\log_{10}\sqrt{x} + \frac{1}{\ln 10} = \frac{\log x \cdot \ln 10 + 1}{\ln 10}$$

22- $y = \log_2(e^{-x}\cos\pi x)$

$$\frac{dy}{dx} = \frac{(e^{-x}\cos\pi x)'}{(e^{-x}\cos\pi x) \cdot \ln 2} = \frac{-e^{-2x} \cdot e^{x} \cdot \cos\pi x + e^{-x} \cdot (-sen\,\pi x) \cdot \pi}{\ln 2 \cdot e^{-x}\cos\pi x} = \frac{-e^{-x} \cdot \cos\pi x - e^{-x} \cdot \pi \cdot sen\,\pi x}{\ln 2 \cdot e^{-x} \cdot \cos\pi x} = \frac{-e^{x}(\cos\pi x + \pi\,sen\,\pi x)}{\ln 2 \cdot e^{x} \cdot \cos\pi x} = \frac{-\cos\pi x - \pi\,sen\,\pi x}{\ln 2 \cdot \cos\pi x} = \frac{-\cos\pi x - \pi\cos\pi x\,tg\,\pi x}{\ln 2 \cdot \cos\pi x} = \frac{-1 - \pi\,tg\,\pi x}{\ln 2}$$

23- $y = x^2\ln 2x$

$$\frac{dy}{dx} = 2x\ln 2x + x^2 \cdot \frac{1}{2x\ln e} \cdot 2 = 2x\ln 2x + x = x(2\ln 2x + 1)$$

$$\frac{d^2y}{dx^2} = 2\ln 2x + 2x \cdot \frac{1}{2x} \cdot 2 + 1 = 2\ln 2x + 3$$

24- $y = \frac{\ln x}{x^2}$

$$\frac{dy}{dx} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \cdot \ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{-2}{x} \cdot x^3 - (1-2\ln x) \cdot 3x^2}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2\ln x}{x^6} = \frac{-5x^2 + 6x^2\ln x}{x^6} = \frac{-5 + 6\ln x}{x^4}$$

25- $y = \ln(x + \sqrt{1+x^2})$

$$\frac{dy}{dx} = \frac{1}{(x+\sqrt{1+x^2}) \cdot \ln e} \cdot \left[1 + \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x\right] = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{x+\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} = (\sqrt{1+x^2})^{-1}$$

$$\frac{d^2y}{dx^2} = -1(\sqrt{1+x^2})^{-2} \cdot \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x = \frac{-1}{1+x^2} \cdot \frac{x}{\sqrt{1+x^2}} = \frac{-1}{1+x^2} \cdot \frac{x\sqrt{1+x^2}}{1+x^2} = \frac{-x\sqrt{1+x^2}}{1+x^2}$$

26- $y = \ln(sec\,x + tg\,x)$

$$\frac{dy}{dx} = \frac{1}{sec\,x\,tg\,x\ln e} \cdot sec\,x\,tg\,x + sec^2x = \frac{tg\,x + sec\,x}{tg\,x} = cotg\,x(tg\,x + sec\,x) = 1 + cossec\,x$$

27- $f(x) = \frac{x}{1 - \ln(x-1)}$

$$\frac{df}{dx} = \frac{1 - \ln(x-1) - x\frac{-1}{x-1}}{[1 - \ln(x-1)]^2} = \frac{1 - \ln(x-1) + x(x-1)^{-1}}{[1 - \ln(x-1)]^2}$$

$D: 1 - \ln(x-1) \neq 0;\ \ln(x-1) \neq 1;\ e^1 \neq x-1;\ D = \{x \in R \mid x \neq e+1\}$

28- $f(x) = \sqrt{2 + \ln x}$

$$\frac{df}{dx} = \frac{1}{2}(2+\ln x)^{-1/2} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{2+\ln x}} = \frac{\sqrt{2+\ln x}}{2x(2+\ln x)}$$

$D: 2 + \ln x \geq 0;\ \ln x \geq -2;\ x \geq e^{-2};\ D = \{x \in R \mid x \geq e^{-2}\}$

29- $f(x) = \ln(x^2 - 2x) = \ln[x(x-2)] = \ln x + \ln(x-2)$

$$\frac{df}{dx} = \frac{1}{x} + \frac{1}{x-2} = \frac{x-2+x}{x(x-2)} = \frac{2x-2}{x(x-2)}$$

$D: x^2-2x>0 \; ; \; x<0 \cup x>2 \; ; \; D=\{x \in \mathbb{R} \mid x<0 \cup x>2\} = (-\infty, 0) \cup (2, +\infty)$

$x^2-2x=0 \; ; \; x \in \{0, 2\}$

30- $f(x) = \ln(\ln x)$

$$\frac{df}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

$D: \ln x > 0 \; ; \; x > e^0 \; ; \; x>1 \; ; \; D = \{x \in \mathbb{R} \mid x>1\}$

31- $f(x) = \frac{\ln x}{x^2}$

$$\frac{df}{dx} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1-2\ln x}{x^3}$$

$$\frac{df(1)}{dx} = 1 - 2\ln 1 = 1$$

32- $f(x) = \ln(1+e^{2x})$

$$\frac{df}{dx} = \frac{1}{1+e^{2x}} \cdot 2e^x \cdot e^x = \frac{2e^{2x}}{1+e^{2x}}$$

$$\frac{df}{dx}(0) = \frac{2}{1+2} = 2/3$$

33- $y = \ln(x^2-3x+1)$

$$\frac{dy}{dx} = \frac{1}{x^2-3x+1} \cdot (2x-3) = \frac{2x-3}{x^2-3x+1}$$

$P(3,0) \; ; \; f'(3) = \frac{3}{9-9+1} = 3$

$t: y-y_0 = m(x-x_0) \; ; \; y-0 = 3(x-3) \; ; \; t: y = 3x-9$

34- $y = x^2 \ln x$

$$\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x = x(2\ln x + 1)$$

$P(1,0) \; ; \; f'(1) = 2\ln 1 + 1 = 1$

$t: y-y_0 = m(x-x_0) \; , \; t: y = x-1$

37- $f(x) = cx + \ln(\cos x)$

$$\frac{df}{dx} = c + \frac{1}{\cos x} \cdot (-\operatorname{sen} x) = c - \operatorname{tg} x$$

$f'(\pi/4) = c - \operatorname{tg} \pi/4 = c - 1 = 6 \; ; \; c = 7$

38- $f(x) = \log_a(3x^2-2)$

$$\frac{df}{dx} = \frac{1}{(3x^2-2) \cdot \ln a} \cdot 6x = \frac{6x}{(3x^2-2)\ln a}$$

$f'(1) = \frac{6}{\ln a} = 3 \; ; \; \ln a = 2 \; ; \; a = e^2$

39- $y = (2x+1)^5 (x^4-3)^6$

$\ln y = 5\ln(2x+1) + 6\ln(x^4-3)$

$$\frac{1}{y} \cdot y' = 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4-3} \cdot 4x^3$$

$$\frac{y'}{y} = \frac{10}{2x+1} + \frac{24x^3}{x^4-3} = \frac{10x^4 - 30 + 48x^4 + 24x^3}{(2x+1)(x^4-3)} = \frac{58x^4 + 24x^3 - 30}{(2x+1)(x^4-3)}$$

$$y' = \frac{(58x^4+24x^3-30)}{(2x+1)(x^4-3)} \cdot (2x+1)^5 (x^4-3)^6 = (58x^4+24x^3-30)(2x+1)^4(x-3)^5$$

40- $y=\sqrt{x}\,e^{x^2}(x^2+1)^{10}$

$\ln y=\frac{1}{2}\ln x+x^2+10\ln(x^2+1)$

$\frac{1}{y}\cdot y'=\frac{1}{2x}+2x+10\cdot\frac{1}{x^2+1}\cdot 2x$

$\frac{y'}{y}=\frac{1}{2x}+2x+\frac{20x}{x^2+1}=\frac{x^2+1+4x^2(x^2+1)+40x^2}{2x(x^2+1)}=\frac{x^2+1+4x^4+4x^2+40x^2}{2x(x^2+1)}=\frac{4x^4+45x^2+1}{2x(x^2+1)}$

$y'=\frac{4x^4+45x^2+1}{2x}\cdot\sqrt{x}\cdot e^{x^2}\cdot(x^2+1)^9$

41- $y=\sqrt{\frac{x-1}{x^4+1}}$

$\ln y=\frac{1}{2}\left[\ln(x-1)-\ln(x^4+1)\right]$

$\frac{1}{y}\cdot y'=\frac{\frac{1}{x-1}-\frac{1}{x^4+1}\cdot 4x^3}{2}=\frac{x^4+1-4x^3(x-1)}{2(x-1)(x^4+1)}=\frac{-3x^4+4x^3+1}{2(x-1)(x^4+1)}$

$y'=\frac{-3x^4+4x^3+1}{2(x-1)(x^4+1)}\cdot\sqrt{\frac{x-1}{x^4+1}}$

42- $y=\sqrt{x}\,e^{x^2-x}(x+1)^{2/3}$

$\ln y=\frac{1}{2}\ln x+x^2-x+\frac{2}{3}\ln(x+1)$

$\frac{1}{y}\cdot y'=\frac{1}{2x}+2x-1+\frac{2}{3(x+1)}=\frac{3(x+1)+12x^2(x+1)-6x(x+1)+4x}{6x(x+1)}$

$y'=\frac{3(x+1)+12x^2(x+1)-6x(x+1)+4x}{6x(x+1)}\cdot\sqrt{x}e^{x^2-x}(x+1)^{2/3}$

43- $y=x^x$

$\ln y=x\ln x$

$\frac{y'}{y}=\ln x+x\cdot\frac{1}{x}=\ln x+1$

$y'=x^x(\ln x+1)$

44- $y=x^{\cos x}$

$\ln y=\cos x\ln x$

$\frac{y'}{y}=-\operatorname{sen}x\ln x+\cos x\,\frac{1}{x}=-\operatorname{sen}x\ln x+\frac{\cos x}{x}=\frac{-x\operatorname{sen}x\ln x+\cos x}{x}$

$y'=x^{\cos x-1}(-x\operatorname{sen}x\ln x+\cos x)$

45- $y=x^{\operatorname{sen}x}$

$\ln y=\operatorname{sen}x\ln x$

$y'=(\operatorname{sen}x\ln x)'\cdot y$

$y'=\left(\cos x\ln x+\operatorname{sen}x\cdot\frac{1}{x}\right)\cdot x^{\operatorname{sen}x}$

$y'=x^{\operatorname{sen}x-1}\cdot(\cos x\ln x+\operatorname{sen}x)$

46- $y=\sqrt{x}^{\,x}$

$\ln y=\frac{x}{2}\ln x$

$\frac{y'}{y}=\frac{\ln x+1}{2}$

$y'=\frac{\sqrt{x}^{\,x}(\ln x+1)}{2}$

47- $y=(\cos x)^x$

$\ln y=x\ln(\cos x)$

$\frac{y'}{y} = \ln(\cos x) + x \cdot \frac{1}{\cos x} \cdot (-\operatorname{sen} x)$

$y' = \left[\ln(\cos x) - x\,tg\,x\right](\cos x)^x$

48- $y = (\operatorname{sen} x)^{\ln x}$

$\ln y = \ln x \cdot \ln(\operatorname{sen} x)$

$\frac{y'}{y} = \frac{1}{x} \cdot \ln(\operatorname{sen} x) + \ln x \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x = \frac{\ln(\operatorname{sen} x)}{x} + \ln x \operatorname{cotg} x$

$y' = (\operatorname{sen} x)^{\ln x}\left[\frac{\ln(\operatorname{sen} x) + x \ln x \operatorname{cotg} x}{x}\right]$

49- $y = (tg\,x)^{1/x}$

$\ln y = \frac{1}{x} \ln(tg\,x)$

$\frac{y'}{y} = \frac{\frac{1}{tg\,x} \cdot \sec^2 x \cdot x - \ln(tg\,x)}{x^2} = \frac{x \cdot \sec x \cdot \operatorname{cossec} x - \ln(tg\,x)}{x^2}$

$y' = (tg\,x)^{1/x}\left[\frac{x \sec x \operatorname{cossec} x - \ln(tg\,x)}{x^2}\right]$

50- $y = (\ln x)^{\cos x}$

$\ln y = \cos x \ln(\ln x)$

$\frac{y'}{y} = -\operatorname{sen} x \cdot \ln(\ln x) + \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{\cos x - x \ln x \cdot \operatorname{sen} x \cdot \ln(\ln x)}{x \ln x}$

$y' = (\ln x)^{\cos x}\left[\frac{\cos x - x \ln x \cdot \operatorname{sen} x \cdot \ln(\ln x)}{x \ln x}\right]$

51- $y = \ln(x^2 + y^2)$

$y' = \frac{1}{x^2 + y^2} \cdot (2x + 2yy')$

$x^2 y' + y^2 y' = 2x + 2yy'$

$x^2 y' + y^2 y' - 2yy' = 2x$

$y'(x^2 + y^2 - 2y) = 2x$

$y' = \frac{2x}{x^2 + y^2 - 2y}$

52- $y^x = x^y$

$x \ln y = y \ln x$

$\ln y + x \cdot \frac{1}{y} \cdot y' = y' \ln x + y \cdot \frac{1}{x}$

$\frac{xy'}{y} - y' \ln x = \frac{y}{x} - \ln y$

$\frac{xy' - yy' \ln x}{y} = \frac{y - x \ln y}{x}$

$y'(x - y \ln x) = \frac{y(y - x \ln y)}{x}$

$y' = \frac{y(y - x \ln y)}{x(x - y \ln x)}$

53- $f(x) = \ln(x-1)$

$f'(x) = \frac{1}{x-1} = (x-1)^{-1} = \frac{1}{(x-1)^1}$

$f''(x) = -(x-1)^{-2} = \frac{-1}{(x-1)^2}$

$f'''(x) = 2(x-1)^{-3} = \frac{2}{(x-1)^3}$

$f''''(x) = -6(x-1)^{-4} = \frac{-6}{(x-1)^4}$

$f^{5}(x) = 24(x-1)^{-5} = \frac{24}{(x-1)^5}$

$f^{6}(x) = -120(x-1)^{-6} = \frac{-120}{(x-1)^6}$

m ímpar

De forma geral, $\frac{d^n f}{dx} = \begin{cases} \frac{(n-1)!}{(x-1)^n}, \text{ se } n \% 2 = 1 \\ \frac{-(n-1)!}{(x-1)^n}, \text{ se } n \% 2 = 0 \end{cases}$; ou seja, $\frac{d^n f}{dx} = \frac{(-1)^{n-1} \cdot (n-1)!}{(x-1)^n}$

m par

54- $y = x^8 \ln x$

$\frac{dy}{dx} = 8x^7 \ln x + x^8 \cdot \frac{1}{x} = 8x^7 \ln x + x^7$

$\frac{d^2y}{dx^2} = 56x^6 \ln x + 8x^7 \cdot \frac{1}{x} + 7x^6 = 56x^6 \ln x + 15x^6$

$\frac{d^3y}{dx^3} = 56 \cdot 6 \cdot x^5 \ln x + 56x^6 \cdot \frac{1}{x} + 15 \cdot 6 \cdot x^5 = 56 \cdot 6x^5 \ln x + x^5(56 + 15 \cdot 6)$

A oitava derivada será $8!\, x \ln x + (7!)' = 8!\, x \ln x + 0$

A nona derivada será $8! \ln x = 8! \cdot \frac{1}{x}$

Portanto, $\frac{d^9y}{dx^9} = \frac{8!}{x} = \frac{40320}{x}$.

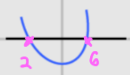
Pg 211 (3.7) João Vaz

1- $f(t) = t^3 - 12t^2 + 36t$

a) $v(t) = \frac{df}{dt} = 3t^2 - 24t + 36$

b) $v(3) = 27 - 72 + 36 = -15\,m/s$

c) $v(t) = 0 \therefore 3(t^2 - 8t + 12) = 0 \therefore t \in \{2, 6\}$

d) $v(t) > 0$ 

$\therefore t \in [0,2) \cup (6,\infty)$

e) $d(8) = d(0\text{-}2) + d(2\text{-}6) + d(6\text{-}8)$

$d(8) = |f(2) - f(0)| + |f(6) - f(2)| + |f(8) - f(6)|$

$d(8) = |32 - 0| + |0 - 32| + |32 - 0| = 32 + 32 + 32 = 96\,m$

g) $a = \frac{dv}{dt} = 6t - 24$

$a(3) = 18 - 24 = -6\,m/s^2$

i) Acelerando: $(a>0 \text{ e } v>0)$ ou $(a<0 \text{ e } v<0)$ (1)

Freiando: $(a>0 \text{ e } v<0)$ ou $(a<0 \text{ e } v>0)$ (2)

(1): $\left.\begin{array}{l} a>0 \therefore 6t-24>0 \therefore t>4 \\ v>0 \therefore t \in [0,2) \cup (6,\infty) \end{array}\right\} t \in (6,\infty)$

$\left.\begin{array}{l} a<0 \therefore t<4 \\ v<0 \therefore t \in (2,6) \end{array}\right\} t \in (2,4)$

Logo, a partícula acelera entre 2 e 4s e a partir de 6s.

(2): $\left.\begin{array}{l} a>0 : t>4 \\ v<0 : t \in (2,6) \end{array}\right\} t \in (4,6)$

$\left.\begin{array}{l} a<0 : t<4 \\ v>0 : t \in [0,2) \cup (6,\infty) \end{array}\right\} t \in [0,2)$

Logo, a partícula freia de $t=0$ até $t=2$ e de $t=4$ a $t=6$.

4- $f(t) = te^{-t/2} = t \cdot (e^t)^{-1/2}$

a) $v(t) = \frac{df}{dt} = e^{-t/2} - t \cdot \frac{1}{2} e^{-3/2} \cdot e^t = e^{-t/2} - \frac{e^{-1/2} \cdot t}{2}$

b) $v(3) = e^{-1,5} - \frac{3e^{-1/2}}{2}$

c) $v(t) = 0 \therefore 2e^{-t/2} = e^{-1/2} \cdot t \therefore t = \frac{2e^{-t/2}}{e^{-1/2}} = 2e^{\frac{-t+1}{2}}$

d) $v(t) > 0 \therefore e^{-t/2} > \frac{e^{-1/2} \cdot t}{2} \therefore t < \frac{2e^{-t/2}}{e^{-1/2}} \therefore t < 2e^{\frac{-t+1}{2}}$

7- a) $v(t) = \frac{dh}{dt} = -9,8t + 24,5$

$v(2) = -19,6 + 24,5 = 4,9\,m/s$

$v(4) = -14,7\,m/s$

b) A altura máxima ocorre na inversão de movimento; ou seja, quando $v=0$

$v(t)=0 \therefore -9{,}8t+24{,}5=0 \therefore t=2{,}5\,s$

c) $h(2{,}5)=32{,}625\,m$

negativa   positiva

d) $h(t)=0 \therefore -4{,}9t^2-24{,}5t+2=0 \therefore t \in \left\{\frac{245+\sqrt{60417}}{-98}, \frac{245-\sqrt{60417}}{-98}\right\}$. Como não há $t<0$, considera-se apenas a 2ª raiz.

e) $v(0{,}0081) \approx 24{,}42\,m/s$

velocidade

8- a) A altura máxima ocorre quando $\frac{dh}{dt}=0$

$\frac{dh}{dt}=-9{,}8t+24{,}5$

$v(t)=0 \therefore t=2{,}5\,s$; $h(2{,}5)=32{,}625\,m$

b) $v^2=v_0^2+2ad$

$v^2=(24{,}5)^2-2\cdot 10\cdot 29{,}4=12{,}25 \therefore v=3{,}5\,m/s$

Chão é o referencial

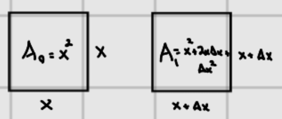
$v^2=(24{,}5)^2+2\cdot 10\cdot(H_{MAX}-29{,}4)=664{,}75 \therefore v\approx 25{,}78\,m/s$

11- a) Como são placas quadradas, $A(x)=x^2$.

$A'(x)=2x \therefore A'(15)=30\,mm$.

b) $A(x)=x^2$

$A'(x)=2x$; $2p=4x$. Portanto, a taxa de variação da área é metade do perímetro.



$2p_0=4x$ $2p_1=4x+4$

$\Delta A=(x+\Delta x)^2-x^2=(x+\Delta x-x)(x+\Delta x+x)=\Delta x(2x+\Delta A)$

$\lim_{\Delta x\to 0}\Delta A=0\cdot(2x+\Delta A)=0$

15- a) $S(r)=4\pi r^2$; $r=20$

$\frac{dS}{dr}=4\pi\cdot 2r=8\pi r$; $S'(20)=160\pi$

b) $S'(40)=320\pi$

c) $S'(60)=480\pi$

S varia linearmente em relação a r: $S'(\alpha r)=\alpha S'(r)$. Isso é ocasionado pela derivada ter grau 1.

23- a) $PV=C$

$V=C/P \therefore V(P)=C/P=C\cdot P^{-1}$

$V'(P)=\frac{-C}{P^2}$

b) No final. O volume decresce de forma proporcional ao quadrado do aumento da pressão.

33- a) $A'(x)=\frac{p'(x)\cdot x-p(x)\cdot 1}{x^2}=\frac{xp'(x)-p(x)}{x^2}=\frac{xp'(x)}{x^2}-\frac{p(x)}{x^2}=\frac{p'(x)}{x}-\frac{A(x)}{x}=\frac{p'(x)-A(x)}{x}$

b) Se $p'(x)$ for maior que $A(x)$, a diferença $p'(x)-A(x)$ será maior que zero. Como x, o número de trabalhadores, é estrit. positivo, $p'(x)>A(x) \Rightarrow A'(x)>0$.

Pg 224 (3.9) João Vaz

1- $V(x) = x^3$

$x(t) = kt + x_0$

$\frac{dx}{dt} = k$

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} = 3x^2k$

2- a) $A(r) = \pi r^2$

$r(t) = kt + r_0$

$\frac{dr}{dt} = k$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} = 2\pi r k$

b) $\frac{dr}{dt} = 1\,m/s$; $r = 30\,m$; $\frac{dA}{dt}$ quando $r = 30\,m$

$\frac{dA}{dt} = 2\pi \cdot 30 \cdot 1 = 60\pi\, m^2/s$

3- $\frac{dl}{dt} = 6\,cm/s$; $\frac{dA}{dt}$ quando $A = 16\,cm^2$?

$A(l) = l^2$. Se $16 = l^2$, $l = \pm 4$; porém considera-se apenas 4 por se tratar de comprimento.

$\frac{dA}{dt} = 2l \cdot \frac{dl}{dt} = 2 \cdot 4 \cdot 6 = 48\,cm^2/s$

4- $\frac{dL}{dt} = 8\,cm/s$; $\frac{dl}{dt} = 3\,cm/s$; $\frac{dA}{dt}$ quando $L = 20\,cm$ e $l = 10\,cm$?

$A(L, l) = L \cdot l$

$\frac{dA}{dt} = \frac{dL}{dt} l + L \cdot \frac{dl}{dt}$

$\frac{dA}{dt} = 8 \cdot 10 + 20 \cdot 3 = 80 + 60 = 140\,cm^2/s$

5- $r = 5\,m$; $\frac{dV}{dt} = 3\,m^3/min$; $\frac{dh}{dt} = ?$

$V = \pi r^2 h = 25\pi \cdot h$

$\frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt}$; $\frac{dh}{dt} = \frac{3}{25\pi}\,m/s$

6- $\frac{dr}{dt} = 4\,mm/s$; $\frac{dV}{dt}$ quando $r = 40\,mm$?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3 \cdot 1600 \cdot 4 = 16 \cdot 1600\pi = 25600\pi$

7- $y = \sqrt{2x+1}$, $y(t) = \sqrt{2x(t)+1}$

a) $\frac{dx}{dt} = 3$; $\frac{dy}{dt}$ quando $x = 4$?

$\frac{dy}{dt} = \frac{1}{2}(2x+1)^{-1/2} \cdot \left(2 \cdot \frac{dx}{dt} + 1\right) = \frac{1}{\sqrt{9}} \cdot 7 = 7/3$

b) $\frac{dx}{dt} = 5$; $\frac{dy}{dt}$ quando $x = 12$

$\frac{dy}{dt} = \frac{1}{\sqrt{25}} \cdot 11 = 11/5$

8- $4x^2(t) + 9y^2(t) = 36$

a) $\frac{dy}{dt} = \frac{1}{3}$, $\frac{dx}{dt}$ quando $x = 2$ e $y = \frac{2}{3}\sqrt{5}$?

$8x \cdot \frac{dx}{dt} + 18y \cdot \frac{dy}{dt} = 0$

$16 \cdot \frac{dx}{dt} + 18 \cdot \frac{2}{3} \cdot \sqrt{5} \cdot \frac{1}{3} = 0$

$\frac{dx}{dt} = \frac{-4\sqrt{5}}{16} = -\sqrt{5}/4$

b) $\frac{dx}{dt} = 3$; $\frac{dy}{dt}$ quando $x = -2$ e $y = \frac{2\sqrt{5}}{3}$

$-16 \cdot 3 + 12\sqrt{5} \cdot \frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{48}{12\sqrt{5}} = \frac{48\sqrt{5}}{60} = \frac{4\sqrt{5}}{5}$

9- $x^2(t) + y^2(t) + z^2(t) = 9$; $\frac{dx}{dt} = 5$; $\frac{dy}{dt} = 4$; $\frac{dz}{dt}$ quando $x = y = 2$ e $z = 1$?

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} + 2z \cdot \frac{dz}{dt} = 0$; $2\left(x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} + z \cdot \frac{dz}{dt}\right) = 0$

$2 \cdot 5 + 2 \cdot 4 + \frac{dz}{dt} = 0$

$\frac{dz}{dt} = -18$

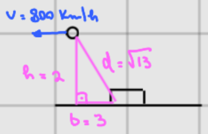
10- $x(t) \cdot y(t) = 8$; quando $x = 4$ e $y = 2$, $\frac{dy}{dt} = 3$ cm/s. Quanto vale $\frac{dx}{dt}$?

$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$

$\frac{dx}{dt} \cdot 2 + 4 \cdot 3 = 0$

$\frac{dx}{dt} = -6$ cm/s

11-



$d^2 = h^2 + b^2$

$b(t) = vt + 3$, sendo b em km, t em h e v em km/h

h é sempre constante

$d^2 = 4 + 9 = 13$; $d = \sqrt{13}$ $d > 0$

$2d \cdot \frac{dd}{dt} = 2b \cdot \frac{db}{dt}$; $\frac{dd}{dt} = \frac{b}{d} \cdot \frac{db}{dt} = \frac{bv}{d}$

Se $b = 3$ km, $v = 800$ km/h e $d = \sqrt{13}$ km: $\frac{dd}{dt} = \frac{3\sqrt{13} \cdot 800}{13}$ km/h $= 665{,}64$ km/h

12- $\frac{dA}{dt} = 1$ cm²/min ; $\frac{dD}{dt}$ quando $r = 5$ cm

$A = \pi r^2 = \pi \cdot \left(\frac{D}{2}\right)^2 = \frac{D^2\pi}{4}$

$\frac{dA}{dt} = \frac{\pi}{4} \cdot 2D \cdot \frac{dD}{dt}$

$\frac{dD}{dt} = \frac{4}{10\pi} = \frac{2}{5\pi}$ cm/s

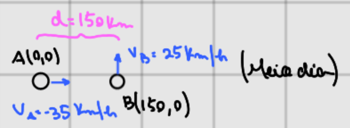
13-

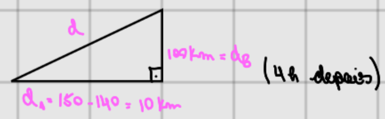
Const.

$\frac{ds}{dt} = \frac{h}{4} \cdot \frac{dd}{dt}$

$\frac{ds}{dt}$ quando $d = 10$ é: $\frac{ds}{dt} = \frac{h}{4} \cdot \frac{dd}{dt} = \frac{1}{3} \cdot 1{,}5 = 0{,}5$ m/s

14-





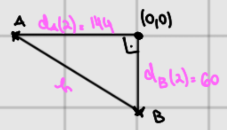
$d^2 = d_A^2 + d_B^2 \rightsquigarrow d = \sqrt{100+10000}$

$$2d \cdot \frac{dd}{dt} = 2d_A \cdot \frac{dd_A}{dt} + 2d_B \cdot \frac{dd_B}{dt}$$

$$2\sqrt{10100} \cdot \frac{dd}{dt} = 2 \cdot 10 \cdot (-35) + 2 \cdot 100 \cdot 25$$

$$\frac{dd}{dt} = \frac{2150}{\sqrt{10100}} \text{ Km/h}$$

15-

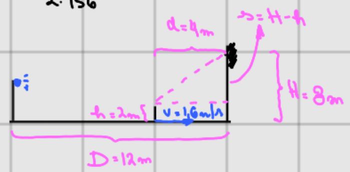


$h^2 = d_A^2 + d_B^2$; $h(2) = 156$

$$2h(2) \cdot \frac{dh}{dt} = 2d_A(2) \cdot \frac{dd_A}{dt} + 2d_B(2) \cdot \frac{dd_B}{dt}$$

$$\frac{dh}{dt} = \frac{2 \cdot 144 \cdot 72 + 2 \cdot 60 \cdot 30}{2 \cdot 156} = 78 \text{ Km/h}$$

16-



$\frac{h}{d} = \frac{s}{D} \therefore \frac{2}{4} = \frac{H-2}{12} \therefore H-2=6 \therefore H=8m$

Quando d, a distância entre o homem e a parede, é 12m, sua sombra mede H-h=6m.

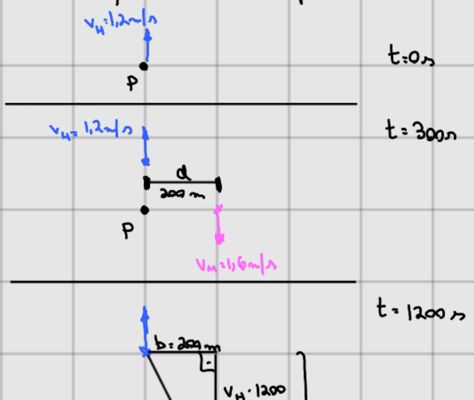
Quando d=12m, sua sombra mede 0m.

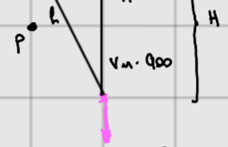
$\frac{h}{d} = \frac{s}{D} \therefore s = \frac{\overset{\text{const.}}{Dh}}{d} = Dh \cdot d^{-1}$

$$\frac{ds}{dt} = D \cdot h \cdot (-1) \cdot d^{-2} \cdot \frac{dd}{dt}$$

$\frac{ds}{dt}$ quando $d=4$: $-12 \cdot 2 \cdot \frac{1}{16} \cdot 1{,}6 = -2{,}4$ m/s

17- dh/dt 15 minutos após a mulher começar a andar?





$b^2 + H^2 = h^2$

$h^2(1200) = b^2(1200) + H^2(1200)$

$h(1200) = \sqrt{40000 + (1440 + 1440)^2}$

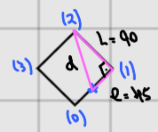
$h(1200) = \sqrt{8334400}$

$\frac{dH}{dt}$ é a soma das velocidades: $1{,}6 + 1{,}2 = 2{,}8\ m/s$

$2b \cdot \underbrace{\frac{db}{dt}}_{0} + 2H \cdot \frac{dH}{dt} = 2h \cdot \frac{dh}{dt}$

Quando $b = 200\ m$, $\frac{dh}{dt} = \frac{H}{h} \cdot \frac{dH}{dt} = \frac{2800}{\sqrt{8334400}} \cdot 2{,}8 \simeq 2{,}71\ m/s$

18- a)



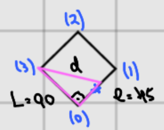
$d^2 = L^2 + \ell^2$; na configuração atual, $d = \sqrt{8100 + 2025}$

$2d \cdot \frac{dd}{dt} = 2L \cdot \underbrace{\frac{dL}{dt}}_{0} + 2\ell \cdot \frac{d\ell}{dt}$

$2\sqrt{10125} \cdot \frac{dd}{dt} = 90 \cdot 24$

$\frac{dd}{dt} = \frac{90 \cdot 24}{2\sqrt{10125}} = 10{,}73$ pés/s

b)



De forma análoga, $\frac{dd}{dt} = 10{,}73$ pés/s

19- $\frac{dh}{dt} = 1\ cm/min$; $\frac{dA}{dt} = 1\ cm^2/min$. $\frac{db}{dt}$ quando $h = 10\ cm$ e $A = 100\ cm^2$?

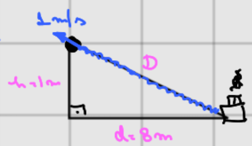
$A(b,h) = b \cdot h$ ∴ se $A = 100$ e $h = 10$, $b = \frac{100}{10} = 10\ cm$

$\frac{dA}{dt} = \frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}$

$1 = \frac{db}{dt} \cdot 10 + 10 \cdot 1$

$\frac{db}{dt} = \frac{-9}{10} = -0{,}9\ cm/min$. Ou seja, a base decresce a uma taxa de $0{,}9\ cm/min$.

20-



$\frac{dd}{dt}$ quando $\frac{dD}{dt} = 1\ m/s$ e $d = 8\ m$?

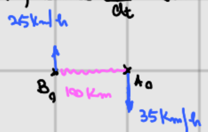
$D^2 = h^2 + d^2$. Nessa configuração, $D = \sqrt{65}\ m$

$2D \cdot \frac{dD}{dt} = 2h \cdot \underbrace{\frac{dh}{dt}}_{0} + 2d \cdot \frac{dd}{dt}$

$2\sqrt{65} \cdot 1 = 16 \cdot \frac{dd}{dt}$

$\frac{dd}{dt} = \frac{\sqrt{65}}{8}\ m/s$

21- dh/dt se $\frac{dH}{dt} = 60\ km/h$, $b = 100\ km$ e $t = 4h$?



$t = 0h$

$t = 4h$

$H^2+b^2=h^2$; na config. atual, $h=\sqrt{67600}$

$2H\cdot\frac{dH}{dt}+2b\cdot\frac{db}{dt}=2h\cdot\frac{dh}{dt}$ 0

$\frac{dh}{dt}=\frac{H}{h}\cdot\frac{dH}{dt}=\frac{240}{\sqrt{67600}}\cdot 60 \simeq 56{,}38\,Km/h$

22- $y=2\,sen(\pi x/2)$; $P_1(1/3, 1)$; $\frac{dx_p}{dt}=\sqrt{10}\ cm/s$. $\frac{dd_{PO}}{dt}=?$

A distância de P a O(0,0) pode ser calculada usando a fórmula da distância entre 2 pontos:

$d_{PO}=\sqrt{(x_P-x_O)^2+(y_P-y_O)^2}=\sqrt{x_P^2+y_P^2}=(x_P^2+y_P^2)^{1/2}$

$y=2\,sen(\pi x/2)$; $\frac{dy}{dt}=2\cos(\pi x/2)\cdot\frac{\pi}{2}\frac{dx}{dt}=\sqrt{10}\,\pi\cos(\pi x/2)$

$\frac{dd_{PO}}{dt}=\frac{1}{2}(x_P^2+y_P^2)^{-1/2}\cdot\left(2x_P\cdot\frac{dx_P}{dt}+2y_P\cdot\frac{dy_P}{dt}\right)$

$\frac{dd_{PO}}{dt}$ no ponto P_1 é: $\frac{1}{2}\left(\frac{1}{9}+1\right)^{-1/2}\cdot\left(\frac{2}{3}\cdot\sqrt{10}+2\cdot\sqrt{10}\,\pi\cos(\pi/6)\right)\simeq 9{,}162$

23- $\frac{dh}{dt}=20\ cm/min$; $\frac{du}{dt}=10\,000\ cm^3/min$; $\frac{db}{dt}=?$; $\frac{dV}{dt}=\frac{db}{dt}-\frac{du}{dt}$

$V=\frac{\pi r^2 h}{3}$

$dV=\frac{\pi r^2}{3}\cdot\frac{dh}{dt}$

$\frac{db}{dt}-\frac{du}{dt}=\frac{\pi r^2}{3}\cdot\frac{dh}{dt}$

$\frac{db}{dt}-10\,000=\frac{\pi\cdot 4\cdot 10^6}{3}\cdot 20$

$\frac{db}{dt}=\frac{80\cdot 10^6\cdot\pi-30\cdot 10^3}{3}$

24- $V=A_\ell C=\frac{b\cdot h}{2}\cdot C=1\cdot 0{,}5\cdot 3=1{,}5\,m^3$; $\frac{dV}{dt}=1{,}2\,m^3/min$; $\frac{dh}{dt}$ quando $h=30\,cm$?

$V(b,h,C)=b\cdot h\cdot C\cdot\frac{1}{2}$

$\frac{dV}{dt}=\left[\left(\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}\right)\cdot C+b\cdot h\cdot\frac{dC}{dt}\right]\cdot\frac{1}{2}=\frac{1}{2}\cdot b\cdot C\cdot\frac{dh}{dt}$ 0 0

$1{,}2=\frac{1}{2}\cdot 1\cdot 6\cdot\frac{dh}{dt}\ \therefore\ \frac{dh}{dt}=\frac{1{,}2}{3}=0{,}4\,m/min$

25-

26-

27- $\frac{dV}{dt}=3\,m^3/min$, $\frac{dh}{dt}$ quando $h=3m$ sendo h sempre igual a r?

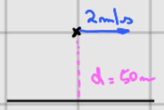
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \cdot \pi \cdot h^3$

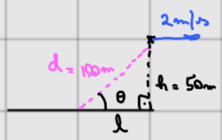
$\frac{dV}{dt} = \frac{\pi}{3} \cdot 3h^2 \cdot \frac{dh}{dt}$

$3 = \frac{\pi}{3} \cdot 27 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{3\pi}$ m/min

28- $\frac{d\theta}{dt}$ quando $d = 100m$?





$h^2 + l^2 = d^2$

$10000 - 2500 = l^2 = 7500 \; ; \; l = \sqrt{7500}$

$tg\,\theta = \frac{h}{l} = h \cdot l^{-1}$

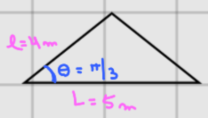
2m/s

$\sec^2\theta \cdot \frac{d\theta}{dt} = 50 \cdot (-1) \cdot (\sqrt{7500})^{-2} \cdot \frac{dl}{dt}$

$\cos\theta = \frac{l}{d} = \frac{\sqrt{7500}}{100}$

$\frac{d\theta}{dt} = \frac{-100}{7500 \cdot \sec^2\theta} = \frac{-100\cos^2\theta}{7500} = -\frac{\cos^2\theta}{75} = \frac{-7500}{750000} = \frac{-1}{100}$ rad/s

29- $\frac{d\theta}{dt} = 0{,}06$ rad/s ; $\frac{dA}{dt}$ quando $\theta = \pi/3$

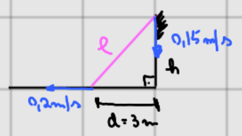


"metade do lado vezes lado vezes seno do ângulo formado"

$A = \frac{1}{2} \cdot L \cdot l \cdot sen\,\theta$

$\frac{dA}{dt} = \frac{L \cdot l}{2} \cdot \cos\theta \cdot \frac{d\theta}{dt} = 10 \cdot \frac{1}{2} \cdot 0{,}06 = 0{,}03\ m^2/s$

31- $l = ?$



l é constante

$h^2 + d^2 = l^2 \; ; \; h = \sqrt{l^2 - 9}$

$2h \cdot \frac{dh}{dt} + 2d \cdot \frac{dd}{dt} = 0$

$2\sqrt{l^2-9} \cdot 0{,}15 + 6 \cdot 0{,}2 = 0$

$\sqrt{l^2-9} = \frac{-1{,}2}{0{,}3} = -4 \therefore l^2 = 25 \therefore l = 5m$

$d = 3 \; ; \; h = 4 \; ; \; l = 5$

33- $PV = C$, sendo C constante. $\frac{dV}{dt}$ quando $P = 150$ KPa, $\frac{dP}{dt} = 20$ KPa/min e $V = 600\ cm^3$?

$P.V = C$

$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$

$\frac{dP}{dt} \cdot V = -P \frac{dV}{dt}$

$20 \cdot 600 = -150 \cdot \frac{dV}{dt}$

$\frac{dV}{dt} = -80\ cm^3/min$

34- $PV^{1,4} = C$, sendo C constante. $P = 80$ KPa, $\frac{dP}{dt} = -10$ KPa/min, $V = 400 cm^3$, $\frac{dV}{dt} = ?$

$PV^{1,4} = C$

$$\frac{dP}{dt} \cdot V^{1,4} + P \cdot 1,4V^{0,4} \cdot \frac{dV}{dt} = 0$$

$$\frac{dV}{dt} = \frac{10 \cdot (400)^{1,4}}{80 \cdot 1,4 \cdot (400)^{0,4}} = \frac{400}{1,4 \cdot 8} \cong 35,7 \text{ cm}^3/\text{min}$$

35. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$; $\frac{dR}{dt}$ quando $R_1 = 80$, $R_2 = 100$, $\frac{dR_1}{dt} = 0,3$ e $\frac{dR_2}{dt} = 0,2$?

$$-R^{-2} \cdot \frac{dR}{dt} = -R_1^{-2} \cdot \frac{dR_1}{dt} - R_2^{-2} \cdot \frac{dR_2}{dt}$$

$$\frac{1}{R} = \frac{1}{80} + \frac{1}{100} = \frac{100+80}{8000} = \frac{180}{8000}$$

$$\frac{dR}{dt} = \frac{\frac{-1}{6400} \cdot 0,3 - \frac{1}{10000} \cdot 0,2}{\left(\frac{18}{800}\right)^2}$$

$$R = \frac{800}{18}$$

$$\frac{dR}{dt} = \frac{-0,3 \cdot 10^4 - 0,2 \cdot 6400}{64 \cdot 10^6} \cdot \left(\frac{800}{18}\right)^2 = \frac{\cancel{64 \cdot 10^4}(-0,3 \cdot 10^4 - 0,2 \cdot 6400)}{324 \cdot \cancel{64 \cdot 10^6}} = \frac{-30-12,8}{324} \cong -0,132 \, \Omega/s$$

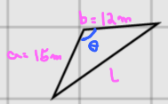
36. $\frac{dL}{dt} = 5 \text{ cm}/10 \cdot 10^6 \text{ anos} = 5 \cdot 10^{-7} \text{ cm/ano}$; $B(w) = 7 \cdot 10^{-3} w^{2/3}$ g ; $w(L) = 0,12 \cdot L^{2,53}$ g; $\frac{dB}{dt}$ quando $L = 18$ cm?

$$\frac{dB}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} w^{-1/3} \cdot \frac{dw}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} \cdot \left[0,12 \cdot (18)^{2,53}\right]^{-1/3} \cdot \frac{dw}{dt}$$

$$\frac{dw}{dt} = 0,12 \cdot 2,53 L^{1,53} \cdot \frac{dL}{dt} = 0,12 \cdot 2,53 \cdot (18)^{1,53} \cdot 5 \cdot 10^{-7}$$

$$\frac{dB}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} \cdot \left[0,12 \cdot (18)^{2,53}\right]^{-1/3} \cdot 0,12 \cdot 2,53 \cdot (18)^{1,53} \cdot 5 \cdot 10^{-7} = 1,045 \cdot 10^{-8} \text{ g/ano} = 0,1045 \text{ g}/10 \cdot 10^6 \text{ anos}$$

37. $\frac{d\theta}{dt} = 2°/\text{min}$; $\frac{dL}{dt}$ quando $\theta = 60°$?



a e b são const.

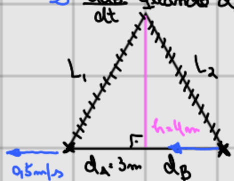
Lei dos cossenos: $L^2 = a^2 + b^2 - 2ab\cos\theta$

Quando $\theta = 60°$, $a = 15$ e $b = 12$: $L = \sqrt{225 + 144 - 90} = \sqrt{279}$

$$2L \cdot \frac{dL}{dt} = 0 + 0 + 2 \cdot 12 \cdot 15 \cdot \text{sen}\,\theta \cdot \frac{d\theta}{dt}$$

$$\frac{dL}{dt} = \frac{12 \cdot 15 \cdot \sqrt{3} \cdot 2}{2\sqrt{279}} = 18,66 \text{ m/min}$$

38. $\frac{dd_B}{dt}$ quando $d_A = 3$ m?



$L_1 = 5$m (pit.) ⟶ $L = L_1 + L_2 \therefore 12 = 5 + L_2 \therefore L_2 = 7$ m

$$h^2 + d_B^2 = L_2^2$$

$$16 + d_B^2 = 49$$

$$d_B = \sqrt{33}$$

$$\begin{cases} h^2 + d_A^2 = L_1^2 & (1) \\ h^2 + d_B^2 = L_2^2 & (2) \end{cases}$$

(1): $h^2 = L_1^2 - d_A^2$

(2): $h^2 = L_2^2 - d_B^2$

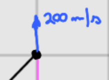
$$L_1^2 - d_A^2 = L_2^2 - d_B^2$$

$$0 - 2d_A \cdot \frac{dd_A}{dt} = 0 - 2d_B \cdot \frac{dd_B}{dt}$$

$$6 \cdot 0,5 = 2\sqrt{33} \cdot \frac{dd_B}{dt}$$

$$\frac{dd_B}{dt} = \frac{3\sqrt{33}}{2 \cdot 33} = \frac{\sqrt{33}}{22} \text{ m/s} \cong 0,26 \text{ m/s}$$

39.



$d = 1500\,m$ (pit.)

$\operatorname{sen}\theta = h/d = 3/5$

$\cos\theta = l/d$

a) $\frac{dd}{dt} = ?$

$d^2 = h^2 + l^2$

$2d\,\frac{dd}{dt} = 2h\,\frac{dh}{dt} + 0$

$\frac{dd}{dt} = \frac{900 \cdot 200}{1500} = 120\ m/s$

b) $\cos\theta = \frac{l}{d} = l \cdot d^{-1}$

$-\operatorname{sen}\theta \cdot \frac{d\theta}{dt} = -1200 \cdot \frac{1}{(1500)^2}$

$\frac{d\theta}{dt} = \frac{1200 \cdot 5}{3 \cdot (1500)^2} \cong 8{,}89 \cdot 10^{-4}\ rad/s$

40- $\frac{d\theta}{dt} = 4$ giros/minuto

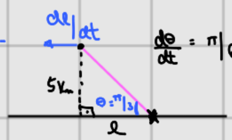
raio do círculo, também vale 3Km

A velocidade para dar uma volta é:

4 voltas/min = 1 volta/15 seg.

Portanto, ele percorre $C = 2\pi R = 6\pi$ Km em 15 segundos: $V = \frac{6\pi}{15}$ Km/s.

41-



$\frac{d\theta}{dt} = \pi/6$ rad/min ; $\frac{dl}{dt} = ?$

$tg\,\theta = 5/l$

$l = 5/\sqrt{3} = \frac{5\sqrt{3}}{3}$ Km

$\cos\theta = \frac{1}{2}$

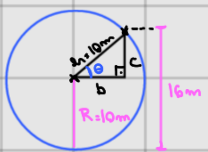
$\sec^2\theta \cdot \frac{d\theta}{dt} = -5 \cdot l^{-2} \cdot \frac{dl}{dt}$

$\frac{1}{\cos^2\theta} \cdot \pi/6 = -5 \cdot \left(\frac{3}{5\sqrt{3}}\right)^2 \cdot \frac{dl}{dt}$

$\frac{4\pi \cdot 25 \cdot 3}{-6 \cdot 5 \cdot 9} = \frac{dl}{dt}$

$\frac{dl}{dt} = \frac{-10\pi}{9}$

42- $\frac{dc}{dt} = ?$



$h^2 = b^2 + c^2$

$0 = 2b \cdot \frac{db}{dt} + 2c \cdot \frac{dc}{dt}$

$C = 16 - 10 = 6$

$\frac{dc}{dt} = -16 \cdot \frac{db}{dt} \cdot \frac{1}{12}$

$b = 8\,m$ (pit.)

$\frac{dc}{dt} = 16 \cdot 6 \cdot \frac{1}{12} = 8\ m/min$

$h = R$ (const.)

$\cos\theta = \frac{b}{h} = b \cdot \frac{1}{h}$

$-\operatorname{sen}\theta = \frac{1}{h} \cdot \frac{db}{dt}$

$\frac{db}{dt} = -h\operatorname{sen}\theta = -h \cdot \frac{c}{h} = -C$

43- $\frac{dr}{dt} = ?$

300

$t = 0$

$t = 1 min$

dist. perc. em 1 min: $d = vt = 300 \cdot \frac{1}{60} = 5 Km$

$r^2 = 1^2 + x^2 - 2 \cdot 1 \cdot x \cos 120 \longrightarrow r^2 = a^2 + x^2 - 2ax \cos 120$

$r^2 = 1 + 5^2 + 5 ; r^2 = 31 Km$

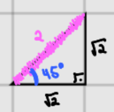
$r = \sqrt{31} Km$

$2r \frac{dr}{dt} = 0 + 2x \cdot \frac{dx}{dt} - 2 \cdot \frac{dx}{dt} \cdot \cos 120$

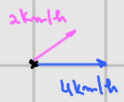
$2\sqrt{31} \frac{dr}{dt} = 10 \cdot 300 + 300$

$\frac{dr}{dt} = \frac{3300}{2\sqrt{31}} = 296,3 Km/h$

44- $\frac{dd}{dt}$ considerando $\theta = 45° = \pi/4$.



$t = 0$



$t = \frac{1}{4} h$

$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = \left(1 - \frac{\sqrt{2}}{4}\right)^2 + \left(0 - \frac{\sqrt{2}}{4}\right)^2 = \left(\frac{4-\sqrt{2}}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{18-8\sqrt{2}}{16} + \frac{2}{16} = \frac{20-8\sqrt{2}}{16} = \frac{5-2\sqrt{2}}{4}$

$2d \cdot \frac{dd}{dt} = 2(x_1 - x_2) \cdot \frac{dx}{dt} + 2(y_1 - y_2) \cdot \frac{dy}{dt}$

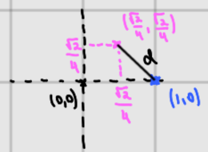
$\frac{dd}{dt} = \left[\left(\frac{4-\sqrt{2}}{2}\right) \cdot (4+\sqrt{2}) + \frac{\sqrt{2}}{2} \cdot \sqrt{2}\right] \cdot \frac{2}{5-2\sqrt{2}}$

$\frac{dx}{dt}$: 4Km/h + componente x da outra velocidade

$\frac{dd}{dt} = \frac{16}{5-2\sqrt{2}} = \frac{16(5+2\sqrt{2})}{25-8} = \frac{16}{17}(5+2\sqrt{2}) Km/h$

$\frac{dy}{dt}$: componente y da velocidade nordeste

45-



r e l não variam...

$2d \frac{dd}{dt} = 2r \cdot \cancel{\frac{dr}{dt}}^{0} + 2l \cdot \cancel{\frac{dl}{dt}}^{0}$

0...

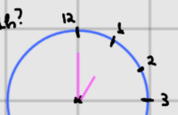
vel. ang.

$V = WR \therefore W = \frac{V}{R} = \frac{7}{100} = 0,07$

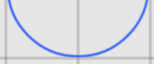
$d^2 = r^2 + l^2$

$2d \cdot \frac{dd}{dt} = 0 +$

46- $\frac{dd}{dt}$ à 1h?



em 1h, o ângulo varia 30°



$\theta = 360/12 = 30°$; $\frac{d\theta}{dt} = 30°/h$

Lei dos cossenos: $d^2 = P^2 + p^2 - 2Pp\cos\theta$

$d^2 = 64 + 16 - 64\frac{\sqrt{3}}{2} = 80 - 32\sqrt{3} = 16(5 - 2\sqrt{3})$

$d = 4\sqrt{5 - 2\sqrt{3}}$

$2d \cdot \frac{dd}{dt} = 0 + 0 + 64 \cdot \operatorname{sen}\theta \cdot \frac{d\theta}{dt}$

$\frac{dd}{dt} = \frac{32 \cdot 30}{8\sqrt{5 - 2\sqrt{3}}} = \frac{120\sqrt{5 - 2\sqrt{3}}}{5 - 2\sqrt{3}} = \frac{120\sqrt{5 - 2\sqrt{3}}\,(5 + 2\sqrt{3})}{13}$ mm/h

Pg 239 (Revisão #3) João Vaz

1- $y = (x^4 - 3x^2 + 5)^3$

$y' = 3(x^4 - 3x^2 + 5)^2 \cdot (4x^3 - 6x)$

2- $y = \cos(\operatorname{tg} x)$

$y' = -\operatorname{sen}(\operatorname{tg} x) \cdot \sec^2 x$

3- $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3}$

$y' = \frac{1}{2} x^{-1/2} - \frac{4}{3} x^{-7/3}$

4- $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$

y' = quociente + cadeia

5- $y = x^2 \operatorname{sen} \pi x$

$y' = 2x \operatorname{sen} \pi x + x^2 \cos \pi x \cdot \pi$

8- $x e^y = y \operatorname{sen} x \therefore y = \dfrac{x e^y}{\operatorname{sen} x}$

$y' = \dfrac{(e^y + x e^y y') \operatorname{sen} x + x e^y \cos x}{(\operatorname{sen} x)^2}$

9- $y = \ln(x \ln x)$

$y' = \dfrac{1}{x \ln x} \cdot (\ln x + 1) = \dfrac{\ln x + 1}{x \ln x}$

10- $y = e^{mx} \cdot \cos mx$

$y' = m e^{mx} \cdot \cos mx - e^{mx} \operatorname{sen} mx \cdot m = e^{mx}(m \cos mx - m \operatorname{sen} mx)$

11- $y = \sqrt{x} \cos \sqrt{x}$

$y' = \frac{1}{2} x^{-1/2} \cdot \cos \sqrt{x} - \sqrt{x} \operatorname{sen} \sqrt{x} \cdot \frac{1}{2} x^{-1/2} = \frac{1}{2} x^{-1/2} (\cos \sqrt{x} - \sqrt{x} \operatorname{sen} \sqrt{x})$

14- $y = \ln \sec x$

$y' = \dfrac{1}{\sec x} \cdot \sec x \operatorname{tg} x$

25- $\operatorname{sen}(xy) = x^2 - y$

$\cos(xy) \cdot (y + xy') = 2x - y'$

$y \cos(xy) + xy' \cos(xy) = 2x - y'$

$y'(x \cos(xy) + 1) = 2x - y \cos(xy) \therefore y' = \dfrac{2x - y \cos(xy)}{x \cos(xy) + 1}$

53- $x^6 + y^6 = 1$

$6x^5 + 6y^5 y' = 0 \therefore y' = \dfrac{-6x^5}{6y^5} = \dfrac{-x^5}{y^5}$

$y'' = \dfrac{-5x^4}{5y^4} = \dfrac{-x^4}{y^4}$

81- $h(x) = f(g(\operatorname{sen} 4x))$

$h'(x) = f'(g(\operatorname{sen} 4x)) \cdot g'(\operatorname{sen} 4x) \cdot \cos 4x \cdot 4$

83- $y = \ln^2(x + 4)$

$y' = 2\ln(x+4) \cdot \frac{1}{x+4}$

$2\ln(x+4) = 0$

$\ln(x+4) = 0 \therefore x+4 = 1 \therefore x = -3$

95- $f(x) = ax^2 + bx + c$; $f'(x) = 2ax + b$

$f(1) = 4$; $f'(-1) = 6$; $f'(5) = -2$

$f(1) = a + b + c = 4$

$f'(-1) = -2a + b = 6$

$f'(5) = 10a + b = -2$

$b = 6 + 2a$; $10a + 6 + 2a = -2$; $12a = -8 \therefore a = -2/3$

$b = 6 - 4/3 = \frac{14}{3}$

$\frac{-2}{3} + \frac{14}{3} + c = 4 \therefore c = 0$

$f(x) = \frac{-2x^2}{3} + \frac{14x}{3} = \frac{-2x^2 + 14x}{3}$

99- a) $\frac{dv}{dh} = \frac{\pi r^2}{3}$ u.v/u.c

b) $\frac{dV}{dr} = \frac{\pi h}{3} \cdot 2r \cdot \cancelto{1}{r'} = \frac{2\pi r h}{3}$

Pg. 255 (4.1) João Vaz

29- $f(x) = 5x^2 + 4x$

$\frac{df}{dx} = 0 \therefore 10x + 4 = 0 ;\ x = -4/10$

30- $f(x) = x^3 + x^2 - x$

$\frac{df}{dx} = 0 \therefore 3x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4+12}}{6} = \frac{-2 \pm 4}{6} \begin{cases} -1 \\ 1/3 \end{cases}$

$x \in \{-1, 1/3\}$

31- $f(x) = 2x^3 - 3x^2 - 36x$

$\frac{df}{dx} = 0 \therefore 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 0$

$x = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} \begin{cases} -3 \\ 2 \end{cases}$

$x \in \{-3, 2\}$

32- $f(x) = 2x^3 + x^2 + 2x$

$\frac{df}{dx} = 0 \therefore 6x^2 + 2x + 2 = 0 \therefore 2(3x^2 + x + 1) = 0$

$x = \frac{-1 \pm \sqrt{1-12}}{6} \begin{cases} \frac{-1+\sqrt{11}}{6} \\ \frac{-1-\sqrt{11}}{6} \end{cases}$

33- $g(t) = t^4 + t^3 + t^2 + 1$

$\frac{dg}{dt} = 0 \therefore 4t^3 + 3t^2 + 2t = 0 \therefore t(4t^2 + 3t + 2) = 0$

$t = \frac{-3 \pm \sqrt{9-32}}{8} \begin{cases} \frac{-3+\sqrt{-23}}{8} \\ \frac{-3-\sqrt{-23}}{8} \end{cases}$

34- $g(t) = |3t - 4|$

$g(t) = |t| = \sqrt{t^2} = (t^2)^{1/2}$

$g'(t) = \frac{1}{2}(t^2)^{-1/2} \cdot 2t = \frac{t}{\sqrt{t^2}} = \frac{t}{|t|}$

$\frac{dg}{dt} = 0 \therefore \frac{3t-4}{|3t-4|} = 0 \therefore t = 4/3$

Como números críticos são aqueles

em que a derivada é igual a zero ou não existe.

não há $\frac{d}{dx}|x|$ quando $x = 0$.

$N_c = \{0, \frac{4}{3}\}$

35- $g(y) = \frac{y-1}{y^2 - y - 1}$

$\frac{dg}{dy} = 0$ ou $\nexists \frac{dg}{dy}$

$\frac{dg}{dy} = 0 \therefore \frac{y^2 - y - 1 - (y-1)(2y-1)}{(y^2 - y - 1)^2} = 0 \therefore y^2 - y - 1 - (2y^2 - y - 2y + 1) = 0 \therefore y^2 + 2y - 2 = 0$

$y = \frac{-2 \pm \sqrt{4+8}}{2} \begin{cases} \frac{-2+\sqrt{12}}{2} \\ \frac{-2-\sqrt{12}}{2} \end{cases}$

$\nexists \frac{dg}{dy} : (y^2 - y - 1) = 0 \therefore y^2 - y - 1 = 0$

$y = \frac{1 \pm \sqrt{5}}{2} \begin{cases} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{cases}$

$N_c = \left\{ \frac{-2-\sqrt{12}}{2}, \frac{1-\sqrt{5}}{2}, \frac{-2+\sqrt{12}}{2}, \frac{1+\sqrt{5}}{2} \right\}$

36- $h(p) = \frac{p-1}{p^2+4}$

$\frac{dh}{dp} = 0$ ou $\nexists \frac{dh}{dp}$

$\frac{dh}{dp} = 0 : \frac{p^2+4-(p-1)2p}{(p^2+4)^2} \therefore p^2+4-2p^2+2p = 0 \therefore -p^2+2p+4 = 0$

$p = \frac{-2 \pm \sqrt{4+16}}{-2} \begin{cases} \frac{-2+\sqrt{20}}{-2} = \frac{2-\sqrt{20}}{2} \\ \frac{-2-\sqrt{20}}{-2} = \frac{2+\sqrt{20}}{2} \end{cases}$

$\nexists \frac{dh}{dp} : (p^2+4)^2 = 0 \therefore p^2+4 = 0 \therefore p = \pm 2$

$N_c = \left\{ -2, \frac{2-\sqrt{20}}{2}, 2, \frac{2+\sqrt{20}}{2} \right\}$

37- $h(t) = t^{3/4} - 2t^{1/4}$

$\frac{dh}{dt} = 0$ ou $\nexists \frac{dh}{dt}$

$\frac{dh}{dt} = 0 \therefore \frac{3}{4} t^{-1/4} - \frac{1}{2} t^{-3/4} = 0 \therefore \frac{3}{4\sqrt[4]{t}} = \frac{1}{2\sqrt[4]{t^3}} \therefore 4\sqrt[4]{t} = 6\sqrt[4]{t^3} \therefore 256t = 1296t^3 ; t(1296t^2 - 256) = 0 ; 81t^2 - 16 = 0 ; t = \frac{4}{9}$

$\nexists \frac{dh}{dt} : 4\sqrt[4]{t} = 0 \therefore t = 0$

$: 2\sqrt[4]{t^3} = 0 \therefore t = 0$

$N_c = \left\{ 0, \frac{4}{9} \right\}$

38- $g(x) = x^{1/3} - x^{-2/3}$

$\frac{dg}{dx} = \frac{1}{3} x^{-2/3} + \frac{2}{3} x^{-5/3} = \frac{1}{3 \cdot \sqrt[3]{x^2}} + \frac{2}{3 \cdot \sqrt[3]{x^5}}$

$\frac{dg}{dx} = 0$ ou $\nexists \frac{dg}{dx}$

$\frac{dg}{dx} = 0 \therefore \frac{1}{3 \cdot \sqrt[3]{x^2}} = \frac{-2}{3 \cdot \sqrt[3]{x^5}} \therefore -2 \cdot \sqrt[3]{x^2} = \sqrt[3]{x^5} \therefore -8x^2 = x^5 \therefore x^2(x^3+8) = 0 \therefore x = \sqrt[3]{-8} = -2$

$\nexists \frac{dg}{dt} : 3 \cdot \sqrt[3]{x^2} = 0 \therefore x = 0$

$: 3 \cdot \sqrt[3]{x^5} = 0 \therefore x = 0$

$N_c = \{ -2, 0 \}$

39- $F(x) = x^{4/5}(x-4)^2 = x^{4/5}(x^2 - 8x + 16) = x^{\frac{14}{5}} - 8x^{\frac{9}{5}} + 16x^{4/5} = \sqrt[5]{x^{14}} - 8 \cdot \sqrt[5]{x^9} + 16 \cdot \sqrt[5]{x^4}$

$\frac{dF}{dx} = \frac{14}{5} x^{9/5} - \frac{72}{5} x^{4/5} + \frac{64}{5} x^{-1/5} = \frac{14 \cdot \sqrt[5]{x^9}}{5} - \frac{72 \cdot \sqrt[5]{x^4}}{5} + \frac{64}{\sqrt[5]{x}}$

$\frac{dF}{dx} = 0$ ou $\nexists \frac{dF}{dx}$

$\frac{dF}{dx} = 0 \therefore x \in \left\{ \frac{8}{7}, 4 \right\}$

$\nexists \frac{dF}{dx} : x = 0$

$N_c = \left\{ 0, \frac{8}{7}, 4 \right\}$

40- $g(\theta) = 4\theta - tg\,\theta$

$\frac{dg}{d\theta} = 4 - sec^2\,\theta$

$\frac{dg}{d\theta} = 0$ ou $\nexists \frac{dg}{d\theta}$

$\frac{dg}{d\theta} = 0 : \sec^2\theta = 4 \therefore \frac{1}{\cos^2\theta} = 4 \therefore \cos\theta = \pm\frac{1}{2} \therefore \theta = \pi/3 + 2k\pi \quad k \in \mathbb{Z}.$

$\nexists \frac{dg}{d\theta} : \nexists \sec^2\theta : \cos^2\theta = 0 \therefore \cos\theta = 0 \therefore \theta = \pi/2 + 2k\pi \quad k \in \mathbb{Z}.$

$N_c = \{\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi\}$

41- $f(\theta) = 2\cos\theta + \text{sen}^2\theta$

$\frac{df}{d\theta} = -2\,\text{sen}\,\theta + 2\,\text{sen}\,\theta\cos\theta$

$\frac{df}{d\theta} = 0$ ou $\nexists \frac{df}{d\theta}$

$\frac{df}{d\theta} = 0 : \text{sen}\,\theta\cos\theta = \text{sen}\,\theta \therefore \cos\theta = 1 \therefore \theta = 0 + 2k\pi = 2k\pi \quad k \in \mathbb{Z}.$

$\nexists \frac{df}{d\theta}$: N/A

$N_c = \{2k\pi\}$

43- $f(x) = x^2 e^{-3x}$

$\frac{df}{dx} = 2xe^{-3x} + x^2 \cdot (-3)e^{-3x-1} \cdot e^x = 2xe^{-3x} - 3x^2e^{-3x}$

$\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore 2xe^{-3x} = 3x^2e^{-3x} \therefore 3x = 2 \therefore x = 2/3$

$\nexists \frac{df}{dx}$: N/A

$N_c = \{2/3\}$

44- $f(x) = x^{-2}\ln x$

$\frac{df}{dx} = -2x^{-3} \cdot \frac{1}{x} = -2x^{-4}$

$\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore \frac{-2}{x^4} = 0 \therefore -2 = 0$: nunca acontece

$\nexists \frac{df}{dx} \therefore x^4 = 0 \therefore x = 0$

$N_c = \{0\}$

47- $f(x) = 3x^2 - 12x + 5$, $[0,3]$

$\frac{df}{dx} = 6x - 12$

N_c: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 : 6x = 12 ; x = 2$

$N_c = \{2\}$

$f(0) = 5$ MÁXIMO

$f(2) = -7$

$f(3) = -22$ MÍNIMO

48- $f(x) = x^3 - 3x + 1$, $[0,3]$

$\frac{df}{dx} = 3x^2 - 3$

Nc: $\frac{df}{dx} = 0 \therefore x = 1$ ~~~ elimina-se a raiz negativa por não pertencer ao intervalo

$f(0) = 1$

$f(1) = -3$ MÍNIMO

$f(3) = 19$ MÁXIMO

49- $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2,3]$

$\frac{df}{dx} = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

Nc: $\frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$

$f(-2) = -3$

$f(-1) = 8$

$f(2) = -19$ MÁXIMO

$f(3) = -8$ MÍNIMO

50- $f(x) = x^3 - 6x^2 + 5$, $[-3,5]$

$\frac{df}{dx} = 3x^2 - 12x$

Nc: $\frac{df}{dx} = 0 \therefore x \in \{0, 4\}$

$f(-3) = -76$ MÍNIMO

$f(0) = 5$ MÁXIMO

$f(4) = -27$

$f(5) = -20$

51- $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, $[-2,3]$

$\frac{df}{dx} = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$

Nc: $\frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$

$f(-2) = 33$ MÁXIMO

$f(-1) = -4$

$f(2) = -31$ MÍNIMO

$f(3) = 28$

52- $f(x) = (x^2 - 1)^3$, $[-1,2]$

$\frac{df}{dx} = 3(x^2 - 1)^2 \cdot 2x$

Nc: $\frac{df}{dx} = 0 \therefore x \in \{-1, 0, 1\}$

$f(-1) = 0$

$f(0) = -1$ MÍNIMO

$f(1) = 0$

$f(2) = 27$ Máximo

53- $f(x) = x + \frac{1}{x}$, $[0{,}2, 4]$

$\frac{df}{dx} = 1 - x^{-2} = 1 - \frac{1}{x^2}$

N_c: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore \frac{1}{x^2} = 1 \therefore x = \pm 1 \rightsquigarrow -1 \notin [0{,}2, 4]$

$\nexists \frac{df}{dx} \therefore x = 0$

$N_c: x \in \{0, 1\}$

$f(0) = \nexists$

$f(0{,}2) = 5{,}2$ Máximo

$f(1) = 2$ Mínimo

$f(4) = \frac{17}{4}$

54- $f(x) = \frac{x}{x^2 - x + 1}$, $[0, 3]$

$\frac{df}{dx} = \frac{x^2 - x + 1 - x(2x - 1)}{(x^2 - x + 1)^2}$

N_c: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore x^2 - x + 1 - 2x^2 + x = 0 \therefore x^2 + 1 = 0 \therefore \nexists$

$\nexists \frac{df}{dx} \therefore x^2 - x + 1 = 0 \therefore \nexists$

$N_c = \emptyset$

$f(0) = 0$ Mínimo

$f(3) = 3/7$ Máximo

55- $f(t) = t\sqrt{4 - t^2}$, $[-1, 2]$

$\frac{df}{dt} = \sqrt{4 - t^2} + \frac{t}{2}(4 - t^2)^{-1/2} \cdot (-2t) = \sqrt{4 - t^2} - \frac{t}{\sqrt{4 - t^2}} = \frac{4 - t^2 - t}{\sqrt{4 - t^2}}$

N_c: $\frac{df}{dt} = 0$ ou $\nexists \frac{df}{dt}$

$\frac{df}{dt} = 0 \therefore -t^2 - t + 4 = 0 \therefore t = \frac{-1 \pm \sqrt{17}}{2}$ $\rightsquigarrow$ a raiz negativa é menor que -1; logo, é eliminada.

$\nexists \frac{df}{dt} \therefore 4 - t^2 = 0 \therefore t = \pm 2 \longrightarrow -2 < -1$, eliminada

$N_c = \{\frac{-1 + \sqrt{17}}{2}, 2\}$

$f(-1) = -\sqrt{3}$ Mínimo

$f(2) = 0$

$f(\frac{-1 + \sqrt{17}}{2}) \cong 1{,}95$ Máximo

63- $a, b > 0 \; ; x \in [0,1]$

$f(x) = x^a (1-x)^b$

$\frac{df}{dx} = ax^{a-1}(1-x)^b + x^a \cdot b(1-x)^{b-1} \cdot (-1) = ax^{a-1}(1-x)^b - x^a \cdot b(1-x)^{b-1}$

$Nc: \frac{df}{dx} = 0 \therefore ax^{a-1}(1-x)^b = x^a \cdot b(1-x)^{b-1}$ ⟶ aleatoriamente, $a = b = 1$

$x^0(1-x) = x \cdot (1-x)^0 \therefore (1-x) = x \therefore 2x = 1 \therefore x = 1/2$

$f(0) = 0$

$f(1/2) = \frac{1}{4}$ MÁXIMO

$f(1) = 0$

67- $f(x) = x\sqrt{x-x^2} = \sqrt{x^2(x-x^2)} = \sqrt{x^3 - x^4}$

$\frac{df}{dx} = \frac{1}{2}(x^3-x^4)^{-1/2}(3x^2-4x^3) = \frac{3x^2-4x^3}{\sqrt{x^3-x^4}}$

$Nc: \frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\left.\begin{array}{l} \frac{df}{dx} = 0 \therefore x^2(3-4x) = 0 \therefore x = 3/4 \\ \nexists \frac{df}{dx} \therefore x^3 - x^4 = 0 \therefore x^3(1-x) = 0 \therefore x = 1 \end{array}\right\} Nc: x \in \{3/4, 1\}$

Note que a raiz $\sqrt{x-x^2} = \sqrt{x(1-x)} = \sqrt{x} \cdot \sqrt{1-x}$.

Para que isso tenha solução em $\mathbb{R}$, $x \geq 0$ e $1-x \geq 0$.

Para satisfazer $x \geq 0$ e $x \leq 1$, $x \in [0,1]$.

$f(0) = 0$ MÍNIMO

$f(3/4) = \frac{3\sqrt{3}}{16}$ MÁXIMO

$f(1) = 0$

69- $v(t) = 999{,}87 - 0{,}06426t + 0{,}0085043t^2 - 0{,}0000679t^3$, $[0,30]$

$\uparrow \rho = \frac{m}{v\downarrow}$: ponto mínimo de volume

$Nc: \frac{dv}{dt} = 0$ ou $\nexists \frac{dv}{dt}$

$\frac{dv}{dt} = -0{,}06426 + 2 \cdot 0{,}0085043t - 3 \cdot 0{,}0000679t^2$

$\frac{dv}{dt} = 0 \therefore t \in \{3.96, 7.53\}$

$Nc = \{3.96\}$

$V(0) = 999{,}87 \text{ cm}^3$

$V(3{,}96) = 999{,}745 \text{ cm}^3$

$V(30) = 1003{,}76 \text{ cm}^3$

A água tem ρ máxima no ponto de volume mínimo, que é atingido em $t \cong 3{,}96$ °C

70- $F(\theta) = \frac{\mu m g}{\mu \operatorname{sen}\theta + \cos\theta}$, $[0, \pi/2]$

$Nc: \frac{dF}{d\theta} = 0$ ou $\nexists \frac{dF}{d\theta}$

$$\frac{dF}{d\theta} = -\mu mg(\mu \operatorname{sen}\theta + \cos\theta) = \frac{-\mu mg}{(\mu \operatorname{sen}\theta + \cos\theta)^2}$$

$$\frac{dF}{d\theta} = 0 \therefore -\mu mg = 0$$

$$\nexists \frac{dF}{d\theta} : (\mu \operatorname{sen}\theta + \cos\theta)^2 = 0 \therefore \mu \operatorname{sen}\theta = -\cos\theta \therefore \mu = -\operatorname{cotg}\theta$$

$$F(0) = \mu mg$$

$$F(\pi/2) = mg$$

Pg 262 (4.2) João Vaz

17- $2x + \cos x = 0$

Suponha que exista a e b, $a \neq b$, tal que $2a + \cos a = 0$ e $2b + \cos b = 0$.

Existiria também $c \in [a,b]$ de forma que $\frac{d}{dc}(2c + \cos c) = \frac{d}{dc} 0 = 0$

$\frac{d}{dc}(2c + \cos c) = 2 - \operatorname{sen} c = 0$. Portanto, $\operatorname{sen} c = 2$, o que é impossível

Assim, a premissa é falsa e a equação tem apenas 1 raiz.

18- $x^3 + e^x = 0$

Suponha que exista a e b, $a \neq b$, que satisfazem essa equação.

Se assim fosse, haveria $c \in [a,b]$ de forma que $\frac{d}{dc}$ da equação fosse zero.

$\frac{d}{dc} = 3c^2 + e^c = 0$. Para tanto, $c^2 = \frac{-e^c}{3}$; o que é impossível, pois $d^2 \geq 0 \; \forall d \in \mathbb{R}$.

Assim, a equação tem apenas uma raiz.

19- $f(x) = x^3 - 15x + c$, $[-2,2]$

$f(-2) = -8 + 30 + c = 22 + c$

$f(2) = 8 - 30 + c = -22 + c$

Pelo TVI, há ao menos um valor de x tal que $f(x) = 0$

Suponha $x_1 \neq x_2$, de forma que $f(x_1) = f(x_2) = 0$.

Se assim fosse, existiria $d \in [x_1, x_2]$ tal que $\frac{df}{dd} = 0$ (Teo. Rolle)

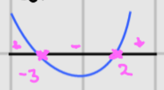
$\frac{df}{dd} = 3d^2 - 15 = 0 \therefore d = \pm\sqrt{5}$

Pg 279 (4.3) João Vaz

9- $f(x)=2x^3+3x^2-36x$

a) $\frac{df}{dx}=6x^2+6x-36=6(x^2+x-6)$

$\frac{df}{dx}=0: x=\frac{-1\pm\sqrt{1^2+24}}{2} < \begin{matrix} 2 \\ -3 \end{matrix}$



b) Nc: $\frac{df}{dx}=0$ ou $\nexists \frac{df}{dx}$

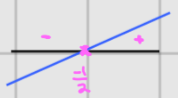
$\frac{df}{dx}=0 \therefore x \in \{-3,2\}$

$f(-3)=81$ Máximo

$f(2)=-44$ Mínimo

c) $\frac{d^2f}{dx}=12x+6$

$12x+6=0 \therefore x=\frac{-1}{2}$



$x \in (-\infty, \frac{-1}{2})$: f(x) tem concavidade para baixo

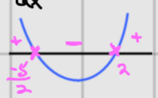
$x \in (\frac{-1}{2}, +\infty)$: f(x) tem concavidade para cima

$x=\frac{-1}{2}$ é um ponto de inflexão

10- $f(x)=4x^3+3x^2-6x+1$

a) $\frac{df}{dx}=12x^2+6x-6=6(2x^2+x-1)$

$\frac{df}{dx}=0: x=\frac{-1\pm\sqrt{1+8}}{4} < \begin{matrix} 2 \\ \frac{-5}{2} \end{matrix}$



b) Nc: $x \in \{-5/2, 2\}$

$f(-5/2)=-96$ Mínimo

$f(2)=54$ Máximo

c) $\frac{d^2f}{dx}=24x+6$

$24x+6=0 \therefore x=-1/4$

$x \in (-\infty, \frac{-1}{4})$: f(x) tem concavidade para baixo

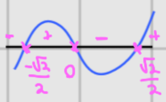
$x \in (-1/4, +\infty)$: $f(x)$ " " " cima

$x = -1/4$: ponto de inflexão

11- $f(x) = x^4 - 2x^2 + 3$

a) $\frac{df}{dx} = 4x^3 - 2x = x(4x^2 - 2)$

$\frac{df}{dx} = 0$: $x = 0$ ou $x = \pm \frac{\sqrt{2}}{2}$



b) Nc: $\frac{df}{dx} = 0$

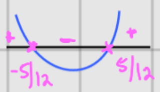
$f(-\sqrt{2}/2) \simeq 7{,}91$ MÁXIMO

$f(0) = 3$ MÍNIMO

$f(\sqrt{2}/2) \simeq 5{,}085$

c) $\frac{d^2f}{dx} = 12x^2 - 2 = 2(6x^2 - 1)$

$\frac{d^2f}{dx} = 0$: $x = \frac{0 \pm \sqrt{0+25}}{12}$ $\begin{cases} 5/12 \\ -5/12 \end{cases}$



$x \in (-\infty, -5/12)$: f é $\cup$

$x \in (-5/12, 5/12)$: f é $\cap$

$x \in (5/12, +\infty)$: f é $\cup$

$x \in \{\pm 5/12\}$ são pontos de inflexão

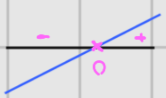
12- $f(x) = \frac{x^2}{x^2+3}$

a) $\frac{df}{dx} = \frac{2x(x^2+3) - x^2 \cdot 2x}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$

$\frac{df}{dx} = 0$

$\frac{df}{dx} = 0$: $x = 0$

Como o denominador é estritamente positivo,



b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Sempre haverá $\frac{df}{dx}$, pois $x^2 + 3 > 0 \; \forall x \in \mathbb{R}$.

Nc = {0}

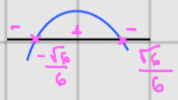
$f(0) = 0$ Mínimo

c) $\frac{d^2f}{dx} = \frac{6(x^2+3)^2 - 12x(x^2+3)\cdot 2x}{(x^2+3)^4}$

$\frac{d^2f}{dx} = 0 \therefore 6(x^2+3)^2 - 24x^2(x^2+3) = 0$

$\therefore (x^2+3)\left[6(x^2+3) - 24x^2\right] = 0$

$\therefore -18x^2 + 3 = 0 \therefore x = \pm \frac{\sqrt{6}}{6}$



$x \in (-\infty, -\sqrt{6}/6) \cup (\sqrt{6}/6, +\infty)$ f é côncava para baixo

$x \in (-\sqrt{6}/6, \sqrt{6}/6)$ f é côncava para cima

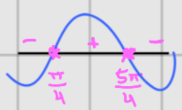
$x \in \{\pm \frac{\sqrt{6}}{6}\}$ são pontos de inflexão

13- $f(x) = \text{sen}\, x + \cos x$, $[0, 2\pi]$

a) $\frac{df}{dx} = \cos x - \text{sen}\, x$

$\frac{df}{dx} = 0 \therefore \cos x = \text{sen}\, x$

$x \in \{\pi/4, 5\pi/4\}$



b) $N_c: \frac{df}{dx} = 0 : x \in \{\pi/4, 5\pi/4\}$

$f(\pi/4) = 1$ Máximo

$f(5\pi/4) = -2$ Mínimo

$f(0) = 1$ Máximo

$f(2\pi) = 1$ Máximo

c) $\frac{d^2f}{dx} = -\text{sen}\, x - \cos x$

$\frac{d^2f}{dx} = 0 \therefore \cos x = -\text{sen}\, x \therefore \text{tg}\, x = -1 \therefore x = \frac{-\pi}{4}$

$x \in (-\infty, -\pi/4)$, f é $\cup$

$x \in (-\pi/4, +\infty)$, f é $\cap$

$x = -\pi/4$ é ponto de inflexão

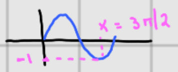
14- $f(x) = \cos^2 x - 2\,\text{sen}\, x$, $[0, 2\pi]$

a) $\frac{df}{dx} = -2\cos x \,\text{sen}\, x - 2\cos x$

$\frac{df}{dx} > 0 \therefore -2\cos x(\text{sen}\,x + 1) > 0 \therefore 2\cos x(\text{sen}\,x+1) < 0 \therefore 2\cos x$ e $\text{sen}\,x + 1$ precisam ter sinais trocados

$\text{sen}\,x + 1 > 0$

$\text{sen}\,x > -1 \longrightarrow 0 \leq x \leq 2\pi;\ x \neq \frac{3\pi}{2}$



Quando $\frac{\pi}{2} \leq x < \frac{3\pi}{2}$, f é crescente

Quando $x < \frac{\pi}{2}$ ou $x > \frac{3\pi}{2}$, f é decrescente.

b) $Nc: \frac{df}{dx} = 0$

$Nc = \{\pi/2, 3\pi/2\}$

$f(\pi/2) = -2$ Mínimo

$f(3\pi/2) = 2$ Máximo

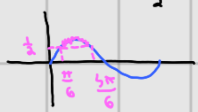
x		$\pi/2$		$3\pi/2$		2π
f'	−	0	+	0	−	
	↘	MIN	↗	MAX	↘	

c) $\frac{d^2f}{dx^2} = -2(\cos^2 x - \text{sen}^2 x - \text{sen}\,x) = -2(1 - \text{sen}^2 x - \text{sen}^2 x - \text{sen}\,x) = 2(2\text{sen}^2 x + \text{sen}\,x - 1) > 0$

$2(2\text{sen}\,x - 1)(\text{sen}\,x + 1) > 0 \therefore (2\text{sen}\,x - 1)$ e $(\text{sen}\,x + 1)$ precisam ter o mesmo sinal

$\text{sen}\,x + 1 > 0 \therefore \text{sen}\,x > -1 \therefore 0 \leq x \leq 2\pi$ e $x \neq \frac{3\pi}{2}$

$2\text{sen}\,x - 1 > 0 \therefore \text{sen}\,x > \frac{1}{2} \therefore$



Quando $\frac{\pi}{6} < x < \frac{5\pi}{6}$, côncava pra cima

Quando $0 \leq x < \pi/6$ ou $\frac{\pi}{6} < x < \frac{3\pi}{2}$ ou $\frac{3\pi}{2} < x \leq 2\pi$, côncava pra baixo

15- $f(x) = e^{2x} + e^{-x}$

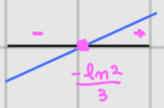
a) $\frac{df}{dx} = 2e^{2x} - e^{-x}$

$\frac{df}{dx} \geq 0 \therefore 2e^{2x} - \frac{1}{e^x} \geq 0$

$\frac{2e^{3x} - 1}{e^x} \geq 0$. Como e^x é sempre positiva,

$2e^{3x} - 1 \geq 0 \therefore x \geq \frac{-\ln 2}{3}$. Seja $a = e^{3x}$, pode-se

escrever a equação como $2a - 1$.



b) $x = -\frac{\ln 2}{3}$ é Mínimo.

c) $\frac{d^2f}{dx^2} = 4e^{2x} + e^{-x}$

$\frac{d^2f}{dx^2} \geq 0 \therefore \frac{4e^{3x} + 1}{e^x} \geq 0 \therefore 4e^{3x} \geq -1 \therefore e^{3x} \geq \frac{-1}{4} \therefore x \geq \frac{\ln(-1/4)}{3}$

$\nexists\, x$ tal que $\frac{d^2f}{dx^2} = 0$.

$x = \frac{-\ln 2}{3}$ é um ponto de inflexão.

f é decrescente em $(-\infty, \frac{-\ln 2}{3})$ e crescente em $(\frac{-\ln 2}{3}, +\infty)$

16- $f(x) = x^2 \ln x$

$x > e^{-1/2}$

a) $\frac{df}{dx} = 2x\ln x + x^2 \cdot \frac{1}{x} = 2x\ln x + x = x(2\ln x + 1)$

$\frac{df}{dx} > 0 \therefore x$ e $(2\ln x + 1)$ tem o mesmo sinal $\therefore x > 0$ e $(2\ln x + 1) > 0 \longrightarrow x > 0$

(2) (1) (2)

$\therefore x < 0$ e $(2\ln x + 1) < 0 \longrightarrow x < e^{-1/2}$

0 $\quad e^{-1/2}$

(1): $\frac{df}{dx} > 0$, f é crescente

(2): $\frac{df}{dx} < 0$, f é decrescente

b) $N_c = \{0, e^{-1/2}\}$

$f(0) = 0$ MÁXIMO

$f(e^{-1/2}) = \frac{-1}{2} e^{-1} = \frac{-1}{2} \cdot \frac{1}{e} = \frac{-1}{2e}$ MÍNIMO

c) $\frac{d^2f}{dx} = 2\ln x + 2 + 1 = 2\ln x + 3$

$\frac{d^2f}{dx} > 0 \therefore x > e^{-3/2}$

$-$ $\quad +$

$e^{-3/2}$

$x \in (-\infty, e^{-3/2})$, f é côncava para baixo

$x \in (e^{-3/2}, +\infty)$, f é côncava para cima

$x = e^{-3/2}$ é ponto de inflexão

17- $f(x) = x^2 - x - \ln x$

a) $\frac{df}{dx} = 2x - 1 - \frac{1}{x} = \frac{2x^2 - x - 1}{x}$

$\frac{df}{dx} > 0 \therefore \frac{2x^2 - x - 1}{x} > 0$

Para que isso ocorra:

- $2x^2 - x - 1 > 0$ e $x > 0 \therefore \left(x < \frac{-1}{2} \text{ ou } x > 1\right)$ e $(x > 0) \therefore x > 1$
- $2x^2 - x - 1 < 0$ e $x < 0 \therefore \left(\frac{-1}{2} < x < 1\right)$ e $(x < 0) \therefore \frac{-1}{2} < x < 0$ ⟶ Como ln só existe para $x > 0$, descarta-se esse caso

Portanto, em $x > 1$, $f(x)$ é crescente; em $x < 1$, f é decrescente

b) $N_c = \{1\}$

$f(1) = 0$ MÍNIMO

1

c) $\frac{d^2f}{dx} = 2 + x^{-2} = 2 + \frac{1}{x^2} = \frac{2x^2 + 1}{x^2}$

$\frac{d^2f}{dx} > 0$: Como $x^2 > 0 \ \forall x \in \mathbb{R}$, resta analisar o numerador:

$2x^2 + 1 > 0 \therefore x^2 > \frac{-1}{2}$. $x^2 > \frac{-1}{2} \ \forall x \in \mathbb{R}$.

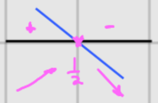
Assim, $\frac{d^2f}{dx}$ é sempre positiva. Portanto, f tem concavidade

para cima $\forall x \in \mathbb{R}$.

18- $f(x) = \sqrt{x}\, e^{-x} = \frac{\sqrt{x}}{e^x}$

a) $\frac{df}{dx} = \frac{1}{2}x^{-1/2}e^{-x} - \sqrt{x}e^{-x} = e^{-x}\left[\frac{1}{2\sqrt{x}} - \sqrt{x}\right]$

$\frac{df}{dx} > 0 \therefore \frac{1-2x}{2\sqrt{x}} > 0 \therefore \underbrace{1-2x}_{a=-2} > 0 \therefore x > \frac{1}{2}$



b) $N_c: \frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore x = \frac{1}{2}$

$\nexists \frac{df}{dx} \therefore x = 0$

$f(1/2) = (2e)^{-1/2}$ MÁXIMO

$f(0) = 0$ MÍNIMO

c) $\frac{d^2f}{dx} = -e^{-x}\left[\frac{1}{2\sqrt{x}} - \sqrt{x}\right] + e^{-x}\left[-\sqrt{x}\cdot x^{-1/2} - \frac{1}{2}x^{-1/2}\right] = e^{-x}\left(\frac{-1}{2\sqrt{x}} + \sqrt{x} - 1 - \frac{1}{2\sqrt{x}}\right) = e^{-x}(\sqrt{x}-1)$

$\frac{d^2f}{dx} > 0 \therefore \sqrt{x}-1 > 0 \therefore \sqrt{x} > 1 \therefore x > 1$

$x \in (1, +\infty)$: f é côncava para cima

$x \in (0, 1)$: f é côncava para baixo

$x = 1$ é ponto de inflexão

19- $f(x) = x^5 - 5x + 3$

$\frac{df}{dx} = 5x^4 - 5$

$\frac{df}{dx} > 0 \therefore x^4 > 1 \therefore x > \pm 1$

$N_c: \frac{df}{dx} = 0: \{-1, 1\}$

$f(-1) = -1 + 5 + 3 = 7$ MÁXIMO

$f(1) = 1 - 5 + 3 = -1$ MÍNIMO

20- $f(x) = \frac{x^2}{x-1}$

$N_c: \frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$

$\frac{df}{dx} = 0 \therefore x(x-2) = 0 \therefore x = 0 \cup x = 2$

$\nexists \frac{df}{dx} \therefore x = 1$

$f(0) = 0$ MÍNIMO

$f(1) = \nexists$

$f(2) = 4$ MÁXIMO

21- $f(x) = \sqrt{x} - \sqrt[4]{x}$

23- $f(x) = \sqrt{x} - \sqrt[4]{x}$

$$\frac{df}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{4}x^{-3/4} = \frac{1}{2\sqrt{x}} - \frac{1}{4\cdot\sqrt[4]{x^3}}$$

$Nc: \frac{df}{dx} = 0$

$2\sqrt{x} = 4\sqrt[4]{x^3} \therefore x^2 = 16x^3 \therefore x^2(16x-1) = 0$

$Nc = \{0, 1/16\}$

$f(0) = 0$ MÁXIMO

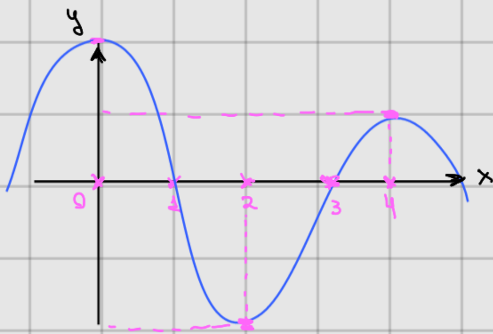
$f(1/16) = \frac{1}{4} - \frac{1}{2} = \frac{1-2}{4} = \frac{-1}{4}$ MÍNIMO

24-

x	$-\infty$		-2		0		∞
y'	+	+		−	∄	−	
y''	−	−	−	−	∄	+	+
y			Máx. local		Assint. vert.		

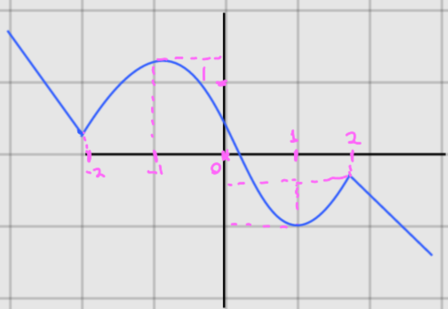
25-

x		0		1		2		3		4	
y'	+	0	−	−	−	0	+	+	+	0	−
y''	−	−	−	0 ou ∄	+	+	+	−	−	−	−
y		MAX LOCAL		Pto. inf.		MIN LOCAL		pto. inf.		MAX LOCAL	



26-

x		−2		−1		0		1		2	
y'	−1		+	0	−	−	−	0	+		−1
y''	0		−	−	−	0 ou ∄	+	+	+	+	0
y				Máx Local		pto. inf.		Mín. Local			

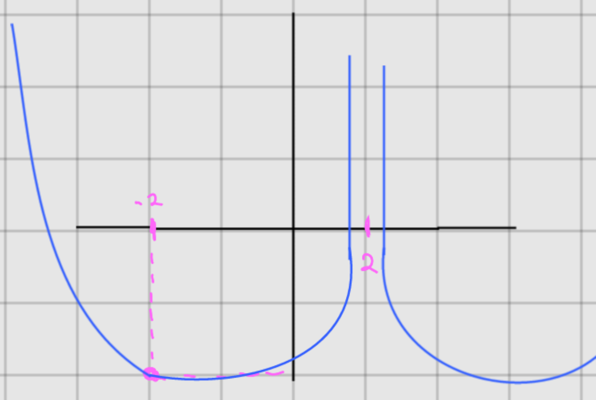


: embora y'' seja igual a zero, não é um ponto de inflexão, pois o "zero" na concavidade é explicado pela inclinação (y') constante.

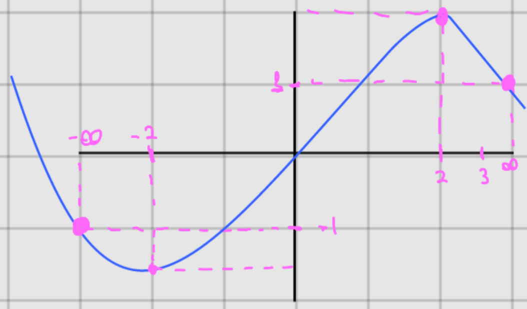
: sinais arbitrariamente escolhidos

27-

x		−2		−1		0		1		2	
y'	−	0	+	+	+	+	+	+	+	∞	−
y''	+	+	+	+	+	+	+	+	+	∄	+
y		MIN LOCAL								Ass. vert.	



29- x	$-\infty$	-2		-1		0		1		2		3	$+\infty$
y'	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	0	$-$	$-$	
y''	$+$	$+$	$+$	$+$	$+$	0	$-$	$-$	$-$	$-$	$-$	0 ou $\nexists$	$+$
y	-1	Mín local				pte. inf.				Máx local		pte inf.	1



33- $f(x) = 2x^3 - 3x^2 - 12x$

a) $\frac{df}{dx} = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$\frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Nc: $\{-1, 2\}$

$f(-1) = -2 - 3 + 12 = 7$ MÁX

$f(2) = 16 - 12 - 24 = -28$ MÍN

c) $\frac{d^2f}{dx} = 12x - 6$

$x \in (-\infty, \frac{1}{2})$ f é $\cap$

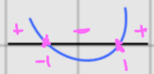
$x \in (1/2, +\infty)$ f é $\cup$

$x = 1/2$ é pto. inf.

34- $f(x) = -x^3 + 3x + 2$

a) $\frac{df}{dx} = -3x^2 + 3$

$\frac{df}{dx} = 0 \therefore x^2 = 1 \therefore x = \pm 1$



b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

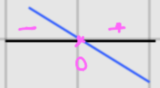
Nc: $\{-1, 1\}$

$f(-1) = 1 - 3 + 2 = 0$ MÍN

$f(1) = -1 + 3 + 2 = 4$ MÁX

c) $\frac{d^2f}{dx} = -6x$

$\frac{df}{dx} = 0 \therefore x = 0$



$x \in (-\infty, 0)$ f é $\cap$

$x \in (0, +\infty)$ f é $\cup$

$x = 0$ é pto. inf.

35- $f(x) = -x^4 + 2x^2 + 2$

a) $\frac{df}{dx} = -4x^3 + 4x = 4x(-x+1)$

$\frac{df}{dx} > 0 \therefore 4x(-x+1) > 0 \longrightarrow$ sinais iguais

$\begin{cases} 4x > 0 \therefore x > 0 \\ (-x+1) > 0 \therefore x < 1 \end{cases}$

$\begin{cases} 4x < 0 \therefore x < 0 \\ (-x+1) < 0 \therefore x > 1 \end{cases}$ ⟶ impossível

$x \in (0,1)$ f é crescente

$x \in (-\infty, 0) \cup (1, +\infty)$ f é decrescente

$x \in \{0, 1\}$ f é constante

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Nc: $\{0, 1\}$

$f(0) = 2$ Mín

$f(1) = -1 + 2 + 2 = 3$ Máx

c) $\frac{d^2f}{dx} = -12x^2 + 4$

$\frac{df}{dx} = 0 \therefore x = \pm \frac{\sqrt{3}}{3}$

$x \in (-\infty, -\sqrt{3}/3) \cup (\sqrt{3}/3, +\infty)$ f é $\cap$

$x \in (-\sqrt{3}/3, \sqrt{3}/3)$ f é $\cup$

$x \in \{-\sqrt{3}/3, \sqrt{3}/3\}$ são ptos. inf.

$4x^2 > 0 \ \forall x \in \mathbb{R}$

36- $g(x) = x^4 + 8x^3 + 200$

a) $\frac{df}{dx} = 4x^3 + 24x^2 = 4x^2(x+6)$

$\frac{df}{dx} > 0 \therefore x + 6 > 0 \therefore x > -6$

$x \in (-\infty, -6)$ f é decrescente

$x \in (-6, 0) \cup (0, +\infty)$ f é crescente

$x \in \{-6, 0\}$ f é constante

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore x \in \{-6, 0\}$

$f(-6) = 1296 - 1278 + 200 = 218$ Máx

$f(0) = 200$ Mín

c) $\frac{d^2f}{dx} = 12x^2 + 48x = 12(x^2 + 4x)$

$x \in (-\infty, -4) \cup (0, +\infty)$ f é $\cup$

$x \in (-4, 0)$ f é $\cap$

$x \in \{-4, 0\}$ são pto. inf.

37- $h(x) = (x+1)^5 - 5x - 2$

a) $\frac{dh}{dx} = 5(x+1)^4 - 5 = 5\left[(x+1)^4 - 1\right]$

$\frac{dh}{dx}$ é sempre maior ou igual a zero.

$\frac{dh}{dx} = 0 \therefore x = -1$

$x \in (-\infty, -1) \cup (-1, +\infty)$ h é crescente

$x = -1$ h é constante

b) Nc: $\frac{dh}{dx} = 0$ ou $\nexists \frac{dh}{dx}$

Nc: $\{-1\}$

$f(-1) = 5 - 2 = 3$

c) $\frac{d^2h}{dx} = 20(x+1)^3 = 20(x+1) \cdot (x+1)^2$

$\frac{d^2h}{dx} > 0 \therefore 20(x+1) > 0 \therefore x > -1$

$\frac{d^2h}{dx} = 0 \therefore x = -1$

$x \in (-\infty, -1)$ h é $\cap$

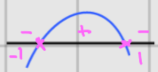
$x \in (-1, +\infty)$ h é $\cup$

$x = -1$ é pto. inf.

38- $h(x) = 5x^3 - 3x^5 = x^3(-x^2+5x)$

a) $\frac{dh}{dx} = 15x^2 - 15x^4 = 15x^2(-x^2+1)$

$\frac{dh}{dx} > 0 \therefore x^2 > 1 \therefore x > \pm 1$



$x \in (-\infty,-1) \cup (1,+\infty)$ h é decresc.

$x \in (-1,0) \cup (0,1)$ h é cresc.

$x \in \{-1,0,1\}$ h é const.

b) Nc: $\frac{dh}{dx} = 0$ ou $\nexists \frac{dh}{dx}$

$\frac{dh}{dx} = 0 \therefore x \in \{-1,0,1\}$

$h(-1) = 4$

$h(0) = 0$ MIN

$h(1) = 6$ MAX

c) $\frac{d^2h}{dx} = 30x - 60x^3 = 30x(-2x^2+1)$

$x^2 = \frac{-1}{-2}$

$x = \pm\frac{\sqrt{2}}{2}$

$x \in (-\infty, -\sqrt{2}/2) \cup (0, \sqrt{2}/2)$ h é $\cup$

$x \in (-\frac{\sqrt{2}}{2}, 0) \cup (\frac{\sqrt{2}}{2}, +\infty)$ h é $\cap$

$x \in \{-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\}$ são ptos inf.

39- $F(x) = x\sqrt{6-x}$

a) $\frac{dF}{dx} = \sqrt{6-x} + \frac{x}{2}(6-x)^{-1/2} \cdot (-1) = \sqrt{6-x} - \frac{x}{2\sqrt{6-x}} = \frac{12-3x}{2\sqrt{6-x}}$

$\frac{dF}{dx} > 0 \therefore 12-3x > 0 \therefore 3x < 12 \therefore x < 4$

$x \in (-\infty, 4)$ f é cresc.

$x = 4$ f é const.

$x \in (4,6)$ f é decresc.

b) Nc: $\frac{dF}{dx} = 0$ ou $\nexists \frac{dF}{dx}$

$\frac{dF}{dx} = 0 \therefore x = 4$

dx

$\nexists \frac{dF}{dx} : x = 6$

$N_c : \{4, 6\}$

$f(4) = 4\sqrt{2}$ MAX

$f(6) = 0$ MIN

40. $G(x) = 5x^{2/3} - 2x^{5/3}$

a) $\frac{dG}{dx} = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3} = \frac{10}{3 \cdot \sqrt[3]{x}} - \frac{10 \cdot \sqrt[3]{x^2}}{3} = \frac{10 - 10x}{3 \cdot \sqrt[3]{x}} = \frac{10}{3} \cdot \frac{-x+1}{\sqrt[3]{x}}$

$\frac{dG}{dx} > 0$

	0		1	
–		–		+
–		+		+
+		–		+

$x \in (-\infty, 0) \cup (1, +\infty)$ G é cresc.

$x \in (0, 1)$ G é decresc.

b) $N_c : \frac{dG}{dx} = 0$ ou $\nexists \frac{dG}{dx}$

$\frac{dG}{dx} = 0 \therefore x = 1$

$\nexists \frac{dG}{dx} : x = 0$

$N_c = \{0, 1\}$

$G(0) = 0$ MIN

$G(1) = 3$ MAX

c) $\frac{d^2G}{dx} = \frac{-10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3} = \frac{-10}{9x \cdot \sqrt[3]{x}} - \frac{20}{9 \cdot \sqrt[3]{x}} = \frac{-90 \cdot \sqrt[3]{x} - 180x\sqrt[3]{x}}{81x\sqrt[3]{x^2}} = \frac{-90}{81} \cdot \frac{\sqrt[3]{x} + 2x\sqrt[3]{x}}{x^3 \cdot \sqrt[3]{x^2}}$

$2x \cdot \sqrt[3]{x} + \sqrt[3]{x} > 0 \therefore \sqrt[3]{x}(2x+1) > 0 \rightsquigarrow x < \frac{-1}{2} \cup x > 0$

sempre ≥ 0.

$x^3 \cdot \sqrt[3]{x^2} > 0 \therefore x^3 > 0 \therefore x > 0$

	-1/2		0	
+		–		+
–		–		+
–		+		+

$x \in (-\infty, -1/2)$ G é $\cap$

$x \in (-1/2, 0) \cup (0, +\infty)$ G é $\cup$

$x \in \{-1/2, 0\}$ são p.i.

41. $C(x) = x^{1/3}(x+4) = x^{4/3} + 4x^{1/3}$

a) $\frac{dC}{dx} = \frac{4}{3}x^{1/3} + \frac{1}{3}x^{-2/3} = \frac{4\cdot\sqrt[3]{x}}{3} + \frac{1}{3\cdot\sqrt[3]{x^2}} = \frac{4x+1}{3\cdot\sqrt[3]{x^2}}$

$\frac{dC}{dx} > 0$ ∴ depende apenas do numerador

$\frac{dC}{dx} > 0$ ∴ $4x+1 > 0$ ∴ $x > -1/4$

$x \in (-\infty, -1/4)$ C é decresc.

$x \in (-1/4, +\infty)$ C é cresc.

$x = -1/4$ C é cte.

b) Nc: $\frac{dC}{dx} = 0$ ou $\nexists \frac{dC}{dx}$

$\frac{dC}{dx} = 0$ ∴ $x = -1/4$

$\nexists \frac{dC}{dx}$ ∴ $x = 0$

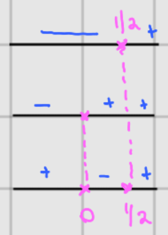
$f(-1/4) \simeq -2,3$ MíN

$f(0) = 0$ MáX

sempre positivo

c) $\frac{d^2C}{dx} = \frac{4}{9}x^{-2/3} - \frac{2}{9}x^{-5/3} = \frac{4}{9\cdot\sqrt[3]{x^2}} - \frac{2}{9x\sqrt[3]{x^2}} = \frac{36x\cdot\sqrt[3]{x^2} - 18\cdot\sqrt[3]{x^2}}{9x^2\cdot\sqrt[3]{x}} = \frac{2\cdot\sqrt[3]{x^2}}{x^2}\cdot\frac{2x-1}{\sqrt[3]{x}}$

$\frac{d^2C}{dx} > 0$



$x \in (-\infty, 0) \cup (1/2, +\infty)$ C é ∪

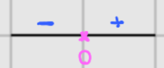
$x \in (0, 1/2)$ C é ∩

$x \in \{0, 1/2\}$ são pontos de inflexão

42- $f(x) = \ln(x^4+27)$

a) $\frac{df}{dx} = \frac{1}{x^4+27}\cdot 4x^3 = \frac{4x^3}{x^4+27}$

O denominador é sempre positivo.



$x \in (-\infty, 0)$ f é decresc.

$x \in (0, +\infty)$ f é cresc.

$x = 0$ f é constt.

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Nc: $\{0\}$

$f(0) = \ln 27$

c) $\frac{d^2f}{dx^2} = \frac{12x^2(x^4+27) - 16x^6}{\ldots}$

c) $\frac{d^2f}{dx} = \frac{[illegible]}{(x^4+27)^2}$

O denominador é sempre positivo.

$\frac{d^2f}{dx} > 0 \therefore 12x^2(x^4+27) > 16x^6 \therefore 3(x^4+27) > 4x^4 \therefore 3x^4 + 3^4 - 4x^4 > 0 \therefore -x^4 > -3^4 \therefore x^4 < 3^4 \therefore x < 3$

$-$ $\quad$ $+$
3

$x \in (-\infty, 3)$ f é $\cap$

$x \in (3, +\infty)$ f é $\cup$

$x = 3$ é pto. infl.

43- $f(\theta) = 2\cos\theta + \cos^2\theta$, $[0, 2\pi]$

a) $\frac{df}{d\theta} = -2\operatorname{sen}\theta - 2\cos\theta \operatorname{sen}\theta$

$\frac{df}{d\theta} > 0 \therefore \cos\theta < -1$

$\cos\theta$ é menor que -1 $\forall\ \theta \neq \pi$

$x \neq \pi$ f é cresc.

$x = \pi$ f é const.

$-\cancel{2}\cancel{\operatorname{sen}\theta} < \cancel{2}\cos\theta\,\cancel{\operatorname{sen}\theta}$

$-1 < \cos\theta$

$\cos\theta > -1 \Rightarrow \frac{df}{d\theta} < 0$???

b)

Pg 279 (4.4) João Vaz

1- a) $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \nexists$

b) $\lim\limits_{x \to a} \frac{f(x)}{p(x)} = \frac{0}{\infty} = 0$

c) $\lim\limits_{x \to a} \frac{h(x)}{p(x)} = \frac{1}{\infty} = 0$

d) $\lim\limits_{x \to a} \frac{p(x)}{f(x)} = \frac{\infty}{0} \nexists$

e) $\lim\limits_{x \to a} \frac{p(x)}{q(x)} = \frac{\infty}{\infty} \nexists$

2- a) $\lim\limits_{x \to a} f(x)p(x) = 0 \cdot \infty \nexists$

b) $\lim\limits_{x \to a} h(x)p(x) = 1 \cdot \infty = \infty$

c) $\lim\limits_{x \to a} p(x)q(x) = \infty \cdot \infty \nexists$

3- a) $\lim\limits_{x \to a} f(x) - p(x) = 0 - \infty = -\infty$

b) $\lim\limits_{x \to a} p(x) - q(x) = \infty - \infty = \infty$

c) $\lim\limits_{x \to a} p(x) + q(x) = \infty + \infty = \infty$

4- a) $\lim\limits_{x \to a} f(x)^{g(x)} = 0^0 \nexists$

b) $\lim\limits_{x \to a} f(x)^{p(x)} = 0^{\infty} = 0$

c) $\lim\limits_{x \to a} h(x)^{p(x)} = 1^{\infty} \nexists$

d) $\lim\limits_{x \to a} p(x)^{f(x)} = \infty^0 \nexists$

e) $\lim\limits_{x \to a} p(x)^{q(x)} = \infty^{\infty} = \infty$

f) $\lim\limits_{x \to a} \sqrt[q(x)]{p(x)} = p(x)^{\frac{1}{q(x)}} = p(x)^0 = \infty^0 \nexists$

5- $\lim\limits_{x \to 2} \frac{f(x)}{g(x)} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 2} \frac{f'(x)}{g'(x)} = \frac{\frac{18}{10}}{\frac{4}{5}} = \frac{18}{10} \cdot \frac{5}{4} = \frac{9}{4}$

6- $\lim\limits_{x \to 2} \frac{f(x)}{g(x)} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 2} \frac{f'(x)}{g'(x)} = \frac{1,5}{1} = 1,5$

7- $\lim\limits_{x \to 1} \frac{x^2-1}{x+1} = \lim\limits_{x \to 1} x-1 = 0$

8- $\lim\limits_{x \to 1} \frac{x^a-1}{x^b-1} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 1} \frac{a x^{a-1}}{b x^{b-1}} = \frac{a}{b}$

9- $\lim\limits_{x \to 1} \frac{x^3-2x^2+1}{x^3-1} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 1} \frac{3x^2-4x}{3x^2} = \frac{-1}{3}$

10- $\lim\limits_{x \to 1/2} \frac{6x^2+5x-4}{4x^2+16x-9} = \frac{1,5+2,5-4}{1+8-9} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to \frac{1}{2}} \frac{12x+5}{8x+16} = \frac{11}{20}$

11- $\lim\limits_{x \to \pi/2^+} \frac{\cos x}{1-\operatorname{sen} x} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to \pi/2^+} \frac{-\operatorname{sen} x}{-\cos x} = \frac{1}{0^+} = +\infty$

12- $\lim\limits_{x \to 0} \frac{\operatorname{sen} 4x}{\operatorname{tg} 5x} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 0} \frac{\cos 4x \cdot 4}{\sec^2 5x \cdot 5} = \frac{4}{5}$

13- $\lim\limits_{t \to 0} \frac{e^{2t}-1}{\operatorname{sen} t} = \frac{0}{0} \overset{LH}{=} \lim\limits_{t \to 0} \frac{2e^t \cdot e^t}{\cos t} = \frac{2}{1} = 2$

14- $\lim\limits_{x \to 0} \frac{x^2}{1-\cos x} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 0} \frac{2x}{\operatorname{sen} x} = \frac{0}{0} \overset{LH}{=} \lim\limits_{x \to 0} \frac{2}{\cos x} = 2$

15- $\lim\limits_{\theta \to \pi/2} \frac{1-\operatorname{sen}\theta}{1+\cos 2\theta} = \frac{0}{0} \overset{LH}{=} \lim\limits_{\theta \to \pi/2} \frac{-\cos\theta}{-\operatorname{sen} 2\theta \cdot 2} = \frac{0}{2} = 0$

16- $\lim\limits_{\theta\to\pi/2} \dfrac{1-\operatorname{sen}\theta}{\operatorname{cossec}\theta} = \lim\limits_{\theta\to\pi/2} \operatorname{sen}\theta(1-\operatorname{sen}\theta) = 1\cdot 0 = 0$

17- $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt{x}} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}}{\frac{x^{-1/2}}{2}} = \lim\limits_{x\to\infty} \dfrac{1}{x}\cdot\dfrac{2}{x^{-1/2}} = \lim\limits_{x\to\infty} \dfrac{1}{x}\cdot 2\sqrt{x} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{x^{-1/2}}{1} = \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{x}} = \dfrac{1}{\infty} = 0$

18- $\lim\limits_{x\to 1} \dfrac{\ln x}{\operatorname{sen}\pi x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{\frac{1}{x}}{\cos\pi x} = \dfrac{1}{1} = 1$

19- $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{1} = \dfrac{1}{0^+} = +\infty$

20- $\lim\limits_{x\to\infty} \dfrac{\ln\ln x}{x} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{1}{\ln x}\cdot\dfrac{1}{x} = \lim\limits_{x\to\infty} \dfrac{1}{\infty}\cdot\dfrac{1}{\infty} = 0\cdot 0 = 0$

21- $\lim\limits_{t\to 1} \dfrac{t^8-1}{t^5-1} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{t\to 1} \dfrac{8t^7}{5t^4} = \lim\limits_{t\to 1} \dfrac{8}{5}t^3 = \dfrac{8}{5}$

22- $\lim\limits_{t\to 0} \dfrac{8^t-5^t}{t} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{t\to 0} \dfrac{8^t\ln 8-5^t\ln 5}{1} = \ln 8 - \ln 5 = \ln(5/8)$

23- $\lim\limits_{x\to 0} \dfrac{\sqrt{1+2x}-\sqrt{1-4x}}{x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{2}(1+2x)^{-1/2}\cdot 2-\frac{1}{2}(1-4x)^{-1/2}\cdot 4}{1} = \lim\limits_{x\to 0} \dfrac{1}{\sqrt{1+2x}} - \dfrac{2}{\sqrt{1-4x}} = \dfrac{-1}{1} = -1$

24- $\lim\limits_{u\to\infty} \dfrac{e^{u/10}}{u^3} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{-9u/10}\cdot e^{u}}{3u^2} = \lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{u/10}}{3u^2} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{u\to\infty} \dfrac{\frac{1}{10}\cdot e^{u/10}}{60u} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{u/10}}{600} = \infty$

25- $\lim\limits_{x\to 0} \dfrac{e^x-1-x}{x^2} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x-1}{2x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x}{2} = \dfrac{1}{2}$

26- $\lim\limits_{x\to 0} \dfrac{\operatorname{senh}x-x}{x^3} = \lim\limits_{x\to 0} \dfrac{\frac{e^x-e^{-x}}{2}-x}{x^3} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\cosh x-1}{3x^2} = \dfrac{\frac{e^x+e^{-x}}{2}-x}{3x^2} = \lim\limits_{x\to 0} \dfrac{\frac{e^x+e^{-x}-2x}{2}}{3x^2} = \lim\limits_{x\to 0} \dfrac{e^x+e^{-x}-2x}{6x^2} = \dfrac{2}{0} \therefore \nexists$

27-

28- $\lim\limits_{x\to 0} \dfrac{x-\operatorname{sen}x}{x-\operatorname{tg}x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{1-\cos x}{1-\sec^2 x} = \dfrac{1}{0} \therefore \nexists$

29- $\lim\limits_{x\to 0} \dfrac{\operatorname{cossec}x}{x} = \lim\limits_{x\to 0} \dfrac{1}{x\operatorname{sen}x} = \dfrac{1}{0} \therefore \nexists$

30- $\lim\limits_{x\to\infty} \dfrac{(\ln x)^2}{2} = \infty$

31- $\lim\limits_{x\to 0} \dfrac{3^x\cdot x}{3^x-1} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{3^x\ln 3\cdot x+3^x\cdot 1}{3^x\ln 3} = \dfrac{1}{\ln 3}$

32- $\lim\limits_{x\to 0} \dfrac{\cos mx-\cos nx}{x^2} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{-\operatorname{sen}mx\cdot m+\operatorname{sen}nx\cdot n}{2x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{-m^2\cos mx+n^2\cos nx}{2} = \dfrac{-m^2+n^2}{2} = \dfrac{n^2-m^2}{2}$

33- $\lim\limits_{x\to 0} \dfrac{x+\operatorname{sen}x}{x+\cos x} = \lim\limits_{x\to 0} \dfrac{x^2-\operatorname{sen}^2x}{(x+\cos x)(x-\operatorname{sen}x)} = \lim\limits_{x\to 0} \dfrac{x^2-1+\cos^2 x}{(x+\cos x)(x-\operatorname{sen}x)} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{2x-2\operatorname{sen}^2x}{(1-\operatorname{sen}x)(x-\operatorname{sen}x)+(x+\cos x)(1-\cos x)} = \dfrac{0}{0} \overset{LH}{=}$

$\lim\limits_{x\to 0} \dfrac{2-4\operatorname{sen}x\cos x}{-\cos x(x-\operatorname{sen}x)+2(1-\operatorname{sen}x)(1-\cos x)+\operatorname{sen}x(x+\cos x)} = \dfrac{2}{0} \therefore \nexists$

34- $\lim\limits_{x\to 0} \dfrac{x}{\operatorname{cotg}4x} = \lim\limits_{x\to 0} x\cdot\operatorname{tg}4x = 0$

35- $\lim\limits_{x\to 1} \dfrac{1-x+\ln x}{1+\cos\pi x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{-1+\frac{1}{x}}{-\operatorname{sen}\pi x\cdot\pi} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{-x^{-2}}{-\pi^2\cos\pi x} = \dfrac{-1}{-\pi^2} = \dfrac{1}{\pi^2} = \pi^{-2}$

36- $\lim\limits_{x\to 0^+} \dfrac{x^x-1}{\ln x+x-1} =$

37- $\lim\limits_{x\to 1} \dfrac{x^a-ax+a-1}{(x-1)^2} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{ax^{a-1}-a}{2(x-1)\cdot 1} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{a(a-1)x^{a-2}}{2} = \dfrac{a(a-1)}{2}$

38- $\lim\limits_{x\to 0} \dfrac{e^x-e^{-x}-2x}{x-\operatorname{sen}x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x+e^{-x}-2}{1-\cos x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x-e^{-x}}{\operatorname{sen}x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x+e^{-x}}{\cos x} = \dfrac{2}{1} = 0$

39- $\lim\limits_{x\to 0} \dfrac{\cos x-1+\frac{1}{2}x^2}{x^4} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{-\operatorname{sen}x+x}{4x^3} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{-\cos x+1}{12x^2} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\operatorname{sen}x}{24x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{24} = \dfrac{1}{24}$

40- $\lim\limits_{x\to a^+} \dfrac{\cos x\ln(x-a)}{\ln(e^x-e^a)} = \dfrac{-\infty}{-\infty} \overset{LH}{=} \lim\limits_{x\to a^+} \dfrac{-\operatorname{sen}x\cdot\frac{1}{x-a}\cdot 1}{(e^x-e^a)^{-1}} = \lim\limits_{x\to a^+} \dfrac{-\operatorname{sen}x}{x-a}\cdot(e^x-e^a) = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to a^+} (e^x-e^a)(-\cos x)-\operatorname{sen}x(e^x-e^a) = 0-0 = 0$

41- $\lim\limits_{x\to\infty} x\operatorname{sen}(\pi/x) = \infty\cdot 0 \overset{PI}{=} \lim\limits_{x\to\infty} \dfrac{\operatorname{sen}(\pi/x)}{\frac{1}{x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{\cos(\pi/x)}{-x^{-2}}\cdot(-\pi x^{-2}) = \lim\limits_{x\to\infty} \dfrac{-\pi x^{-2}\cos(\pi/x)}{-x^{-2}} = \lim\limits_{x\to\infty} \pi\cos(\pi/x) = \pi$

42- $\lim\limits_{x\to\infty} \sqrt{x}\,e^{-x/2} = \lim\limits_{x\to\infty} \dfrac{\sqrt{x}}{\sqrt{e^x}} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{2}x^{-1/2}}{\frac{1}{2}(e^x)^{-1/2}\cdot e^x} = \lim\limits_{x\to\infty} \dfrac{x^{-1/2}}{e^{1/2}} = \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{x}\cdot\sqrt{e}} = 0$

43- $\lim\limits_{x\to 0} \operatorname{cotg}2x\cdot\operatorname{sen}6x = \dfrac{\operatorname{sen}6x}{\operatorname{tg}2x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\cos 6x\cdot 6}{\sec^2 2x\cdot 2} = \lim\limits_{x\to 0} 3\cos 6x\cdot\cos^2 2x = 3$

44- $\lim\limits_{x\to 0^+} \operatorname{sen}x\ln x = 0\cdot(-\infty) \overset{PI}{=} \lim\limits_{x\to 0^+} \dfrac{\operatorname{sen}x}{\frac{1}{\ln x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{\cos x}{-\ln x^{-2}\cdot\frac{1}{x}} = \lim\limits_{x\to 0^+} \dfrac{x\cos x}{-\ln^2 x} =$

45- $\lim\limits_{x\to\infty} x^3\cdot e^{-x^2} = \lim\limits_{x\to\infty} \dfrac{x^3}{e^{x^2}} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{3x^2}{2x\,e^{x^2}} = \lim\limits_{x\to\infty} \dfrac{3x}{2e^{x^2}} = \dfrac{\infty}{\infty} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{3}{4x\,e^{x^2}} = \dfrac{3}{\infty} = 0$

46- $\lim\limits_{x\to\infty} x\,tg(1/x) = \infty\cdot 0 \overset{PI}{=} \lim\limits_{x\to\infty} \dfrac{tg(1/x)}{\frac{1}{x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{-sec^2(1/x)\cdot x^{-2}}{-x^{-2}} = \lim\limits_{x\to\infty} \dfrac{1}{\cos^2\left(\frac{1}{x}\right)} = 1$

47- $\lim\limits_{x\to 1^+} \ln x\cdot tg(\pi x/2) = 0\cdot\infty \overset{PI}{=} \lim\limits_{x\to 1^+} \dfrac{\ln x}{cotg(\pi x/2)} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1^+} \dfrac{\frac{1}{x}}{-cossec^2(\pi x/2)\cdot\pi/2} = \lim\limits_{x\to 1^+} \dfrac{2}{-\pi x\, cossec^2(\pi x/2)} = \dfrac{-2}{\pi}$

48- $\lim\limits_{x\to\frac{\pi}{2}^-} \cos x\cdot sec\,5x = \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{\cos x}{\frac{1}{sec\,5x}} = \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{\cos x}{\cos 5x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to\frac{\pi}{2}^-} \dfrac{-sen\,x}{-5\,sen\,5x} = \dfrac{1}{5}$

49- $\lim\limits_{x\to 1} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right) = \lim\limits_{x\to 1} \dfrac{x\ln x - x + 1}{\ln x(x-1)} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{\ln x}{\frac{1}{x}(x-1)+\ln x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1} \dfrac{\frac{1}{x}}{-x^{-2}(x-1)+\frac{1}{x}+\frac{1}{x}} = \dfrac{1}{0+1+1} = \dfrac{1}{2}$

50- $\lim\limits_{x\to 0} (cossec\,x - cotg\,x) = \lim\limits_{x\to 0} \dfrac{1}{sen\,x} - \dfrac{\cos x}{sen\,x} = \lim\limits_{x\to 0} \dfrac{1-\cos x}{sen\,x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{1+sen\,x}{\cos x} = 1$

51- $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x-1}\right) = \lim\limits_{x\to 0^+} \dfrac{e^x-1-x}{x(e^x-1)} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{e^x-1}{e^x-1+xe^x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{e^x}{e^x+e^x+xe^x} = \dfrac{1}{2}$

52- $\lim\limits_{x\to 0} \left(cotg\,x - \dfrac{1}{x}\right) = \lim\limits_{x\to 0} \left(\dfrac{\cos x}{sen\,x} - \dfrac{1}{x}\right) = \lim\limits_{x\to 0} \dfrac{x\cos x - sen\,x}{x\,sen\,x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{\cos x - x\,sen\,x - \cos x}{sen\,x + x\cos x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} \dfrac{-sen\,x-(sen\,x+x\cos x)+sen\,x}{\cos x+\cos x - x\,sen\,x} = \dfrac{0}{2} = 0$

53- $\lim\limits_{x\to\infty} (x-\ln x) = \lim\limits_{x\to\infty} x\left(1-\dfrac{\ln x}{x}\right)$. Note que $\lim\limits_{x\to\infty} \dfrac{\ln x}{x} = \dfrac{\infty}{\infty} \overset{LH}{=} \dfrac{1}{x} = 0$. Portanto, $\lim\limits_{x\to\infty} x\left(1-\dfrac{\ln x}{x}\right) = \infty(1-0) = \infty$

54- $\lim\limits_{x\to 1^+} \left[\ln(x^7-1) - \ln(x^5-1)\right] = \lim\limits_{x\to 1} \ln\left(\dfrac{x^7-1^7}{x^5-1^5}\right) = \lim\limits_{x\to 1} \ln\left(\dfrac{(x-1)(x^6+x^5\cdot 1+x^4\cdot 1^2+x^3\cdot 1^3+x^2\cdot 1^4+x\cdot 1^5+1^6)}{(x-1)(x^4+x^3\cdot 1+x^2\cdot 1^2+x\cdot 1^3+1^4)}\right) = \ln\left(\dfrac{7}{5}\right) = \ln 7 - \ln 5$

55- $\lim\limits_{x\to 0^+} x^{\sqrt{x}} = a$

$\ln a = \lim\limits_{x\to 0^+}\left[\sqrt{x}\cdot\ln x\right] = \lim\limits_{x\to 0^+} \dfrac{\sqrt{x}}{\frac{1}{\ln x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{2}x^{-1/2}}{-(\ln x)^{-2}} = \lim\limits_{x\to 0^+} \dfrac{-\ln^2 x}{2\sqrt{x}} = \lim\limits_{x\to 0^+} \dfrac{\sqrt{x}}{\frac{2x}{\ln^2 x}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{2}x^{-1/2}}{\frac{2\ln^2 x - 2x\cdot 2\cdot\ln x\cdot\frac{1}{x}}{\ln^4 x}} = \lim\limits_{x\to 0^+} \dfrac{1}{2\sqrt{x}}\cdot\dfrac{\ln^3 x}{2\ln x - 4}$

$\ln a = \lim\limits_{x\to 0^+}\left[\dfrac{1}{2\sqrt{x}}\cdot\dfrac{\ln^3 x}{2\ln x-4}\right]$

56- $\lim\limits_{x\to 0^+} (tg\,2x)^x = a$

$\ln a = \lim\limits_{x\to 0^+} x\ln(tg\,2x) = 0\cdot(-\infty) \overset{PI}{=} \lim\limits_{x\to 0^+} \dfrac{x}{\frac{1}{\ln(tg\,2x)}} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0^+} \dfrac{1}{-\ln^{-2}(tg\,2x)\cdot sec^2 2x\cdot 2} = \lim\limits_{x\to 0^+} \dfrac{-\ln(tg\,2x)^2}{2\,sec^2\,2x} = \dfrac{+\infty}{2} = +\infty$

$\ln a = \infty\,;\ a = e^{\infty} = \infty$

$\lim\limits_{x\to 0^+} (tg\,2x)^x = \infty$

57- $\lim\limits_{x\to 0} (1-2x)^{1/x} = a$

$\ln a = \lim\limits_{x\to 0} \dfrac{1}{x}\ln(1-2x) = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 0} -x^{-2}\cdot\ln(1-2x) + \dfrac{1}{x}\cdot\dfrac{1}{1-2x}\cdot(-2) = \lim\limits_{x\to 0} \dfrac{-\ln(1-2x)}{x^2} - \dfrac{2}{x(1-2x)} = \lim\limits_{x\to 0} \dfrac{(-x+2x^2)\ln(1-2x) - 2x^2}{x^3-2x^3} = \dfrac{0}{0} \overset{LH}{=}$

58- $\lim\limits_{x\to\infty} \left(1+\dfrac{a}{x}\right)^{bx} = c$

$\ln c = \lim\limits_{x\to\infty}\left[bx\cdot\ln\left(1+\dfrac{a}{x}\right)\right] = \infty\cdot 0 \overset{PI}{=} \lim\limits_{x\to\infty}\left[\dfrac{b\ln(1+a/x)}{\frac{1}{x}}\right] = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to\infty} \dfrac{b\cdot\frac{1}{1+a/x}\cdot(-ax^{-2})}{-x^{-2}} = \lim\limits_{x\to\infty} \dfrac{b\cdot a}{1+a/x} = ab$

$\ln c = ab,\ c = e^{ab}$

$\lim\limits_{x\to\infty} \left(1+\dfrac{a}{x}\right)^{bx} = e^{ab}$

59- $\lim\limits_{x\to 1^+} x^{\frac{1}{1-x}} = a$

$\ln a = \lim\limits_{x\to 1^+} \dfrac{1}{x-1}\ln x = \lim\limits_{x\to 1^+} \dfrac{\ln x}{1-x} = \dfrac{0}{0} \overset{LH}{=} \lim\limits_{x\to 1^+} \dfrac{\frac{1}{x}}{-1} = -1$

$\ln a = -1\,;\ a = e^{-1}$

$\lim\limits_{x\to 1^+} x^{\frac{1}{x-1}} = e^{-1}$

60- $\lim_{x\to\infty} x^{\ln 2/(1+\ln x)} = \infty^0 = a$

$\ln a = \lim_{x\to\infty} \frac{\ln 2}{1+\ln x} \ln x = 0\cdot\infty \overset{PI}{=} \lim_{x\to\infty} \frac{\frac{\ln 2}{1+\ln x}}{\frac{1}{\ln x}} = \frac{0}{0} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{2^{-1}}{x^{-1}}}{x} = \lim_{x\to\infty} \frac{\frac{x}{2}}{x} = \lim_{x\to\infty} \frac{x}{2}\cdot\frac{1}{x} = \frac{1}{2}$

$\ln a = \frac{1}{2};\ a = \sqrt{e}$

$\lim_{x\to\infty} x^{\ln 2)(1+\ln x)} = \sqrt{e}$

61- $\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^0 = a$

$\ln a = \lim_{x\to\infty} \frac{1}{x}\cdot\ln x = \lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln a = 0;\ a = e^0 = 1$

$\lim_{x\to\infty} x^{\frac{1}{x}} = 1$

62- $\lim_{x\to\infty} (e^x+x)^{1/x} = \infty^0 = a$

$\ln a = \lim_{x\to\infty} \frac{1}{x}\ln(e^x+x) = \lim_{x\to\infty} \frac{\ln(e^x+x)}{x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{e^x+x}\cdot(e^x+1)}{1} = \lim_{x\to\infty} \frac{e^x+1}{e^x+x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{e^x+1} = \lim_{x\to\infty} \frac{e^x}{e^x(1+\frac{1}{e^x})} = \frac{1}{1} = 1$

$\ln a = 1;\ a = e$

$\lim_{x\to\infty} (e^x+x)^{1/x} = e$

63- $\lim_{x\to 0^+} (4x+1)^{\operatorname{cotg} x} = 1^{\frac{1}{0}} = a$

$\ln a = \lim_{x\to 0^+} \operatorname{cotg} x\cdot\ln(4x+1) = \lim_{x\to 0^+} \frac{\cos x\cdot\ln(4x+1)}{\operatorname{sen} x} = \frac{0}{0} \overset{LH}{=} \lim_{x\to 0^+} \frac{-\operatorname{sen} x\ln(4x+1)+\cos x\,\frac{1}{4x+1}\cdot 4}{\cos x} = \lim_{x\to 0^+} \frac{4}{1} = 4$

$\ln a = 4;\ a = e^4$

$\lim_{x\to 0^+} (4x+1)^{\operatorname{cotg} x} = e^4$

64- $\lim_{x\to 1^+} (2-x)^{\operatorname{tg}(\frac{\pi x}{2})} = 1^{+\infty} = a$

$\ln a = \lim_{x\to 1^+} \operatorname{tg}(\frac{\pi x}{2})\cdot\ln(2-x) = \infty\cdot 0 \overset{PI}{=} \lim_{x\to 1^+} \frac{\ln(2-x)}{\operatorname{cotg}(\frac{\pi x}{2})} = \frac{0}{0} \overset{LH}{=} \lim_{x\to 1^+} \frac{\frac{1}{2-x}}{-\operatorname{cossec}^2(\frac{\pi x}{2})\cdot\frac{\pi}{2}} = \lim_{x\to 1^+} \frac{-2\operatorname{sen}^2(\frac{\pi x}{2})}{\pi(2-x)} = \frac{-2}{\pi}$

$\ln a = \frac{-2}{\pi};\ a = e^{-2/\pi} = \sqrt[\pi]{e^{-2}}$

$\lim_{x\to 1^+} (2-x)^{\operatorname{tg}(\frac{\pi x}{2})} = e^{-2/\pi}$

65- $\lim_{x\to 0^+} (\cos x)^{1/x^2} = 1^{\frac{1}{0}} = a$

$\ln a = \lim_{x\to 0^+} \frac{1}{x^2}\ln(\cos x) = \lim_{x\to 0^+} \frac{\ln(\cos x)}{x^2} = \frac{0}{0} \overset{LH}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cos x}\cdot(-\operatorname{sen} x)}{2x} = \lim_{x\to 0^+} \frac{-\operatorname{sen} x}{2x\cos x} = \frac{0}{0} \overset{LH}{=} \lim_{x\to 0^+} \frac{-\cos x}{2\cos x - 2x\operatorname{sen} x} = \frac{-1}{2}$

$\ln a = \frac{-1}{2};\ a = e^{-1/2}$

$\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}} = e^{-1/2}$

66- $\lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1} = \infty^{\infty} = a$

$\ln a = \lim_{x\to\infty}\left[(2x+1)\ln\left(\frac{2x-3}{2x+5}\right)\right] = \lim_{x\to\infty}\left[(2x+1)\ln\left(\frac{x(2-3/x)}{x(2+5/x)}\right)\right] = \lim_{x\to\infty}\left[(2x+1)\cdot\ln 1\right] = \lim_{x\to\infty}\left[\frac{\ln 1}{\frac{1}{2x+1}}\right] = \frac{0}{0} \overset{LH}{=} \lim_{x\to\infty} \frac{0}{(-1)(2x+1)^{-2}\cdot 2} = \lim_{x\to\infty} \frac{0\cdot\sqrt{2x+1}}{-2} = 0$

$\ln a = 0;\ a = e^0 = 1$

$\lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1} = 1$

73- $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}{1} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ LOOP

$\lim_{x\to\infty} \frac{x(\sqrt{x^2+1})}{x^2+1} = \lim_{x\to\infty} \frac{x(\sqrt{x^2+1})}{x(x+1/x)} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2(1+1/x^2)}}{x} = \lim_{x\to\infty} \sqrt{1} = 1$

0 0

74- $\lim \frac{\sec x}{\ } = \lim \frac{1}{\ }\cdot\frac{\cos x}{\ } = \lim \operatorname{cossec} x = 1$

$x \to (\pi/2)^- \quad tg\,x \qquad x \to (\pi/2)^- \quad \cos x \qquad sen\,x \quad x \to (\pi/2)^-$

77- $\lim\limits_{n \to \infty} A_0 \cdot \left(1 + \frac{r}{n}\right)^{nt} = A_0 \cdot 1^{\infty} = a$

$\ln a = \lim\limits_{n \to \infty} \left[\ln A_0 + nt \ln\left(1 + \frac{r}{n}\right) \right] = \lim\limits_{n \to \infty} \left[nt \ln\left(1 + \frac{r}{n}\right) \right] + \ln A_0 = \lim\limits_{n \to \infty} (\infty \cdot 0) + \ln A_0 \overset{PI}{=} \lim\limits_{n \to \infty} \left[\frac{\ln(1 + r/n)}{\frac{1}{nt}} \right] + \ln A_0 = \lim\limits_{n \to \infty} \left(\frac{0}{0}\right) + \ln A_0 \overset{LH}{=} \ln A_0 + \lim\limits_{n \to \infty} \left[\frac{(1 + r/n)^{-1} \cdot (-r n^{-2})}{-t^{-1} n^{-2}} \right]$

$\ln a = \ln A_0 + \lim\limits_{n \to \infty} \frac{rt}{1 + r/n} = \ln A_0 + rt$

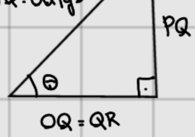
$a = e^{\ln A_0 + rt} = e^{\ln A_0} \cdot e^{rt} = A_0 e^{rt}$

$\lim\limits_{n \to \infty} A_0 \left(1 + \frac{r}{n}\right)^{nt} = A_0 e^{rt}$

área do setor circular com $r = OR$ e $\alpha = \theta$

$tg\,\theta = \frac{PQ}{OQ}$ $\qquad OR = 2OQ$

$PQ = OQ\,tg\,\theta$



82- $A(\theta) = C(\theta) - \Delta PQR = \frac{\theta}{2\pi} \cdot \pi \cdot (OR)^2 - 2B(\theta) = \frac{\theta \pi (OR)^2 - 4\pi B(\theta)}{2\pi}$

$B(\theta) = \frac{QR \cdot PQ}{2} = \frac{OQ \cdot PQ}{2} = \frac{OQ^2\,tg\,\theta}{2}$

$A(\theta) = \theta \cdot 2OQ^2 - OQ^2\,tg\,\theta = OQ^2(2\theta - tg\,\theta)$

$\lim\limits_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)} = \lim\limits_{\theta \to 0^+} \frac{2OQ^2(2\theta - tg\,\theta)}{OQ^2\,tg\,\theta} = \lim\limits_{\theta \to 0^+} \frac{4\theta - 2\,tg\,\theta}{tg\,\theta} = \frac{4 \cdot 0 - 2 \cdot 1}{1} = -2$

Pg 287 (4.5) João Vaz 1-54

1- $y = x^3 + x = x(x^2+1)$

domínio: $D = \mathbb{R}$

interseções: $(0,0)$

assíntotas vert.: $\varnothing$

assíntotas horiz.: $\varnothing$

Crescimento:

$$\frac{dy}{dx} = 3x^2 + 1$$

$x \in \mathbb{R}$, y é crescente

máximos e mínimos: $N_c = \varnothing$

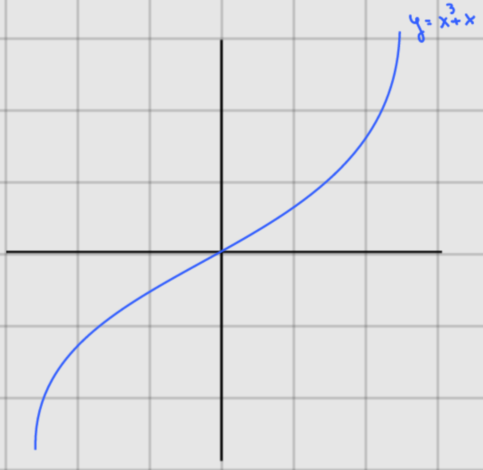
Concavidade:

$$\frac{d^2y}{dx} = 6x$$

$x \in (-\infty, 0)$ y é $\cap$

$x = 0$ é pto. infl.

$x \in (0, +\infty)$ y é $\cup$



2- $y = x^3 + 6x^2 + 9x = x(x^2 + 6x + 9)$

domínio: $D = \mathbb{R}$

interseções: $(0,0)$; $(-3,0)$

assíntotas vert.: $\varnothing$

assíntotas horiz.: $\varnothing$

Crescimento:

$$\frac{dy}{dx} = 3x^2 + 12x + 9 = 3(x^2 + 4x + 3)$$

+ − +
−3 −1

$x \in (-\infty, -3) \cup (1, +\infty)$ y é cresc.

$x \in (-3, -1)$ y é decresc.

$x \in \{-3, -1\}$ y é cte.

máx. e mín.: $Nc = \{-3, -1\}$

$f(-3) = 0$ MAX

$f(-1) = -4$ MIN

Conc.:

$\frac{dy}{dx} = 3(2x+4) = 6x+12$

$x \in (-\infty, -1/2)$ y é $\cap$

$x \in (-1/2, +\infty)$ y é $\cup$

$x = -1/2$ é pto. infl.

3- $y = 2 - 15x + 9x^2 - x^3 = -x^3 + 9x^2 - 15x + 2$

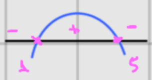
dom.: $D = \mathbb{R}$

inters.: $(0,2)$; $(2,0)$; $\left(\frac{7-3\sqrt{5}}{2}, 0\right)$; $\left(\frac{7+3\sqrt{5}}{2}, 0\right)$

assíntotas: $\emptyset$

Crescimento:

$\frac{dy}{dx} = -3x^2 + 18x - 15$



$x \in (-\infty, 1) \cup (5, +\infty)$ y é decresc.

$x \in (1,5)$ y é cresc.

$x \in \{1,5\}$ y é cte.

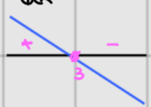
máx e mín.: $N_c = \{1,5\}$

$f(1) = -5$ MIN

$f(5) = 27$ MAX

Concavidade

$\frac{d^2y}{dx^2} = -6x + 18$



$x \in (-\infty, 3)$ y é $\cup$

$x \in (3, +\infty)$ y é $\cap$

$x = 3$ é pto. infl.

4- $y = -x^4 + 8x^2$

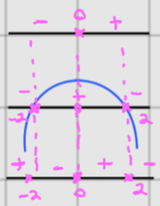
dom.: $\mathbb{R}$

inters.: $(0,0)$; $(-2\sqrt{2}, 0)$; $(2\sqrt{2}, 0)$

assíntotas: ∅

crescimento:

$$\frac{dy}{dx} = -4x^3 + 16x = x(-4x^2+16)$$



$x \in (-\infty, -2) \cup (0, 2)$ y é cresc.

$x \in (-2, 0) \cup (2, +\infty)$ y é decresc.

$x \in \{-2, 0, 2\}$ y é cte.

máx. e mín.: $N_c = \{-2, 0, 2\}$

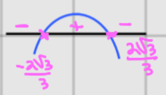
$f(-2) = 16$ MAX

$f(0) = 0$ MIN

$f(2) = 16$ MAX

conc.

$$\frac{d^2y}{dx^2} = -12x^2 + 16$$



$x \in \left(-\infty, -\frac{2\sqrt{3}}{3}\right) \cup \left(\frac{2\sqrt{3}}{3}, +\infty\right)$ y é $\cap$

$x \in \left(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$ y é $\cup$

$x \in \left\{\pm\frac{2\sqrt{3}}{3}\right\}$ são ptos. infl.

5. $y = x(x-4)^3$

dom.: $\mathbb{R}$

interc.: $(0,0)$, $(4,0)$

assíntotas: ∅

crescimento:

$$\frac{dy}{dx} = (x-4)^3 + 3x(x-4)^2 = (x-4)^2(x-4+3x) = \underbrace{(x-4)^2}_{\geq 0\ \forall x \in \mathbb{R}}(4x-4) = 4(x-4)^2(x-1)$$

$(x-4)^2 = 0 \therefore x = 4$

$x - 1 = 0 \therefore x = 1$

$x \in (-\infty, 1)$ y é decresc.

$x \in (1,4) \cup (4,+\infty)$ y é cresc.

$x \in \{1,4\}$ y é cte.

máx. e mín.: $N_c = \{1,4\}$

$f(1) = -27$ Min

$f(4) = 0$ Máx

Conc.

$$\frac{d^2y}{dx^2} = 3(x-4)^2 + 3(x-4)^2 + 6x(x-4) = 3(x-4)(x-4+x-4+2x) = 3(x-4)(4x-8) = 12(x-4)(x-2) = 12(x^2-6x+8)$$

$x \in (-\infty,2) \cup (4,+\infty)$ y é $\cup$

$x \in (2,4)$ y é $\cap$

$x \in \{2,4\}$ são ptos. infl.

Pg 300 (4.7) João Vaz 19 - 40

10- $P(i) = \frac{100i}{i^2+i+4}$; $i \geq 0$.

$\frac{dP}{di} = \frac{100(i^2+i+1) - 100i \cdot (2i+1)}{(i^2+i+4)^2}$

A primeira derivada será zero quando o numerador for zero

$100i^2 + 100i + 100 = 200i^2 + 100i$

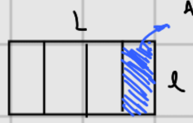
$100i^2 = 100$

$i = \pm 1$ ⟶ $i = -1$ está fora do domínio.

P é máxima com $i = 1$.

11-

$A/4 = \frac{Ll}{4} = A_p$



$2(L+l) = 300$; $L + l = 150$; $L = 150 - l$

Máxima de $\frac{Ll}{4}$?

$A = L \cdot l = 150l - l^2$; $0 \leq l \leq 150$

Como as áreas parciais são sempre 1/4 da área total,

maximizando-se A, maximiza-se, também, as Ap.

$\frac{dA}{dl} = 150 - 2l$

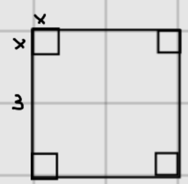
$\frac{dA}{dl} = 0 \therefore l = 75$

$A(0) = A(150) = 0$

$A(75) = 5625\ m^2$

Portanto, a maior Ap é $1406,25\ m^2$

12-



$A = (3-2x)^2$

$V = A_b \cdot h = A \cdot x = (3-2x)^2 \cdot x$, $0 \leq x \leq 1,5$

$\frac{dV}{dx} = 2(3-2x) \cdot (-2) \cdot x + (3-2x)^2 = (3-2x)(-4x+3-2x) = (3-2x)(-6x+3) = -18x+9+12x^2-6x = 12x^2 - 24x + 9 = 3(4x^2 - 8x + 3)$

Para $\frac{dV}{dx} = 0$, $x = \frac{8 \pm \sqrt{64-48}}{8} = \frac{8 \pm 4}{8}$ ⟨ $3/2$; $1/2$.

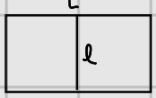
$V(0) = 0$

$V(1/2) = 2$

$V(1,5) = 0$

Portanto, o maior volume é $2\ m^3$, atingido quando $x = 1/2\ m$

13-



$A = L\ell = 15\cdot10^3$; $L = \frac{15\cdot10^3}{\ell}$; $0 < \ell \leq L \leq 15\cdot10^3$

O custo depende do perímetro:

$C = 3\ell + 2L$

$C(\ell) = 3\ell + \frac{3\cdot10^4}{\ell} = 3\ell + 3\cdot10^4\cdot\ell^{-1}$

$\frac{dC}{d\ell} = 3 - \frac{3\cdot10^4}{\ell^2} = \frac{3\ell^2 - 3\cdot10^4}{\ell^2}$

Para que a derivada seja 0,

$3\ell^2 = 3\cdot10^4$

$\ell = \pm 100 \longrightarrow -100$ não está no domínio

$C(15\cdot10^3) = 45\cdot10^3 + 2 = 45\,002$

$C(100) = 300 + 300 = 600$

Portanto, o menor custo é 600, quando $\ell = 100$ e $L = 150$ m.

14- $V = \ell^2\cdot h = 32\cdot10^3$; $h = \frac{32\cdot10^3}{\ell^2}$; $0 < \ell \leq 80\sqrt{5}$

Para minimizar o uso de material,

minimiza-se a área.

$A = A_b + A_\ell = \ell^2 + 4\ell h = \ell^2 + \frac{128\cdot10^3}{\ell} = \ell^2 + 128\cdot10^3\cdot\ell^{-1}$

$\frac{dA}{d\ell} = 2\ell - 128\cdot10^3\ell^{-2}$

$\frac{dA}{d\ell} = 0 \therefore \ell^3 = 256\cdot10^3 \therefore \ell = \sqrt[3]{256\cdot10^3} = 10\cdot\sqrt[3]{64\cdot4} = 40\sqrt[3]{4}$

$A(40\cdot\sqrt[3]{4}) = 9111\ m^2$

$A(80\sqrt{5}) = 32\,002\ m^2$

Portanto, quando $\ell = 40\cdot\sqrt[3]{4}$ (≈ 63), o custo é mínimo.

15- $A_b + A_\ell = 1200$

$\ell^2 + 4\ell h = 1200$; $h = \frac{1200 - \ell^2}{4\ell} = 300\ell^{-1} - \frac{\ell}{4}$ $\quad 0 < \ell \leq \sqrt{1200}$

$V = \ell^2 h = \ell^2\left(300\ell^{-1} - \frac{\ell}{4}\right) = 300\ell - \frac{\ell^3}{4} = \frac{-\ell^3 + 1200\ell}{4}$

$\frac{dV}{d\ell} = \frac{-3\ell^2 + 1200}{4}$

$\frac{dV}{d\ell} = 0 \therefore \ell^2 = 400$; $\ell = \pm 20 \longrightarrow -20 \notin D$.

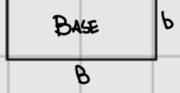
$V(0) = 0$

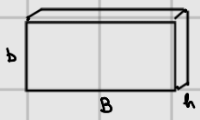
$V(20) = 16\,000\ cm^3$

$V(\sqrt{1200}) = 0$

Portanto, o maior volume possível é $16\cdot10^3\ cm^3$, atingido quando $\ell = 20$ cm

16- $V = A_b\cdot h$





$A_b = 2(B+b)$

$h = B+b$

$V = 2(B+b)^2 = 10\,m^3$

$(B+b)^2 = 5$

reunir
as variáveis

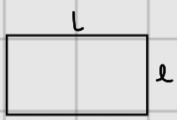
$B+b = \sqrt{5} \rightsquigarrow B = \sqrt{5} - b$

$C = 10(B+b) + 6(2bh + 2Bh) = 10(B+b) + 12h(B+b) = 10(B+b) + 12(B+b)^2 = 10\sqrt{5} + 60$

Mínimo custo?

17-

18- a)



b)

$A = L \cdot \ell \quad 0 < \ell < L < A$

$P = 2(L+\ell) \therefore L = \frac{P}{2} - \ell = \frac{P-2\ell}{2}$

$A(\ell) = \frac{P-2\ell}{2} \cdot \ell = \frac{\ell P - 2\ell^2}{2}$

$\frac{dA}{d\ell} = \frac{P-4\ell}{2}$

$\frac{dA}{d\ell} = 0 \therefore 4\ell = P ;\ \ell = P/4$

Esse é um máximo local, pois $\frac{d^2A}{d\ell^2} = -2$

$\ell = \frac{P}{4} ;\ L = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$. Os quatro lados são iguais e a área vale $\frac{P^2}{16}$.

19- $y = f(x) = 2x+3$. Seja x_1 a coordenada x do ponto mais próximo da origem

$d_{yo} = \sqrt{(x_1-0)^2+(y_1-0)^2} = \sqrt{x_1^2 + f^2(x_1)} = \sqrt{x_1^2 + (2x_1+3)^2}$

$\frac{dd_{yo}}{dx_1} = \frac{1}{2}\left[x_1^2+(2x_1+3)^2\right]^{-1/2} \cdot \left[2x_1 + 2(2x_1+3)\cdot 2\right]$

$= \frac{x_1 + 4x_1 + 6}{\sqrt{x_1^2+(2x_1+3)^2}}$

Para que essa derivada seja 0, o numerador deve ser 0 (pois o denominador é $>0\ \forall x_1 \in \mathbb{R}$).

$5x_1 + 6 = 0 \therefore x_1 = -6/5$.

O ponto $(-6/5, 6/10)$ é o mais próximo de $(0,0)$ dos que são passados por $y = 2x+3$.

20- $y = f(x) = \sqrt{x}$. Seja x_1 a coordenada x do ponto mais próximo da origem.

$d_{yo} = \sqrt{(x_1-3)^2 + f^2(x_1)} = \sqrt{(x_1-3)^2 + x_1}$

$\frac{dd_{yo}}{dx_1} = \frac{1}{2}\left[(x_1-3)^2+1\right]^{-1/2} \cdot \left[2(x_1-3)+1\right]$

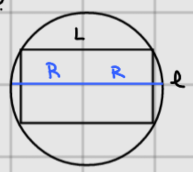
$= \frac{2x_1 - 5}{2\sqrt{(x_1-3)^2+1}}$

$\frac{dd_{yo}}{dx_1} = 0 \therefore x_1 = 5/2$

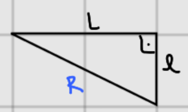
O ponto mais próximo é $(5/2, f(5/2))$, ou seja, $(5/2, \sqrt{10}/2)$.

21- $4x^2 + y^2 = 4$. Seja $P(x_1, f(x_1))$ o ponto mais distante de $A(1,0)$

23- A_{max} ?



A diagonal do retângulo vale R.



$R^2 = L^2 + \ell^2 \therefore L = \sqrt{R^2-\ell^2}$; $R > L \geq \ell > 0$, ou seja, L e $\ell \in (0,R)$

$A(\ell) = L \cdot \ell = \sqrt{R^2-\ell^2} \cdot \ell$

$\frac{dA}{d\ell} = \frac{1}{2} \cdot (R^2-\ell^2)^{-1/2} \cdot (-2\ell) \cdot \ell + \sqrt{R^2-\ell^2}$

$= -\ell^2 + \sqrt{R^2-\ell^2} = -\ell^2 + R^2 - \ell^2 = R^2 - 2\ell^2$

$\sqrt{R^2-\ell^2}$ $\qquad$ $\sqrt{R^2-\ell^2}$ $\qquad$ $\sqrt{R^2-\ell^2}$

$$\frac{dA}{d\ell} = 0 \therefore \ell = \frac{R\sqrt{2}}{2}$$

$$A\left(\frac{R\sqrt{2}}{2}\right) = \sqrt{R^2 - \frac{R^2}{2}} \cdot \frac{R\sqrt{2}}{2}$$

$$= \frac{R\sqrt{2}}{2} \cdot \frac{R\sqrt{2}}{2} = \frac{R}{2}$$

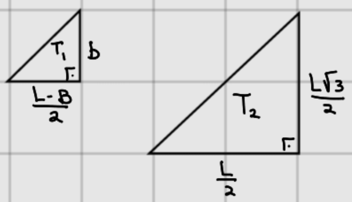
As maiores dimensões são $L = \ell = \frac{R\sqrt{2}}{2}$; que resultam em um retângulo de área $R/2$ u.a.

24-

25-

B e $b \in (0, L)$

$A = B \cdot b$



$$T_1 \sim T_2 \therefore \frac{\frac{L-B}{2}}{\frac{L}{2}} = \frac{b}{\frac{L\sqrt{3}}{2}} \therefore \frac{2(L-B)}{2L} = \frac{2b}{L\sqrt{3}} \therefore L\sqrt{3}(L-B) = 2bL$$

$$\therefore b = \frac{\sqrt{3}(L-B)}{2}$$

$$A(B) = B \cdot b = \frac{B\sqrt{3}(L-B)}{2} = \frac{\sqrt{3}LB - B^2\sqrt{3}}{2}$$

$$\frac{dA}{dB} = \frac{\sqrt{3}L - 2B\sqrt{3}}{2}$$

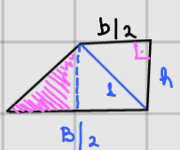
$$\frac{dA}{dB} = 0 \therefore B = L/2$$

$$b = \frac{L\sqrt{3}}{4}$$

A maior área é $\frac{L^2\sqrt{3}}{6}$, atingida quando os lados do retângulo medem $L/2$ e $L\sqrt{3}/4$.

26-

$$A = \frac{(B+b)h}{2} = \frac{(2+b)h}{2} \; ; \; B = 2, \; b < B$$



$$h^2 + \frac{b^2}{4} = 1^2$$

$$h = \sqrt{1 - \frac{b^2}{4}} = \frac{\sqrt{4-b^2}}{2}$$

$$A(b) = \frac{(2+b)\sqrt{4-b^2}}{4}$$

$$\frac{dA}{db} = \frac{\sqrt{4-b^2} + (2+b)\cdot\frac{1}{2}\cdot(4-b^2)^{-1/2}\cdot(-2b)}{4} = \frac{\sqrt{4-b^2} + \frac{\quad}{\sqrt{4-b^2}}}{4} = \frac{4-b^2-2b-b^2}{4\sqrt{4-b^2}} = \frac{-b^2-b+2}{2\sqrt{4-b^2}}$$

O sinal da derivada depende do numerador:

$$\frac{dA}{db} = 0 \therefore -b^2-b+2=0 \therefore b < {-2 \atop 1} \; ; -2 \notin D.$$

Portanto, a maior área é $A(1) = \frac{3\sqrt{3}}{4}$, atingida quando $b = 1$.

27- raio = r

$$A = \frac{bh}{2}$$

$$\frac{b^2}{4} + h^2 = l^2$$

$$h^2 = l^2 - \frac{b^2}{4} \therefore h = \frac{\sqrt{4l^2-b^2}}{2}$$

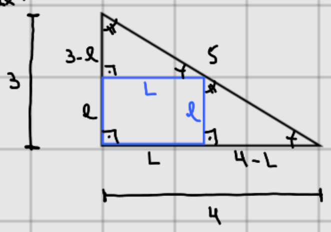
$$A(b) = \frac{1}{4}\cdot b\sqrt{4l^2-b^2}$$

$$\frac{dA}{db} = \frac{1}{4}\cdot\sqrt{4l^2-b^2} + \frac{b}{2}(4l^2-b^2)^{-1/2}\cdot(-2b) = \frac{\sqrt{4l^2-b^2}}{4} - \frac{b^2}{\sqrt{4l^2-b^2}} = \frac{(4l^2-b^2)-4b^2}{4\sqrt{4l^2-b^2}} = \frac{4l^2-5b^2}{4\sqrt{4l^2-b^2}}$$

$$\frac{dA}{db} = 0 \therefore 4l^2 = 5b^2 \therefore b = \frac{2l\sqrt{5}}{5}$$

A maior área é $A\left(\frac{2l\sqrt{5}}{5}\right) = \frac{2l^2}{5}$.

28- Amáx?



$$A = L\cdot l$$

$$L \in (0,4) \; ; \; l \in (0,3)$$

Os três triângulos são congruentes:

$$\frac{l}{3-l} = \frac{4-L}{L} \therefore L\cdot l = (4-L)(3-l) = 12-4l-3L+Ll$$

$$Ll = 12-4l-3L+Ll \therefore -4l-3L = -12 \therefore 4l+3L = 12$$

$$L = \frac{12-4l}{3}$$

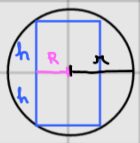
$$A(l) = l\cdot\frac{12-4l}{3} = \frac{12l-4l^2}{3}$$

$$\frac{dA}{dl} = \frac{12-8l}{3}$$

$$\frac{dA}{dl} = 0 \therefore l = \frac{3}{2}$$

A maior área possível é $A(3/2) = 3\ cm^2$; $L = 2\ cm$

29- Vmáx?



$V_{esf} = \frac{4}{3}\pi r^2$

$V_{cil} = \pi R^2 h$

A diagonal do cilindro vale r:

$R^2 + h^2 = r^2$

$h = \sqrt{r^2 - R^2}$

$V(R) = \pi R^2 \sqrt{r^2 - R^2}$

$\frac{dV}{dR} = 2\pi R\sqrt{r^2-R^2} + \pi R^2 \cdot \frac{1}{2}\cdot (r^2-R^2)^{-1/2}\cdot(-2R) = 2\pi R\sqrt{r^2-R^2} - \frac{\pi R^3}{\sqrt{r^2-R^2}} = \frac{2\pi R(r^2-R^2) - \pi R^3\sqrt{r^2-R^2}}{\sqrt{r^2-R^2}}$

O sinal da derivada depende apenas do numerador:

$R^2 = y$

$\frac{dV}{dR} = 0 \therefore \left[2\pi R(r^2-R^2)\right]^2 = \left[\pi R^3\sqrt{r^2-R^2}\right]^2 \therefore 4\cancel{\pi^2}\cancel{R^2}(r^2-R^2)^{\cancel{2}} = \cancel{\pi^2}R^{\cancel{6}\,4}\cancel{(r^2-R^2)} \therefore 4(r^2-R^2) = R^4 \therefore 4r^2 - 4R^2 = R^4 \therefore 4r^2 - 4y = y^2 \therefore y^2 + 4y - 4r^2 = 0 \therefore y = \frac{-4 \pm\sqrt{16+16r^2}}{2} = \frac{-4\pm4\sqrt{1+r^2}}{2}$

Como r é maior que zero, descarta-se a raiz negativa. Assim, $R = \left(\frac{-4+4\sqrt{1+r^2}}{4}\right)^2$.

$V(R) = \frac{1}{16}\pi\cdot\left(\sqrt{r^2+1}-1\right)^4\cdot\sqrt{r^2-16\left(\sqrt{r^2+1}-1\right)^4}$

30-

$V_{cil} = \pi \cdot R^2 \cdot H$

O triângulo T é congruente à metade do corte transversal do cone (o triângulo externo). Assim:

$\frac{h}{H} = \frac{r}{r-R} \therefore rH = hr - hR \therefore H = \frac{hr - hR}{r}$

$V(R) = \pi R^2 \cdot \frac{hr - hR}{r} = \frac{\pi R^2 hr - \pi R^3 h}{r}$

$\frac{dV}{dR} = \frac{2\pi R hr - 3\pi R^2 h}{r}$

Como r é sempre maior que 0,

$\frac{dV}{dR} = 0 \therefore 2\cancel{\pi}\cancel{R}\cancel{h}r = 3\cancel{\pi}R^{\cancel{2}}\cancel{h} \therefore R = \frac{2r}{3}$

O maior volume é $V\left(\frac{2r}{3}\right) = \frac{4\pi r^2 h}{27}$. Isso ocorre quando $H = \frac{h}{3}$.

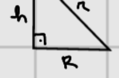
31-

$A_S = A_l + 2A_b = 2\pi Rh + 2\pi R^2$

$R \in [0, r]$.

Seja R o raio do cilindro e h sua altura.

Estando o cilindro inscrito,



$h^2 = r^2 - R^2 \therefore h = \sqrt{r^2 - R^2}$

$A_s(R) = 2\pi R\sqrt{r^2-R^2} + 2\pi R^2$

$\frac{dA_s}{dR} = 2\pi\sqrt{r^2-R^2} + 2\pi R \cdot \frac{1}{2}(r^2-R^2)^{-1/2} \cdot (-2R) + 4\pi R = 2\pi\sqrt{r^2-R^2} - \frac{2\pi R^2}{\sqrt{r^2-R^2}} + 4\pi R = \frac{2\pi(r^2-R^2) - 2\pi R^2 + 4\pi R\sqrt{r^2-R^2}}{\sqrt{r^2-R^2}}$ $R^2 = y$

Para $\frac{dA_s}{dR} = 0 \therefore r^2 + 2R\sqrt{r^2-R^2} = 2R^2 \therefore 2R^2 - r^2 = 2R\sqrt{r^2-R^2} \therefore 4R^4 - 4R^2r^2 + r^4 = 4R^2r^2 - 4R^4 \therefore 8R^4 - 8R^2r^2 + r^4 = 0 \therefore 8y^2 - 8yr^2 + r^4 = 0 \therefore$

$y = \frac{8r^2 \pm \sqrt{64r^4 - 32r^4}}{16} = \frac{8r^2 \pm 4r^2\sqrt{2}}{16} = \frac{r^2(2 \pm \sqrt{2})}{4}$

Como $0 < R < r$, descarta-se a raiz positiva. Assim, $R = \frac{r^2(2-\sqrt{2})}{4}$.

$A_s(R) = \frac{1}{8}(2-\sqrt{2})^2 \pi r^4 + \frac{1}{2}(2-\sqrt{2})\pi\sqrt{r^2 - \frac{1}{16}(2-\sqrt{2})^2 \cdot r^4} \cdot r^2$

32- A_{max}?

$P = \frac{C}{2} + 2L + \ell = \frac{\pi\ell + 2\ell}{2} + 2L = 10$

$\pi\ell + 2\ell + 4L = 20 \therefore L = \frac{20 - \pi\ell - 2\ell}{4}$

$A = \frac{C}{2} + L\ell = \frac{\pi\ell^2}{8} + L\ell$

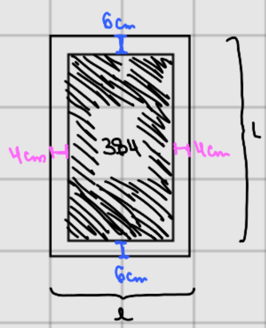
$A(\ell) = \frac{\pi\ell^2}{8} + \ell\left(\frac{20-\pi\ell-2\ell}{4}\right) = \frac{\pi\ell^2}{8} + \frac{20\ell - \pi\ell^2 - 2\ell^2}{4} = \frac{-4\ell^2 - \pi\ell^2 + 40\ell}{8}$

$\frac{dA}{d\ell} = \frac{-8\ell - 2\pi\ell + 40}{8} = \frac{-4\ell - \pi\ell + 20}{4}$

$\frac{dA}{d\ell} = 0 \therefore 4\ell + \pi\ell = 20 \therefore \ell = \frac{20}{4+\pi}$

$A\left(\frac{20}{4+\pi}\right) = \frac{100}{4+\pi} \cong 14{,}0025\ m^2$

33- A_{min}? Retângulo $\ell \cdot L$.



$A_p = (\ell-8)(L-12) = 384 = L\ell - 12\ell - 8L + 96 = 384 \therefore L\ell - 8L = 288 + 12\ell \therefore L = \frac{288 + 12\ell}{\ell - 8}$

$A = L \cdot \ell$

$A(\ell) = \frac{12\ell^2 + 288\ell}{\ell - 8}$

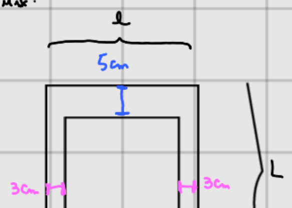
$\frac{dA}{d\ell} = \frac{(24\ell + 288)(\ell-8) - (12\ell^2 + 288\ell)}{(\ell-8)^2} = \frac{12\ell^2 - 192\ell - 2304}{(\ell-8)^2}$

$\frac{dA}{d\ell} = 0 \therefore 12(\ell^2 - 16\ell - 192) = 0 \therefore \ell <^{-8}_{24}$

Como $\ell > 0$, descarta-se a raiz negativa.

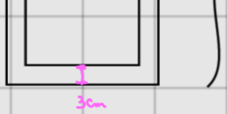
Assim $\ell = 24$, $L = 36$ e $A(24) = 864\ cm^2$

34- A_{max}?



$A_T = L \cdot \ell = 900$

$A_I = (L-8)(\ell-6) = L\ell - 6L - 8\ell + 48$



$L = \frac{900}{\ell}$

$A(\ell) = 900 - 5600\ell^{-1} - 8\ell + 48$

$\frac{dA}{d\ell} = 5600\,\ell^{-2} - 8 = \frac{5600 - 8\ell^2}{\ell^2}$

$\frac{dA}{d\ell} = 0 \therefore \ell = \pm\sqrt{700}$. $-\sqrt{700} \notin D$.

$\ell = \sqrt{700}$; $L = \frac{900}{\sqrt{700}}$; $A_I = 525\,cm^2$

35- $q + t = 10$; $t = 10 - q$ $\qquad 0 \leq q \leq t \leq 10$

$A_T = A_q + A_t$

$A_{\Delta q} = \frac{\ell^2\sqrt{3}}{4}$

$A_{\square} = L^2$

$L = \frac{q}{4}$; $\ell = \frac{t}{3} = \frac{10-q}{3}$

$A_T(q) = \frac{q^2 + 4\left(\frac{10-q}{3}\right)^2\sqrt{3}}{16} = \frac{9q^2 + 4\sqrt{3}(10-q)^2}{144}$

$\frac{dA_T}{dq} = \frac{18q - 8\sqrt{3}(10-q)}{144}$

$\frac{dA_T}{dq} = 0 \therefore 18q = 80\sqrt{3} - 8q\sqrt{3} \therefore q = \frac{80\sqrt{3}}{18 - 8\sqrt{3}}$

36-

37- $V = \pi r^2 h \therefore h = \frac{V}{\pi r^2} = V\cdot(\pi r^2)^{-1}$

$A = \pi r^2 + 2\pi r h$

$A(r) = \pi r^2 + 2\pi r \cdot V \cdot (\pi r^2)^{-1}$

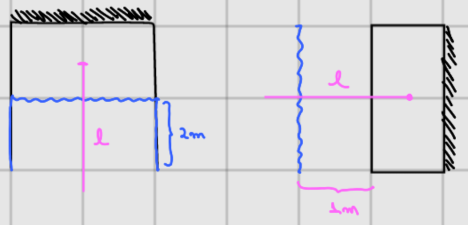
$\frac{dA}{dr} = 2\pi r + 2\pi V(\pi r^2)^{-1} - 2\pi r V(\pi r^2)^{-2} \cdot 2r = \frac{2\pi^2 r^3 + 2\pi V - 4V}{\pi r^2}$

$\frac{dA}{dr} = 0 \therefore 2\pi^2 r^3 = V(2\pi - 4) \therefore r_1 = \sqrt[3]{\frac{V(\pi-2)}{\pi^2}}$ cm

$A(r_1) = \frac{(3\pi-2)V}{\sqrt[3]{\ldots}}$

$\sqrt{\pi(\pi-2)}$

38-



$h > 2$

$l^2 = h^2 + b^2 \therefore l = (h^2+b^2)^{1/2}$

Os triângulos T_1 e externo são semelhantes:

$$\frac{h}{h-2} = \frac{b}{1} \therefore b = \frac{h}{h-2}$$

$$l(h) = \left[h^2 + \left(\frac{h}{h-2}\right)^2\right]^{1/2}$$

$$\frac{dl}{dh} = \frac{1}{2}\left[h^2 + \left(\frac{h}{h-2}\right)^2\right]^{-1/2} \cdot \left(2h + \underbrace{\frac{2h}{h-2} \cdot \frac{h-2-h}{(h-2)^2}}_{\frac{2h(h-1)}{h-2}}\right) = \frac{\frac{-4h(h-1)}{(h-2)^3}}{2\sqrt{h^2 + (h/h-2)^2}} = \frac{-4h(h-1)}{2(h-2)^3\sqrt{h^2 + (h/h-2)^2}}$$

Para que a derivada seja 0, $h \in \{0, 1\}$

40- $V = \frac{\pi r^2 h}{3} = 27 \therefore h = \frac{81}{\pi r^2}$

A_{min}?

$A = \pi r^2 + \pi r g$

$g^2 = h^2 + r^2 \therefore g = \sqrt{h^2 + r^2}$

$$g = \sqrt{\left(\frac{81}{\pi r^2}\right)^2 + r^2} = \frac{\sqrt{3^8 + \pi^2 r^6}}{\pi r^2}$$

$$A(r) = \pi r^2 + \frac{\pi r \sqrt{3^8 + \pi^2 r^6}}{\pi r^2} = \pi r^2 + \underbrace{r^{-1} \cdot \sqrt{3^8 + \pi^2 r^6}}_{3^8 + \pi^2 r^6 = a}$$

$$\frac{dA}{dr} = 2\pi r - r^{-2}\sqrt{3^8 + \pi^2 r^6} + \cancel{r^{-1}} \cdot \frac{1}{\cancel{2}}(3^8 + \pi^2 r^6)^{-1/2} \cdot \cancel{6}3\pi^2 r^{\cancel{5}4} = \frac{2\pi r^3 - \sqrt{a}}{r^2} + \frac{3\pi^2 r^4}{\sqrt{a}} = \frac{2\pi r^3\sqrt{a} - a + 2\pi^2 r^6}{r^2\sqrt{a}}$$

Para que a derivada seja 0,

$2\pi r^3\sqrt{a} + 2\pi^2 r^6 = a$

$2\pi r^3\sqrt{3^8 + \pi^2 r^6} + \cancel{2}\pi^2 r^6 = 3^8 + \cancel{\pi^2 r^6}$

a

$2\pi r^3\sqrt{a} = 3^8 - \pi^2 r^6$ $\qquad 6 = 3\cdot 2 \therefore 3^8\cdot 6 = 2\cdot 3^9$

$4\pi^2 r^6 a = 3^{16} - 2\pi^2 r^6\cdot 3^8 + \pi^4 r^{12}$

$4\pi^2 r^6\cdot 3^8 + \overset{3}{\cancel{4}}\pi^4 r^{12} = 3^{16} - 2\pi^2 r^6 \cdot 3^8 \cancel{+ \pi^4 r^{12}}$

$2\cdot 3^9\cdot \pi^2 r^6 + \pi^4 r^{12} = 3^{16} \longrightarrow \pi^2 r^6 = x$

$x^2 + 2\cdot 3^9\cdot x - 3^{16} = 0$

$x^2 + 19683x - 43046721 = 0$

Como se trata de medidas, descarta-se a raiz negativa.

$x \approx 1986,5$

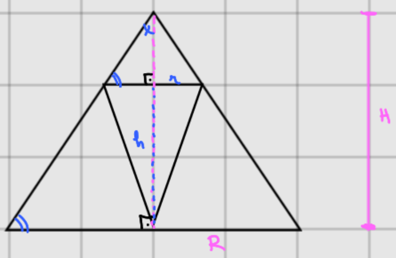
$r = \frac{\sqrt{x}}{\pi} \simeq 14,187$ cm

$h = 0,012$ cm

$A(14,187) = 1264,65$ cm^2

41- $C_1: H, R$; $C_2: h, r$

$0 \leq h \leq H$



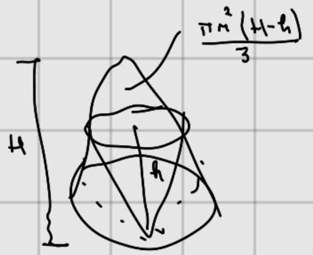
Por semelhança:

$\frac{H}{H-h} = \frac{R}{r}$

$h = \frac{H(R-r)}{R}$

$V = \frac{\pi r^2 h}{3} \therefore V(h) = \frac{\pi r^2 H(R-r)}{3R}$

$\frac{dV}{dh} = 0 \therefore \pi r^2 HR = \pi r^2 Hr$



42- $F = \frac{\mu m g}{\mu \operatorname{sen}\theta + \cos\theta} = \mu m g\cdot(\mu\operatorname{sen}\theta + \cos\theta)^{-1}$ $\qquad 0 \leq \theta \leq 2\pi$

$\frac{dF}{d\theta} = -\mu m g(\mu\operatorname{sen}\theta + \cos\theta)^{-2}\cdot(\mu\cos\theta - \operatorname{sen}\theta) = \frac{-\mu^2 m g\cos\theta + \mu m g\operatorname{sen}\theta}{(\mu\operatorname{sen}\theta + \cos\theta)^2}$

$\frac{dF}{d\theta} = 0 \therefore \operatorname{sen}\theta = \mu\cos\theta \therefore \mu = \operatorname{tg}\theta \therefore \theta = \arctan\mu$

43- $P(R) = \frac{E^2\cdot R}{(R+r)^2}$; $R \geq 0$

$\frac{dP}{dR} = \frac{E^2(R+r)^2 - E^2R\cdot 2(R+r)}{(R+r)^4}$

$\frac{dP}{dR} = 0 \therefore E^2(R+r)^2 = 2E^2R(R+r) \therefore 2R = R + r \therefore R = r$

$P(0) = 0$

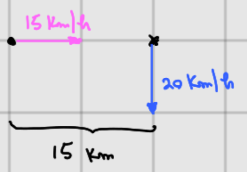
$P(r) = \frac{E^2 r}{4r^2} = \frac{E^2}{4r}$

44- a) $\frac{dE}{dv} = 3av^2\cdot\frac{L}{v-u} - \frac{av^3L}{(v-u)^2} = \frac{3av^2L(v-u) - av^3L}{(v-u)^2}$

$\frac{dE}{dv} = 0 \therefore v = 3v - 3u \therefore 2v = 3u \therefore v = \frac{3}{2}u$

$$E(3u/2) = a\cdot\frac{9}{8}u^3\cdot 2Lu = \frac{9}{4}aLu^4$$

46- 14h:



15h: