Pág. 89 (2.2) João G. G. A. Vaz

1- a) O valor de f(x) se aproxima de 5 quando x se aproxima de 2.

b) Sim. $\lim_{x \to a} f(x)$ não necessariamente é igual a f(a).

2- a) O limite pela esquerda de f(x), quando $x \approx 1$, é 3.

b) O limite pela direita de f(x), quando $x \approx 1$, é 7.

c) Não. Só existe limite se, e somente se, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

3- a) O limite de f(x), quando x tende a -3, tende a infinito.

b) O limite de f(x) pela direita, quando $x \approx 4$, tende a menos infinito.

4- a) 3

b) 1

c) $\nexists$, pois $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$.

d) 3

e) 4

f) $\nexists$, pois o $4 \notin D$.

5- a) 2

b) 1

c) 4

d) $\nexists$, pois os limites laterais são diferentes

e) 3

6- a) 4

b) 4

c) 4

d) $\nexists$, pois $-3 \notin D$.

e) 1

f) -1

g) $\nexists$, pois os limites laterais são diferentes

h) 1

i) 2

j) $\nexists$, pois $2 \notin D$.

k) 3

l) 3

7- a) -1

b) -2

c) $\nexists$, pois os limites laterais são diferentes.

d) 2

e) 0

f) $\nexists$, pois os limites laterais são distintos.

g) 1

h) 3

8- a) $-\infty$

b) $+\infty$

c) $-\infty$

d) $+\infty$

e) $x=-3$; $x=2$; $x=5$

9- a) $-\infty$

b) $+\infty$

c) $+\infty$

d) $-\infty$

e) $+\infty$

f) $x=-7$, $x=-3$, $x=0$, $x=6$

10- a) 150 ; 300

29- $\lim\limits_{x \to -3^+} \frac{x+2}{x+3} = \lim\limits_{x \to -3^+} \frac{-1}{0^+} = -\infty$

31- $\lim\limits_{x \to 1} \frac{2-x}{(x-1)^2} = \lim\limits_{x \to 1} \frac{2-1}{(1-1)^2} = \frac{1}{0} = \infty$

33- $\lim_{x \to 3^+} \ln(x^2-9) = \lim_{x \to 3^+} \ln[(x-3)(x+3)] = \lim_{x \to 3^+} \ln(x-3) + \ln(x+3) = \ln 0 + \ln 6$

Pg 99 (2.3) João G. G. A. Vaz

1- a) $\lim\limits_{x\to 2}\left[f(x)+5g(x)\right] = \lim\limits_{x\to 2} f(x) + 5\cdot\lim\limits_{x\to 2} g(x) = 4 - 10 = -6$

b) $\lim\limits_{x\to 2}\left[g(x)\right]^3 = \left[\lim\limits_{x\to 2} g(x)\right]^3 = (-2)^3 = -8$

c) $\lim\limits_{x\to 2}\sqrt{f(x)} = \sqrt{\lim\limits_{x\to 2} f(x)} = 2$

d) $\lim\limits_{x\to 2}\dfrac{3f(x)}{g(x)} = \dfrac{3\cdot\lim\limits_{x\to 2} f(x)}{\lim\limits_{x\to 2} g(x)} = \dfrac{3\cdot 4}{-2} = -6$

e) $\lim\limits_{x\to 2}\dfrac{g(x)}{h(x)} = \dfrac{\lim\limits_{x\to 2} g(x)}{\lim\limits_{x\to 2} h(x)} = \dfrac{-2}{0} \rightsquigarrow \nexists \lim\limits_{x\to 2}\dfrac{g(x)}{h(x)}$, pois $\lim\limits_{x\to 2} h(x) = 0 \rightsquigarrow$ indeterminação

f) $\lim\limits_{x\to 2}\dfrac{g(x)h(x)}{f(x)} = \dfrac{\lim\limits_{x\to 2} g(x)\cdot\lim\limits_{x\to 2} h(x)}{\lim\limits_{x\to 2} f(x)} = \dfrac{-2\cdot 0}{4} = \dfrac{0}{4} = 0$

2- a) $\lim\limits_{x\to 2}\left[f(x)+g(x)\right] = \lim\limits_{x\to 2} f(x) + \lim\limits_{x\to 2} g(x) = 2 + 0 = 2$

b) $\lim\limits_{x\to 1}\left[f(x)+g(x)\right] = \lim\limits_{x\to 1} f(x) + \lim\limits_{x\to 1} g(x) \longrightarrow \nexists \lim\limits_{x\to 1}\left[f(x)+g(x)\right]$, pois $\nexists \lim\limits_{x\to 1} g(x)$: $\lim\limits_{x\to 1^-} g(x) \neq \lim\limits_{x\to 1^+} g(x)$

c) $\lim\limits_{x\to 0}\left[f(x)g(x)\right] = \lim\limits_{x\to 0} f(x)\cdot\lim\limits_{x\to 0} g(x) = 0\cdot 1{,}5 = 0$

d) $\lim\limits_{x\to -1}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{\lim\limits_{x\to -1} f(x)}{\lim\limits_{x\to -1} g(x)} = \dfrac{-1}{0} \longrightarrow \nexists \lim\limits_{x\to -1}\dfrac{f(x)}{g(x)}$, pois $\lim\limits_{x\to -1} g(x) = 0$.

e) $\lim\limits_{x\to 2}\left[x^3 f(x)\right] = \lim\limits_{x\to 2} x^3\cdot\lim\limits_{x\to 2} f(x) = 8\cdot 2 = 16$

f) $\lim\limits_{x\to 1}\sqrt{3+f(x)} = \sqrt{\lim\limits_{x\to 1}\left[3+f(x)\right]} = \sqrt{\lim\limits_{x\to 1} 3 + \lim\limits_{x\to 1} f(x)} = \sqrt{3+1} = 2$

3- $\lim\limits_{x\to -2}(3x^4+2x^2-x+1) = \lim\limits_{x\to -2}(3\cdot 16+2\cdot 4+2+1) = \lim\limits_{x\to -2} 59 = 59$

4- $\lim\limits_{x\to -1}(x^4-3x)(x^2+5x+3) = \lim\limits_{x\to -1}(1+3)(1-5+3) = \lim\limits_{x\to -1} -4 = -4$

5- $\lim\limits_{t\to -2}\dfrac{t^4-2}{2t^2-3t+2} = \dfrac{14}{4+6+2} = \dfrac{14}{12} = \dfrac{7}{6}$

6. $\lim\limits_{u\to -2}\sqrt{u^4+3u+6} = \sqrt{\lim\limits_{u\to -2} u^4+3u+6} = \sqrt{16-6+6} = \sqrt{16} = 4$

7- $\lim\limits_{x\to 8}(1+\sqrt[3]{x})(2-6x^2+x^3) = (1+\lim\limits_{x\to 8}\sqrt[3]{x})(2+\lim\limits_{x\to 8} -6x^2+x^3) = (1+2)(2+128) = 3\cdot 130 = 390$

8- $\lim\limits_{t\to 2}\left[\dfrac{t^2-2}{t^3-3t+5}\right]^3 = \left[\lim\limits_{t\to 2}\left(\dfrac{t^2-2}{t^3-3t+5}\right)\right]^3 = \left(\dfrac{2}{7}\right)^3 = \dfrac{8}{343}$

9- $\lim\limits_{x\to 2}\sqrt{\dfrac{2x^2+1}{3x-2}} = \sqrt{\lim\limits_{x\to 2}\dfrac{2x^2+1}{3x-2}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

10- a) A função $f(x) = \dfrac{x^2+x-6}{x-2}$ é idêntica à $g(x)$ - definida como $g(x) = x+3$ -, exceto no ponto $x=2$; no qual $f(2) = \dfrac{0}{0}$ e $g(2) = 5$.

b) Embora sejam diferentes no ponto citado, para valores de x próximos de 2, $f(x)$ se aproxima de 5. Portanto, diz-se que os limites são iguai

11- $\lim\limits_{x\to 2} \frac{x^2+x-6}{x-2} = \lim\limits_{x\to 2} \frac{(x+3)(x-2)}{(x-2)} = 5$

12- $\lim\limits_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4} = \lim\limits_{x\to -4} \frac{(x+1)(x+4)}{(x-1)(x+4)} = \lim\limits_{x\to -4} \frac{x+1}{x-1} = \frac{-3}{-5} = \frac{3}{5}$

13- $\lim\limits_{x\to 2} \frac{x^2-x+6}{x-2} = \nexists$, pois $S(x^2-x+6) \notin \mathbb{R}$.

14- $\lim\limits_{x\to -1} \frac{x^2-4x}{x^2-3x-4} = \lim\limits_{x\to -1} \frac{x(x-4)}{(x-4)(x+1)} = \lim\limits_{x\to -1} \frac{x}{x+1} = \nexists$

15- $\lim\limits_{t\to -3} \frac{t^2-9}{2t^2+7t+3} = \lim\limits_{t\to -3} \frac{(t+3)(t-3)}{(t+0,5)(t+3)} = \lim\limits_{t\to -3} \frac{t+3}{t+0,5} = \frac{0}{-2,5} = 0$

16- $\lim\limits_{x\to -1} \frac{2x^2+3x+1}{x^2-2x-3} = \lim\limits_{x\to -1} \frac{(x+0,5)(x+1)}{(x-3)(x+1)} = \lim\limits_{x\to -1} \frac{x+0,5}{x-3} = \frac{-0,5}{-4} = \frac{1}{8}$

17- $\lim\limits_{h\to 0} \frac{(-5+h)^2-25}{h} = \lim\limits_{h\to 0} \frac{25-10h+h^2-25}{h} = \lim\limits_{h\to 0} \frac{h(-10+h)}{h} = \lim\limits_{h\to 0} -10+h = -10$

18- $\lim\limits_{h\to 0} \frac{(2+h)^3-8}{h} = \lim\limits_{h\to 0} \frac{(4+4h+h^2)(2+h)-8}{h} = \lim\limits_{h\to 0} \frac{(8+4h+8h+4h^2+2h^2+h^3)-8}{h} = \lim\limits_{h\to 0} \frac{h^3+6h^2+12h}{h} = \lim\limits_{h\to 0} h^2+6h+12 = 12$

19- $\lim\limits_{x\to -2} \frac{x+2}{x^3+8} = \lim\limits_{x\to -2} \frac{x+2}{(x+2)(x^2-2x+4)} = \lim\limits_{x\to -2} \frac{1}{x^2-2x+4} = \frac{1}{12}$

$(a^3+b^3) = (a+b)(a^2-ab+b^2)$

20- $\lim\limits_{t\to 1} \frac{t^4-1}{t^3-1} = \lim\limits_{t\to 1} \frac{(t^2+1)(t^2-1)}{(t-1)(t^2+t+1)} = \lim\limits_{t\to 1} \frac{(t^2+1)(t-1)(t+1)}{(t-1)(t^2+t+1)} = \lim\limits_{t\to 1} \frac{(t^2+1)(t+1)}{t^2+t+1} = \frac{(1+1)(1+1)}{1+1+1} = \frac{4}{3}$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

21- $\lim\limits_{h\to 0} \frac{\sqrt{9+h}-3}{h} = \lim\limits_{h\to 0} \frac{\sqrt{9+h}-3}{h} \cdot \left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right) = \lim\limits_{h\to 0} \frac{9+h-9}{h(\sqrt{9+h}+3)} = \lim\limits_{h\to 0} \frac{1}{\sqrt{9+h}+3} = \frac{1}{3+3} = \frac{1}{6}$

22- $\lim\limits_{u\to 2} \frac{\sqrt{4u+1}-3}{u-2} = \lim\limits_{u\to 2} \frac{\sqrt{4u+1}-3}{u-2} \cdot \left(\frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}\right) = \lim\limits_{u\to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)} = \lim\limits_{u\to 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)} = \frac{4}{\sqrt{4\cdot 2+1}+3} = \frac{4}{6} = \frac{2}{3}$

23- $\lim\limits_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim\limits_{x\to -4} \frac{(x+4)}{4x} \cdot \frac{1}{(4+x)} = \lim\limits_{x\to -4} \frac{1}{4x} = \frac{1}{-16} = \frac{-1}{16}$

24- $\lim\limits_{x\to -1} \frac{x^2+2x+1}{x^4-1} = \lim\limits_{x\to -1} \frac{x^2+2x+1}{(x^2-1)(x^2+1)} = \lim\limits_{x\to -1} \frac{(x+1)^2}{(x+1)(x-1)(x^2+1)} = \lim\limits_{x\to 1} \frac{x+1}{(x-1)(x^2+1)} = \nexists$

25- $\lim\limits_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \lim\limits_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} \cdot \left(\frac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\right) = \lim\limits_{t\to 0} \frac{1+t-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim\limits_{t\to 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim\limits_{t\to 0} \frac{2}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$

26- $\lim\limits_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim\limits_{t\to 0} \left(\frac{t^2}{t^3+t^2}\right) = \lim\limits_{t\to 0} \frac{1}{t+1} = \frac{1}{1} = 1$

27- $\lim\limits_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2} = \lim\limits_{x\to 16} \frac{4-\sqrt{x}}{x(16-x)} \cdot \left(\frac{4+\sqrt{x}}{4+\sqrt{x}}\right) = \lim\limits_{x\to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})} = \lim\limits_{x\to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16\cdot 8} = \frac{1}{128}$

28- $\lim\limits_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} = \lim\limits_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim\limits_{h\to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \lim\limits_{h\to 0} \frac{3-3-h}{3(3+h)} \cdot \frac{1}{h} = \lim\limits_{h\to 0} \frac{-1}{3(3+h)} = \frac{-1}{9}$

29- $\lim\limits_{t\to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t}\right) = \lim\limits_{t\to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}} = \lim\limits_{t\to 0} \frac{1-(1+t)}{t\sqrt{1+t}(1+\sqrt{1+t})} = \lim\limits_{t\to 0} \frac{-1}{\sqrt{1+t}+1+t} = \lim\limits_{t\to 0} \frac{-1}{\sqrt{1}+1+0} = \frac{-1}{2}$

30- $\lim\limits_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim\limits_{x\to -4} \frac{\sqrt{x^2+9}-5}{(x+4)} \cdot \left(\frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}\right) = \lim\limits_{x\to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)} = \lim\limits_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-8}{10} = \frac{-4}{5}$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

diferença de cubos ou desenvolve e simplifica

31- $\lim\limits_{h\to 0} \frac{(x+h)^3-x^3}{h} = \lim\limits_{h\to 0} \frac{(x^2+2hx+h^2)(x+h)-x^3}{h} = \lim\limits_{h\to 0} 3x^2+3xh+h^2 = 3x^2$

32- $\lim\limits_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim\limits_{h\to 0} \frac{x^2-(x+h)^2}{x^2(x+h)^2} \cdot \frac{1}{h} = \lim\limits_{h\to 0} \frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2} \cdot \frac{1}{h} = \lim\limits_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$

Pg. 117 (2.5) João G. G. A. Vaz

11- $g(2)=6$; $f(2)=?$

$$\lim_{x\to 2}\left[3f(x)+f(x)g(x)\right]=36$$

$$3\lim_{x\to 2}f(x)+6\lim_{x\to 2}f(x)=36 \ ; \ \lim_{x\to 2}f(x)\cdot(3+6)=36$$

$\lim_{x\to 2}f(x)=4 \rightsquigarrow f(2)=4$, pois f é contínua

12- $f(x)=x^2+\sqrt{7-x}$

$$\lim_{x\to 4}f(x)=16+\sqrt{3}$$

$$f(4)=4^2+\sqrt{7-4}=16+\sqrt{3}$$

13- $f(x)=(x+2x^3)^4$

$$\lim_{x\to -1}f(x)=(-1-2)^4=(-3)^4=3^4=81$$

$$f(-1)=(-1-2)^4=(-3)^4=3^4=81$$

14- $h(t)=\dfrac{2t-3t^2}{1+t^3}$

$$\lim_{t\to 1}\frac{2t-3t^2}{1+t^3}=\frac{2\cdot 1-3\cdot 1}{2}=\frac{-1}{2}$$

17- $f(x)=\dfrac{1}{x+2}$

$$\lim_{x\to -2}\frac{1}{x+2}=\frac{1}{0} \ ; \ \nexists \lim_{x\to -2}f(x)$$

$f(-2)=\frac{1}{0}$; $\nexists f(-2)$

$f(x)$ em $x=-2$ é descontínua.

19- $f(x)=\begin{cases} e^x, \text{ se } x<0 \\ x^2, \text{ se } x\geq 0 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to 0}f(x)=0^2=0 \\ f(0)=0 \end{array}\right\} \lim_{x\to 0}f(x)=f(x)$$

$f(x)$ é contínua em $x=0$

20- $f(x)=\begin{cases} \dfrac{x^2-x}{x^2-1}, \text{ se } x\neq 1 \\ 1, \text{ se } x=1 \end{cases}$

$$\left.\begin{array}{l} \lim_{x\to 1}f(x)=\lim_{x\to 1}\dfrac{x^2-x}{x^2-1}=\lim_{x\to 1}\dfrac{x(x-1)}{(x-1)}=\lim_{x\to 1}x=1 \\ f(1)=1 \end{array}\right\} \lim_{x\to 1}f(x)=f(1)$$

$f(x)$ é contínua em $x=1$.

f $\begin{cases} \cos x, \text{ se } x<0 \\ \end{cases}$

21- $f(x) = \begin{cases} 0, \text{ se } x = 0 \\ 1 - x^2, \text{ se } x > 0 \end{cases}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} \cos x = \cos 0 = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} 1 - x^2 = 1 - 0 = 1$

$\nexists \lim\limits_{x \to 0} f(x)$

$f(0) = 0$

$f(0)$ é descontínua em $x = 0$

45- Para que $f(x)$ seja contínua em (a, b), $f(x) = \lim\limits_{x \to \alpha} f(x)$ para todo $\alpha \in (a, b)$.

$f(0) = c \cdot 0^2 + 2 \cdot 0 = 0$

$f(1) = c \cdot 1^2 + 2 \cdot 1 = c + 2$

$f(2) = 2^3 - 2c = 8 - 2c$

$\lim\limits_{x \to 2^-} f(x) = cx^2 + 2x = 4c + 4$

$\lim\limits_{x \to 2^+} f(x) = x^3 - cx = 8 - 2c$

Para o limite existir, os limites laterais devem ser iguais.

$4c + 4 = 8 - 2c$; $6c = 4$; $c = 4/6 = 2/3$.

Para que a função seja contínua em todas as partes, $c = 2/3$.

46- $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, \text{ se } x < 2 \\ ax^2 - bx + 3, \text{ se } 2 \leq x < 3 \\ 2x - a + b, \text{ se } x \geq 3 \end{cases}$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) \therefore \lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2} = \lim\limits_{x \to 2} ax^2 - bx + 3 \therefore \lim\limits_{x \to 2} x + 2 = \lim\limits_{x \to 2} ax^2 - bx + 3 \therefore \lim\limits_{x \to 2} ax^2 - bx + 3 = 4 \therefore 4a - 2b = 1$

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) \therefore \lim\limits_{x \to 3} ax^2 - bx + 3 = \lim\limits_{x \to 3} 2x - a + b \therefore 9a - 3b + 3 = 6 - a + b$; $9a - 3b = -a + b + 3$; $10a - 4b = 3$

$4a - 2b = 1$

$10a - 4b = 3$

$a = 1/2$; $b = -1/2$

Pg 128 (2.6) João G. G. A. Vaz

15- $\lim\limits_{x\to\infty} \frac{1}{2x+3} = \lim\limits_{x\to\infty} \frac{1}{x(2+\frac{3}{x})} = \lim\limits_{x\to\infty} \frac{1}{2x} = 0$

17- $\lim\limits_{x\to-\infty} \frac{1-x-x^2}{2x^2-7} = \lim\limits_{x\to-\infty} \frac{x(-x-1+\frac{1}{x})}{x(2x-\frac{7}{x})} = \lim\limits_{x\to-\infty} \frac{-x-1}{2x} = \lim\limits_{x\to-\infty} \frac{x(-1-\frac{1}{x})}{2x} = \lim\limits_{x\to-\infty} \frac{-1}{2} = \frac{-1}{2}$

19- $\lim\limits_{t\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2} = \frac{t(\frac{\sqrt{t}}{t}+t)}{t(2-t)} = \frac{\sqrt{\frac{1}{t}}+t}{2-t} = \frac{t}{t(\frac{2}{t}-1)} = -1$

21- $\lim\limits_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)} = \lim\limits_{x\to\infty} \frac{4x^4+4x^2+1}{(x^2-2x+1)(x^2+x)} = \lim\limits_{x\to\infty} \frac{x^4(4+\frac{4}{x^2}+\frac{1}{x^4})}{x^4(1-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3})} = 4$

23- $\lim\limits_{x\to\infty} \frac{\sqrt{9x^6-x}}{x^3+1} = \lim\limits_{x\to\infty} \frac{\sqrt{x^6(9-\frac{1}{x^5})}}{x^3(1+\frac{1}{x^3})} = \lim\limits_{x\to\infty} \frac{x^3\sqrt{9-\frac{1}{x^5}}}{x^3(1+\frac{1}{x^3})} = \frac{3}{1} = 3$

25- $\lim\limits_{x\to\infty} (\sqrt{9x^2+x}-3x) = \lim\limits_{x\to\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x} = \lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2(9+\frac{1}{x})}+3x} = \lim\limits_{x\to\infty} \frac{x}{x(\sqrt{9+\frac{1}{x}}+3)} = \frac{1}{6}$

Pg 137 (2.7) João G. G. A. Vaz

5- $y = -3x^2 + 4x$; $(2,-4)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-3(x+h)^2 + 4(x+h) - (-3x^2 + 4x)}{h} = \lim_{h \to 0} \frac{-3h^2 - 6xh + 4h}{h} = \lim_{h \to 0} -3h - 6x + 4 = -6x + 4$$

$f'(2) = -12 + 4 = -8$

6- $y = x^3 - 3x + 1$; $(2,3)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - 3(x+h) + 1 - (x^3 - 3x + 1)}{h} = \lim_{h \to 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x - 3x - 3h + 1 - x^3 + 3x - 1}{h} = \lim_{h \to 0} \frac{h^3 + 3x^2h + 3xh^2 - 3h}{h} = 3x^2 - 3$$

$f'(2) = 12 - 3 = 9$

7- $y = \sqrt{x}$; $(1,1)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2x}$$

$f'(1) = \frac{1}{2}$

8- $y = \frac{2x+1}{x+2}$; $(1,1)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{2(x+h)+1}{(x+h)+2} - \frac{2x+1}{x+2}}{h} = \lim_{h \to 0} \frac{(x+2)(2x+2h+1) - (x+h+2)(2x+1)}{(x+2)(x+h+2)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{2x^2 + 2xh + x + 4x + 4h + 2 - 2x^2 - x - 2xh - h - 4x - 2}{x^2 + 4x + xh + 2h + 4} \cdot \frac{1}{h} = \frac{3}{x^2 + 4x + 4} = \frac{3}{(x+2)^2}$$

$f'(1) = \frac{3}{3^2} = \frac{1}{3}$

15- $d = \frac{1}{t^2}$; $d = vt$; $v(t) = \frac{1}{t^3}$

$v(1) = 1$ m/s ; $v(2) = \frac{1}{8}$ m/s ; $v(3) = \frac{1}{9}$ m/s ; $v(a) = \frac{1}{a^3}$ m/s

Pg 164 (3.1) João G. G. A. Vaz

3- $f(x) = 186{,}5 \qquad f'(x) = 0$

4- $f(x) = \sqrt{30} \qquad f'(x) = 0$

5- $f(x) = 5x - 1 \qquad f'(x) = 5$

6- $F(x) = -4x^{10} \qquad F'(x) = -40x^9$

7- $f(x) = x^3 - 4x + 6 \qquad f'(x) = 3x^2 - 4$

8- $f(t) = 1{,}4t^5 - 2{,}5t^2 + 6{,}7 \qquad f'(t) = 7t^4 - 5t$

9- $g(x) = x^2(1-2x) \qquad g'(x) = 2x(1-2x) + x^2 \cdot (-2) = 2x - 4x^2 - 2x^2 = -6x^2 + 2x$

10- $h(x) = (x-2)(2x+3) \qquad h'(x) = u'v + uv' = 1(2x+3) + (x-2)(2) = 2x+3+2x-4 = 4x-1$

11- $y = x^{-2/5} \qquad y' = \frac{-2}{5} \cdot x^{-7/5}$

12- $B(y) = cy^{-6} \qquad B'(y) = -6y^{-7}$

13- $A(s) = \frac{-12}{s^5} \qquad A'(s) = \frac{u'v - uv'}{v^2} = \frac{0 \cdot s^5 - 12 \cdot 5s^4}{s^{10}} = \frac{-60s^4}{s^{10}} = -5s^{-6} = \frac{-60}{s^6} = -60s^{-6}$

14- $y = x^{5/3} - x^{2/3} \qquad y' = \frac{5}{3}x^{2/3} - \frac{2}{3}x^{-1/3} = \frac{5x^{2/3} - 2x^{-1/3}}{3}$

15- $R(a) = (3a+1)^2 \qquad R'(a) = 18a + 6$

16- $h(t) = \sqrt[4]{t} - 4e^t \qquad h'(t) = \frac{1}{4} \cdot t^{-3/4} - (0 \cdot e^t + 4e^t) = \frac{1}{4}t^{-3/4} - 4e^t$

17- $S(p) = \sqrt{p} - p \qquad \sqrt{x^3} \qquad S'(p) = \frac{1}{2} \cdot p^{-1/2} - 1 = \frac{\sqrt{\frac{1}{p}} - 2}{2}$

18- $y = \sqrt{x}(x-1) = \sqrt{x} \cdot x - x \qquad y' = \frac{1}{3}x^{-2/3} - 1$

19- $y = 3e^x + \frac{4}{\sqrt[3]{x}} \qquad y' = 3e^x + \frac{0 - 4 \cdot \frac{1}{3}x^{-2/3}}{x^{2/3}} = 3e^x - \frac{4x^{-2/3}}{3x^{2/3}} = \frac{9e^x - 4x^{-4/3}}{3}$

$\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$

20- $S(R) = 4\pi R^2 \qquad S'(R) = f'g + fg' = 0 \cdot R^2 + 4\pi \cdot 2R = 8\pi R$

21- $h(u) = Au^3 + Bu^2 + Cu \qquad h'(u) = 3Au^2 + 2Bu + C$

$\sqrt{\frac{x^3}{4}} \qquad \sqrt{4x} \qquad \sqrt{\frac{9}{4x}} = \frac{3}{2\sqrt{x}} = \frac{3\sqrt{x}}{2x}$

22- $y = \frac{\sqrt{x} + x}{x^2} \qquad y' = \frac{f'g - fg'}{g^2} = \frac{\frac{1}{2}(-3\sqrt{x^3} - 2x^2)}{x^4} = \frac{-3\sqrt{x^3} - 2x^2}{2x^4}$

23- $y = \frac{x^2 + 4x + 3}{\sqrt{x}} \qquad y' = \frac{f'g - fg'}{g^2} = \frac{(2x+4)(\sqrt{x}) - (x^2+4x+3)(\frac{1}{2} \cdot x^{-1/2})}{x} = \left(\sqrt{4x^3} + \sqrt{16x} - \sqrt{\frac{1}{4x}} \cdot x^2 - \sqrt{\frac{1}{4x}} \cdot 4x - \sqrt{\frac{1}{4x}} \cdot 3\right) \cdot \frac{1}{x} = \frac{1}{x}\left(2x\sqrt{x} + 4x - \frac{x\sqrt{x}}{2} - 2\sqrt{x} - \frac{3\sqrt{x}}{2x}\right) = \frac{3\sqrt{x}}{2} + 4 - \frac{2\sqrt{x}}{x} - \frac{3\sqrt{x}}{2x^2}$

24- $g(u) = \sqrt{2}u + \sqrt{3u} \qquad g'(u) = \left((\sqrt{2})' \cdot u + \sqrt{2} \cdot (u)'\right) + \left((\sqrt{3})' \cdot \sqrt{u} + (\sqrt{3} \cdot \sqrt{u}')\right) = (\sqrt{2}) + (\sqrt{3} \cdot \frac{1}{2} \cdot u^{-\frac{1}{2}}) = \sqrt{2} + \frac{\sqrt{3} \cdot \sqrt{\frac{1}{u}}}{2} = \frac{2\sqrt{2} + \sqrt{\frac{3}{u}}}{2}$

25- $j(x) = x^{2{,}4} + e^{2{,}4} \qquad j'(x) = 2{,}4x^{1{,}4}$

26- $K(n) = e^n + n^e \qquad K'(n) = e^n + en^{(e-1)} = e^n + \frac{en^e}{n}$

27- $H(x) = (x + x^{-1})^3 \qquad H'(x) = 3x^2 + 3 - 3x^{-2} - 3x^{-4}$

28- $y = ae^v + \frac{b}{v} + \frac{c}{v^2} \qquad y' = ae^v + \frac{(0 \cdot v - b)}{v^2} + \frac{(0 \cdot v^2 - 2c)}{v^4} = ae^v - \frac{b}{v^2} - \frac{2c}{v^4} = ae^v - \frac{v^2b - 2c}{v^4}$

$\left[\frac{f}{g}\right]' = \frac{f'g - fg'}{g^2}$

29- $u = \sqrt[5]{t} + 4\sqrt{t^5} \qquad u' = \frac{1}{5} \cdot t^{-4/5} + 4 \cdot \frac{5}{2}t^{3/2} = \frac{t^{-\frac{4}{5}}}{5} + 10t^{\frac{3}{2}}$

30- $v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 \qquad v' = 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3}$

31- $z = \frac{A}{y^{10}} + Be^{y}$ $\qquad$ $z' = -Ay^{-11} + Be^{y}$

32- $y = e^{x+1} + 1$ $\qquad$ $y' = e \cdot e^{x} + 1 = e \cdot e^{x} = e^{x+1}$

33- $y = \sqrt[4]{x}$ $\qquad$ $y' = \frac{1}{4} x^{-3/4}$

$f'(1) = \frac{1}{4}$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 1 = \frac{1}{4}(x-1)$

$t: y = \frac{x+3}{4}$

34- $y = x^4 + 2x^2 - x$ $\qquad$ $y' = 4x^3 + 4x$

$f'(1) = 4 + 4 = 8$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 2 = 8(x-1)$

$t: y = 8x - 6$

35- $y = x^4 + 2e^x$ $\qquad$ $y' = 4x^3 + 2e^x$

$f'(0) = 2$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 2 = 2(x-0)$

"capota e inverte o sinal"

$t: y = 2x + 2$ $\qquad$ $N: y = \frac{-x}{2} + 2$

36- $y = x^2 - x^4$ $\qquad$ $y' = 2x - 4x^3$

$f'(1) = 2 - 4 = -2$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 0 = -2(x-1)$

$t: y = -2x + 2$ $\qquad$ $N: y = \frac{x}{2} + 2$

37- $y = 3x^2 - x^3$ $\qquad$ $y' = 6x - 3x^2$

$f'(1) = 6 - 3 = 3$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 2 = 3(x-1)$

$t: y = 3x - 1$

38- $y = x - \sqrt{x}$ $\qquad$ $y' = 1 - \frac{1}{2} x^{-1/2}$

$f'(1) = 1 - \frac{1}{2} = \frac{1}{2}$ $\qquad$ $y - y_0 = m(x - x_0) \rightsquigarrow y - 0 = \frac{1}{2}(x-1)$

$t: y = \frac{x-1}{2}$

39- $f(x) = x^4 - 2x^3 + x^2$ $\qquad$ $f'(x) = 4x^3 - 6x^2 + 2x$

40- $f(x) = x^5 - 2x^3 + x - 1$ $\qquad$ $f'(x) = 5x^4 - 6x^2 + 1$

43- $f(x) = 10x^{10} + 5x^5 - x$ $\qquad$ $f'(x) = 100x^9 + 25x^4 - 1$ $\qquad$ $f''(x) = 900x^8 + 100x^3$

44- $G(r) = \sqrt{r} + \sqrt[3]{r}$ $\qquad$ $G'(r) = \frac{1}{2} r^{-1/2} + \frac{1}{3} r^{-2/3}$ $\qquad$ $G''(r) = \frac{-1}{4} r^{-3/2} - \frac{2}{9} r^{-5/3}$

45- $f(x) = 2x - 5x^{3/4}$ $\qquad$ $f'(x) = 2 - \frac{15}{4} x^{-1/4}$ $\qquad$ $f''(x) = \frac{15}{16} x^{-5/4}$

46- $f(x) = e^x - x^3$ $\qquad$ $f'(x) = e^x - 3x^2$ $\qquad$ $f''(x) = e^x - 6x$

51- $y = 2x^3 + 3x^2 - 12x + 1$; pontos onde a tangente é horizontal

Para a tangente ser horizontal, $f'(x) = 0$

$f'(x) = 6x^2 + 6x - 12 = 0$; $x \in \{-2, 1\}$.

Nos pontos $P_1(x_1, f(x_1))$ e $P_2(x_2, f(x_2))$, a tangente é horizontal; ou seja, $P_1(-2, 21)$ e $P_2(1, -6)$

52- Para que a reta tangente seja horizontal, $f'(x) = 0$.

$f'(x) = e^x - 2$. $e^x - 2 = 0$; $e^x = 2$; $x = \ln 2$. Portanto, em $x = \ln 2$, o gráfico de $f(x)$ tem tangente horizontal.

53- $y = 2e^x + 3x + 5x^3$; $y' = 2e^x + 3 + 15x^2$

$y' = 2$. Logo, $2e^x + 15x^2 = -1$. No entanto, isso é impossível.

Não importa o valor que x assuma, $2e^x + 15x^2$ é estritamente positivo.

$2e^x$ e $15x^2$ serão SEMPRE positivos.

54- $r: y = x\sqrt{x}$ $\quad r': y' = x'\sqrt{x} + x(\sqrt{x})' = \sqrt{x} + \frac{1}{2}x \cdot x^{-1/2}$

$s: y = 3x + 1$ $\quad r \parallel s$ se e somente se $m_r = m_s$.

$\sqrt{x} + \frac{x}{2} \cdot \sqrt{\frac{1}{x}} = 3$; $\sqrt{x} + \frac{\sqrt{x}}{2} = 3$; $\frac{3\sqrt{x}}{2} = 3$; $\sqrt{x} = 2$; $x = 4$

$P_1(4, f(4))$; $P_1(4, 8)$. $t: y - y_0 = m(x - x_0) = y - 8 = 3(x - 4)$; $t: y = 3x - 12 + 8$; $t: y = 3x - 4$

55- $r: y = x^3 + 1$; $r': y' = 3x^2$; $s: y = 12x + 1$

t_1 e t_2 são paralelas a s.

$3x^2 = 12$; $x^2 = 4$; $x = \pm 2$

$t_1: y = 12x + 2$; $t_2: y = 12x - 2$

56- $r: y = 1 + 2e^x - 3x$; $r': y' = 2e^x - 3$

$s: y = 3x - 5$; $t \parallel s$.

$r': y' = 2e^x - 3 = 3$; $2e^x = 6$; $e^x = 3$; $x = \ln 3$.

No ponto $(\ln 3, 0)$, a reta tangente t é paralela à reta s.

$t: y - y_0 = m(x - x_0): y - 0 = 3(x - \ln 3)$

$t: y = 3x - 3\ln 3$

57- $r: y = x^2 - 5x + 4$; $r': y' = 2x - 5$

$s: y = \frac{x-5}{3}$; Para que a normal à reta tangente de r seja paralela a s, $m_m = m_s = \frac{1}{3}$.

coef. linear

$m: y = \frac{x}{3} + b$

Coef. ang. da normal

derivada = coef. ang. da tg

A normal é perpendicular à tangente. Logo, $\frac{1}{3} \cdot r' = -1$, $r' = -3$.

$(1, f(1))$

$2x - 5 = -3$; $2x = 2$; $x = 1$. No ponto $(1, 0)$ a normal é paralela à reta s.

$m: y - y_0 = m(x - x_0) \therefore m: y - 0 = \frac{1}{3}(x - 1)$; $m: y = \frac{x-1}{3}$

58- $r: y = x - x^2$; $r': y' = 1 - 2x$

$P(1, 0)$: $m_t = -1$; $t: y - y_0 = m(x - x_0) \therefore y - 0 = -1(x + 1) \therefore t: y = -x - 1$

$m: y = x + b$; $0 = 1 + b$; $b = -1$; $m: y = x - 1$

$x - x^2 = x - 1$; $-x^2 = -1$; $x^2 = 1$; $x = \pm 1$. A reta m intercepta a parábola r em $(1, f(1))$ e em $(-1, f(-1))$.

Ou seja, em $P_1(1, 0)$ e em $P_2(-1, -2)$.

59- $r: y = x^2$; $r': y' = 2x$

$P(0, -4)$: $m_t = 0$; $t: y - y_0 = m(x - x_0) \therefore y + 0 = 0(x - 0) \therefore t: y = 0$

64. $y'' + y' - 2y = x^2$

$y = Ax^2 + Bx + C$

$y'' + 2Ax + B - 2(Ax^2 + Bx + C) = x^2$

$2A + 2Ax + B - 2Ax^2 - 2Bx - 2C = x^2$

77. $\lim\limits_{x \to 1} \dfrac{x^{1000} - 1}{x - 1} = \left[f(x) = x^{1000}\right] (1)' = 1000x^{999} = 1000 \cdot 1^{999} = 1000$

Pg 171 (3.2) João G. G. A. Vaz

3- $f(x) = (x^3+2x)e^x$

$f'(x) = (3x^2+2)e^x + (x^3+2x)e^x = e^x(x^3+3x^2+2x+2)$

4- $g(x) = \sqrt{x}\,e^x$

$g'(x) = \frac{1}{2}x^{-1/2}e^x + \sqrt{x}e^x = e^x\left(\frac{x^{-1/2}}{2} + x^{1/2}\right)$

5- $y = \frac{e^x}{x^2}$

$y' = \frac{e^x x^2 - e^x 2x}{x^4} = \frac{e^x(x^2-2x)}{x^4} = \frac{e^x(x-2)}{x^3}$

6- $y = \frac{e^x}{1+x}$

$y' = \frac{e^x(1+x) - e^x(1)}{(1+x)^2} = \frac{e^x(1+x-1)}{(1+x)^2} = \frac{e^x \cdot x}{(1+x)^2}$

7- $g(x) = \frac{3x-1}{2x+1}$

$g'(x) = \frac{3(2x+1) - (3x-1)(2)}{(2x+1)^2} = \frac{6x+3-6x+2}{(2x+1)^2} = \frac{5}{(2x+1)^2}$

8- $f(t) = \frac{2t}{4+t^2}$

$f'(t) = \frac{2(4+t^2) - 2t(2t)}{(4+t^2)^2} = \frac{8+2t^2-4t^2}{(4+t^2)^2} = \frac{-2t^2+8}{(4+t^2)^2}$

9- $H(u) = (u-\sqrt{u})(u+\sqrt{u})$

$H'(u) = (u^2-u)' = 2u-1$

10- $J(v) = (v^3-2v)(v^{-4}+v^{-2})$

$J'(v) = (3v^2-2)(v^{-4}+v^{-2}) + (v^3-2v)(-4v^{-5}-2v^{-3})$

$J'(v) = -4v^{-2}-2+8v^{-4}+4v^{-2} = 8v^{-4}-2$

11- $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y+5y^3)$

$F'(y) = (y^{-2}-3y^{-4})'(y+5y^3) + (y^{-2}-3y^{-4})(1+15y^2)$

$F'(y) = (-2y^{-3}+12y^{-5})(y+5y^3) + (y^{-2}+15-3y^{-4}-45y^{-2})$

$F'(y) = -2y^{-2}-10+12y^{-4}+60y^{-2}+y^{-2}+15-3y^{-4}-45y^{-2}$

$F'(y) = 14y^{-2}+9y^{-4}+5$

regra da prod. regra da cadeia $(e^z)^2$

12- $f(z) = (1-e^z)(z+e^z) = z+e^z-ze^z-e^{2z}$

$f'(z) = 1-ze^z-2e^{2z}$

$f'(z) = e^z(-z-e^z+1+e^z) = e^z(-z+1)$

13- $y = \frac{x^3}{1-x^2}$

$y' = \left[3x^2(1-x^2) - x^3(-2x)\right] \cdot \frac{1}{(1-x^2)^2}$

$y' = \frac{3x^2-3x^4+2x^4}{(1-x^2)^2} = \frac{3x^2-x^4}{(1-x^2)^2}$

14- $y = \frac{x+1}{x^3+x-2}$

$y' = \left[1(x^3+x-2) - (x+1)(3x^2+1)\right] \cdot \frac{1}{(x^3+x-2)^2}$

$y'=\left[x^3+x-2-(3x^3+x+3x^2+1)\right]\cdot\dfrac{1}{(x^3+x-2)^2}$

$y'=\dfrac{x^3+x-2-3x^3-x-3x^2-1}{(x^3+x-2)^2}=\dfrac{-2x^3-3x^2-3}{(x^3+x-2)^2}$

15- $y=\dfrac{t^2+2}{t^4-3t^2+1}$

$y'=\dfrac{2t(t^4-3t^2+1)-(t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2}=\dfrac{(2t^5-6t^3+2t)-(4t^5-6t^3+8t^3-12t)}{(t^4-3t^2+1)^2}=\dfrac{2t^5-6t^3+2t-4t^5+6t^3-8t^3+12t}{(t^4-3t^2+1)^2}=\dfrac{-2t^5-8t^3+14t}{(t^4-3t^2+1)^2}$

16- $y=\dfrac{t}{(t-1)^2}$

$y'=\dfrac{1(t-1)^2-t(2t-2)}{(t-1)^4}=\dfrac{(t-1)^2-2t^2+2t}{(t-1)^4}=\dfrac{-t^2+1}{(t-1)^4}$

$p^{3/2}$

17- $y=e^p(p+p\sqrt{p})$

$y'=e^p(p+p\sqrt{p})+e^p(1+\frac{3}{2}\sqrt{p})=e^p\left[p+1+\sqrt{p}(p+\frac{3}{2})\right]$

18- $y=\dfrac{1}{s+ke^s}$

$y'=\dfrac{0\cdot g'-1(1+ke^s)}{(s+ke^s)^2}=\dfrac{-1-ke^s}{(s+ke^s)^2}$

19- $y=\dfrac{v^3-2v\sqrt{v}}{v} \rightarrow 2v^{3/2}$

$3v^{3/2}$

$y'=\dfrac{(3v^2-3\sqrt{v})\cdot v-(v^3-2v^{3/2})\cdot 1}{v^2}=\dfrac{3v^3-3v\sqrt{v}-v^3+2v^{3/2}}{v^2}=\dfrac{2v^3-v^{3/2}}{v^2}=2v-v^{-1/2}=2v-\sqrt{\frac{1}{v}}=2v-\dfrac{\sqrt{v}}{v}$

20- $z=w^{3/2}(w+ce^w)$

$z'=\frac{3}{2}w^{1/2}(w+ce^w)+w^{3/2}(1+ce^w)$

$ce^w\cdot w^{1/2}(3+2w)$

$z'=\dfrac{3w^{3/2}+3w^{1/2}ce^w}{2}+w^{3/2}+w^{3/2}ce^w=\dfrac{3w^{3/2}+3w^{1/2}ce^w+2w^{3/2}+2w^{3/2}ce^w}{2}=\dfrac{5w^{3/2}+ce^w(3w^{1/2}+2w^{3/2})}{2}=\dfrac{w^{1/2}\left[5w+ce^w(3+2w)\right]}{2}=\sqrt{w}\cdot\dfrac{ce^w(3+2w)+5w}{2}$

21- $f(t)=\dfrac{2t}{2+\sqrt{t}}$ $\qquad t^{1/2}\quad \frac{1}{2}-\frac{2}{2}=\frac{-1}{2}$

$f'(t)=\dfrac{t(2+\sqrt{t})-2t(t^{-1/2})}{(2+\sqrt{t})^2}=\dfrac{2t+t\sqrt{t}-2t^{1/2}}{(2+\sqrt{t})^2}=\dfrac{2t+t^{3/2}-2t^{1/2}}{(2+\sqrt{t})^2}=t\cdot\dfrac{2+\sqrt{t}-2t^{-1/2}}{(2+\sqrt{t})^2}$

22- $g(t)=\dfrac{t-\sqrt{t}}{t^{1/3}}$

$g'(t)=\left[(1-\frac{1}{2}t^{-1/2})(t^{1/3})-(t-t^{1/2})(\frac{1}{3}t^{-2/3})\right]\cdot\dfrac{1}{t^{2/3}}$

$g'(t)=\left[t^{1/3}-\dfrac{t^{-1/6}}{2}-\left(\dfrac{t^{1/3}-t^{-1/6}}{3}\right)\right]\cdot\dfrac{1}{t^{2/3}}=\dfrac{4t^{1/3}-t^{-1/6}}{6t^{2/3}}$

23- $f(x)=\dfrac{A}{B+Ce^x}$

$f'(x)=\dfrac{0\cdot g'-Ce^x}{(B+Ce^x)^2}=\dfrac{-Ce^x}{(B+Ce^x)^2}$

24- $f(x)=\dfrac{1-xe^x}{x+e^x}$

$f'(x)=\dfrac{(-xe^x)(x+e^x)-(1-xe^x)(1+e^x)}{(x+e^x)^2}=\dfrac{-x^2e^x-xe^{2x}-(1+e^x-xe^x-xe^{2x})}{(x+e^x)^2}=\dfrac{-x^2e^x-xe^{2x}-1-e^x+xe^x+xe^{2x}}{(x+e^x)^2}=\dfrac{e^x(-x^2+x-1)-1}{(x+e^x)^2}$

25- $f(x)=\dfrac{x}{x+\frac{c}{x}}$

$f'(x)=\dfrac{1(x+\frac{c}{x})-x(1-\frac{c}{x^2})}{(x+\frac{c}{x})^2}=\dfrac{\frac{x^2+c}{x}-x+\frac{c}{x}}{(x+\frac{c}{x})^2}=\dfrac{\frac{x^2+c}{x}-\frac{x^2+c}{x}}{(x+\frac{c}{x})^2}=\dfrac{0}{\left(\frac{x^2+c}{x}\right)^2}=0$

26- $f(x)=\dfrac{ax+b}{cx+d}$

$f'(x)=\dfrac{a(cx+d)-(ax+b)c}{(cx+d)^2}=\dfrac{acx+ad-cax-cb}{(cx+d)^2}=\dfrac{ad-cb}{(cx+d)^2}$

27- $f(x)=x^4e^x$

$f'(x)=4x^3e^x+x^4e^x=e^x(x^4+4x^3)=x^3e^x(x+4)$

$f''(x)=e^x(x^4+4x^3)+e^x(4x^3+12x^2)=e^x(x^4+8x^3+12x^2)=x^2e^x(x^2+8x+12)$

28- $f(x)=x^{5/2}e^x$

$$f'(x) = \frac{5}{2}x^{3/2}e^x + x^{5/2}e^x = \frac{5e^x}{2}\left(x^{5/2} + x^{3/2}\right)$$

$$f''(x) = \frac{5e^x}{2}\left(x^{5/2} + x^{3/2}\right) + \frac{5e^x}{2}\left(\frac{5}{2}x^{3/2} + \frac{3}{2}x^{1/2}\right) = \frac{5e^x}{2}\left(\frac{2x^{5/2} + 2x^{3/2} + 5x^{3/2} + 3x^{1/2}}{2}\right) = \frac{5e^x}{4}\left(2x^{5/2} + 7x^{3/2} + 3x^{1/2}\right) = \frac{5x^{1/2}e^x}{4}\left(2x^2 + 7x + 3\right)$$

29. $f(x) = \dfrac{x^2}{1+2x}$

$$f'(x) = \frac{2x(1+2x) - 2x^2}{(1+2x)^2} = \frac{2x + 4x^2 - 2x^2}{(1+2x)^2} = \frac{2x + 2x^2}{(1+2x)^2} = \frac{2x(1+x)}{(1+2x)^2}$$

$$f''(x) = \frac{(2+4x)(1+2x)^2 - (2x+2x^2)(4+8x)}{(1+2x)^4} = \frac{(2+4x)\left[(1+2x)^2 - 2(2x+2x^2)\right]}{(1+2x)^4} = \left[2\cancel{(1+2x)}\left(1 + \cancel{4x} + \cancel{4x^2} - \cancel{4x} - \cancel{4x^2}\right)\right] \cdot \frac{1}{(1+2x)^{\cancel{4}\,3}} = \frac{2}{(1+2x)^3}$$

30. $f(x) = \dfrac{x}{x^2-1}$

$$f'(x) = \frac{1(x^2-1) - x \cdot 2x}{(x^2-1)^2} = \frac{-x^2-1}{(x^2-1)^2}$$

$$f''(x) = \frac{-2(x^2-1)^2 - (-x^2-1)(4x^3-4x)}{(x^2-1)^4} = \frac{-2(x^4-2x^2+1) - (-x^2-1)(x^4-2x^2+1)}{(x^2-1)^4} = \frac{(x^4-2x^2+1)(-2+x^2+1)}{(x^2-1)^4} = \frac{-2x^4+x^6+x^4-4x^2-2x^4-2x^2-2+x^2+1}{(x^2-1)^4} = \frac{x^6-3x^4-5x^2-1}{(x^2-1)^4}$$

31. $y = \dfrac{x^2-1}{x^2+x+1}$

$$y' = \frac{2x(x^2+x+1) - (x^2-1)(2x+1)}{(x^2+x+1)^2} = \frac{2x^3+2x^2+2x-(2x^3+x^2-2x-1)}{x^4+2x^3+3x^2+2x+1} = \frac{2x^3+2x^2+2x-2x^3-x^2+2x+1}{x^4+2x^3+3x^2+2x+1} = \frac{x^2+4x+1}{x^4+2x^3+3x^2+2x+1}$$

$P(1,0)$; $f'(1) = 2/3$

$t: y - y_0 = m(x - x_0) \therefore y - 0 = \frac{2}{3}(x-1)$; $t: y = \dfrac{2x-2}{3}$

32. $y = \dfrac{e^x}{x}$

$$y' = \frac{e^x \cdot x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$$

$P(1, e)$; $f'(1) = 0$

$t: y - y_0 = m(x - x_0) \therefore y - e = 0(x-1) \therefore y - e = 0 \therefore y = e$; $t: y = e$

33. $y = 2xe^x$

$y' = 2 \cdot e^x + 2x \cdot e^x = e^x(2+2x)$

$P(0,0)$; $f'(0) = 2 = m_t \therefore m_m \cdot m_t = -1$; $m_m = -1/2$

$t: y - y_0 = m(x - x_0)$; $y - 0 = 2(x-0)$; $t: y = 2x$

$m: y - y_0 = m(x - x_0)$; $y - 0 = -1/2(x-0)$; $m: y = \dfrac{-x}{2}$

34. $y = \dfrac{2x}{x^2+1}$

$$y' = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-2x^2}{(x^2+1)^2} = \frac{2}{x^4+2x^2+1}$$

$P(1,1)$; $f'(1) = \dfrac{2}{1+2+1} = \dfrac{2}{4} = \dfrac{1}{2} = m_t$; $m_m = -2$

$t: y - y_0 = m(x - x_0)$; $y - 1 = \frac{1}{2}(x-1)$; $t: \dfrac{x+1}{2}$

$m: y - y_0 = m(x - x_0) \therefore y - 1 = -2(x-1)$; $m: y = -2x - 1$

36. $y = \dfrac{x}{1+x^2}$

$$y' = \frac{1(1+x^2) - x(2x)}{(1+x^2)^2} = \frac{1+x^2-2x^2}{1+2x^2+x^4} = \frac{-x^2+1}{x^4+2x^2+1}$$

$P(3, 0.3)$; $f'(3) = \dfrac{-8}{100} = -0{,}08$

$t: y - y_0 = m(x - x_0)$; $y - 0{,}3 = \dfrac{-8}{100}(x-3)$; $t: y = \dfrac{-8x-6}{100}$

42. $g(x) = \dfrac{x}{e^x}$

$g^{2}(x) = \frac{x^2}{e^{2x}}$; $g^{3}(x) = \frac{x^3}{e^{3x}}$

$g^{n}(x) = \frac{x^n}{e^{nx}}$

43- a) $(fg)'(5) = f'(5)g(5) + f(5)g'(5) = 6\cdot(-3) + 1\cdot 2 = -18+2 = -16$

b) $\left(\frac{f}{g}\right)'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{g^2(5)} = \frac{6\cdot(-3)-1\cdot 2}{9} = \frac{-18-2}{9} = -20/9$

c) $\left(\frac{g}{f}\right)'(5) = \frac{g'(5)f(5) - g(5)f'(5)}{f^2(5)} = \frac{2\cdot 1 - (-3)\cdot 5}{1} = \frac{2+15}{1} = 17$

44- a) $h(2) = 5f(2) - 4g(2)$

$h'(2) = 5f'(2) - 4g'(2) = -10 - 28 = -38$

b) $h(2) = f(2)g(2)$

$h'(2) = f'(2)g(2) + f(2)g'(2) = -2\cdot 4 + (-3)\cdot 7 = -8-21 = -29$

c) $h(2) = \frac{f(2)}{g(2)}$

$h'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g^2(2)} = \frac{-2\cdot 4 - (-3)\cdot 7}{16} = \frac{-8+21}{16} = \frac{13}{16}$

d) $h(2) = \frac{g(2)}{1+f(2)}$

$h'(2) = \frac{g'(2)(1+f(2)) - g(2)f'(2)}{(1+f(2))^2} = \frac{7\cdot(-2) - 4\cdot(-2)}{4} = \frac{(-2)(7-4)}{4} = \frac{-6}{4} = \frac{-3}{2}$

45. $f(x) = e^x g(x)$

$f'(0) = e^0 \cdot g(0) + e^0 \cdot g'(0) = e^0\cdot 2 + e^0 \cdot 5 = 7e^0 = 7$

46- $f(x) = \frac{h(x)}{x}$; $f'(2) = ?$

$f'(2) = \frac{-3\cdot 2 - 4\cdot 1}{4} = \frac{-6-4}{4} = \frac{-10}{4} = -5/2$

48- $f'(x) = x^2 f(x)$

$f''(x) = 2x f(x) + x^2 f'(x)$

$f''(2) = 4\cdot 10 + 4\cdot(4\cdot 10) = 40(4+1) = 200$

52- d) $y = \frac{1+x f(x)}{\sqrt{x}}$

$y' = \frac{f'(x)\cdot\sqrt{x} - (1+x f(x))\frac{1}{2}x^{-1/2}}{x} = \frac{f'(x)\sqrt{x} - \frac{x^{-1/2}\cdot\sqrt{x} f(x)}{2}}{x} = \frac{2f'(x)\sqrt{x} - f(x)}{2x}$

Pg 178 (3.3) João G. G. A. Vaz

1- $f(x) = 3x^2 - 2\cos x$

$f'(x) = 6x + 2\operatorname{sen} x$

2- $f(x) = \sqrt{x}\operatorname{sen} x$

$f'(x) = \frac{x^{-1/2}}{2} \cdot \cos x$

3- $f(x) = \operatorname{sen} x + \frac{\operatorname{cotg} x}{2}$

$f'(x) = \cos x + \frac{-\operatorname{cossec}^2 x \cdot 2 - \operatorname{cotg} x \cdot 0}{4} = \frac{4\cos x - 2\operatorname{cossec}^2 x}{4}$

4- $y = 2\sec x - \operatorname{cossec}$

$y' = 2 \cdot \sec x \cdot \operatorname{tg} x + \operatorname{cossec} x \cdot \operatorname{cotg} x$

5- $g(t) = t^3 \cos t$

$g'(t) = 3t^2 \cdot \cos t + t^3 \cdot (-\operatorname{sen} t) = t^2(3\cos t - t \cdot \operatorname{sen} t)$

6- $g(t) = 4\sec t + \operatorname{tg} t$

$g'(t) = 4 \cdot \sec t \cdot \operatorname{tg} t + \sec^2 t = \sec t(4 \cdot \operatorname{tg} t + \sec t)$

7- $h(\theta) = \operatorname{cossec}\theta + e^\theta \cdot \operatorname{cotg}\theta$

$h'(\theta) = -\operatorname{cossec}\theta \cdot \operatorname{cotg}\theta + e^\theta \cdot \operatorname{cotg}\theta + e^\theta \cdot (-\operatorname{cossec}^2\theta)$

$h'(\theta) = -\operatorname{cossec}\theta(\operatorname{cotg}\theta + e^\theta \cdot \operatorname{cossec}\theta) + e^\theta \cdot \operatorname{cotg}\theta = -\operatorname{cossec}\theta(\operatorname{cotg} + e^\theta \operatorname{cossec}\theta) + e^\theta \cdot \operatorname{cossec}\theta \cdot \cos\theta$

$h'(\theta) = -\operatorname{cossec}\theta(\operatorname{cotg}\theta + e^\theta \operatorname{cossec}\theta - e^\theta \cos\theta) = -\operatorname{cossec}\theta[\operatorname{cotg}\theta + e^\theta(\operatorname{cossec}\theta - \cos\theta)]$

8- $y = e^u(\cos u + cu)$

$y' = e^u(\cos u + cu) + e^u(-\operatorname{sen} u + c) = e^u[\cos u - \operatorname{sen} u + c(1+u)]$

9- $y = \frac{x}{2 - \operatorname{tg} x}$

$y' = \frac{(2 - \operatorname{tg} x) - x(-\sec^2 x)}{(2 - \operatorname{tg} x)^2} = \frac{2 - \operatorname{tg} x + x\sec^2 x}{(2 - \operatorname{tg} x)^2}$

10- $y = \operatorname{sen}\theta\cos\theta$

$y' = \cos\theta \cdot (-\operatorname{sen}\theta) = -\operatorname{sen}\theta\cos\theta$

11- $f(\theta) = \frac{\sec\theta}{1 + \sec\theta}$

$f'(\theta) = \frac{\sec\theta\operatorname{tg}\theta(1 + \sec\theta) - \sec\theta \cdot \sec\theta\operatorname{tg}\theta}{(1 + \sec\theta)^2}$

$f'(\theta) = [\sec\theta \cdot \operatorname{tg}\theta(1 + \sec\theta - \sec\theta)] \cdot \frac{1}{(1 + \sec\theta)^2}$

$f'(\theta) = \frac{\sec\theta\operatorname{tg}\theta}{(1 + \sec\theta)^2}$

12- $y = \frac{\cos x}{1 - \operatorname{sen} x}$

$y' = \frac{-\operatorname{sen} x \cdot (1 - \operatorname{sen} x) - \cos x(-\cos x)}{(1 - \operatorname{sen} x)^2}$

$y' = \frac{-\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{(1 - \operatorname{sen} x)^2} = \frac{1}{(1 - \operatorname{sen} x)}$

13- $y = \frac{t \cdot \operatorname{sen} t}{1 + t}$

$$y' = \frac{\cos t(1+t) - t\,\text{sen}\,t \cdot 1}{(1+t)^2} = \frac{\cos t + t\cos t - t\cdot \text{sen}\,t}{(1+t)^2} = \frac{\cos t(1+t) - t\,\text{sen}\,t}{(1+t)^2}$$

14- $y = \dfrac{1-\sec x}{tg x}$

$$y' = \frac{-\sec x\, tg x \cdot tg x - (1-\sec x)\cdot \sec^2 x}{tg^2 x}$$

$$y' = \frac{-\sec x\, tg^2 x - \sec^2 x + \sec^3 x}{tg^2 x} = \frac{-\sec x\left[tg^2 x + \sec(1-\sec)\right]}{tg^2 x}$$

15- $f(x) = xe^x \cdot \text{cossec}\,x$

$$f'(x) = (xe^x)' \cdot \text{cossec}\,x - xe^x \cdot \text{cossec}\,x\, \text{cotg}\,x$$

$$f'(x) = (e^x + xe^x)\cdot \text{cossec}\,x - xe^x\, \text{cossec}\,x\, \text{cotg}\,x$$

$$f'(x) = \text{cossec}\,x\,(e^x + xe^x - xe^x\,\text{cotg}\,x) = \text{cossec}\,x \cdot e^x(1 + x - x\,\text{cotg}\,x)$$

16- $y = x^2\,\text{sen}\,x\, tg x$

$$y' = (x^2\,\text{sen}\,x)' \cdot tg x + (x^2\,\text{sen}\,x)\cdot \sec^2 x = (2x\,\text{sen}\,x + x^2\cdot\cos x)\cdot tg x + x^2\,\text{sen}\,x\,\sec^2 x$$

$$y' = 2x\cdot\text{sen}\,x\cdot tg x + x^2\cdot\text{sen}\,x + x^2\cdot\text{sen}\,x\cdot\sec^2 x = \text{sen}\,x\cdot x(2tgx + x + x\cdot\sec^2 x) = x\cdot\text{sen}\,x\left[2tgx + x(1+\sec^2 x)\right]$$

17- $f(x) = \text{cossec}\,x = \dfrac{1}{\text{sen}\,x}$

$$f'(x) = \frac{0\cdot\text{sen}\,x - \cos x}{\text{sen}^2 x} = \frac{-\cos x}{\text{sen}^2 x} = \text{cotg}\,x \cdot \frac{-1}{\text{sen}\,x} = -\text{cossec}\,x\cdot\text{cotg}\,x$$

18- $f(x) = \sec x = \dfrac{1}{\cos x}$

$$f'(x) = \frac{0\cdot\cos x + \text{sen}\,x}{\cos^2 x} = \frac{\text{sen}\,x}{\cos^2 x} = tgx\cdot\frac{1}{\cos x} = tgx\cdot\sec x$$

19- $f(x) = \text{cotg}\,x = \dfrac{\cos x}{\text{sen}\,x}$

$$f'(x) = \frac{-\text{sen}^2 x - \cos^2 x}{\text{sen}^2 x} = \frac{-1(\text{sen}^2 x + \cos^2 x)}{\text{sen}^2 x} = \frac{-1}{\text{sen}^2 x} = -\text{cossec}^2 x$$

20- $f(x) = \text{sen}\,x$ (modificada) $\quad \text{sen}\,x\cos h + \text{sen}\,h\cos x$

limite trig. fundamental $= 1$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\text{sen}(x+h) - \text{sen}\,x}{h} = \lim_{h\to 0}\frac{\text{sen}\,x\cos h + \text{sen}\,h\cos x - \text{sen}\,x}{h} = \lim_{h\to 0}\frac{\text{sen}\,x(\cos h - 1)}{h} + \lim_{h\to 0}\frac{\text{sen}\,h\cos x}{h}$$

$-\text{sen}^2 h$

$$f'(x) = \text{sen}\,x\cdot\lim_{h\to 0}\left[\frac{\cos h - 1}{h}\right] + \lim_{h\to 0}\left[\frac{\text{sen}\,h\cdot\cos x}{h}\right] = \text{sen}\,x\cdot\lim_{h\to 0}\left[\frac{\cos^2 h - 1}{h(\cos h + 1)}\right] + \lim_{h\to 0}\left[\frac{\text{sen}\,h\cos x}{h}\right] = \text{sen}\,x\cdot\lim_{h\to 0}\left[\frac{\text{sen}\,h}{h}\cdot\frac{-\text{sen}\,h}{\cos h + 1}\right] + \lim_{h\to 0}\left[\frac{\text{sen}\,h}{h}\cdot\cos x\right]$$

$$f'(x) = \text{sen}\,x\cdot\lim_{h\to 0}[0\cdot 1] + \lim_{h\to 0}[\cos x\cdot 1] = \cos x \therefore (\text{sen}\,x)' = \cos x$$

21- $y = \sec x = \dfrac{1}{\cos x}$

$$y' = \frac{0 - (-\text{sen}\,x)}{\cos^2 x} = \frac{\text{sen}\,x}{\cos^2 x} = tgx\cdot\sec x$$

$P(\pi/3, 2)$; $f'(\pi/3) = \sqrt{3}\cdot 2 = 2\sqrt{3}$

$t: y - y_0 = m(x - x_0)$; $y - 2 = 2\sqrt{3}(x - \pi/3)$; $t: y = \dfrac{6\sqrt{3}x - 2(\sqrt{3}\pi + 3)}{3}$

22- $y = e^x\cos x$

$y' = e^x\cdot\cos x + e^x\,\text{sen}\,x$

$P(0,1)$; $f'(0) = 0$

$t: y - y_0 = m(x - x_0)$; $y - 1 = 0\cdot x$; $y - 1 = 0$; $t: y = 1$

23- $y = \cos x - \text{sen}\,x$

$y' = -\text{sen}\,x - \cos x$

$P(\pi, -1)$; $f'(\pi) = -0 - (-1) = 1$

$t: y-y_0 = m(x-x_0)$; $y+1 = 1(x-\pi)$; $t: y = x-\pi-1$

24- $y = x + tgx = x + \frac{sen\,x}{cos\,x}$

$y' = 1 + \frac{cos^2 x + sen^2 x}{cos^2 x} = 1 + \frac{1}{cos^2 x} = 1 + sec^2 x$

$P(\pi, \pi)$; $f'(\pi) = 1+1 = 2$

$t: y-y_0 = m(x-x_0)$; $y-\pi = 2(x-\pi)$; $t: y = 2x-\pi$

27- a) $f(x) = sec\,x - x = \frac{1}{cos\,x} - x$

$f'(x) = \frac{sen\,x}{cos^2 x} - 1 = tgx \cdot \frac{1}{cos\,x} - 1 = tgx \cdot sec\,x - 1$

29- $H(\theta) = \theta\, sen\,\theta$

$H'(\theta) = sen\,\theta + \theta\, cos\,\theta$

$H''(\theta) = cos\,\theta - sen\,\theta$

32- $f(\pi/3) = 4$; $f'(\pi/3) = -2$

$g(x) = f(x)\, sen\,x$; $h(x) = \frac{cos\,x}{f(x)}$

a) $g'(\pi/3) = ?$

$g'(x) = f'(x) \cdot sen\,x + f(x) cos\,x$

$g'(\pi/3) = f'(\pi/3) \cdot sen\,\pi/3 + f(\pi/3) \cdot cos\,\pi/3 = -2 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} = \frac{-2\sqrt{3}}{2} + 2 = \frac{-2\sqrt{3}+4}{2}$

b) $h'(\pi/3) = ?$

$h'(x) = \frac{-sen\,x \cdot f(x) - cos\,x \cdot f'(x)}{f(x)} = \frac{-\frac{\sqrt{3}}{2} \cdot 4 + \frac{1}{2} \cdot 2}{16} = \frac{-2\sqrt{3}+1}{16}$

33- $f(x) = x + 2sen\,x$

$f'(x) = 1 + 2cos\,x$

$1 + 2cos\,x = 0$; $cos\,x = -1/2$

$x = \pi - \pi/3 = \frac{2\pi}{3} + 2k\pi \mid k \in \mathbb{Z}$

34- $f(x) = e^x cos\,x$

$f'(x) = e^x cos\,x - e^x sen\,x$

$e^x(cos\,x - sen\,x) = 0$; $cos\,x = sen\,x$

$x = \frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z}$

35- a) $v = x'(t)$; $a = x''(t)$

$v(t) = 8 \cdot cos\,t$

$a(t) = -8 \cdot sen\,t$

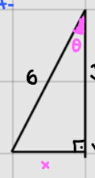
b) $x(2\pi/3) = 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$ cm

$v(2\pi/3) = 8 \cdot (-1/2) = -4$ cm/s

$a(2\pi/3) = -8 \cdot \frac{\sqrt{3}}{2} = -4\sqrt{3}\ cm/s^2$

Está se movendo para a esquerda (e acelerando).

37- $\operatorname{sen}\theta = \frac{x}{6}$, se $\theta = \pi/3$: $\frac{\sqrt{3}}{2} = \frac{x}{6} \therefore x = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$



De modo geral, $x(\theta) = 6\operatorname{sen}\theta$. A velocidade é a derivada da posição.

$v(\theta) = 6\cos\theta$. Quando $\theta = \pi/3$, $v(\pi/3) = 6 \cdot 1/2 = 3\ m/s$.

39- $\lim_{x\to 0} \frac{\operatorname{sen}3x}{x} = \lim_{x\to 0} \frac{\operatorname{sen}(2x+x)}{x} = \lim_{x\to 0} \frac{\operatorname{sen}x(3-4\operatorname{sen}^2x)}{x} = 1 \cdot \lim_{x\to 0} 3-4\operatorname{sen}^2x = 1 \cdot 3 = 3$

40- $\lim_{x\to 0} \frac{\operatorname{sen}4x}{\operatorname{sen}6x} = \lim_{x\to 0} \frac{4\operatorname{sen}x\cos x(1-2\operatorname{sen}^2x)}{2\operatorname{sen}x\cos x(3-4\operatorname{sen}^2x)(1-4\operatorname{sen}^2x)} = \lim_{x\to 0} \frac{2-4\operatorname{sen}^2x}{3-12\operatorname{sen}^2x-4\operatorname{sen}^2x+16\operatorname{sen}^2x} = \frac{2-4\operatorname{sen}^2x}{3} = \frac{2}{3}$

41- $\lim_{t\to 0} \frac{tg6t}{\operatorname{sen}2t} = \lim_{t\to 0} \frac{\frac{\operatorname{sen}6t}{\cos 6t}}{2\operatorname{sen}t\cos t} = \lim_{t\to 0} \frac{\operatorname{sen}6t}{12\operatorname{sen}t\cos t\cos 6t} = \lim_{t\to 0} \frac{2\operatorname{sen}t\cos t(3-4\operatorname{sen}^2t)(1-4\operatorname{sen}^2t)}{12\operatorname{sen}t\cos t\cos 6t} = \lim_{t\to 0} \frac{3}{6\cdot\cos 6t} = \frac{1}{2}$

42- $\lim_{\theta\to 0} \frac{\cos\theta-1}{\operatorname{sen}\theta} = \lim_{\theta\to 0} \frac{\cos^2\theta-1}{\operatorname{sen}\theta(\cos\theta+1)} = \lim_{\theta\to 0} \frac{-\operatorname{sen}^2\theta}{\operatorname{sen}\theta(\cos\theta+1)} = \lim_{\theta\to 0}\left[\frac{-\operatorname{sen}\theta}{\operatorname{sen}\theta} \cdot \frac{\operatorname{sen}\theta}{\cos\theta+1}\right] = -1 \cdot \frac{0}{2} = 0$

43- $\lim_{x\to 0} \frac{\operatorname{sen}3x}{5x^3-4x} = \lim_{x\to 0} \frac{\operatorname{sen}x(3-4\operatorname{sen}^2x)}{x(5x^2-4)} = \lim_{x\to 0}\left[\frac{\operatorname{sen}x}{x} \cdot \frac{3-4\operatorname{sen}^2x}{5x^2-4}\right] = 1 \cdot \frac{-3}{4} = \frac{-3}{4}$

44- $\lim_{x\to 0} \frac{\operatorname{sen}3x \cdot \operatorname{sen}5x}{x^2} =$

45- $\lim_{\theta\to 0} \frac{\operatorname{sen}\theta}{\theta+tg\theta}$

46- $\lim_{x\to 0} \frac{\operatorname{sen}x^2}{x}$

47- $\lim_{x\to \pi/4} \frac{1-tg\,x}{\operatorname{sen}x-\cos x}$

48- $\lim_{x\to 1} \frac{\operatorname{sen}(x-1)}{x^2+x-2}$

49- $\frac{d^{99}}{dx^{99}}\operatorname{sen}x = ?$

$(\operatorname{sen}x)' = \cos x$ (1)

$(\cos x)' = -\operatorname{sen}x$ (2)

$(-\operatorname{sen}x)' = -\cos x$ (3)

$(-\cos x)' = \operatorname{sen}x$ (0)

As derivadas são periódicas. Em 99, terão 24 ciclos completos e 3 transformações.

Logo, $\frac{d^{99}}{dx^{99}}\operatorname{sen}x = -\cos x$.

50- $(x\operatorname{sen}x)' = \operatorname{sen}x + x\cos x$

$(\operatorname{sen}x + x\cos x)' = \cos x + \cos x - x\operatorname{sen}x = 2\cos x - x\operatorname{sen}x$ (1)

$(2\cos x - x\operatorname{sen}x)' = -2\operatorname{sen}x + (-\operatorname{sen}x + x\cos x) = -3\operatorname{sen}x + x\cos x$ (2)

$(-3\operatorname{sen}x + x\cos x)' = -3\cos x + (\cos x - x\operatorname{sen}x) = -2\cos x - x\operatorname{sen}x$ (3)

$(-2\cos x - x\operatorname{sen}x)' = 2\operatorname{sen}x + (-\operatorname{sen}x - x\cos x) = \operatorname{sen}x - x\cos x$ (4)

$(\operatorname{sen}x - x\cos x)' = \cos x + (-\cos x + x\operatorname{sen}x) = x\operatorname{sen}x$ (0)

De forma análoga ao exercício anterior, $\frac{d^{35}}{dx^{35}}x\operatorname{sen}x$ resulta em $x\operatorname{sen}x$, após 7 ciclos completos.

53- a) $(tg\,x)' = \sec^2x$

b) $(\sec x)' = \sec x \cdot tg\,x$

c) $(\operatorname{sen} x + \cos x)' = \cos x - \operatorname{sen} x$

Pg 186 (3.4) João Vaz

1- $y = \text{sen}(4x)$

$f(x) = \text{sen}(x)$; $g(x) = 4x$; $f(g(x)) = \text{sen}(4x)$

$y' = f'(g(x)) \cdot g'(x) = \text{sen}'(4x) \cdot (4x)' = \cos(4x) \cdot 4 = 4\cos(4x)$

2- $y = \sqrt{4+3x}$

$f(x) = \sqrt{4+x}$; $g(x) = 3x$; $f(g(x)) = \sqrt{4+3x}$

$y' = f'(g(x)) \cdot g'(x) = \left[(4+x)^{1/2}\right]' \cdot (3x)' = \frac{1}{2}(4+x)^{-1/2} \cdot 3 = \sqrt{\frac{1}{4+x}} \cdot \frac{3}{2} = \sqrt{\frac{9}{16+4x}}$

3- $y = (1-x^2)^{10}$

$f(x) = x^{10}$; $g(x) = 1-x^2$; $f(g(x)) = (1-x^2)^{10}$

$y' = f'(g(x)) \cdot g'(x) = \left[(1-x^2)^{10}\right]' \cdot (1-x^2)' = 10(1-x^2)^9 \cdot (-2x) = -20x(1-x^2)^9$

4- $y = \text{tg}(\text{sen}\,x)$

$f(x) = \text{tg}\,x$; $g(x) = \text{sen}\,x$; $f(g(x)) = \text{tg}(\text{sen}\,x)$

$y' = f'(g(x)) \cdot g'(x) = \left[\text{tg}(\text{sen}\,x)\right]' \cdot \text{sen}'(x) = \sec^2(\text{sen}\,x) \cdot \cos x = \cos x \cdot \sec^2(\text{sen}\,x)$

5- $y = e^{\sqrt{x}}$

$f(x) = e^x$; $g(x) = \sqrt{x}$; $f(g(x)) = e^{\sqrt{x}}$

$y' = f'(g(x)) \cdot g'(x) = \left[e^{\sqrt{x}}\right]' \cdot (\sqrt{x})' = \sqrt{x}\, e^{\sqrt{x}-1} \cdot \frac{1}{2} x^{-1/2} = \frac{e^{\sqrt{x}}}{2e}$

6- $y = \sqrt{2-e^x}$

$f(x) = \sqrt{x}$; $g(x) = 2-e^x$; $f(g(x)) = \sqrt{2-e^x}$

$y' = f'(g(x)) \cdot g'(x) = \left[\sqrt{2-e^x}\right]' \cdot (2-e^x)' = \frac{1}{2}(2-e^x)^{-1/2} \cdot (-e^x) = \frac{-e^x}{2\sqrt{2-e^x}} = \frac{-e^x\sqrt{2-e^x}}{2(2-e^x)} = \frac{\sqrt{-2e^{2x}+e^{3x}}}{4-2e^x}$

7- $F(x) = (x^4+3x^2-2)^5$

$F'(x) = 5(x^4+3x^2-2)^4 \cdot (4x^3+6x)$

8- $F(x) = (4x-x^2)^{100}$

$F'(x) = 100(4x-x^2)^{99} \cdot (4-2x)$

9- $F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{1/4}$

$F'(x) = \frac{1}{4}(1+2x+x^3)^{-3/4} \cdot (2+3x^2)$

10- $f(x) = (1+x^4)^{2/3}$

$f'(x) = \frac{2}{3}(1+x^4)^{-1/3} \cdot 4x^3 = \frac{8x^3}{3} \cdot (1+x^4)^{-1/3}$

11- $g(t) = \frac{1}{(t^4+1)^3}$

$g'(t) = \frac{0 \cdot (t^4+1) - 1 \cdot 3(t^4+1)^2 \cdot 4t^3}{(t^4+1)^6} = \frac{-12t^3(t^4+1)^2}{(t^4+1)^6} = \frac{-12t^3}{(t^4+1)^4}$

12- $f(t) = \sqrt[3]{1+\text{tg}\,t} = (1+\text{tg}\,t)^{1/3}$

$f'(t) = \frac{1}{3}(1+\text{tg}\,t)^{-1/3} \cdot \sec^2 t = \frac{\sec^2 t}{3\sqrt[3]{1+\text{tg}\,t}}$

13- $y = \cos(a^3 + x^3)$

$y' = -\operatorname{sen}(a^3 + x^3) \cdot 3x^2 = -3x^2 \operatorname{sen}(a^3 + x^3)$

14- $y = a^3 + \cos^3 x$

$y' = 3\cos^2 x \cdot (-\operatorname{sen}^3 x) = -3\cos^2 x \operatorname{sen}^3 x$

15- $y = x e^{-kx} = \frac{x}{e^{kx}}$

$y' = \frac{0 \cdot e^{kx} - x e^{kx}}{e^{2kx}} = -x e^{kx} \cdot e^{-2kx} = -x e^{-kx}$

16- $y = e^{-2t} \cdot \cos 4t$

$y' = e^{-2t} \cdot \cos 4t + e^{-2t} \cdot \operatorname{sen} 4t \cdot 4 = e^{-2t}\left[\cos 4t + 4 \operatorname{sen} 4t\right]$

17- $f(x) = (2x-3)^4 (x^2+x+1)^5$

$f'(x) = 4 \cdot (2x-3)^3 \cdot 2 + 5(x^2+x+1)^4 \cdot (2x+1) = 8 \cdot (2x-3)^3 + 5 \cdot (2x+1) \cdot (x^2+x+1)^4$

18- $g(x) = (x^2+1)^3 (x^2+2)^6$

$g'(x) = 3 \cdot (x^2+1)^2 \cdot 2x + 6 \cdot (x^2+2)^5 \cdot 2x = 6x\left[(x^2+1)^2 + 2 \cdot (x^2+2)^5\right]$

19- $h(t) = (t+1)^{2/3} \cdot (2t^2-1)^3$

$h'(t) = \frac{2}{3}(t+1)^{-1/3} \cdot 1 + 3(2t^2-1)^2 \cdot 4t = \frac{2}{3}\left[(t+1)^{-1/3} + 18t(2t^2-1)^2\right]$

20- $F(t) = (3t-1)^4 (2t+1)^{-3}$

$F'(t) = 4(3t-1)^3 \cdot 3 - 3(2t+1)^{-4} \cdot 2 = 6\left[2(3t-1)^3 - (2t+1)^{-4}\right]$

21- $y = \left(\frac{x^2+1}{x^2-1}\right)^3$

$y' = 3 \cdot \left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \left(\frac{2x(x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2}\right) = 6x\left(\frac{x^2+1}{x^2-1}\right) \cdot \left(\frac{-2}{(x^2-1)^2}\right) = \frac{-12x(x^2+1)}{(x^2-1)^3}$

22- $f(s) = \sqrt{\frac{s^2+1}{s^2+4}}$

$f'(s) = \frac{1}{2}\left(\frac{s^2+1}{s^2+4}\right)^{-1/2} \cdot \left[\frac{2s(s^2+4) - (s^2-1) \cdot 2s}{(s^2+4)^2}\right] = \left(\frac{s^2+1}{s^2+4}\right)^{-1/2} \cdot \frac{5s^2}{(s^2+4)^2}$

23- $y = \sqrt{1 + 2e^{3x}}$

$y' = \frac{1}{2}(1+2e^{3x})^{-1/2} \cdot 2e^{3x} = \frac{2e^{3x}}{2\sqrt{1+2e^{3x}}} = \frac{e^{3x}}{\sqrt{1+2e^{3x}}} = \frac{e^{3x}\sqrt{1+2e^{3x}}}{1+2e^{3x}}$

24- $y = 10^{1-x^2}$

$y' = (1-x^2)10^{-x^2} \cdot (-2x) = -2x(1-x^2)10^{-x^2}$

25- $y = 5^{-1/x}$

$y' = (-1/x) \cdot 5^{\frac{-1}{x}-1} \cdot \frac{0 \cdot x + 1 \cdot 1}{x^2} = \frac{-5^{\frac{-1-x}{x}}}{x} \cdot \frac{2}{x^2} = \frac{2 \cdot (-5)^{\frac{-1-x}{x}}}{x^3}$

26- $G(y) = \frac{(y-1)^4}{(y^2+2y)^5} = \frac{4(y-1)^3 \cdot (y^2+2y)^5 - (y-1)^4 \cdot 5 \cdot (y^2+2y)^4}{(y^2+2y)^{10}} = \frac{(y-1)^3 \cdot (y^2+2y)^4\left[4y(y+2) - 5(y-1)\right]}{(y^2+2y)^{10}} = \frac{(y-1)^3\left[4y(y+2) - 5(y-1)\right]}{(y^2+2y)^6}$

27- $y = \dfrac{x}{\sqrt{x^2+1}}$

$y' = \dfrac{\sqrt{x^2+1} - x\left(\sqrt{x^2+1}\right)'}{x^2+1} = \dfrac{\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right]}{x^2+1} = \dfrac{2(x^2+1)^{1/2} - 2x^2 \cdot (x^2+1)^{-1/2}}{2(x^2+1)} = \left[\dfrac{2(x^2+1)\left[(x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}\right]}{2(x^2+1)}\right] = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}$

28- $y = \dfrac{e^{u} - e^{-u}}{e^{u} + e^{-u}}$

$y' = \dfrac{(e^{u} - e^{-u}) \cdot (e^{u} + e^{-u}) - (e^{u} - e^{-u})(e^{u} + e^{-u})}{(e^{u} + e^{-u})^2} = 0$

29- $F(t) = e^{t \,\mathrm{sen}\, 2t}$

$F'(t) = e^{t\,\mathrm{sen}\,2t} \cdot (t \cdot \mathrm{sen}\,2t)' = e^{t\,\mathrm{sen}\,2t} \cdot t\cos 2t \cdot (2t)' = e^{t\,\mathrm{sen}\,2t} \cdot t\cos 2t \cdot 2 = 2t\cos 2t \cdot e^{t\,\mathrm{sen}\,2t}$

30- $F(v) = \left(\dfrac{v}{v^3+1}\right)^6$

$F'(v) = 6\left(\dfrac{v}{v^3+1}\right)^5 \cdot \dfrac{v^3+1 - v \cdot 3v^2}{(v^3+1)^2} = 6 \cdot \left(\dfrac{v}{v^3+1}\right)^5 \cdot \left[\dfrac{4v^3+1}{(v^3+1)^2}\right]$

31- $y = \mathrm{sen}(\mathrm{tg}\, 2t)$

$y' = \cos(\mathrm{tg}\,2t) \cdot \sec^2 2t \cdot 2 = 2 \cdot \sec^2 2t \cdot \cos(\mathrm{tg}\,2t)$

32- $y = \sec^2(m\theta)$

$y' = 2\sec(m\theta) \cdot m = 2m\sec(m\theta)$

33- $y = 2^{\mathrm{sen}(\pi x)}$

$y' = \mathrm{sen}(\pi x) \cdot 2^{\mathrm{sen}(\pi x) - 1} \cdot \cos(\pi x) \cdot \pi = \dfrac{\pi\,\mathrm{sen}(\pi x)\cos(\pi x) \cdot 2^{\mathrm{sen}(\pi x)}}{2}$

34- $y = x^2 \cdot e^{-1/x} = x^2 e^{-x^{-1}}$

$y' = 2x \cdot e^{-1/x} + x^2 \cdot e^{-x^{-1}} \cdot (-1) \cdot (-x^{-2}) = 2x \cdot e^{-x^{-1}} + e^{-x^{-1}} = e^{-x^{-1}}(2x+1)$

35- $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)$

$y' = -\mathrm{sen}\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right) \cdot \left[\dfrac{-e^{2x}(1+e^{2x}) - (1-e^{2x})e^{2x}}{(1+e^{2x})^2}\right] = -\mathrm{sen}\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right) \cdot \left[\dfrac{-e^{2x} \cdot 2}{(1+e^{2x})^2}\right]$

36- $y = \sqrt{1+xe^{-2x}} = (1+xe^{-2x})^{1/2}$

$y' = \frac{1}{2}(1+xe^{-2x})^{-1/2} \cdot xe^{-2x} \cdot (-2) = -xe^{-2x}(1+xe^{-2x})^{-1/2}$

37- $y = \mathrm{cotg}^2(\mathrm{sen}\,\theta)$

$y' = 2\mathrm{cotg}(\mathrm{sen}\,\theta) \cdot \left[-\mathrm{cossec}^2(\mathrm{sen}\,\theta)\right] \cdot \cos\theta = -2 \cdot \dfrac{\cos(\mathrm{sen}(\theta))}{\mathrm{sen}^3(\mathrm{sen}(\theta))} \cdot \cos\theta = \dfrac{-2\cos\theta \cdot \cos(\mathrm{sen}(\theta))}{\mathrm{sen}^3(\mathrm{sen}(\theta))}$

38- $y = e^{k\,\mathrm{tg}\sqrt{x}}$

$y' = e^{k\,\mathrm{tg}\sqrt{x}} \cdot k\sec^2\sqrt{x} \cdot \frac{1}{2}x^{-1/2} = \dfrac{e^{k\,\mathrm{tg}\sqrt{x}} \cdot k\sec^2\sqrt{x} \cdot x^{-1/2}}{2}$

39- $f(t) = tg(e^t) + e^{tg\,t}$

$f'(t) = sec^2(e^t) \cdot e^t + e^{tg\,t} \cdot sec^2 t$

40- $y = sen(sen(sen(x)))$

$y' = cos(sen(sen(x))) \cdot cos(sen(x)) \cdot cos\,x$

41- $f(t) = sen^2(e^{sen^2 t})$

$f'(t) = 2sen(e^{sen^2 t}) \cdot cos(e^{sen^2 t}) \cdot e^{sen^2 t} \cdot 2sen\,t \cdot cos\,t = 4sen(e^{sen^2 t}) \cdot cos(e^{sen^2 t}) \cdot e^{sen^2 t} \cdot sen\,t \cdot cos\,t$

42- $y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \left[x + (x + x^{1/2})^{1/2}\right]^{1/2}$

$y' = \frac{1}{2}\left[x + (x + x^{1/2})^{1/2}\right]^{-1/2} \cdot \left[1 + \frac{1}{2}(x + x^{1/2})\right]^{-1/2} \cdot \left[1 + \frac{1}{2}x^{-1/2}\right]$

43- $g(x) = (2ra^{rx} + n)^p$

$g'(x) = p(2ra^{rx} + n)^{p-1} \cdot ($

44- $y = 2^{3^{x^2}}$

$y' = 3^{x^2} \cdot 2^{3^{x^2} - 1} \cdot x^2 \cdot 3^{x^2 - 1} \cdot 2x = 2x^2 \cdot 2^{3^{x^2} - 1} \cdot 2^{2x^2 - 1}$

45- $y = cos\left[\sqrt{sen(tg(\pi x))}\right]$

$y' = sen\left[sen(tg(\pi x))^{1/2}\right] \cdot \frac{1}{2} \cdot sen\left[tg(\pi x)\right]^{-1/2} \cdot cos\left[tg(\pi x)\right] \cdot sec^2(\pi x) \cdot \pi = \frac{\pi}{2} \cdot sen\left[sen(tg(\pi x))^{1/2}\right] \cdot sen\left[tg(\pi x)\right]^{-1/2} \cdot cos\left[tg(\pi x)\right] \cdot sec^2(\pi x)$

46- $y = \left[x + (x + sen^2 x)^3\right]^4$

$y' = 4\left[x + (x + sen^2 x)^3\right]^3 \cdot \left[1 + 3(x + sen^2 x) \cdot (1 + 2sen\,x \cdot cos\,x)\right]$

47- $y = cos(x^2)$

$y' = -sen(x^2) \cdot 2x$

$y'' = -cos(x^2) \cdot 2x \cdot 2x + (-sen(x^2)) \cdot 2 = -4x^2 cos(x^2) - 2sen(x^2) = -2\left[2x^2 cos(x^2) + sen(x^2)\right]$

48- $y = cos^2 x$

$y' = 2cos\,x \cdot sen\,x$

$y'' = -2sen\,x \cdot sen\,x + 2cos\,x \cdot cos\,x = -2sen^2 x + 2cos^2 x = 2(-sen^2 x + cos^2 x) = 2(-1 + cos^2 x + cos^2 x) = -2 + 4cos^2 x$

49- $y = e^{\alpha x} \cdot sen\,\beta x$

$y' = e^{\alpha x} \cdot sen\,\beta x + e^{\alpha x} cos\,\beta x \cdot \beta = e^{\alpha x}(sen\,\beta x + \beta cos\,\beta x)$

$y'' = e^{\alpha x}(sen\,\beta x + \beta cos\,\beta x) + e^{\alpha x}\left[cos\,\beta x \cdot \beta + \beta \cdot (-sen\,\beta x) \cdot \beta\right] = e^{\alpha x}\left[2\beta cos\,\beta x + sen\,\beta x \cdot (1 - \beta^2)\right]$

50- $y = e^{e^x}$

$y' = e^{e^x} \cdot e^x$

$y'' = e^{e^x} \cdot e^x + e^{e^x} \cdot e^x = 2 \cdot e^{e^x} \cdot e^x$

51- $y = (1 + 2x)^{10}$

$y' = 10(1 + 2x)^9 \cdot 2 = 20 \cdot (1 + 2x)^9$

$f'(0) = 20$, $P(0,1)$

$t: y - y_0 = m(x - x_0) \therefore y - 1 = 20(x - 0) \therefore t: y = 20x + 1$

52- $y = \sqrt{1+x^3}$

$y' = \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2$; $P(2,3)$

$f'(2) = \frac{1}{2} \cdot \frac{1}{\sqrt{9}} \cdot 12 = 2$

$t: y - y_0 = m(x - x_0) \therefore y - 3 = 2(x-2) \therefore t: y = 2x - 1$

53- $y = \text{sen}(\text{sen}\, x)$

$y' = \cos(\text{sen}\, x) \cdot \cos x$; $P(\pi, 0)$

$f'(\pi) = 1 \cdot (-1) = -1$

$t: y - y_0 = m(x - x_0) \therefore y = -1(x - \pi) \therefore t: y = -x + \pi$

54- $y = \text{sen}\, x + \text{sen}^2 x$

$y' = \cos x + 2\,\text{sen}\, x \cdot \cos x = \cos x(1 + 2\,\text{sen}\, x)$

$f'(0) = 1(1 + 2 \cdot 0) = 1$

$t: y - y_0 = m(x - x_0) \therefore y - 0 = m(x - 0) \therefore t: y = x$

59- $f(x) = 2\,\text{sen}\, x + \text{sen}^2 x$

$f'(x) = 2\cos x + 2\,\text{sen}\, x \cdot \cos x = 2\cos x(1 + \text{sen}\, x)$

$2\cos x(1 + \text{sen}\, x) = 0 \therefore 2\cos x = 0$ ou $(1 + \text{sen}\, x) = 0$

$2\cos x = 0 \therefore \cos x = 0 \therefore x = \pi/2 + 2k\pi \mid k \in \mathbb{Z}$.

$1 + \text{sen}\, x = 0 \therefore \text{sen}\, x = -1 \therefore x = \frac{3\pi}{2} + 2k\pi \mid k \in \mathbb{Z}$.

Para a reta tangente ser horizontal, $S = \{\pi/2 + 2k\pi,\ 3\pi/2 + 2k\pi\} \mid k \in \mathbb{Z}$.

60- $y = \text{sen}\, 2x - 2\,\text{sen}\, x$

$y' = \cos 2x \cdot 2 - 2\cos x = 2(\cos 2x - \cos x)$

$2(\cos 2x - \cos x) = 0 \therefore \cos 2x = \cos x$

$\cos 2x = \cos x \cdot \cos x - \text{sen}\, x\, \text{sen}\, x = \cos^2 x - \text{sen}^2 x = -2\,\text{sen}^2 x + 1$

$\cos x = -2\,\text{sen}^2 x + 1 \therefore$ Para a reta tangente ser horizontal, $\{x \in \mathbb{R} \mid \cos x = -2\,\text{sen}^2 x + 1\}$

61- $F(x) = f(g(x))$

$F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$

72- $f(x) = x \cdot g(x^2)$

$f'(x) = x g'(x^2) \cdot 2x = 2x^2 g'(x^2)$

$f''(x) = x g''(x^2) \cdot 2x \cdot 2 = 4x^2 g''(x^2)$

77- $y = \cos 2x$

$y' = -\text{sen}\, 2x \cdot 2 = -2\,\text{sen}\, 2x$ (1)

$y'' = -2\cos 2x \cdot 2 = -4\cos 2x$ (2)

$y''' = 4 \operatorname{sen} 2x \cdot 2 = 8 \operatorname{sen} 2x$ (3)

$y'''' = 8 \cos 2x \cdot 2 = 16 \cos 2x$ (4)

$y''''' = -16 \operatorname{sen} 2x \cdot 2 = -32 \operatorname{sen} 2x$ (5)

$y^{6} = -32 \cos 2x \cdot 2 = -64 \cos 2x$ (6)

$y^{7} = 64 \operatorname{sen} 2x \cdot 2 = 128 \operatorname{sen} 2x$ (7)

$$\frac{d^{50}}{dx} = -2^{50} \cos 2x$$

78- $f(x) = xe^{-x}$

$f'(x) = x \cdot e^{-x} \cdot (-1) = -xe^{-x}$

$f''(x) = -x \cdot e^{-x} \cdot (-1) = xe^{-x}$

$$\frac{d^{1000}}{dx} f(x) = xe^{-x}$$

79- $v(t) = s'(t)$

$$v(t) = \frac{1}{4} \cos(10\pi t) \cdot 10\pi = \frac{5\pi}{2} \cos(10\pi t)$$