

Pg 186 (3.4) \rightarrow Jaro Vaz

1- $y = \operatorname{sen}(4x)$

$$f(x) = \operatorname{sen}(x); g(x) = 4x; f(g(x)) = \operatorname{sen}(4x)$$

$$y' = f'(g(x)) \cdot g'(x) = \operatorname{sen}'(4x) \cdot (4x)' = \cos(4x) \cdot 4 = 4\cos(4x)$$

2- $y = \sqrt{4+3x}$

$$f(x) = \sqrt{4+x}; g(x) = 3x; f(g(x)) = \sqrt{4+3x}$$

$$y' = f'(g(x)) \cdot g'(x) = \left[(4+3x)^{\frac{1}{2}} \right]' \cdot (3x)' = \frac{1}{2} (4+3x)^{-\frac{1}{2}} \cdot 3 = \sqrt{\frac{1}{4+3x}} \cdot \frac{3}{2} = \frac{3}{2} \sqrt{\frac{1}{4+3x}}$$

3- $y = (1-x^2)^{10}$

$$f(x) = x^{10}; g(x) = 1-x^2; f(g(x)) = (1-x^2)^{10}$$

$$y' = f'(g(x)) \cdot g'(x) = \left[(1-x^2)^{10} \right]' \cdot (1-x^2)' = 10(1-x^2)^9 \cdot (-2x) = -20x(1-x^2)^9$$

4- $y = \operatorname{tg}(\operatorname{sen}x)$

$$f(x) = \operatorname{tg}x; g(x) = \operatorname{sen}x; f(g(x)) = \operatorname{tg}(\operatorname{sen}x)$$

$$y' = f'(g(x)) \cdot g'(x) = [\operatorname{tg}(\operatorname{sen}x)]' \cdot \operatorname{sen}'(x) = \sec^2(\operatorname{sen}x) \cdot \cos x = \cos x \cdot \sec^2(\operatorname{sen}x)$$

5- $y = e^{\sqrt{x}}$

$$f(x) = e^x; g(x) = \sqrt{x}; f(g(x)) = e^{\sqrt{x}}$$

$$y' = f'(g(x)) \cdot g'(x) = \left[e^{\sqrt{x}} \right]' \cdot (\sqrt{x})' = \sqrt{x} e^{\sqrt{x}-1} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

6- $y = \sqrt{2-e^x}$

$$f(x) = \sqrt{x}; g(x) = 2-e^x; f(g(x)) = \sqrt{2-e^x}$$

$$y' = f'(g(x)) \cdot g'(x) = \left[\sqrt{2-e^x} \right]' \cdot (2-e^x)' = \frac{1}{2} (2-e^x)^{-\frac{1}{2}} \cdot (-e^x) \cdot \frac{-e^x}{2\sqrt{2-e^x}} = \frac{e^x \sqrt{2-e^x}}{2(2-e^x)} = \frac{\sqrt{-2e^x + e^{2x}}}{4-2e^x}$$

7- $F(x) = (x^4 + 3x^2 - 2)^5$

$$F'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot (4x^3 + 6x)$$

8- $F(x) = (4x-x^2)^{100}$

$$F'(x) = 100(4x-x^2)^{99} \cdot (4-2x)$$

9- $F(x) = \sqrt[4]{1+2x+x^3} \cdot (1+2x+x^3)^{\frac{11}{4}}$

$$F'(x) = \frac{1}{4} (1+2x+x^3)^{-\frac{3}{4}} \cdot (2+3x^2)$$

10- $f(x) = (1+x^4)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3} (1+x^4)^{-\frac{1}{3}} \cdot 4x^3 = \frac{8x^3}{3} \cdot (1+x^4)^{-\frac{1}{3}}$$

11- $g(t) = \frac{1}{(t^4+1)^3}$

$$g'(t) = \frac{0 \cdot (t^4+1) - 1 \cdot 3(t^4+1)^2 \cdot 4t^3}{(t^4+1)^6} = \frac{-12t^3(t^4+1)^2}{(t^4+1)^6} = \frac{-12t^3}{(t^4+1)^4}$$

12- $f(t) = \sqrt[3]{1+tgt} = (1+tgt)^{\frac{1}{3}}$

$$f'(t) = \frac{1}{3} (1+tgt)^{-\frac{2}{3}} \cdot \sec^2 t \cdot \frac{\sec^2 t}{3\sqrt[3]{1+tgt}^2}$$

$y = \cos(\alpha + x^3)$

$y' = -\sin(\alpha + x^3) \cdot 3x^2 = -3x^2 \sin(\alpha + x^3)$

$14- y = \alpha^3 + \cos^3 x$

$y' = 3 \cos^2 x \cdot (-\sin x) = -3 \cos^2 x \sin x$

$15- y = x e^{-kx} = \frac{x}{e^{kx}}$

$y' = \frac{0 \cdot e^{-kx} - x e^{-kx}}{e^{2kx}} = -x e^{-kx} \cdot e^{-2kx} = -x e^{-3kx}$

$16- y = e^{-2t} \cdot \cos 4t$

$y' = e^{-2t} \cdot \cos 4t + e^{-2t} \cdot -\sin 4t \cdot 4 = e^{-2t} [\cos 4t + 4 \sin 4t]$

$17- f(x) = (2x-3)^4 (x^2+x+1)^6$

$f'(x) = 4 \cdot (2x-3)^3 \cdot 2 + 6 \cdot (x^2+x+1)^5 \cdot (2x+1) = 8 \cdot (2x-3)^3 + 5 \cdot (2x+1) \cdot (x^2+x+1)^4$

$18- g(x) = (x^2+1)^3 (x^2+2)^6$

$g'(x) = 3 \cdot (x^2+1)^2 \cdot 2x + 6 \cdot (x^2+2)^5 \cdot 2x = 6x [(x^2+1)^2 + 2 \cdot (x^2+2)^4]$

$19- h(t) = (t+1)^{2/3} \cdot (2t^2-1)^3$

$h'(t) = \frac{2}{3} (t+1)^{-1/3} \cdot 1 + 3(2t^2-1)^2 \cdot 4t = \frac{2}{3} \left[(t+1)^{-1/3} + 18t(2t^2-1)^2 \right]$

$20- F(t) = (3t-1)^4 (2t+1)^{-3}$

$F'(t) = 4(3t-1)^3 \cdot 3 - 3(2t+1)^{-4} \cdot 2 = 6 \left[2(3t-1)^3 - (2t+1)^{-4} \right]$

$21- y = \left(\frac{x^2+1}{x^2-1} \right)^3$

$y' = 3 \cdot \left(\frac{x^2+1}{x^2-1} \right)^2 \cdot \left(\frac{2x(x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2} \right) = 6x \left(\frac{x^2+1}{x^2-1} \right) \cdot \left(\frac{-2}{(x^2-1)^2} \right) = \frac{-12x(x^2+1)}{(x^2-1)^3}$

$22- f(n) = \sqrt{\frac{n^2+1}{n^2+4}}$

$f'(n) = \frac{1}{2} \left(\frac{n^2+1}{n^2+4} \right)^{-1/2} \cdot \left[\frac{2n(n^2+4) - (n^2+1) \cdot 2n}{(n^2+4)^2} \right] = \left(\frac{n^2+1}{n^2+4} \right)^{-1/2} \cdot \frac{5n^2}{(n^2+4)^2}$

$23- y = \sqrt{1+2e^{3x}}$

$y' = \frac{1}{2} (1+2e^{3x})^{-1/2} \cdot 2e^{3x} \cdot \frac{2e^{3x}}{2\sqrt{1+2e^{3x}}} = \frac{e^{3x}}{\sqrt{1+2e^{3x}}} = \frac{e^{3x}\sqrt{1+2e^{3x}}}{1+2e^{3x}}$

$24- y = 10^{-x^2}$

$y' = (1-x^2) 10^{-x^2} \cdot (-2x) = -2x(1-x^2) 10^{-x^2}$

$25- y = s^{-1/x}$

$y' = (-1/x) \cdot s^{\frac{-1}{x-1}} \cdot \frac{0 \cdot x + 1 \cdot 1}{x^2} = \frac{-\frac{1-x}{x}}{x} \cdot \frac{2}{x^2} = \frac{2 \cdot (-1)}{x^3}$

$26- G(y) = \frac{(y-1)^4}{(y^2+2y)^5} = \frac{4(y-1)^3 \cdot (y^2+2y)^5 \cdot (y-1)^4 \cdot 5 \cdot (y^2+2y)^4}{(y^2+2y)^{10}} = \frac{(y-1)^3 \cdot (y^2+2y)^4 \cdot [4y(y+2) - 5(y-1)]}{(y^2+2y)^6} = \frac{(y-1)^3 [4y(y+2) - 5(y-1)]}{(y^2+2y)^6}$



$$27- y = \frac{x}{\sqrt{x^2+1}}$$

$$y' = \frac{\sqrt{x^2+1} - x(\sqrt{x^2+1})'}{x^2+1} = \frac{\sqrt{x^2+1} - x\left[\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x\right]}{x^2+1} = \frac{2(x^2+1)^{1/2} - 2x^2 \cdot (x^2+1)^{-1/2}}{2(x^2+1)} = \frac{2(x^2+1)\left[(x^2+1)^{1/2} - x^2(x^2+1)^{-3/2}\right]}{2(x^2+1)} = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}$$

$$28- y = \frac{e^{4x} - e^{-4x}}{e^{4x} + e^{-4x}}$$

$$y' = \frac{(e^{4x} - e^{-4x}) \cdot (e^{4x} + e^{-4x}) - (e^{4x} + e^{-4x})(e^{4x} - e^{-4x})}{(e^{4x} + e^{-4x})^2} = 0$$

$$29- F(t) = e^{t \cos 2t}$$

$$F'(t) = e^{t \cos 2t} \cdot (t \cos 2t)' = e^{t \cos 2t} \cdot t \cos 2t \cdot (2t)' = e^{t \cos 2t} \cdot t \cos 2t \cdot 2 = 2t \cos 2t \cdot e^{t \cos 2t}$$

$$30- F(v) = \left(\frac{v}{v^3+1} \right)^6$$

$$F'(v) = 6 \left(\frac{v}{v^3+1} \right)^5 \cdot \frac{v^3+1-v \cdot 3v^2}{(v^3+1)^2} = 6 \cdot \left(\frac{v}{v^3+1} \right)^5 \cdot \left[\frac{4v^3+1}{(v^3+1)^2} \right]$$

$$31- y = \tan(\operatorname{tg} 2t)$$

$$y' = \cos(\operatorname{tg} 2t) \cdot \sec^2 2t \cdot 2 = 2 \cdot \sec^2 2t \cdot \cos(\operatorname{tg} 2t)$$

$$32- y = \sec^2(m\theta)$$

$$y' = 2m \sec(m\theta) \cdot m = 2m \sec(m\theta)$$

$$33- y = 2^{\tan(\pi x)}$$

$$y' = \tan(\pi x) \cdot 2^{\tan(\pi x)-1} \cdot \cos(\pi x) \cdot \pi = \pi \tan(\pi x) \cos(\pi x) \cdot 2^{\tan(\pi x)-1}$$

$$34- y = x^2 \cdot e^{-1/x} = x^2 e^{-x^{-1}}$$

$$y' = 2x \cdot e^{-1/x} + x \cdot e^{-x^{-1}} \cdot (-1) \cdot (-x^{-2}) = 2x \cdot e^{-x^{-1}} \cdot e^{-x^{-1}} (2x+1)$$

$$35- y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$$

$$y' = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \cdot \left[\frac{-2x(1)e^{2x} - (1-e^{2x})^2 e^{2x}}{(1+e^{2x})^2} \right] = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \cdot \left[\frac{-e^{2x} \cdot 2}{(1+e^{2x})^2} \right]$$

$$36- y = \sqrt{1+x e^{-2x}} = (1+x e^{-2x})^{1/2}$$

$$y' = \frac{1}{2} (1+x e^{-2x})^{-1/2} \cdot x e^{-2x} \cdot (-2) = -x e^{-2x} (1+x e^{-2x})^{-1/2}$$

$$37- y = \cot^2(\operatorname{sen} \theta)$$

$$y' = 2 \cot(\operatorname{sen} \theta) \cdot [-\operatorname{cosec}^2(\operatorname{sen} \theta)] \cdot \cos \theta = -2 \cdot \frac{\cos(\operatorname{sen} \theta)}{\operatorname{sen}^3(\operatorname{sen} \theta)} \cdot \cos \theta = \frac{-2 \cos \theta \cdot \cos(\operatorname{sen} \theta)}{\operatorname{sen}^3(\operatorname{sen} \theta)}$$

$$38- y = e^{ktg \sqrt{x}}$$

$$y' = e^{ktg \sqrt{x}} \cdot k \sec^2 \sqrt{x} \cdot \frac{1}{2} x^{-1/2} = \frac{e^{ktg \sqrt{x}} \cdot k \sec^2 \sqrt{x} \cdot x^{-1/2}}{2}$$

$$3x - f(t) = \frac{1}{2}t^2 + e^{tg t}$$

$$f'(t) = \frac{1}{2}t^2(e^t) \cdot e^t + e^{tg t} \cdot \frac{1}{2}t^2 \cdot e^{tg t}$$

$$40 - y = \operatorname{sen}(\operatorname{sen}(\operatorname{sen}(x)))$$

$$y' = \cos(\operatorname{sen}(\operatorname{sen}(x))) \cdot \cos(\operatorname{sen}(x)) \cdot \cos x$$

$$41 - f(t) = \operatorname{sen}^2(e^{\operatorname{sen}^2 t})$$

$$f'(t) = 2\operatorname{sen}(e^{\operatorname{sen}^2 t}) \cdot \cos(e^{\operatorname{sen}^2 t}) \cdot e^{\operatorname{sen}^2 t} \cdot 2\operatorname{sen} t \cdot \cos t = 4\operatorname{sen}(e^{\operatorname{sen}^2 t}) \cdot \cos(e^{\operatorname{sen}^2 t}) \cdot e^{\operatorname{sen}^2 t} \cdot \operatorname{sen} t \cdot \cos t$$

$$42 - y = \sqrt{x + \sqrt{x + \sqrt{x}}} = \sqrt{x + (x + x^{1/2})^{1/2}}^{1/2}$$

$$y' = \frac{1}{2} \left[x + (x + x^{1/2})^{1/2} \right]^{-1/2} \cdot \left[1 + \frac{1}{2}(x + x^{1/2})^{-1/2} \right] \cdot \left[1 + \frac{1}{2}x^{-1/2} \right]$$

$$43 - g(x) = (2\operatorname{sen} x + m)^3$$

$$g'(x) = p(2\operatorname{sen} x + m)^{p-1} \cdot ($$

$$44 - y = 2^{3x^2}$$

$$y' = 3 \cdot 2^{3x^2-1} \cdot x^2 \cdot 3^{x^2-1} \cdot 2x = 2x \cdot 2^{3x^2-1} \cdot 2^{x^2-1}$$

$$45 - y = \operatorname{cos} \left[\sqrt{\operatorname{sen}(\operatorname{tg}(mx))} \right]$$

$$y' = \operatorname{sen} \left[\sqrt{\operatorname{sen}(\operatorname{tg}(mx))} \right] \cdot \frac{1}{2} \cdot \sqrt{\operatorname{sen}(\operatorname{tg}(mx))}^{-1/2} \cdot \cos \left[\operatorname{tg}(mx) \right] \cdot \operatorname{sec}^2(mx) \cdot \pi = \frac{\pi}{2} \cdot \operatorname{sen} \left[\sqrt{\operatorname{sen}(\operatorname{tg}(mx))} \right] \cdot \operatorname{sen} \left[\operatorname{tg}(mx) \right] \cdot \cos \left[\operatorname{tg}(mx) \right] \cdot \operatorname{sec}^2(mx)$$

$$46 - y = \sqrt{x + (x + \operatorname{sen}^2 x)^3}^4$$

$$y' = 4 \left[x + (x + \operatorname{sen}^2 x)^3 \right]^3 \cdot \left[1 + 3(x + \operatorname{sen}^2 x) \cdot (1 + 2\operatorname{sen} x \cdot \operatorname{cos} x) \right]$$

$$47 - y = \operatorname{cos}(x^2)$$

$$y' = -\operatorname{sen}(x^2) \cdot 2x$$

$$y'' = -\operatorname{cos}(x^2) \cdot 2x \cdot 2x + (-\operatorname{sen}(x^2)) \cdot 2 = -4x^2 \operatorname{cos}(x^2) - 2\operatorname{sen}(x^2) = -2 \left[2x^2 \operatorname{cos}(x^2) + \operatorname{sen}(x^2) \right]$$

$$48 - y = \operatorname{cos}^2 x$$

$$y' = 2\operatorname{cos} x \cdot \operatorname{sen} x$$

$$y'' = -2\operatorname{sen} x \cdot \operatorname{sen} x + 2\operatorname{cos} x \cdot \operatorname{cos} x = -2\operatorname{sen}^2 x + 2\operatorname{cos}^2 x = 2(-\operatorname{sen}^2 x + \operatorname{cos}^2 x) = 2(-1 + \operatorname{cos}^2 x + \operatorname{cos}^2 x) = -2 + 4\operatorname{cos}^2 x$$

$$49 - y = e^{ax} \cdot \operatorname{sen} bx$$

$$y' = e^{ax} \cdot \operatorname{sen} bx + e^{ax} \cdot \operatorname{cos} bx \cdot b = e^{ax} \cdot (\operatorname{sen} bx + b \operatorname{cos} bx)$$

$$y'' = e^{ax} \cdot (\operatorname{sen} bx + b \operatorname{cos} bx) + e^{ax} \cdot [\operatorname{cos} bx \cdot b + b \cdot (-\operatorname{sen} bx) \cdot b] = e^{ax} \left[2b \operatorname{cos} bx + \operatorname{sen} bx \cdot (-b^2) \right]$$

$$50 - y = e^{ex}$$

$$y' = e^{ex} \cdot e^x$$

$$y'' = e^x \cdot e^x + e^x \cdot e^x = 2e^x \cdot e^x$$

$$51 - y = (1+2x)^{10}$$

$$y = 10(1+2x)^9 \cdot 2 = 20 \cdot (1+2x)^9$$

$$f'(0) = 20, P(0,1)$$

$$t: y - y_0 = m(x - x_0) \Rightarrow y - 1 = 20(x - 0) \Rightarrow t: y = 20x + 1$$

$$52- y = \sqrt{1+x^2}$$

$$y' = \frac{1}{2} (1+x^2)^{-1/2} \cdot 2x^2; P(2,3)$$

$$f'(2) = \frac{1}{2} \cdot \frac{1}{\sqrt{5}} \cdot 12 = 2$$

$$t: y - y_0 = m(x - x_0) \Rightarrow y - 3 = 2(x - 2) \Rightarrow t: y = 2x - 1$$

$$53- y = \sin(\pi \cos x)$$

$$y' = \cos(\pi \cos x) \cdot \cos x; P(\pi, 0)$$

$$f'(\pi) = 1 \cdot (-1) = -1$$

$$t: y - y_0 = m(x - x_0) \Rightarrow y = -1(x - \pi) \Rightarrow t: y = -x + \pi$$

$$54- y = \sin x + \sin^2 x$$

$$y' = \cos x + 2\sin x \cdot \cos x = \cos x (1 + 2\sin x)$$

$$f'(0) = 1(1+2 \cdot 0) = 1$$

$$t: y - y_0 = m(x - x_0) \Rightarrow y = 1(x - 0) \Rightarrow t: y = x$$

$$55- f(x) = 2\sin x + \sin^2 x$$

$$f'(x) = 2\cos x + 2\sin x \cdot \cos x = 2\cos x (1 + \sin x)$$

$$2\cos x (1 + \sin x) = 0 \Rightarrow 2\cos x = 0 \text{ ou } (1 + \sin x) = 0$$

$$2\cos x = 0 \Rightarrow \cos x = 0 \Rightarrow x = \pi/2 + k\pi \mid k \in \mathbb{Z}.$$

$$1 + \sin x = 0 \Rightarrow \sin x = -1 \Rightarrow x = \frac{3\pi}{2} + 2k\pi \mid k \in \mathbb{Z}.$$

Para a reta tangente não horizontal, $S = \{\pi/2 + 2k\pi, 3\pi/2 + 2k\pi\} \mid k \in \mathbb{Z}$.

$$56- y = \sin 2x - 2\sin x$$

$$y' = \cos 2x \cdot 2 - 2\cos x = 2(\cos 2x - \cos x)$$

$$2(\cos 2x - \cos x) = 0 \Rightarrow \cos 2x = \cos x$$

$$\cos 2x = \cos x \cdot (\cos x - \sin x) \Rightarrow \cos^2 x - \sin^2 x = -2\sin^2 x + 1$$

$$\cos x = -2\sin^2 x + 1 \Rightarrow \text{Para a reta tangente não horizontal, } \{x \in \mathbb{R} \mid \cos x = -2\sin^2 x + 1\}$$

$$61- F(x) = f(g(x))$$

$$F'(5) = f'(g(5)) \cdot g'(5) = f'(-2) \cdot 6 = 4 \cdot 6 = 24$$

$$72- f(x) = x \cdot g(x^2)$$

$$f'(x) = xg'(x^2) \cdot 2x + 2x^2g''(x^2)$$

$$f''(x) = xg''(x^2) \cdot 2x \cdot 2 + 4x^3g''(x^2)$$

$$77- y = \cos 2x$$

$$y' = -\sin 2x \cdot 2 = -2\sin 2x \quad (1)$$

$$y'' = -2\cos 2x \cdot 2 = -4\cos 2x \quad (2)$$

$$y''' = 4 \cdot \sin 2x \cdot 2 = 8 \sin 2x \quad (3)$$

$$y'''' = 8 \cos 2x \cdot 2 = 16 \cos 2x \quad (4)$$

$$y'''' = -16 \sin 2x \cdot 2 = -32 \sin 2x \quad (5)$$

$$y'''' = -32 \cos 2x \cdot 2 = -64 \cos 2x \quad (6)$$

$$y'''' = 64 \sin 2x \cdot 2 = 128 \sin 2x \quad (7)$$

$$\frac{d^{50}}{dx^{50}} = -2^{50} \cos 2x$$

7B- $f(x) = x e^{-x}$

$$f'(x) = x \cdot e^{-x} \cdot (-1) = -x e^{-x}$$

$$f''(x) = -x \cdot e^{-x} \cdot (-1) = x e^{-x}$$

$$\frac{d^{1000}}{dx^{1000}} f(x) = x e^{-x}$$

7a- $v(t) = r(t)$

$$v(t) = \frac{1}{4} \cos(10\pi t) \cdot 10\pi = \frac{5\pi}{2} \cos(10\pi t)$$

8)- a) $\frac{dB}{dt} > 0,35 \cos\left(\frac{2\pi t}{5,11}\right) \cdot \frac{2\pi}{5,11}$

b) $B(1) = 0,35 \cdot 0,39 \cdot 1,16 = 0,158$

a) $x^4 + 2x^2 + 3x^2 = 4$

$$y + y' + 2 + 6x = 0$$

$$y' = -y - 6x - 2$$

b) $x^4 + 2x^2 + 3x^2 = 4$

$$y = \frac{4 - 2x - 3x^2}{x}$$

$$y' = \frac{(1-2-6x) \cdot x - (4-2x-3x^2)}{x^2} = \frac{-2x - 6x^2 - 4 + 2x + 3x^2}{x^2} = \frac{-3x^2 - 4}{x^2}$$

$$c) y' = \frac{1}{x} \left(\frac{4-2x-3x^2}{x} \right) - 6x - 2 = \frac{-4+2x+3x^2}{x^2} - \frac{6x^2 - 2x}{x} = \frac{-3x^2 - 4}{x^2}$$

2- a) $4x^2 + 9y^2 = 36$

$$8x + 18y \cdot y' = 0$$

$$y' = \frac{-8x}{18y} = \frac{-4x}{9y}$$

b) $y = \frac{\sqrt{36 - 4x^2}}{3} = \frac{(36 - 4x^2)^{1/2}}{3}$

$$y' = \frac{1}{2}(36 - 4x^2) \cdot (-2x) \cdot 3 - (36 - 4x^2)^{1/2} \cdot 0 \\ 9$$

$$y' = \frac{-8x \cdot (36 - 4x^2)^{1/2}}{6} = \frac{-4x \cdot (36 - 4x^2)^{1/2}}{3}$$

$$c) y' = \frac{-4x}{9y} = \frac{-4x}{9 \cdot \frac{\sqrt{36 - 4x^2}}{3}} = \frac{-4x}{3 \cdot \sqrt{36 - 4x^2}} = \frac{-4x \cdot (36 - 4x^2)^{1/2}}{3}$$

3- a) $\frac{1}{x} + \frac{1}{y} = 1$

$$\frac{0-1}{x^2} + \frac{0-y'}{y^2} = 0$$

$$\frac{-1 \cdot y'}{x^2 y^2} = 0$$

$$-x^2 y' = y^2$$

$$y' = \frac{-y^2}{x^2} = -\left(\frac{y}{x}\right)^2$$

b) $\frac{1}{y} = 1 - \frac{1}{x} = \frac{x-1}{x}$

$$y = \frac{x-1}{x}$$

$$y' = \frac{1 \cdot x - (x-1)}{x^2} = \frac{x - x + 1}{x^2} = \frac{1}{x^2}$$

$$c) y' = -\left(\frac{x-1}{x}\right)^2 = -\left(\frac{x-1}{x^2}\right)^2 = \frac{y^2 - 2x + 1}{x^4}$$

4- a) $\cos x + \sqrt{y} = 5$

$$-\sin x + \frac{1}{2} y^{-1/2} y' = 0$$

$$y' = \frac{2 \sin x}{y^{1/2}}$$

b) $y = (5 - \cos x)^2$

$$y' = 2(5 - \cos x) \cdot \sin x + 2\sin x(5 - \cos x)$$

$$\text{c)} y' = \frac{2\sin x}{(5 - \cos x)^2} = \frac{2\sin x \cdot (5 - \cos x)}{(5 - \cos x)^2}$$

$$5- x^3 + y^3 = 1$$

$$3x^2 + 3y^2 \cdot y' = 0$$

$$y' = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}$$

$$6- 2\sqrt{x} + \sqrt{y} = 3$$

$$x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot y' = 0$$

$$y' = \frac{-2x^{-1/2}}{y^{-1/2}}$$

$$7- x^2 + xy - y^2 = 4$$

$$2x + y + x \cdot y' - 2y \cdot y' = 0$$

$$2x + y + y'(x - 2y) = 0$$

$$y' = \frac{-2x - y}{x - 2y}$$

$$8- 2x^3 + x^2y - xy^3 = 2$$

$$6x^2 + 2xy + x^2y' - (y^3 + 3xy^2 \cdot y') = 0$$

$$6x^2 + 2xy + x^2y' - y^3 - 3xy^2 \cdot y' = 0$$

$$6x^2 + 2xy + y'(x^2 - 3xy^2) - y^3 = 0$$

$$y' = \frac{y^3 - 6x^2 - 2xy}{x^2 - 3xy^2}$$

$$9- x^4(x+y) = y^2(3x-y)$$

$$x^5 + x^4y + 3xy^2 - y^3$$

$$5x^4 + 4x^3y + x^4y' = 3y(y + 2xy') - 3y^2y'$$

$$x^4y' + 3y^2y' - 3y(y + 2xy') = 5x^4 - 4x^3y$$

$$x^4y' + 3y^2y' - 3y^2 - 6xyy' = -5x^4 - 4x^3y$$

$$x^4y' + 3y^2y' - 6xyy' = -5x^4 - 4x^3y + 3y^2$$

$$y'(x^4 + 3y^2 - 6xy) = -5x^4 - 4x^3y + 3y^2$$

$$y' = \frac{-5x^4 - 4x^3y + 3y^2}{x^4 + 3y^2 - 6xy}$$

$$10- xe^y = x - y$$

$$xe^y + x e^y \cdot y' = 1 - y'$$

$$-y' - xe^y \cdot y' = xe^y - 1$$

$$y'(-1 - xe^y) = xe^y - 1$$

$$y' = \frac{xe^y - 1}{-1 - xe^y}$$

$$11- x^2y^2 + x \operatorname{sen} y = 4$$

$$2x^2y^2 + x^2 \cdot 2y \cdot y' + 2xy + x\cos y \cdot y' = 0$$

$$y(x^2 \cdot 2y + x\cos y) = -2x^2y - 2xy$$

$$y' = \frac{-2x^2y - 2xy}{2x^2y + x\cos y}$$

$$12- 1+x = \tan(xy^2)$$

$$1 = \cos(xy^2) \cdot (y^2 + x \cdot 2y \cdot y')$$

$$1 = y^2 \cos(xy^2) + 2xyy' \cos(xy^2)$$

$$y' = \frac{1 - y^2 \cos(xy^2)}{2xy \cos(xy^2)}$$

$$13- 4\cos x \cdot \sin y = 1$$

$$-4\sin x \cdot \sin y + 4\cos x \cdot \cos y \cdot y' = 0$$

$$4\cos x \cdot \cos y \cdot y' = 4\sin x \cdot \sin y$$

$$y' = \frac{-\sin x \sin y}{\cos x \cos y} = \operatorname{tg} x \cdot \operatorname{tg} y$$

$$14- e^y \cdot \tan x = x + xy'$$

$$e^y \cdot y' \cdot \tan x + e^y \cdot \cos x = 1 + y + xy'$$

$$e^y \cdot y' \cdot \tan x - xy' = 1 + y - e^y \cos x$$

$$y'(e^y \cdot \tan x - x) = 1 + y - e^y \cos x$$

$$y' = \frac{1 + y - e^y \cos x}{e^y \tan x - x}$$

$$15- e^{xy} = x - y$$

$$\ln e^{xy} = \ln(x - y)$$

$$\frac{x}{y} = \ln(x - y)$$

$y = \ln(x - y) \rightarrow y \neq x$ para diferentes; considerar ramos funções

$$e^y = x - y$$

$$e^y \cdot y' = 1 - y'$$

$$y' = \frac{1 - y'}{e^y}$$

$$\frac{y - y'}{y^2} = \frac{1 - y'}{x - y}$$

$$(y - y')(x - y) \cdot y^2 - y^2 \cdot y'$$

$$xy - y^2 - x^2y + xy^2 = y^2 - y^2y'$$

$$-xy' + y^2y' = y^2 - xy + y^2$$

$$y'(-x + y^2) = 2y^2 - xy$$

$$y' = \frac{2y^2 - xy}{y^2 - x}$$

$$16- \sqrt{x+y} = 1 + x^2y^2$$

$$\frac{1}{2}(x+y)^{-1/2} \cdot (1+y') = 2xy \cdot (y + xy')$$

$$(x+y)^{-1/2} + y^1 \cdot (x+y)^{-1/2} = 4xy^2 + 4x^2yy^1$$

$$y^1 \cdot (x+y)^{-1/2} - 4x^2y \cdot y^1 = 4xy^2 - (x+y)^{-1/2}$$

$$y^1 \left[(x+y)^{-1/2} - 4x^2y \right] = 4xy^2 - (x+y)^{-1/2}$$

$$y^1 = \frac{4xy^2 - (x+y)^{-1/2}}{(x+y)^{-1/2} - 4x^2y}$$

$$17- \tan^{-1}(x^2y) = x + xy^2$$

$$\cot^{-1}(x^2y) = x + xy^2$$

$$- \operatorname{cosec}^2(x^2y) \cdot (2xy + x^2y^1) = 1 + y^2 + x \cdot 2y \cdot y^1$$

$$- 2xy \operatorname{cosec}^2(x^2y) - x^2y^1 \cdot \operatorname{cosec}^2(x^2y) = 1 + 2xy \cdot y^1 + y^2$$

$$- x^2y^1 \cdot \operatorname{cosec}^2(x^2y) - 2xy \cdot y^1 = 1 + y^2 + 2xy \operatorname{cosec}^2(x^2y)$$

$$y^1 \left[-x^2 \operatorname{cosec}^2(x^2y) - 2xy \right] = 1 + 2xy \operatorname{cosec}^2(x^2y) + y^2$$

$$y^1 = \frac{1 + 2xy \operatorname{cosec}^2(x^2y) + y^2}{-x^2 \operatorname{cosec}^2(x^2y) - 2xy}$$

$$18- x \operatorname{sen} y + y \operatorname{sen} x = 1$$

$$\operatorname{sen} y + x \cos y \cdot y^1 + y^1 \operatorname{sen} x + y \cdot \operatorname{cos} x = 0$$

$$y^1 (\operatorname{cos} y + \operatorname{sen} x) = -y \operatorname{cos} x - \operatorname{sen} y$$

$$y^1 = \frac{-y \operatorname{cos} x - \operatorname{sen} y}{x \cos y + \operatorname{sen} x}$$

$$19- e^x \operatorname{cos} x = 1 + \operatorname{sen}(xy)$$

$$e^x \cdot y^1 \cdot \operatorname{cos} x - e^x \operatorname{sen} x = \operatorname{cos}(xy) \cdot (y + xy^1)$$

$$e^x \cdot y^1 \cdot \operatorname{cos} x - e^x \operatorname{sen} x = y \operatorname{cos}(xy) + xy^1 \operatorname{cos}(xy)$$

$$e^x \cdot y^1 \cdot \operatorname{cos} x - xy^1 \operatorname{cos}(xy) = y \operatorname{cos}(xy) + e^x \operatorname{sen} x$$

$$y^1 \left[e^x \operatorname{cos} x - x \operatorname{cos}(xy) \right] = y \operatorname{cos}(xy) + e^x \operatorname{sen} x$$

$$y^1 = \frac{y \operatorname{cos}(xy) + e^x \operatorname{sen} x}{e^x \operatorname{cos} x - x \operatorname{cos}(xy)}$$

$$20- \tan(x-y) = \frac{y}{1+x^2}$$

$$\sec^2(x-y) \cdot (1-y^1) = \frac{y \cdot (1+x^2) - 2xy}{(1+x^2)^2}$$

$$\sec^2(x-y) \cdot (1+x^2)^2 - y^1 \cdot \sec^2(x-y) \cdot (1+x^2)^2 - y^1 \cdot (1+x^2) = -2xy$$

$$y^1 \left[-\sec^2(x-y) \cdot (1+x^2)^2 - (1+x^2) \right] = -2xy - \sec^2(x-y) \cdot (1+x^2)^2$$

$$y^1 = \frac{-2xy - \sec^2(x-y) \cdot (1+x^2)^2}{-\sec^2(x-y) \cdot (1+x^2)^2 - (1+x^2)} = \frac{2xy + \sec^2(x-y) \cdot (1+x^2)^2}{\sec^2(x-y) \cdot (1+x^2)^2 + (1+x^2)}$$

$$21- f(x) + x^2 f'(x) = 10$$

$$f(1) = 2$$

$$f'(1) = ?$$

$$y + x^2 y^3 = 10$$

$$y^1 + 2xy^3 + x^2 \cdot 3y^2 \cdot y^1 = 0$$

$$y'(1+x^2 \cdot 3y^2) = -2xy^3$$

$$y' = \frac{-2xy^3}{1+x^2 \cdot 3y^2}$$

$$f'(1) = \frac{-2 \cdot 1 \cdot 8}{1+1 \cdot 12} = \frac{-16}{12} = \frac{-4}{3}$$

$$22. y(x) + x \operatorname{sen} y(x) = x^2$$

$$y'(0) = ?$$

$$y + x \operatorname{sen} y = x^2$$

$$y' + x \operatorname{cos} y + x \cdot \operatorname{cos} y \cdot y' = 2x$$

$$y'(1+x \operatorname{cos} y) = 2x - x \operatorname{cos} y$$

$$y' = \frac{2x - x \operatorname{cos} y}{1+x \operatorname{cos} y}$$

$$y'(0) = \frac{0 - x \operatorname{cos} 0}{1+0 \cdot 1} = \frac{0}{1} = 0$$

$$23. x^4 y^2 - x^3 y + 2x^2 y^3 = 0$$

$$4x^3 y^2 + x^4 y \cdot y' - (3x^2 y + x^3 y') + 2y^3 + 2x \cdot 3y^2 \cdot y' = 0$$

$$4x^3 y^2 + x^4 y \cdot y' - 3x^2 y - x^3 y' + 2y^3 + 6x^2 y^2 \cdot y' = 0$$

$$x^4 y y' - x^3 y' + 6x^2 y^2 \cdot y' = -4x^3 y^2 + 3x^2 y - 2y^3$$

$$y'(x^4 y - x^3 + 6x^2 y^2) = -4x^3 y^2 + 3x^2 y - 2y^3$$

$$y' = \frac{-4x^3 y^2 + 3x^2 y - 2y^3}{x^4 y - x^3 + 6x^2 y^2}$$

$$24. y \operatorname{sec} x = x \operatorname{tg} y$$

$$y' \operatorname{sec} x + y \cdot \operatorname{tg} x \cdot \operatorname{sec} x = \operatorname{tg} y + x \cdot \operatorname{sec}^2 y \cdot y'$$

$$y' \operatorname{sec} x - x \operatorname{sec}^2 y \cdot y' = \operatorname{tg} y - y \operatorname{tg} x \operatorname{sec} x$$

$$y' (\operatorname{sec} x - x \operatorname{sec}^2 y) = \operatorname{tg} y - y \operatorname{tg} x \operatorname{sec} x$$

$$y' = \frac{\operatorname{tg} y - y \operatorname{tg} x \operatorname{sec} x}{\operatorname{sec} x - x \operatorname{sec}^2 y}$$

$$25. n: y \operatorname{sen} 2x = x \operatorname{cos} 2y$$

$$n: y' \operatorname{sen} 2x + y \operatorname{cos} 2x \cdot 2 = \operatorname{cos} 2y - x \operatorname{sen} 2y \cdot 2y'$$

$$n: y' \operatorname{sen} 2x + 2xy' \operatorname{sen} 2y = \operatorname{cos} 2y - 2y \operatorname{cos} 2x$$

$$n: y' (\operatorname{sen} 2x + 2x \operatorname{sen} 2y) = \operatorname{cos} 2y - 2y \operatorname{cos} 2x$$

$$n: y' = \frac{\operatorname{cos} 2y - 2y \operatorname{cos} 2x}{\operatorname{sen} 2x + 2x \operatorname{sen} 2y}$$

$$P(\pi/2, \pi/4); f'(\pi/2) = \frac{0+\pi/4}{0+\pi/2} = \frac{\pi}{2} \cdot \frac{1}{\pi} = \frac{1}{2}$$

$$t: y_0 = m(x_0); y - \pi/4 = \frac{1}{2}(x - \pi/2), t: y = \frac{2x-\pi}{4} + \pi/4 = x/2$$

$$26. n: \operatorname{sen}(x+y) = 2x - 2y$$

$$n: \operatorname{cos}(x+y)(1+y') = 2 - 2y'$$

$$n: \cos(x+y) + y' \cos(x+y) = 2 - 2y'$$

$$n: y' \cos(x+y) + 2y' = 2 - \cos(x+y)$$

$$n: y' \left[\cos(x+y) + 2 \right] = 2 - \cos(x+y)$$

$$n: y' \frac{2 - \cos(x+y)}{\cos(x+y) + 2}$$

$$P(\pi, \pi); f'(\pi) = \frac{2-1}{1+2} = \frac{1}{3}$$

$$t: y - y_0 = m(x - x_0); t: y - \pi = \frac{1}{3}(x - \pi); t: y = \frac{x - 2\pi}{3}$$

$$27- n: x^2 + xy + y^2 = 3$$

$$n: 2x + y + xy' + 2y \cdot y' = 0$$

$$n: y'(x + 2y) = -2x - y$$

$$n: y' = \frac{-2x - y}{x + 2y}$$

$$P(1, 1); f'(1) = \frac{-2-1}{1+2 \cdot 1} = \frac{-3}{3} = -1$$

$$t: y - y_0 = m(x - x_0); t: y - 1 = -1(x - 1); t: y = -x + 2$$

$$28- n: x^2 + 2xy - y^2 + x = 2$$

$$n: 2x + 2y + 2x \cdot y' - 2y \cdot y' + 1 = 0$$

$$n: y'(2x - 2y) = -2x - 2y - 1$$

$$n: y' = \frac{-2x - 2y - 1}{2x - 2y}$$

$$P(1, 2); f'(1) = \frac{-2-4-1}{2-4} = \frac{-7}{2} = -7/2$$

$$t: y - y_0 = m(x - x_0); t: y - 2 = \frac{7}{2}(x - 1); t: y = \frac{7x + 7}{2}$$

$$29- n: x^2 + y^2 = (2x^2 + 2y^2 - x^2)^2$$

$$n: 2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x^2) \cdot (4x + 4y \cdot y' - 2x)$$

$$n: 2x + 2y \cdot y' = 2(8x^3 + 8x^2y \cdot y' - 4x^3 + 8x^2y + 8y^3 \cdot y' - 4x^2y^2 - 4x^3 - 4x^2y^2 + 2x^3)$$

$$n: 2x + 2y \cdot y' = 8x^2y \cdot y' + 8x^2y + 16y^3 \cdot y' + 4x^3$$

$$n: 2y \cdot y' - 8x^2y \cdot y' - 16y^3 \cdot y' = 8x^2y + 4x^3 - 2x$$

$$n: y'(2y - 8x^2y - 16y^3) = 8x^2y + 4x^3 - 2x$$

$$n: y' = \frac{8x^2y + 4x^3 - 2x}{2y - 8x^2y - 16y^3}$$

$$P(0, 1/2); f'(0) = \frac{0+0-0}{1-0-2} = \frac{0}{-1} = 0$$

$$t: y - y_0 = m(x - x_0); t: y - 1/2 = 0(x - 0); t: y = 1/2$$

$$30- n: x^{2/3} + y^{2/3} = 4$$

$$n: \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot y' = 0$$

$$n: y' = \frac{-2x^{-1/3} \cdot \frac{3}{3}}{3 \cdot 2y^{-1/3}} = \frac{-x^{-1/3}}{2y^{1/3}}$$

$$P(-3\sqrt[3]{3}, 1); f'(-3\sqrt[3]{3}) = \frac{1}{27}$$

$$t: y - 1 = \frac{1}{27}(x + 3\sqrt[3]{3}); t: y = \frac{1}{27}x + 1 + \frac{1}{27}(x + 3\sqrt[3]{3})$$

$$31- \text{ Given: } 2(x^2 + y^2)^2 = 25(x^2 - y^2)$$

$$\text{Let: } x^2 = m, y^2 = n \Rightarrow 2(2m+2n)(2m-2n) = 25(2m-2n)$$

$$\text{Let: } 4(2m^2 + 2mn + 2m^2 - 2n^2) = 50m - 50n$$

$$\text{Let: } 8m^2 + 8mn + 50n = 50m - 8m^2 - 8n^2$$

$$\text{Let: } y' = 8x^2y + 8y^3 + 50y = 50x - 8x^3 - 8xy^2$$

$$\text{Let: } y' = \frac{50x - 8x^3 - 8xy^2}{8x^2y + 8y^3 + 50y}$$

$$P(3,1), f'(3) = \frac{150 - 216 - 24}{72 + 8 + 50} = -\frac{9}{13}$$

$$t \cdot y - y_0 = m(x - x_0), t \cdot y - 1 = \frac{-9}{13}(x - 3), t \cdot y = \frac{-9x + 40}{13}$$

$$32- \text{ Given: } y^2(y^2 - 4) = x^2(x^2 - 5)$$

$$\text{Let: } 2yy'(y^2 - 4) + y^2 \cdot 2yy' = 2x(x^2 - 5) + x^2 \cdot 2x$$

$$\text{Let: } y' = 2y^3 - 8y + 2y^3 = 2x^3 - 10x + 2x^3$$

$$\text{Let: } y' = \frac{4x^3 - 10x}{4y^3 - 8y} = \frac{2x^3 - 5x}{2y^3 - 4y}$$

$$35- x^2 + y^2 = 9$$

$$18x + 2yy' = 0$$

$$y' = \frac{9x}{y}$$

$$y'' = \frac{9y - 9xy'}{y^2}$$

$$36- \sqrt{x + \sqrt{y}} = 1$$

$$\frac{1}{2}x^{1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = \frac{2x^{-1/2}}{2 \cdot y^{-1/2}} = \frac{x^{-1/2}}{y^{-1/2}}$$

$$y'' = \frac{\frac{1}{2}x^{-3/2}y^{-1/2} + \frac{1}{2}y^{-3/2}x^{-1/2}}{2y^{-1}} = \frac{-x^{-3/2}y^{-1/2} + y^{-3/2}x^{-1/2}}{2y^{-1}}$$

$$37- x^3 + y^3 = 1$$

$$3x^2 + 3y^2y' = 0$$

$$y' = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}$$

$$y'' = \frac{-2x^3 + x^2 \cdot 2y^1}{y^4} = \frac{-2x^3 + 2x^2y^1}{y^4}$$

$$38- x^4 + y^4 = 1$$

$$4x^3 + 4y^3y' = 0$$

$$y' = \frac{-x^3}{y^3}$$

$$y'' = \frac{-3x^2y^3 + 3y^3y'}{y^6} = \frac{-3x^2y^3 + 3x^3y^1}{y^6}$$

$$39- xy + e^x = e$$

$$y + xy' + e^x y' = 0$$

$$y(x+e^y) = -y$$

$$y' = \frac{-y}{x+e^y}$$

$$y'' = \frac{y'(x+e^y) + y(1+e^y y')}{(x+e^y)^2}$$

Quando $x=0$, $e^y = e$; $y=1$.

$$f(0) = 1$$

$$f'(0) = \frac{-1}{0+e} = -\frac{1}{e}$$

$$f''(0) = \frac{\frac{1}{e}(0+e)+1(1-1)}{e^2} = \frac{1+0}{e^2} = \frac{1}{e^2}$$

40. $x^2 + xy + y^3 = 1$

$$2x+y+x'y'+3y^2y'=0$$

$$y'(x+3y^2) = -2x-y$$

$$y' = \frac{-2x-y}{x+3y^2}$$

$$y'' = \frac{(-2-y)(x+3y^2) - (-2x-y)(1+6y)y'}{(x+3y^2)^2} = \frac{-2x-6y^2-y'y-3y^3 - (-2x-12xyy'-y-6y^3)y'}{(x+3y^2)^2} = \frac{-6y^2-y'y+12xyy'+y+3y^3y'}{(x+3y^2)^2}$$

$$y'' = \frac{[-12yy' - (y-x)y' + (12yy' + 12xyy' + 12xy^2) + y' + 6yy^3y' + 3y^2y']}{(x+3y^2)^2}$$

41- a) $y(y^2-1)(y-2) = x(x-1)(x-2)$

$$y(y^3-2y^2-y+2) = x(x^2-2x-x+2)$$

$$y^4-2y^3-y^2+2y = x^3-3x^2+2x$$

$$4y^3y' - 6y^2y' - 2yy' + 2y = 3x^2 - 6x + 2$$

$$y'(4y^3 - 6y^2 - 2y + 2) = 3x^2 - 6x + 2$$

$$y' = 0 \text{ se e somente se, } 3x^2 - 6x + 2 = 0$$

$$x = \frac{6 \pm \sqrt{36-24}}{6} \cdot \frac{1}{6} = \frac{6 \pm 2\sqrt{3}}{6} \begin{cases} \frac{3+\sqrt{3}}{3} \\ \frac{3-\sqrt{3}}{3} \end{cases}$$

A tangente é horizontal em 2 pontos.

b) $y = \frac{3x^2 - 6x + 2}{4y^3 - 6y^2 - 2y + 2}$

$$P_1(0,1), f'(0) = \frac{0-0+2}{4-6-2+2} = \frac{2}{-2} = -1$$

$$t_1: y - y_0 = m(x - x_0); y - 1 = -1(x - 0); t_1: y = -x + 1$$

$$P_2(0,2); f'(0)$$

$$42- \text{ d) } 2y + y^{-3} = x - 2x + x^3$$

$$6y^2 \cdot y' + 2y \cdot y' - 5y^4 \cdot y' = 4x^3 - 6x^2 + 2x$$

$$y'(6y^2 + 2y - 5y^4) = 4x^3 - 6x^2 + 2x$$

$$y' = \frac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4}$$

$$4x^3 - 6x^2 + 2x = 0$$

$$x(4x^2 - 6x + 2) = 0$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{8} \quad \begin{cases} 6+2/8=1 \\ 6-2/8=\frac{1}{2} \end{cases}$$

Para a tangentes horizontais, $x \in \{0, \frac{1}{2}, 1\}$.

$$43- 2(x^2 y^2)^2 = 25(x^2 - y^2)$$

$$y' = \frac{50x - 8x^3 - 8xy^2}{8x^2 y + 8y^3 + 50y}$$

$$y' = 0; 50x - 8x^3 - 8xy^2 = 0$$

$$8xy^2 = 50x - 8x^3$$

$$y = \sqrt{\frac{50-8x^2}{8}}$$

$$44- m: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$n': \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

$$n': y' = \frac{-2xb^2}{a^2ay} = \frac{-xb^2}{ya^2}$$

$$P(x_0, y_0) \rightsquigarrow t: y - y_0 = m(x - x_0)$$

$$t: y - y_0 = \frac{-xb^2}{ya^2}(x - x_0)$$

$$t: \frac{-x_0 b^2 x}{y_0 a^2} + \frac{-x^2 b^2}{y_0 a^2} + y_0 - y = 0$$

$$45- m: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$n': \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$

$$n': y' = \frac{2x b^2}{a^2 2y} = \frac{xb^2}{ya^2}$$

$$P(x_0, y_0) \rightsquigarrow t: y - y_0 = m(x - x_0)$$

$$t: y - y_0 = \frac{xb^2}{ya^2}(x - x_0)$$

$$t: y = \frac{xb^2(x - x_0) + y_0 a^2 y}{ya^2}$$

$$46- \sqrt{x} + \sqrt{y} = \sqrt{c}$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot y' = 0$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{2x}$$

$$47- y = \tan^{-1} \sqrt{x}$$

$$\frac{dy}{dx} = -\operatorname{cosec}^2 \sqrt{x} \cdot \frac{1}{2}x^{-1/2} = -\operatorname{cosec}^2 \sqrt{x} \cdot \frac{\sqrt{x}}{2x}$$

$$50- y = \sqrt{\tan^{-1} x}$$

$$\frac{dy}{dx} = \frac{1}{2} (\operatorname{tg}^2 x) \cdot 1 - \operatorname{cosec}^2 x = -\operatorname{tg} x \cdot \operatorname{cosec}^2 x$$

51. $y = \operatorname{sen}^{-1}(2x+1)$

$$\frac{dy}{dx} = \operatorname{cosec}(2x+1) \cdot \operatorname{ctg}(2x+1) \cdot 2 = 2 \operatorname{cosec}(2x+1) \operatorname{ctg}(2x+1)$$

52. $g(x) = \sqrt{x^2 - 1} \cdot \operatorname{arc}^{-1} x$

$$\frac{dg}{dx} = \frac{1}{2} (x^2 - 1)^{-1/2} \cdot 2x \cdot \operatorname{arc}^{-1} x - (x^2 - 1)^{1/2} \cdot \operatorname{sen} x = \frac{\operatorname{cos} x \cdot x}{\sqrt{x^2 - 1}} - \frac{\operatorname{sen} x}{\sqrt{x^2 - 1}} = \frac{x \operatorname{cos} x - \operatorname{sen} x}{\sqrt{x^2 - 1}}$$

* 53.

54. $y = \operatorname{tg}^{-1}(x - \sqrt{1+x^2})$

$$\frac{dy}{dx} = -\operatorname{cosec}^2(x - \sqrt{1+x^2}) \left[1 - \frac{1}{2} (1+x^2) \cdot 2x \right] = -\operatorname{cosec}^2(x - \sqrt{1+x^2}) \frac{(1+x^2) - x^2}{\sqrt{1+x^2}}$$

55. $f(t) = \operatorname{ctg}^{-1}(t) + \operatorname{ctg}^{-1}(t^2)$

$$\frac{df}{dt} = \operatorname{sec}^2 t + \operatorname{sec}^2 t^2 \cdot (-t^2) = \operatorname{sec}^2 t [1 - t^2]$$

* 56.

57. $y = x \cdot \operatorname{arc}^{-1} x + \sqrt{1-x^2}$

$$\frac{dy}{dx} = \operatorname{cosec} x - x \cdot \operatorname{cosec} x \operatorname{ctg} x + \frac{1}{2} (1-x^2) \cdot 2x = \operatorname{cosec} x \left[1 - \frac{x^2 \operatorname{ctg} x}{\sqrt{1-x^2}} \right] = \operatorname{cosec} x \frac{[\sqrt{1-x^2} - x^2 \operatorname{ctg} x]}{\sqrt{1-x^2}}$$

58. $y = \operatorname{sec}(\operatorname{cosec} t)$

$$\frac{dy}{dt} = \operatorname{sec}(\operatorname{cosec} t) \cdot \operatorname{tg}(\operatorname{cosec} t) \cdot (-\operatorname{cosec} t) \cdot \operatorname{ctg} t = -\operatorname{sec}(\operatorname{cosec} t) \cdot \operatorname{cosec} t \cdot \operatorname{tg}(\operatorname{cosec} t) \cdot \operatorname{ctg} t$$

75. $x^2y^2 + xy = 2 \implies x(xy^2 + y) = 2$

$$2xy^2 + x^2y^2 + y + xy = 0$$

$$y'(2y+x) = -2xy^2 - y$$

$$y' = \frac{-2xy^2 - y}{2y+x} = -1$$

$$-2xy^2 - y = -2y - x$$

$$2xy^2 + y = 2y + x$$

$$y = 2xy^2 - x = x(2y^2 - 1)$$

$$x = \frac{y}{2y^2 - 1}$$

$$\left(\frac{y}{2y^2 - 1} \right) \left| \frac{y^3}{2y^2 - 1} - y \right| = 2$$

$$\frac{y}{2y^2 - 1} \cdot \frac{y^3 + y(2y^2 - 1)}{2y^2 - 1} = 2$$

$$\frac{y^6 + y^4(2y^2 - 1)}{4y^4 - 4y^2 + 1} = 2$$

$$8y^6 - 8y^4 + 2 = y^4 + 2y^4 - y^2$$

$$7y^4 - 9y^2 + 2 = 0$$

$$z = y^2$$

$$z^2 - 9z + 2 = 0$$

$$z = \frac{9 \pm \sqrt{81-56}}{14} \quad \begin{cases} z \\ 4|14 = 2|7 \end{cases}$$

$$y = \frac{9 \pm 1}{2} = \left\{ \frac{5}{2}, \frac{7}{2} \right\}$$

$$x = \frac{y}{2y-1}$$

$$y_1 = -1 \quad x_1 = 1 \quad P_1(1, -1)$$

$$y_2 = 1 \quad x_2 = 1 \quad P_2(1, 1)$$

$$y_3 = -\sqrt{\frac{2}{7}} \quad x_3 = \frac{-\sqrt{14}}{3} \quad P_3\left(-\frac{\sqrt{14}}{3}, -\sqrt{\frac{2}{7}}\right)$$

$$y_4 = \sqrt{\frac{2}{7}} \quad x_4 = \frac{\sqrt{14}}{3} \quad P_4\left(\frac{\sqrt{14}}{3}, \sqrt{\frac{2}{7}}\right)$$

Pg 201 (3.6) JOJO VAC

2- $f(x) = x \ln x - x = x(\ln x - 1)$

$$\frac{dy}{dx} = \ln x + x \left(\frac{1}{x} \right) = \ln x + \frac{x}{x} = \ln x + 1$$

3- $f(x) = \ln(\ln x)$

$$\frac{dy}{dx} = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$$

4- $f(x) = \ln(\sin^2 x)$

$$\frac{dy}{dx} = \frac{1}{\sin^2 x \ln e} \cdot (\sin^2 x)' = \frac{2 \sin x \cdot \cos x}{\sin^2 x} = \frac{2 \cos x}{\sin x} = 2 \operatorname{ctg} x$$

5- $f(x) = \sqrt[5]{\ln x} = (\ln x)^{1/5}$

$$\frac{dy}{dx} = \frac{1}{5} (\ln x)^{-1/5} \cdot \frac{1}{x} = \frac{(\ln x)^{-1/5}}{x}$$

6- $f(x) = \ln \sqrt[5]{x} = \ln(x^{1/5}) = \frac{1}{5} \ln x$

$$\frac{dy}{dx} = \frac{1}{5} \cdot \frac{1}{x} = \frac{1}{5x} = (5x)^{-1}$$

7- $f(x) = \log_{10}(x^3+1)$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

$$\frac{dy}{dx} = \frac{(x^3+1)'}{(x^3+1) \ln 10} = \frac{3x^2}{\ln 10 (x^3+1)}$$

8- $f(x) = \log_{10}(xe^x)$

$$\frac{dy}{dx} = \frac{(xe^x)'}{(xe^x) \ln 10} = \frac{e^x + xe^x}{xe^x \ln 10} = \frac{e^x(1+x)}{xe^x \ln 10} = \frac{1+x}{x \ln 10}$$

9- $f(x) = \ln x \ln 5x$

$$\frac{dy}{dx} = \cos x \ln 5x + \ln x \cdot \frac{(5x)'}{5x \ln e} = \cos x \ln 5 + \frac{\ln x}{5 \ln e}$$

10- $f(v) = \frac{v}{1+v \ln v}$

$$\frac{dy}{dv} = \frac{(1+v \ln v)-v(\frac{1}{1+v \ln v})}{(1+v \ln v)^2} = \frac{1+v \ln v - 1}{(1+v \ln v)^2} = \frac{\ln v}{(1+v \ln v)^2}$$

11- $g(x) = \ln(x\sqrt{x^2+1})$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2+1} \cdot \ln e} \cdot \left[\sqrt{x^2+1} + x \cdot \frac{1}{2} (x^2+1)^{-1/2} \cdot 2x \right] = \frac{\sqrt{x^2+1} + (x^2+1)^{-1/2}}{x^2\sqrt{x^2+1}}$$

12- $f(x) = \ln(x + \sqrt{x^2+1})$

$$\frac{dy}{dx} = \frac{(x+\sqrt{x^2+1})'}{(x+\sqrt{x^2+1}) \ln e} = \frac{\frac{1}{2} [2+(x^2+1)^{-1/2} \cdot 2x]}{x+\sqrt{x^2+1}} = \frac{2x(x^2+1)^{-1/2}}{2x+2\sqrt{x^2+1}}$$

13- $G(y) = \ln\left(\frac{(2y+1)^5}{y^2+1}\right) = 5 \ln(2y+1) - \frac{1}{2} \ln(y^2+1)$

$$\frac{dG}{dy} = \frac{5}{(2y+1) \ln e} - \frac{1}{2} \cdot \frac{2y}{(y^2+1) \ln e} = \frac{10}{2y+1} - \frac{2}{y^2+1} = \frac{10(y^2+1) - 2(2y+1)}{(2y+1)(y^2+1)}$$

14- $g(n) = n^2 \ln(2n+1)$

$$\frac{dy}{dx} = 2n \ln(2n+1) + n^2 \cdot \frac{2}{(2n+1) \ln e} = 2n \ln(2n+1) + \frac{2n^2}{2n+1} = \frac{(2n+1)2n \ln(2n+1) + 2n^2}{2n+1} = \frac{2n[(2n+1)\ln(2n+1) + n]}{2n+1}$$

15- $F(x) = \ln(\ln x)$

$$\frac{dF}{dx} = \frac{(\ln x)'}{\ln x \ln e} = \frac{1}{\ln x} = \frac{1}{x \ln e}$$

16- $y = \ln|1+t-t^2|$

$$\frac{dy}{dt} = \frac{(1+t-t^2)}{(1+t-t^2) \ln t} = \frac{1-3t}{1+t-t^2}$$

17- $y = \operatorname{tg}[\ln(ax+b)]$

$$\frac{dy}{dx} = \sec^2[\ln(ax+b)] \cdot \frac{(ax+b)^1}{(ax+b)\ln^2} + \operatorname{sec}^2[\ln(ax+b)] \cdot \frac{a}{ax+b} = \operatorname{sec}^2[\ln(ax+b)] \cdot \frac{a}{ax+b}$$

18- $y = \ln(\cos(\ln x))$

$$\frac{dy}{dx} = \frac{1}{\cos(\ln x) \ln^2} \cdot [-\operatorname{sen}(\ln x) \cdot \frac{1}{x}] = \frac{-\operatorname{sen}(\ln x)}{\cos(\ln x) \cdot x} = \frac{-\operatorname{tg}(\ln x)}{x}$$

19- $y = \ln(e^{-x} + xe^{-x})$

$$\frac{dy}{dx} = \frac{(e^{-x} + xe^{-x})'}{(e^{-x} + xe^{-x}) \cdot \ln e} = \frac{-e^{-x} \cdot e^{-x} + e^{-x} \cdot (-e^{-x})}{e^{-x} + xe^{-x}} = \frac{-e^{-x} - xe^{-x}}{e^{-x} + xe^{-x}}$$

20- $H(z) = \ln\left(\left(\frac{a^2-3^2}{a^2+3^2}\right)^{1/2}\right) = \frac{1}{2}[\ln(a^2-3^2) - \ln(a^2+3^2)]$

$$\frac{dH}{dz} = \frac{1}{2} \left[\frac{-2z}{(a^2-3^2)\ln e} - \frac{2z}{(a^2+3^2)\ln e} \right] = \frac{(a^2+3^2)(-2z) - (a^2-3^2)2z}{2(a^2-3^2)(a^2+3^2)} = \frac{2z(-a^2-3^2-a^2+3^2)}{2(a^2-3^2)(a^2+3^2)} = \frac{-2z(a^2+3^2)}{(a^2-3^2)}$$

21- $y = 2 \times \log_{10} \sqrt{x}$

$$\frac{dy}{dx} = 2 \log_{10} \sqrt{x} + 2x \cdot \frac{\frac{1}{2}x^{-1/2}}{(\sqrt{x}) \cdot \ln 10} = 2 \log_{10} \sqrt{x} + \frac{x^{1/2}}{x^{1/2} \cdot \ln 10} = 2 \log_{10} \sqrt{x} + \frac{1}{\ln 10} = \frac{\log_{10} \sqrt{x} + 1}{\ln 10}$$

22- $y = \log_2(e^x \cos \pi x)$

$$\frac{dy}{dx} = \frac{(e^x \cos \pi x)'}{(e^x \cos \pi x) \cdot \ln 2} = \frac{-e^x \cdot e^x \cos \pi x + e^x \cdot (-\pi \operatorname{sen} \pi x) \cdot \pi}{\ln 2 \cdot e^x \cos \pi x} = \frac{-e^x (\cos \pi x + \pi \operatorname{sen} \pi x)}{\ln 2 \cdot e^x \cos \pi x} = \frac{-\cos \pi x - \pi \operatorname{sen} \pi x}{\ln 2 \cdot \cos \pi x} = \frac{-\cos \pi x - \pi \operatorname{sen} \pi x + \operatorname{tg} \pi x}{\ln 2 \cdot \cos \pi x} = \frac{-1 - \pi^2 \operatorname{tg} \pi x}{\ln 2}$$

23- $y = x^2 \ln 2x$

$$\frac{dy}{dx} = 2x \ln 2x + x^2 \cdot \frac{1}{2x \cdot \ln 2} \cdot 2 = 2x \ln 2x + x = x(2 \ln 2x + 1)$$

$$\frac{d^2y}{dx^2} = 2 \ln 2x + 2x \cdot \frac{1}{2x} \cdot 2 + 1 = 2 \ln 2x + 3$$

24- $y = \frac{\ln x}{x^2}$

$$\frac{dy}{dx} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \cdot \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{2}{x} \cdot x^3 - (1-2 \ln x) \cdot 3x^2}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2 \ln x}{x^6} = \frac{-5x^2 + 6x^2 \ln x}{x^6} = \frac{-5 + 6 \ln x}{x^4}$$

25- $y = \ln(x + \sqrt{1+x^2})$

$$\frac{dy}{dx} = \frac{1}{(x + \sqrt{1+x^2}) \cdot \ln e} \left[1 + \frac{1}{2} (1+x^2) \cdot 2x \right] = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}} \right) = \frac{1}{x + \sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} = (\sqrt{1+x^2})^{-1}$$

$$\frac{d^2y}{dx^2} = -(\sqrt{1+x^2})^{-2} \cdot \frac{1}{2} (1+x^2)^{-1/2} \cdot 2x = \frac{-1}{1+x^2} \cdot \frac{x}{\sqrt{1+x^2}} = \frac{-x\sqrt{1+x^2}}{1+x^2} = \frac{-x\sqrt{1+x^2}}{1+x^2}$$

26- $y = \ln(\sec x + \operatorname{tg} x)$

$$\frac{dy}{dx} = \frac{1}{\sec x + \operatorname{tg} x} \cdot \sec x \operatorname{tg} x + \sec^2 x = \frac{\operatorname{tg} x + \sec x}{\operatorname{tg} x} \cdot \operatorname{cosec} x / (\operatorname{tg} x + \sec x) = 1 + \operatorname{cosec} x$$

27- $f(x) = \frac{x}{1 - \ln(x-1)}$

$$\frac{df}{dx} = \frac{1 - \ln(x-1) - x \cdot \frac{-1}{x-1}}{[(1 - \ln(x-1))^2]} = \frac{1 - \ln(x-1) + x(x-1)^{-1}}{[(1 - \ln(x-1))^2]}$$

II: $1 - \ln(x-1) + 0 ; \ln(x-1) \neq 1 ; e^t \neq x-1 ; \text{II: } \exists x \in \mathbb{R} \mid x > e+1 \}$

28- $f(x) = \sqrt{2 + \ln x}$

$$\frac{df}{dx} = \frac{1}{2} (2 + \ln x)^{-1/2} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{2+\ln x}} = \frac{\sqrt{2+\ln x}}{2x(2+\ln x)}$$

II: $2 + \ln x > 0 ; \ln x > -2 ; x > e^2 ; \text{II: } \exists x \in \mathbb{R} \mid x > e^2 \}$

29- $f(x) = \ln(x^2 - 2x) = \ln[x(x-2)] = \ln x + \ln(x-2)$

$$\frac{df}{dx} = \frac{1}{x} + \frac{1}{x-2} = \frac{x-2+x}{x(x-2)} = \frac{2x-2}{x(x-2)}$$

$$D: x^2 - 2x > 0 ; x < 0 \cup x > 2 ; D = \{x \in \mathbb{R} \mid x < 0 \cup x > 2\} = (-\infty, 0) \cup (2, +\infty)$$

$$x^2 - 2x = 0 ; x \in [0, 2]$$

$$\begin{array}{c} + \\ \text{U} \\ - \\ \circ \\ 2 \end{array}$$

$$30. f(x) = \ln(\ln x)$$

$$\frac{df}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

$$D: \ln x > 0 ; x > e^0 ; x > 1 ; D = \{x \in \mathbb{R} \mid x > 1\}$$

$$31. f(x) = \frac{\ln x}{x^2}$$

$$\frac{df}{dx} = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$$

$$\frac{df(1)}{dx} = 1 - 2 \ln 1 = 1$$

$$32. f(x) = \ln(1 + e^{2x})$$

$$\frac{df}{dx} = \frac{1}{1 + e^{2x}} \cdot 2e^{2x} \cdot 2 = \frac{2e^{2x}}{1 + e^{2x}}$$

$$\frac{df(0)}{dx} = \frac{2}{1+2} = \frac{2}{3}$$

$$33. y = \ln(x^2 - 3x + 1)$$

$$\frac{dy}{dx} = \frac{1}{x^2 - 3x + 1} \cdot (2x - 3) = \frac{2x - 3}{x^2 - 3x + 1}$$

$$P(1, 0) ; F'(1) = \frac{3}{9 - 9 + 1} = 3$$

$$t: y_0 - y_0 = m(x - x_0) ; y_0 - 0 = 3(x - 1) ; t: y = 3x - 9$$

$$34. y = x^2 \ln x$$

$$\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x = x(2 \ln x + 1)$$

$$P(1, 0) ; F'(1) = 2 \ln 1 + 1 = 1$$

$$t: y_0 - y_0 = m(x - x_0) ; t: y = x - 1$$

$$35. f(x) = Cx + \ln(\cos x)$$

$$\frac{df}{dx} = C + \frac{1}{\cos x} \cdot (-\sin x) = C - \tan x$$

$$F(\pi/4) = C - \tan \pi/4 = C - 1 = 6 ; C = 7$$

$$36. f(x) = \log_a(3x^2 - 2)$$

$$\frac{df}{dx} = \frac{1}{(3x^2 - 2) \ln a} \cdot 6x = \frac{6x}{(3x^2 - 2) \ln a}$$

$$F'(1) = \frac{6}{\ln a} = 3 ; \ln a = 2 ; a = e^2$$

$$37. y = (2x+1)^5 (x^4 - 3)^6$$

$$\ln y = 5 \ln(2x+1) + 6 \ln(x^4 - 3)$$

$$\frac{1}{y} \cdot y' = 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4 - 3} \cdot 4x^3$$

$$\frac{y'}{y} = \frac{10}{2x+1} + \frac{24x^3}{x^4 - 3} = \frac{10(2x+1)^4 + 24x^3(2x+1)^4}{(2x+1)(x^4 - 3)} = \frac{58x^4 + 24x^3 - 30}{(2x+1)(x^4 - 3)}$$

$$\frac{y'}{y} = \frac{(58x^4 + 24x^3 - 30) \cdot (2x+1)^5 (x^4 - 3)^6}{(2x+1)(x^4 - 3)} = (58x^4 + 24x^3 - 30)(2x+1)^4 (x^4 - 3)^5$$

$$40- y = \sqrt{x} e^x (x+1)^{10}$$

$$\ln y = \frac{1}{2} \ln x + x + 10 \ln(x+1)$$

$$\frac{1}{y} \cdot y' = \frac{1}{2x} + 2x + 10 \cdot \frac{1}{x+1} \cdot 2x$$

$$\frac{y'}{y} = \frac{1}{2x} + 2x + \frac{20x}{x+1} = \frac{x^2+4x^2(x+1)+40x^2}{2x(x+1)} = \frac{x^2+4x^2+4x^2+40x^2}{2x(x+1)} = \frac{4x^2+44x^2+1}{2x(x+1)}$$

$$y' = \frac{4x^2+44x^2+1}{2x} \cdot \sqrt{x} \cdot e^x (x+1)^9$$

$$41- y = \sqrt{\frac{x-1}{x^4+1}}$$

$$\ln y = \frac{1}{2} [\ln(x-1) - \ln(x^4+1)]$$

$$\frac{1}{y} \cdot y' = \frac{\frac{1}{x-1} - \frac{1}{x^4+1} \cdot 4x^3}{2} = \frac{x^4+1 - 4x^3(x-1)}{2(x-1)(x^4+1)} = \frac{-3x^4+4x^3+1}{2(x-1)(x^4+1)}$$

$$y' = \frac{-3x^4+4x^3+1}{2(x-1)(x^4+1)} \cdot \sqrt{\frac{x-1}{x^4+1}}$$

$$42- y = \sqrt{x} e^{x^2-x} (x+1)^{2/3}$$

$$\ln y = \frac{1}{2} \ln x + x^2 - x + \frac{2}{3} \ln(x+1)$$

$$\frac{1}{y} \cdot y' = \frac{1}{2x} + 2x - 1 + \frac{2}{3(x+1)} = \frac{3(x+1) + 12x^2(x+1) - 6x(x+1) + 4x}{6x(x+1)}$$

$$y' = \frac{3(x+1) + 12x^2(x+1) - 6x(x+1) + 4x}{6x(x+1)} \cdot \sqrt{x} e^{x^2-x} (x+1)^{2/3}$$

$$43- y = x^x$$

$$\ln y = x \ln x$$

$$\frac{y'}{y} = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

$$y' = x \ln x (\ln x + 1)$$

$$44- y = x^{\cos x}$$

$$\ln y = \cos x \ln x$$

$$\frac{y'}{y} = -\sin x \ln x + \cos x \frac{1}{x} = -\sin x \ln x + \frac{\cos x}{x} = \frac{-x \sin x \ln x + \cos x}{x}$$

$$y' = x^{\cos x - 1} (-x \sin x \ln x + \cos x)$$

$$45- y = x^{\sin x}$$

$$\ln y = \sin x \ln x$$

$$y' = (\sin x \ln x)' \cdot y$$

$$y' = (\cos x \ln x + \sin x \cdot \frac{1}{x}) \cdot x^{\sin x}$$

$$y' = x^{\sin x - 1} (\cos x \ln x + \sin x)$$

$$46- y = \sqrt{x^x}$$

$$\ln y = \frac{x \ln x}{2}$$

$$\frac{y'}{y} = \frac{\ln x + 1}{2}$$

$$y' = \frac{\sqrt{x}(\ln x + 1)}{2}$$

$$47- y = (\cos x)^x$$

$$\ln y = x \ln(\cos x)$$

$$y' = \ln(\cos x) + x \cdot \frac{1}{\cos x} \cdot (-\sin x)$$

$$y' = [\ln(\cos x) - x \tan x] (\cos x)^x$$

$$48. y = (\sin x)^{\ln x}$$

$$\ln y = \ln x \cdot \ln(\sin x)$$

$$y' = \frac{1}{x} \cdot \ln(\sin x) + \ln x \cdot \frac{1}{\sin x} \cdot \cos x = \frac{\ln(\sin x)}{x} + \ln x \cot x$$

$$y' = \frac{\ln(\sin x)}{x} \left[\frac{\ln(\sin x) + \ln x \cot x}{x} \right]$$

$$49. y = (\tan x)^{\ln x}$$

$$\ln y = \frac{1}{x} \ln(\tan x)$$

$$y' = \frac{1}{x^2} \tan x \cdot \sec^2 x \cdot x - \ln(\tan x) = \frac{x \sec x \cdot \csc x - \ln(\tan x)}{x^2}$$

$$y' = \frac{(\tan x)^{\ln x}}{x^2} \left[x \sec x \csc x - \ln(\tan x) \right]$$

$$50. y = (\ln x)^{\cos x}$$

$$\ln y = \cos x \cdot \ln(\ln x)$$

$$y' = -\sin x \cdot \ln(\ln x) + \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{\cos x - x \ln x \cdot \csc x \cdot \ln(\ln x)}{x \ln x}$$

$$y' = \frac{\cos x}{x \ln x} \left[\cos x - x \ln x \cdot \csc x \cdot \ln(\ln x) \right]$$

$$51. y = \ln(x^2 + y^2)$$

$$y' = \frac{1}{x^2 + y^2} \cdot (2x + 2yy')$$

$$x^2 y' + y^2 y' = 2x + 2yy'$$

$$x^2 y' + y^2 y' - 2yy' = 2x$$

$$y'(x^2 + y^2 - 2y) = 2x$$

$$y' = \frac{2x}{x^2 + y^2 - 2y}$$

$$52. y = x^y$$

$$x \ln y = y \ln x$$

$$\ln y + x \cdot \frac{1}{y} y' = y \ln x + y \cdot \frac{1}{x}$$

$$\frac{xy'}{y} - y \ln x = \frac{1}{x} - \ln y$$

$$\frac{xy' + y \ln x}{y} = \frac{1}{x} - \ln y$$

$$y' \left(x - y \ln x \right) = \frac{y(1 - x \ln y)}{x}$$

$$y' = \frac{y(y - x \ln y)}{x(x - y \ln x)}$$

$$53. f(x) = \ln(x-1)$$

$$f'(x) = \frac{1}{x-1} = (x-1)^{-1} = \frac{1}{(x-1)}$$

$$f''(x) = -(x-1)^{-2} = \frac{-1}{(x-1)^2}$$

$$f'''(x) = 2(x-1)^{-3} = \frac{2}{(x-1)^3}$$

$$f'''(x) = -6(x-1)^{-4} = \frac{-6}{(x-1)^4}$$

$$f''(x) = 24(x-1)^{-5} = \frac{24}{(x-1)^5}$$

$$f'(x) = -120(x-1)^{-6} = \frac{-120}{(x-1)^6}$$

De forma general, $\frac{d^m f}{dx^m} = \begin{cases} \frac{(m-1)!}{(x-1)^m}, & \text{si } m \geq 1, 2, \dots \\ \frac{-(m-1)!}{(x-1)^m}, & \text{si } m = 0 \end{cases}$; vea seja, $\frac{d^m f}{dx^m} = \frac{(-1)^{m-1} \cdot (m-1)!}{(x-1)^m}$

m jamón

m pan

$$59- y = x^8 \ln x$$

$$\frac{dy}{dx} = 8x^7 \ln x + x^8 \cdot \frac{1}{x} = 8x^7 \ln x + x^7$$

$$\frac{d^2y}{dx^2} = 56x^6 \ln x + 8x^7 \cdot \frac{1}{x} + 7x^6 = 56x^6 \ln x + 15x^6$$

$$\frac{d^3y}{dx^3} = 56 \cdot 6 \cdot x^5 \ln x + 56x^6 \cdot \frac{1}{x} + 15 \cdot 6 \cdot x^5 = 56 \cdot 5 \cdot 6 \ln x + x^5 (56 + 15 \cdot 6)$$

$$\text{A tercera derivada será } 8! \times \ln x + (7!)^2 = 8! \times \ln x + 0$$

$$\text{A cuarta derivada será } 8! \ln x = 8! \cdot \frac{1}{x}$$

$$\text{Por tanto, } \frac{d^4y}{dx^4} = \frac{8!}{x} = \frac{40320}{x}.$$

Pg 211 (3.7) Joso Vz

4- $f(t) = t^3 - 12t^2 + 36t$

a) $v(t) = \frac{df}{dt} = 3t^2 - 24t + 36$

b) $v(3) = 27 - 72 + 36 = -9 \text{ m/s}$

c) $v(t) = 0 \Rightarrow 3(t^2 - 8t + 12) = 0 \Rightarrow t \in \{2, 6\}$

d) $v(t) > 0 \quad \begin{array}{c} \text{---} \\ \text{---} \\ 2 \quad 6 \end{array} \quad \therefore t \in [0, 2) \cup (6, \infty)$

e) $d(8) = d(0-2) + d(2-6) + d(6-8)$

$d(8) = |f(2) - f(0)| + |f(6) - f(2)| + |f(8) - f(6)|$

$d(8) = |32 - 0| + |0 - 32| + |32 - 0| = 32 + 32 + 32 = 96 \text{ m}$

g) $\alpha = \frac{dv}{dt} = 6t - 24$

$\alpha(3) = 18 - 24 = -6 \text{ m/s}^2$

i) Acelerando: $(\alpha > 0 \wedge v > 0)$ ou $(\alpha < 0 \wedge v < 0)$ (1)

Frenando: $(\alpha > 0 \wedge v < 0)$ ou $(\alpha < 0 \wedge v > 0)$ (2)

(1): $\alpha > 0 \Rightarrow 6t - 24 > 0 \Rightarrow t > 4 \quad \left. \begin{array}{l} \\ v > 0 \Rightarrow t \in [0, 2) \cup (6, \infty) \end{array} \right\} t \in (6, \infty)$

$\alpha < 0 \quad \left. \begin{array}{l} t < 4 \\ v < 0 \Rightarrow t \in (2, 6) \end{array} \right\} t \in (2, 4)$

(2): $\alpha > 0 \quad \left. \begin{array}{l} t > 4 \\ v < 0 \Rightarrow t \in (2, 6) \end{array} \right\} t \in (4, 6)$

$\alpha < 0 \quad \left. \begin{array}{l} t < 4 \\ v > 0 \Rightarrow t \in [0, 2) \cup (6, \infty) \end{array} \right\} t \in [0, 2)$

Logo, a partícula acelera entre 2 e 4s e a partir de 6s.

4- $f(t) = t e^{-t/2} + t \cdot (e^{-t})^{1/2}$

a) $v(t) = \frac{df}{dt} = e^{-t/2} - t \cdot \frac{1}{2} e^{-3/2} \cdot e^{-t} + e^{-t/2} - \frac{1}{2} e^{-t/2}$

b) $v(3) = e^{-1.5} - \frac{3e^{-1.5}}{2}$

c) $v(t) = 0 \Rightarrow 2e^{-t/2} - t \cdot \frac{2e^{-t/2}}{e^{-t/2}} + t = \frac{2e^{-t/2}}{e^{-t/2}} - 2e^{\frac{-t+1}{2}}$

d) $v(t) > 0 \Rightarrow e^{-t/2} > \frac{e^{-t}}{2} \Rightarrow t < \frac{2e^{-t/2}}{e^{-t/2}} \Rightarrow t < 2e^{\frac{-t+1}{2}}$

e) $v(t) = \frac{dh}{dt} = -9,8t + 29,5$

$v(2) = -19,6 + 29,5 = 9,9 \text{ m/s}$

$v(4) = -14,7 \text{ m/s}$

b) A altura máxima ocorre no instante de lançamento; ou seja, quando $v=0$

$$v(t) = 0 \therefore -9,8t + 24,5 = 0 \therefore t = 2,5 \text{ s}$$

c) $h(2,5) = 32,625 \text{ m}$

negative positive

d) $h(t) = 0 \therefore -4,9t^2 - 24,5t + 2 = 0 \therefore t \in \left\{ \frac{-24,5 + \sqrt{6041}}{-9,8}, \frac{-24,5 - \sqrt{6041}}{-9,8} \right\}$. Como não há $t < 0$, considerar-se apenas a 2ª raiz.

e) $v(0,008) \approx 24,42 \text{ m/s}$ velocidade

8- a) A altura máxima ocorre quando $\frac{dh}{dt} = 0$

$$\frac{dh}{dt} = -9,8t + 24,5$$

$$v(t) = 0 \therefore t = 2,5 \text{ s}, h(2,5) = 32,625 \text{ m}$$

b) $V^2 = V_0^2 + 2ad$

$$V^2 = (24,5)^2 - 2 \cdot 10 \cdot 29,4 = 12,25 \therefore V = 3,5 \text{ m/s}$$

Chão é o referencial

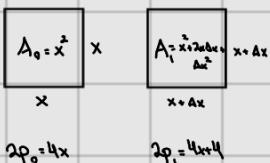
$$V^2 = (24,5)^2 + 2 \cdot 10 \cdot (H_{\max} - 29,4) = 664,75 \therefore V = 25,78 \text{ m/s}$$

11- a) Como não podemos quadrados, $A(x) = x^2$.

$$A'(x) = 2x \therefore A'(15) = 30 \text{ mm}.$$

b) $A(x) = x^2$

$A'(x) = 2x$; $2p = 4x$. Portanto, a taxa de variação da área é metade do perímetro.



$$\Delta A = (x + \Delta x)^2 - x^2 = (x + \Delta x - x)(x + \Delta x + x) = \Delta x(2x + \Delta x)$$

$$\lim_{\Delta x \rightarrow 0} \Delta A = 0 \cdot (2x + \Delta x) = 0$$

15- a) $S(n) = 4\pi n^2$; $n = 20$

$$\frac{dS}{dn} = 4\pi \cdot 2n = 8\pi n; S'(20) = 160\pi$$

b) $S'(40) = 320\pi$

c) $S'(60) = 480\pi$

S varia linearmente em relação a n : $S'(dn) = dS'(n)$. Taxa é proporcional ao diâmetro terceira.

22- a) $PV = C$

$$V = C/P \therefore V(P) = C/P = C \cdot P^{-1}$$

$$V'(P) = -\frac{C}{P^2}$$

b) No final, o volume decrece de forma proporcional ao quadrado da pressão.

33- a) $A'(x) = \frac{p'(x) \cdot x - p(x) \cdot 1}{x^2} = \frac{x p'(x) - p(x)}{x^2} = \frac{x^2 p'(x)}{x^2} - \frac{p(x)}{x^2} = \frac{p'(x)}{x} - \frac{p(x)}{x} = \frac{p'(x) - p(x)}{x}$

b) Se $p'(x)$ for maior que $A'(x)$, a diferença $p'(x) - A'(x)$ será maior que zero. Como x , o número de trabalhadores, é estritamente positivo, $p'(x) > A'(x) \Rightarrow A'(x) < 0$.

1- $v(x) = x^3$

$x(t) = kt + x_0$

$\frac{dx}{dt} = k$

$\frac{dv}{dt} = 3x^2 \cdot \frac{dx}{dt} = 3x^2 k$

2- a) $A(n) = \pi n^2$

$x(t) = kt + x_0$

$\frac{dn}{dt} = k$

$\frac{dA}{dt} = 2\pi n \cdot \frac{dn}{dt} = 2\pi nk$

b) $\frac{dn}{dt} = 1 \text{ m/s}$; $n = 30 \text{ m}$; $\frac{dA}{dt}$ quando $n = 30 \text{ m}$

$\frac{dA}{dt} = 2\pi \cdot 30 \cdot 1 = 60\pi \text{ m}^2/\text{s}$

3- $\frac{dl}{dt} = 6 \text{ cm/s}$; $\frac{dA}{dt}$ quando $A = 16 \text{ cm}^2$?

$A(l) = l^2$. Se $16 = l^2$, $l = 4$; para m. considerar-se apenas 4 para tratar de comprimento.

$\frac{dA}{dt} = 2l \cdot \frac{dl}{dt} = 2 \cdot 4 \cdot 6 = 48 \text{ cm}^2/\text{s}$

4- $\frac{dL}{dt} = 8 \text{ cm/s}$; $\frac{dl}{dt} = 3 \text{ cm/s}$; $\frac{dA}{dt}$ quando $L = 20 \text{ cm}$ e $l = 10 \text{ cm}$?

$A(L, l) = L \cdot l$

$\frac{dA}{dt} = \frac{dL}{dt}l + L \cdot \frac{dl}{dt}$

$\frac{dA}{dt} = 8 \cdot 10 + 20 \cdot 3 = 80 + 60 = 140 \text{ cm}^2/\text{s}$

5- $x = 5 \text{ m}$; $\frac{dv}{dt} = 3 \text{ m}^2/\text{min}$; $\frac{dh}{dt} = ?$

$V = \pi r^2 h = 25\pi \cdot h$

$\frac{dv}{dt} = 25\pi \cdot \frac{dh}{dt}$; $\frac{dh}{dt} = \frac{3}{25\pi} \text{ m/s}$

6- $\frac{dx}{dt} = 4 \text{ mm/s}$; $\frac{dv}{dt}$ quando $r = 40 \text{ mm}$?

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3 \cdot 1600 \cdot 4 = 16 \cdot 1600\pi = 25600\pi$

7- $y = \sqrt{2x+1}$, $y(t) = \sqrt{2x(t)+1}$

a) $\frac{dy}{dt} = 3$; $\frac{dy}{dt}$ quando $x = 4$?

$\frac{dy}{dt} = \frac{1}{2}(2x+1)^{-1/2} \cdot (2 \cdot \frac{dx}{dt} + 1) = \frac{1}{2} \cdot 7 \cdot 7/3$

b) $\frac{dx}{dt} = 5$; $\frac{dy}{dt}$ quando $x = 12$

$\frac{dy}{dt} = \frac{1}{\sqrt{2x}} \cdot 11 = 11/5$

8- $4x^2(t) + 9y^2(t) = 36$

$$a) \frac{dy}{dt} = \frac{1}{3}, \frac{dx}{dt} \text{ quando } x=2 \text{ e } y=\frac{2\sqrt{5}}{3}$$

$$8x \cdot \frac{dx}{dt} + 18y \cdot \frac{dy}{dt} = 0$$

$$16 \cdot \frac{dx}{dt} + 18 \cdot \frac{2}{3} \cdot \sqrt{5} \cdot \frac{1}{3} = 0$$

$$\frac{dx}{dt} = \frac{-4\sqrt{5}}{16} = -\frac{\sqrt{5}}{4}$$

$$b) \frac{dx}{dt} = 3; \frac{dy}{dt} \text{ quando } x=-2 \text{ e } y=\frac{2\sqrt{5}}{3}$$

$$-16 \cdot 3 + 12\sqrt{5} \cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{48}{12\sqrt{5}} = \frac{4\sqrt{5}}{60} = \frac{\sqrt{5}}{15}$$

$$9- x''(t) + y'(t) + z''(t) \cdot 9; \frac{dx}{dt} = 5; \frac{dy}{dt} = 4; \frac{dz}{dt} \text{ quando } x=y=2 \text{ e } z=1?$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} + 2z \cdot \frac{dz}{dt} = 0; 2(x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} + z \cdot \frac{dz}{dt}) = 0$$

$$2 \cdot 5 + 2 \cdot 4 + \frac{dz}{dt} = 0$$

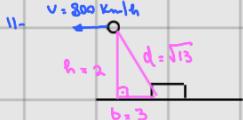
$$\frac{dz}{dt} = -18$$

$$10- x(t) \cdot y(t) = 8; \text{ quando } x=4 \text{ e } y=2, \frac{dy}{dt} = 3 \text{ cm/s}. \text{ Quante vale } \frac{dx}{dt}?$$

$$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} \cdot 2 + 4 \cdot 3 = 0$$

$$\frac{dx}{dt} = -6 \text{ cm/s}$$



$$a^2 = b^2 + b^2$$

$$b(t) = \sqrt{t+3}, \text{ donde } b \text{ em km, } t \text{ em h e } v \text{ em km/h}$$

b é sempre constante

$$\rightarrow \frac{db}{dt} > 0$$

$$d^2 = 4 \cdot 0 = 13; d = \sqrt{13}$$

$$2d \cdot \frac{dd}{dt} = 2b \cdot \frac{db}{dt}, \frac{dd}{dt} = \frac{b}{d} \cdot \frac{db}{dt} = \frac{bv}{d}$$

$$\text{Se } b = 3 \text{ km, } v = 800 \text{ km/h e } d = \sqrt{13} \text{ km: } \frac{dd}{dt} = \frac{3\sqrt{13} \cdot 800}{13} \text{ km/h} = 665,64 \text{ km/h}$$

$$12- \frac{ds}{dt} = 1 \text{ cm}^2/\text{min}; \frac{dD}{dt} \text{ quando } r = 5 \text{ cm}$$

$$A = \pi r^2 = \pi \cdot \left(\frac{D}{2}\right)^2 = \frac{D^2 \pi}{4}$$

$$\frac{dA}{dt} = \frac{\pi}{4} \cdot 2D \cdot \frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{1}{10\pi} = \frac{2}{5\pi} \text{ cm/s}$$

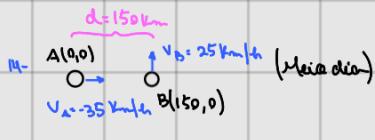
13-



compr.

$$\frac{dr}{dt} = \frac{h}{4} \cdot \frac{dh}{dt}$$

$$\frac{dr}{dt} \text{ quando } dh/dt = 10 \text{ e: } \frac{dr}{dt} = \frac{h}{4} \cdot \frac{dh}{dt} = \frac{1}{4} \cdot \frac{10}{3} = 0,5 \text{ m/s}$$

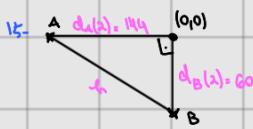


$$d^2 = d_A^2 + d_B^2 \Rightarrow d = \sqrt{100 + 10000}$$

$$\frac{dd}{dt} = \frac{2d_A \cdot dd_A}{dt} + \frac{2d_B \cdot dd_B}{dt}$$

$$2\sqrt{10000} \cdot \frac{dd}{dt} = 2 \cdot 10 \cdot (-35) + 2 \cdot 100 \cdot 25$$

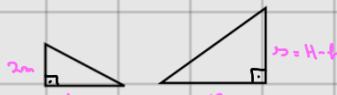
$$\frac{dd}{dt} = \frac{-2150}{\sqrt{10000}} \text{ km/h}$$



$$h^2 = d_A^2 + d_B^2; h(2) = 156$$

$$2h(t) \cdot \frac{dh}{dt} = 2d_A(t) \cdot \frac{dd_A}{dt} + 2d_B(t) \cdot \frac{dd_B}{dt}$$

$$\frac{dh}{dt} = \frac{2 \cdot 144 \cdot 32 + 2 \cdot 60 \cdot 30}{2 \cdot 156} = 78 \text{ km/h}$$



$$\frac{h}{d} = \frac{v}{D} \therefore \frac{2}{12} = \frac{H-h}{12} \therefore H-2 = 6 \therefore H = 8 \text{ m}$$

Quando d , a distância entre o homem e a parede, é 12 m, sua sombra mede $H-h = 6$ m.

Quando $d=12 \text{ m}$, sua sombra mede 0 m.

Certo?

$$\frac{h}{d} = \frac{v}{D} \therefore v = \frac{Dh}{d} = Dh \cdot \frac{1}{d}$$

$$\frac{dh}{dt} = D \cdot v \cdot (-1) \cdot d^{-2} \cdot \frac{dd}{dt}$$

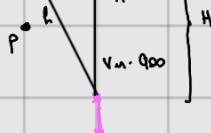
$$\frac{dh}{dt} \text{ quando } d=12 \cdot -12 \cdot 2 \cdot \frac{1}{16} \cdot 1,6 = -2,4 \text{ m/s}$$

P- $\frac{dh}{dt}$ 15 minutos depois da mulher começar a andar?



$t = 1200 \text{ s}$





$$l^2(1200) = b^2(1200) + h^2(1200)$$

$$h(1200) = \sqrt{40000 + (1440 + 1440)^2}$$

$$h(1200) = \sqrt{8334400}$$

$$b^2 + h^2 = l^2$$

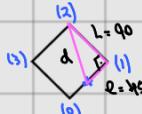
$\frac{dh}{dt}$ é a soma das velocidades: $1,6 + 1,2 = 2,8 \text{ m/s}$

$$2b \cdot \frac{db}{dt} + 2h \cdot \frac{dh}{dt} = 2l \cdot \frac{dl}{dt}$$

$$\text{Quando } b = 200 \text{ m, } \frac{db}{dt} = \frac{h}{l} \cdot \frac{dh}{dt} = \frac{2800}{\sqrt{8334400}} \cdot 2,8 = 2,71 \text{ m/s}$$

18-

a)



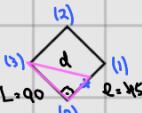
$$d^2 = L^2 + l^2; \text{ na configuração atual, } d = \sqrt{8100 + 2025}$$

$$2d \cdot \frac{dd}{dt} = 2L \cdot \frac{dl}{dt} + 2l \cdot \frac{dl}{dt}$$

$$2\sqrt{10125} \cdot \frac{dd}{dt} = 90 \cdot 24$$

$$\frac{dd}{dt} = \frac{90 \cdot 24}{2\sqrt{10125}} = 10,73 \text{ pés/s}$$

b)



$$\text{De forma similar, } \frac{dd}{dt} = 10,73 \text{ pés/s}$$

$$19- \frac{db}{dt} = \text{cm/min}; \frac{dh}{dt} = 1 \text{ cm/min} \cdot \frac{db}{dt} \text{ quando } h=10 \text{ cm e } A=100 \text{ cm}^2?$$

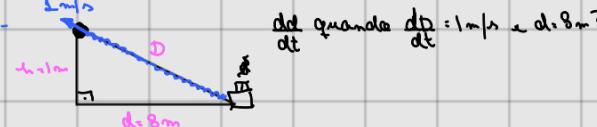
$$A(b, h) = b \cdot h \therefore \text{se } A=100 \text{ e } h=10, b = \frac{100}{10} = 10 \text{ cm}$$

$$\frac{da}{dt} = \frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}$$

$$L = db \cdot 10 + 10 \cdot L$$

$$\frac{db}{dt} = \frac{a}{10} = -0,9 \text{ cm/min. (ou seja, a base decresce a uma taxa de } 0,9 \text{ cm/min.)}$$

20-



$$\text{Se } \frac{dD}{dt} = 1 \text{ m/s, } \text{ qual é } \frac{dh}{dt} \text{ quando } \frac{db}{dt} = 1 \text{ m/s e } b=8 \text{ m?}$$

$$D^2 = b^2 + h^2. \text{ Na sua configuração, } D = \sqrt{65} \text{ m}$$

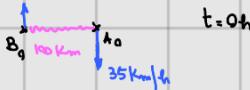
$$2D \cdot \frac{dD}{dt} = 2h \cdot \frac{dh}{dt} + 2b \cdot \frac{db}{dt}$$

$$2\sqrt{65} \cdot 1 = 16 \cdot \frac{dh}{dt}$$

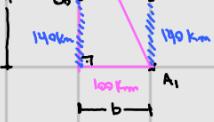
$$\frac{dh}{dt} = \frac{\sqrt{65}}{8} \text{ m/s}$$

$$21- \frac{dh}{dt} \text{ se } \frac{dh}{dt} = 60 \text{ km/h, } b=100 \text{ km e } t=4h?$$

$$25 \text{ km/h}$$



$$t=4h$$



$$h^2 + b^2 = l^2; \text{ na config. atual, } l = \sqrt{17000}$$

24. $\frac{dh}{dt} + 2b \cdot \frac{db}{dt} = 2l \cdot \frac{dl}{dt}$

$$\frac{dh}{dt} = \frac{l}{h} \cdot \frac{dl}{dt} = \frac{240}{100} \cdot 60 \approx 56,38 \text{ km/h}$$

22. $y = 2\sin(\pi x/2); P(1/3, 1); \frac{dy}{dt} = \sqrt{10} \text{ cm/s}; \frac{ddy}{dt} = ?$

A distância de P a O(0,0) pode ser calculada usando a fórmula da distância entre 2 pontos:

$$d_{PO} = \sqrt{(x_p - x_0)^2 + (y_p - y_0)^2} = \sqrt{x_p^2 + y_p^2} = (x_p^2 + y_p^2)^{1/2}$$

$$\frac{dd_{PO}}{dt} = \frac{1}{2} (x_p^2 + y_p^2)^{-1/2} \cdot (2x_p \cdot \frac{dx_p}{dt} + 2y_p \cdot \frac{dy_p}{dt})$$

$$\frac{dd_{PO}}{dt} \text{ na ponto P, é: } \frac{1}{2} \left(\frac{1}{9} + 1 \right) \cdot \left(\frac{2}{3} \cdot \sqrt{10} + 2 \cdot \sqrt{10} \cdot \cos(\pi/6) \right) \approx 9,162$$

23. $\frac{dh}{dt} = 20 \text{ cm/min}; \frac{du}{dt} = 10000 \text{ cm}^3/\text{min}; \frac{db}{dt} = ?; \frac{dV}{dt} = \frac{db}{dt} \cdot \frac{du}{dt}$



$$V = \frac{\sqrt{3}}{3} b^2 h$$

$$dV = \frac{\sqrt{3}}{3} b^2 \frac{dh}{dt}$$

$$\frac{db - du}{dt} = \frac{\sqrt{3}}{3} \cdot \frac{dh}{dt}$$

$$\frac{db}{dt} - \frac{du}{dt} = \frac{\pi \cdot 4 \cdot 10^6}{3} \cdot 20$$

$$\frac{db}{dt} = \frac{80 \cdot 10^6 \pi - 30 \cdot 10^3}{3}$$

24. $V = A_2 C = \frac{b \cdot h}{2} \cdot C = 1 \cdot 0,5 \cdot 3 = 1,5 \text{ m}^3; \frac{dh}{dt} = 1,2 \text{ m/min}; \frac{db}{dt} \text{ quando } h = 30 \text{ cm?}$

$$V(b, h, C) = b \cdot h \cdot C \cdot \frac{1}{2}$$

$$\frac{dV}{dt} = \left[\left(\frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt} \right) \cdot C + b \cdot h \cdot \frac{dC}{dt} \right] \cdot \frac{1}{2} = \frac{1}{2} \cdot b \cdot C \cdot \frac{dh}{dt}$$

$$1,2 \cdot \frac{1}{2} \cdot 1 \cdot 0,5 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1,2}{0,75} = 0,4 \text{ m/min}$$

25.

26.

27. $\frac{dV}{dt} = 3 \text{ m}^3/\text{min}, \frac{dh}{dt} \text{ quando } h = 3 \text{ m} \text{ rende } h \text{ sempre igual a } n?$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \cdot \pi \cdot h^3$$

$$\frac{dV}{dt} = \frac{\pi}{3} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$3 = \frac{\pi}{3} \cdot 27 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{3\pi} \text{ m/min}$$

28- $\frac{d\theta}{dt}$ quando $d=100 \text{ m}$?



$$h^2 + l^2 = d^2$$

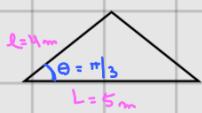
$$10000 - 2500 = l^2 = 7500; l = \sqrt{7500}$$

$$\tan \theta = \frac{h}{l} = \frac{h \cdot l'}{l^2}$$

$$\sec^2 \theta \cdot \frac{d\theta}{dt} = 50 \cdot (1) \cdot (\sqrt{7500})^2 \cdot \frac{dl}{dt}$$

$$\frac{d\theta}{dt} = \frac{-100}{7500 \cdot \sec^2 \theta} = \frac{-100 \cdot \cos^2 \theta}{7500} = \frac{-\cos^2 \theta}{75} = \frac{-7500}{750000} = \frac{-1}{100} \text{ rad/s}$$

29- $\frac{d\theta}{dt} = 0,06 \text{ rad/s}$; $\frac{dA}{dt}$ quando $\theta = \pi/3$



"método da base regras lado regras
/ soma das regras formadas"

$$A = \frac{1}{2} \cdot L \cdot l \cdot \sin \theta$$

$$\frac{dA}{dt} = \frac{L \cdot l}{2} \cdot \cos \theta \cdot \frac{d\theta}{dt} = 10 \cdot \frac{1}{2} \cdot 0,06 = 0,03 \text{ m}^2/\text{s}$$

31- $l = ?$



$$h^2 + d^2 = l^2; h = \sqrt{l^2 - d^2}$$

$$2h \cdot \frac{dh}{dt} + 2d \cdot \frac{dd}{dt} = 0$$

$$2\sqrt{l^2 - 9} \cdot 0,15 + 6 \cdot 0,2 = 0$$

$$\sqrt{l^2 - 9} = -1,2 = -4 \therefore l^2 = 25 \therefore l = 5$$

$$d = 3; h = 4; l = 5$$

33- $PV = C$, sendo C constante. $\frac{dV}{dt}$ quando $P = 150 \text{ kPa}$, $\frac{dP}{dt} = 20 \text{ kPa/min}$ e $V = 600 \text{ cm}^3$?

$$P \cdot V = C$$

$$\frac{dP}{dt} \cdot V + P \cdot \frac{dV}{dt} = 0$$

$$\frac{dP}{dt} \cdot V = -P \frac{dV}{dt}$$

$$20 \cdot 600 = -150 \cdot \frac{dV}{dt}$$

$$\frac{dV}{dt} = -80 \text{ cm}^3/\text{min}$$

34- $PV^n = C$, sendo C constante. $P = 80 \text{ kPa}$, $\frac{dP}{dt} = 10 \text{ kPa/min}$, $V = 400 \text{ cm}^3$, $\frac{dV}{dt} = ?$

$$PV^n = C$$

$$\frac{dP}{dt} \cdot V^{1/4} + P \cdot 1,14V^{0,1} \cdot \frac{dV}{dt} = 0$$

$$\frac{dV}{dt} = \frac{10 \cdot (1400)^{1/4}}{80 \cdot 1,14 \cdot (1400)^{0,1}} = \frac{400}{14 \cdot 8} \approx 35,7 \text{ cm}^3/\text{min}$$

35. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$; $\frac{dR}{dt}$ quando $R_1 = 80$, $R_2 = 100$, $\frac{dR_1}{dt} = 0,3$ e $\frac{dR_2}{dt} = 0,2$?

$$-R^2 \cdot \frac{dR}{dt} = -R_1^{-2} \cdot \frac{dR_1}{dt} - R_2^{-2} \cdot \frac{dR_2}{dt}$$

$$\frac{1}{R} = \frac{1}{80} + \frac{1}{100} = \frac{100+80}{8000} = \frac{180}{8000}$$

$$\frac{dR}{dt} = \frac{-1}{(180)^2} \cdot 0,3 - \frac{1}{(800)^2} \cdot 0,2$$

$$R = \frac{800}{18}$$

$$\frac{dR}{dt} = \frac{-0,3 \cdot 10 - 0,2 \cdot 6400}{64 \cdot 10^6} \cdot \left(\frac{800}{18}\right)^2 = \frac{64 \cdot 10^6 \cdot (-0,3 \cdot 10 - 0,2 \cdot 6400)}{324 \cdot 64 \cdot 10^6} = \frac{-30 - 128}{324} \approx -0,132 \text{ min}^{-1}$$

36. $\frac{dl}{dt} = 5 \text{ cm}/\text{min} \cdot 10^6 \text{ amper} = 5 \cdot 10^{-7} \text{ cm}/\text{ampe}; B(w) = 7 \cdot 10^{-3} w^{2/3} \text{ g}; w(l) = 0,12 \cdot l^{5/3} \text{ g}; \frac{dB}{dt}$ quando $l = 18 \text{ cm}$?

$$\frac{dB}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} w^{-1/3} \cdot \frac{dw}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} \cdot \left[0,12 \cdot (18)^{5/3}\right]^{-1/3} \cdot \frac{dw}{dt}$$

$$\frac{dw}{dt} = 0,12 \cdot 2,53 l \cdot \frac{dl}{dt} = 0,12 \cdot 2,53 \cdot (18) \cdot 5 \cdot 10^{-7}$$

$$\frac{dB}{dt} = 7 \cdot 10^{-3} \cdot \frac{2}{3} \cdot \left[0,12 \cdot (18)^{5/3}\right]^{-1/3} \cdot 0,12 \cdot 2,53 \cdot (18)^{1/3} \cdot 5 \cdot 10^{-7} \approx 1,045 \cdot 10^{-8} \text{ g}/\text{ampe} = 1,045 \text{ g}/10 \cdot 10^6 \text{ amper}$$

37. $\frac{de}{dt} = 2^\circ/\text{min}; \frac{dl}{dt}$ quando $\theta = 60^\circ$?

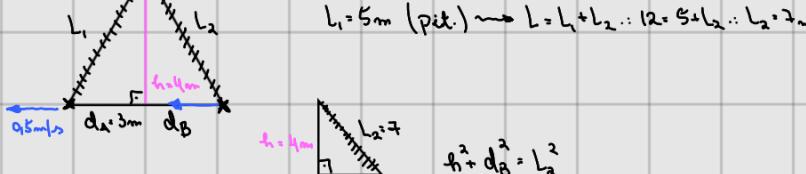


Quando $\theta = 60^\circ$, $a = 15$ e $b = 12$: $L = \sqrt{225 + 144 - 90} = \sqrt{279}$

$$2L \cdot \frac{dL}{dt} = 0 + 0 + 2 \cdot 12 \cdot 15 \cdot \sin \theta \cdot \frac{d\theta}{dt}$$

$$\frac{dL}{dt} = \frac{12 \cdot 15 \cdot \sqrt{3} \cdot 2}{2\sqrt{279}} = 18,66 \text{ m/min}$$

38. $\frac{dd_A}{dt}$ quando $d_A = 3 \text{ m}$?



$$\begin{cases} h^2 + d_A^2 = L_1^2 & (1) \\ h^2 + d_B^2 = L_2^2 & (2) \end{cases}$$

$$16 + d_B^2 = 49$$

$$(1): h^2 = L_1^2 - d_A^2$$

$$(2): h^2 = L_2^2 - d_B^2$$

$$L_1^2 - d_A^2 = L_2^2 - d_B^2$$

$$0 - 2d_A \cdot \frac{dd_A}{dt} = 0 - 2d_B \cdot \frac{dd_B}{dt}$$

$$6 \cdot 0,5 \cdot 2\sqrt{33} \cdot \frac{dd_B}{dt}$$

$$\frac{dd_B}{dt} = \frac{3\sqrt{33}}{2 \cdot 33} = \frac{\sqrt{33}}{22} \text{ m/s} \approx 0,26 \text{ m/s}$$

$$200 \text{ m/s}$$



$$d = 1500 \text{ m (pit.)}$$

$$\sin \theta = \frac{h}{d} = \frac{3}{5}$$

$$\text{a) } \frac{dl}{dt} = ?$$

$$\cos \theta = \frac{l}{d}$$

$$d^2 = h^2 + l^2$$

$$\frac{2d \cdot dl}{dt} = 2h \frac{dh}{dt} + 0$$

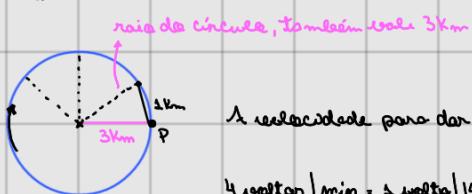
$$\frac{dl}{dt} = \frac{900 \cdot 200}{1500} = 120 \text{ m/s}$$

$$\text{b) } \cos \theta = \frac{l}{d} = \frac{4}{5}$$

$$-\sin \theta \cdot \frac{dl}{dt} = -1200 \cdot \frac{1}{(1500)^2}$$

$$\frac{dl}{dt} = \frac{1200 \cdot 5}{3 \cdot (1500)^2} \approx 8,89 \cdot 10^{-4} \text{ rad/s}$$

$$\text{* 40- } \frac{dl}{dt} = 4 \text{ giro/s | minute}$$



A velocidade para dar uma volta é:

$$4 \text{ voltas/min} = 2 \text{ voltas/15 seg.}$$

Pontanto, ele percorre $C = 2\pi R = 6\pi \text{ Km em 15 segundos: } v = \frac{6\pi}{15} \text{ Km/s.}$

$$\text{41- } \frac{dl}{dt} = \pi/6 \text{ rad/min; } \frac{dl}{dt} = ?$$

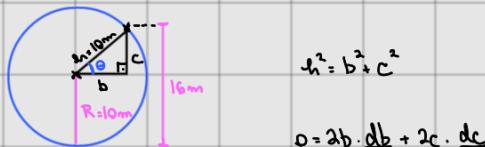
$$\tan \theta = 5/l \quad \sec^2 \theta \cdot \frac{dl}{dt} = -5 \cdot l^2 \cdot \frac{dl}{dt}$$

$$l = 5/\sqrt{3} = \frac{5\sqrt{3}}{3} \text{ Km} \quad \frac{1}{\cos^2 \theta} \cdot \frac{\pi}{6} = -5 \cdot \left(\frac{3}{5\sqrt{3}}\right)^2 \frac{dl}{dt}$$

$$\cos \theta = \frac{1}{2} \quad \frac{4\pi \cdot 25 \cdot 3}{-6 \cdot 5 \cdot 9} = \frac{dl}{dt}$$

$$\frac{dl}{dt} = -\frac{10}{9}\pi$$

$$\text{42- } \frac{dc}{dt} = ?$$



$$h^2 = b^2 + c^2$$

$$0 = 2b \cdot \frac{db}{dt} + 2c \cdot \frac{dc}{dt}$$

$$\frac{dc}{dt} = -16 \cdot \frac{db}{dt} \cdot \frac{1}{12}$$

$$\frac{dc}{dt} = 16 \cdot 6 \cdot \frac{1}{12} = 8 \text{ m/min}$$

$$h = R \text{ (const.)}$$

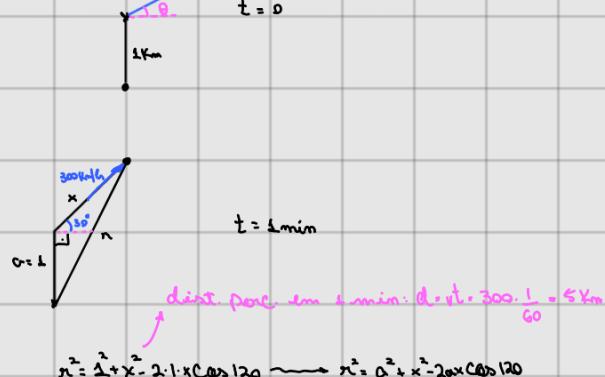
$$\cos \theta = \frac{b}{h} = b \cdot \frac{1}{h}$$

$$-\sin \theta = \frac{1}{h} \cdot \frac{db}{dt}$$

$$\frac{db}{dt} = -h \sin \theta = -h \cdot \frac{c}{h} = -c$$

$$\text{43- } \frac{dr}{dt} = ?$$

300 m



distanță parcursă în 1 min: $d = vt = 300 \cdot \frac{1}{60} = 5 \text{ km}$

$$r^2 = 1 + x^2 - 2 \cdot 1 \cdot x \cos 120^\circ \rightarrow r^2 = a^2 + x^2 - 2ax \cos 120^\circ$$

$$r^2 = 2 + 5 + 5; r^2 = 31 \text{ km}^2$$

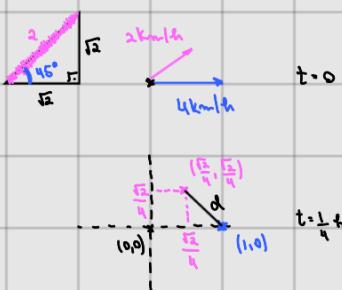
$$r = \sqrt{31} \text{ km}$$

$$2r \frac{dr}{dt} = 0 + 2x \cdot \frac{dx}{dt} - 2 \cdot x \cdot \frac{dx}{dt} \cdot \cos 120^\circ$$

$$2\sqrt{31} \frac{dr}{dt} = 10 \cdot 300 + 300$$

$$\frac{dr}{dt} = \frac{3300}{2\sqrt{31}} = 296,3 \text{ km/h}$$

* 44- $\frac{dd}{dt}$ considerăm că $\theta = 45^\circ = \pi/4$.



$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = \left(1 - \frac{\sqrt{2}}{4}\right)^2 + \left(0 - \frac{\sqrt{2}}{4}\right)^2 = \left(\frac{4-\sqrt{2}}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{18-8\sqrt{2}}{16} + \frac{2}{16} = \frac{20-8\sqrt{2}}{16} = \frac{5-2\sqrt{2}}{4}$$

$$2d \cdot \frac{dd}{dt} = 2(x_1 - x_2) \cdot \frac{dx}{dt} + 2(y_1 - y_2) \cdot \frac{dy}{dt}$$

$$\frac{dd}{dt} = \left[\left(\frac{4-\sqrt{2}}{4} \right) \cdot (4 + \sqrt{2}) + \frac{\sqrt{2}}{2} \cdot \sqrt{2} \right] \cdot \frac{2}{5-2\sqrt{2}}$$

$$\frac{dd}{dt} = \frac{16}{5-2\sqrt{2}} = \frac{16(5+2\sqrt{2})}{25-8} = \frac{16(5+2\sqrt{2})}{17} \text{ km/h}$$

$\Delta v = 4 \text{ km/h} + \text{componentă x din alta velocitate}$

$\Delta v = \text{componentă y din velocitatea nordică}$

* 45-



vel. ang.

$$v = WR \Rightarrow \omega = \frac{v}{R} = \frac{7}{100} = 0,07$$

$$d^2 = r^2 + l^2$$

$$2d \cdot \frac{dd}{dt} = 0 +$$

$$2d \cdot \frac{dd}{dt} = 2r \cdot \frac{dr}{dt} + 2l \cdot \frac{dl}{dt}$$

$\rightarrow 0 \dots$

* 46- $\frac{dd}{dt} = ?$



în 12, a îngrijoră iesirea 30°

$$\theta = 360^\circ |_{12} = 30^\circ ; \frac{d\theta}{dt} = 30^\circ / h$$

Bei den Winkelbeschleunigungen: $d^2 \rho + \rho^2 \cdot 2 \ddot{\rho} \cos \theta$

$$d^2 = 64 + 16 - 64 \frac{\sqrt{3}}{2} = 80 - 32\sqrt{3} = 16(5 - 2\sqrt{3})$$

$$d = 4\sqrt{5 - 2\sqrt{3}}$$

$$2d \cdot \frac{dd}{dt} = 0 + 0 + 64 \cdot \tan 0 \cdot \frac{d\theta}{dt}$$

$$\frac{dd}{dt} = \frac{32 \cdot 30}{8\sqrt{5-2\sqrt{3}}} = \frac{120\sqrt{5-2\sqrt{3}}}{5-2\sqrt{3}} = \frac{120\sqrt{5-2\sqrt{3}}(5+2\sqrt{3})}{13} \text{ mm/h}$$

$$1- y = (x^4 - 3x^2 + 5)^3$$

$$y' = 3(x^4 - 3x^2 + 5)^2 \cdot (4x^3 - 6x)$$

$$2- y = \cos(\tan x)$$

$$y' = -\sin(\tan x) \cdot \sec^2 x$$

$$3- y = \frac{\sqrt{x} + \frac{1}{\sqrt[3]{x^4}}}{3\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3}$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3}$$

$$4- y = \frac{3x-2}{\sqrt{2x+1}}$$

$y = \text{Quociente} + \text{Cociente}$

$$5- y = x^2 \tan mx$$

$$y' = 2x \sec^2 mx + x^2 \sec mx \cdot m$$

$$6- x e^x \cdot y \sec mx \therefore y = \frac{x e^x}{\sec mx}$$

$$y = \frac{(e^x + x e^x) \sec mx + x e^x \sec mx}{(\sec mx)^2}$$

$$7- y = \ln(x \ln x)$$

$$y' = \frac{1}{x \ln x} \cdot (\ln x + 1) = \frac{\ln x + 1}{x \ln x}$$

$$8- y = e^{mx} \cdot \cos mx$$

$$y' = m e^{mx} \cdot \cos mx - e^{mx} \cdot \sec mx \cdot m = e^{mx} (m \cos mx - m \sec mx)$$

$$9- y = \sqrt{x} \cos \sqrt{x}$$

$$y' = \frac{1}{2}x^{-1/2} \cos \sqrt{x} - \sqrt{x} \cos \sqrt{x} \cdot \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2} (\cos \sqrt{x} - \sqrt{x} \cos \sqrt{x})$$

$$10- y = \ln \sec x$$

$$y' = \frac{1}{\sec x} \cdot \sec x \tan x$$

$$11- \tan(y) = x^2 y$$

$$\cos(xy) \cdot (y + xy') = 2x - y'$$

$$y \cos(xy) + x y' \cos(xy) = 2x - y'$$

$$y'(\ln \cos(xy) + 1) = 2x - y \cos(xy) \therefore y' = \frac{2x - y \cos(xy)}{\ln \cos(xy) + 1}$$

$$12- x^6 + y^6 = 1$$

$$6x^5 + 6y^5 y' = 0 \therefore y' = \frac{-6x^5}{6y^5} = \frac{-x^5}{y^5}$$

$$y'' = \frac{-5x^4}{5y^4} = \frac{-x^4}{y^4}$$

$$13- f_n(x) = f(g(\ln \cos 4x))$$

$$f'(x) = f'(g(\ln \cos 4x)) \cdot g'(\ln \cos 4x) \cdot \cos 4x \cdot 4$$

$$14- y = \ln^2(x+1)$$

$$y = 2\ln(x+4) \cdot \frac{1}{x+4}$$

$$2\ln(x+4) = 0 \therefore x+4 = 1 \therefore x = -3$$

$$\text{95- } f(x) = ax^2 + bx + c; f'(x) = 2ax + b$$

$$f(1) = 1; f(-1) = 6; f(5) = -2$$

$$f(1) = a + b + c = 1$$

$$f(-1) = -2a + b = 6$$

$$f(5) = 10a + b = -2$$

$$b = 6 + 2a; 10a + 6 + 2a = -2; 12a = -8 \therefore a = -2/3$$

$$b = 6 - 4/3 = 14/3$$

$$\frac{-2}{3} + \frac{14}{3} + c = 1 \therefore c = 0$$

$$f(x) = \frac{-2x^2 + 14x}{3} = \frac{-2x^2 + 14x}{3}$$

$$\text{96- a) } \frac{dV}{dt} = \frac{\pi r^2}{3} \cdot u \cdot c$$

$$\text{b) } \frac{dV}{dt} = \frac{\pi h}{3} \cdot 2r \cdot r' = \frac{2\pi rh}{3}$$

29- $f(x) = 5x^2 + 4x$

$\frac{df}{dx} = 0 \therefore 10x + 4 = 0 \therefore x = -\frac{4}{10}$

30- $f(x) = x^3 + x^2 - x$

$\frac{df}{dx} = 0 \therefore 3x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{14+12}}{6} = \frac{-2 \pm 4}{6} \leftarrow \begin{array}{l} -1 \\ 1/3 \end{array}$

$x \in \{-1, 1/3\}$

31- $f(x) = 2x^3 - 3x^2 - 36x$

$\frac{df}{dx} = 0 \therefore 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 0$

$x = \frac{-1 \pm \sqrt{1+241}}{2} = \frac{-1 \pm 5}{2} \leftarrow \begin{array}{l} -3 \\ 2 \end{array}$

$x \in \{-3, 2\}$

32- $f(x) = 2x^3 + x^2 + 2x$

$\frac{df}{dx} = 0 \therefore 6x^2 + 2x + 2 = 0 \therefore 2(3x^2 + x + 1) = 0$

$x = \frac{-1 \pm \sqrt{1-12}}{6} \leftarrow \begin{array}{l} -1+\sqrt{11} \\ -1-\sqrt{11} \end{array} \overline{\frac{6}{6}}$

33- $g(t) = t^4 + t^3 + t^2 + t + 1$

$\frac{dg}{dt} = 0 \therefore 4t^3 + 3t^2 + 2t + 0 \therefore t(4t^2 + 3t + 2) = 0$

$t = \frac{-3 \pm \sqrt{9-32}}{8} \leftarrow \begin{array}{l} -3+\sqrt{-23} \\ -3-\sqrt{-23} \end{array} \overline{\frac{8}{8}}$

34- $g(t) = |3t-4|$

$g(t) = |t| = \sqrt{t^2} = (t^2)^{1/2}$

$g'(t) = \frac{1}{2}(t^2)^{1/2} \cdot 2t = \frac{t}{\sqrt{t^2}} \cdot \frac{t}{|t|}$

$\frac{dg}{dt} = 0 \therefore \frac{3t-4}{|3t-4|} = 0 \therefore t = 4/3$

Portanto os números críticos são aqueles

em que o denominador é igual a zero ou não existe.

p não tem $\frac{d}{dx}|x|$ quando $x=0$.

$N_c = \{0, \frac{4}{3}\}$

35- $g(y) = \frac{y-1}{y^2-y-1}$

$\frac{dg}{dy} = 0 \text{ se } \neq \frac{dy}{dy}$

$\frac{dg}{dy} = 0 \therefore \frac{y^2-y-1-(y-1)(2y)}{(y^2-y-1)^2} = 0 \therefore y^2-y-1-2y^2+y-2y+1 = 0 \therefore y^2+2y-2 = 0$

$y = \frac{-2 \pm \sqrt{4+8}}{2} \leftarrow \begin{array}{l} -2+\sqrt{12} \\ -2-\sqrt{12} \end{array} \overline{\frac{2}{2}}$

$$\frac{dy}{dx} = (y^2 - y - 1) = 0 \therefore y^2 - y - 1 = 0$$

$$y = \frac{1 \pm \sqrt{5}}{2} \quad \begin{cases} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{cases}$$

$$N_c = \left\{ \frac{-2+\sqrt{12}}{2}, \frac{1-\sqrt{5}}{2}, \frac{-2+\sqrt{12}}{2}, \frac{1+\sqrt{5}}{2} \right\}$$

$$36. h(p) = \frac{p-1}{p^2+4}$$

$$\frac{dh}{dp} = 0 \text{ au } \neq \frac{dh}{dp}$$

$$\frac{dh}{dp} = 0 \therefore p^2 + 4 - (p^2 + 1)2p = 0 \therefore p^2 + 4 - 2p^2 - 2p = 0 \therefore -p^2 + 2p + 4 = 0$$

$$p = \frac{-2 \pm \sqrt{4+16}}{2} \quad \begin{cases} \frac{-2+\sqrt{20}}{2} = \frac{2-\sqrt{20}}{2} \\ \frac{-2-\sqrt{20}}{2} = \frac{2+\sqrt{20}}{2} \end{cases}$$

$$\cancel{\frac{dh}{dp}} : (p^2 + 4)^2 = 0 \therefore p^2 + 4 = 0 \therefore p = \pm 2$$

$$N_c = \left\{ -2, \frac{2-\sqrt{20}}{2}, 2, \frac{2+\sqrt{20}}{2} \right\}$$

$$37. h(t) = t^{1/4} - 2t^{1/4}$$

$$\frac{dh}{dt} = 0 \text{ au } \neq \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0 \therefore \frac{3}{4}t^{-1/4} - \frac{1}{2}t^{-3/4} = 0 \therefore \frac{3}{4\sqrt[4]{t}} = \frac{1}{2\sqrt[4]{t^3}} \therefore 4\sqrt[4]{t} = 6\sqrt[4]{t^3} \therefore 256t = 1296t^3; t(1296t^3 - 256) = 0; 8t^3 - 16 = 0; t = \frac{4}{9}$$

$$\cancel{\frac{dh}{dt}} : 4\sqrt[4]{t} = 0 \therefore t = 0$$

$$\therefore 2\sqrt[4]{t^3} = 0 \therefore t = 0$$

$$N_c = \left\{ 0, \frac{4}{9} \right\}$$

$$38. g(x) = \frac{x^{1/3} - 2}{x}$$

$$\frac{dg}{dx} = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3\sqrt[3]{x^2}} + \frac{2}{3\sqrt[3]{x^5}}$$

$$\frac{dg}{dx} = 0 \text{ au } \neq \frac{dg}{dx}$$

$$\frac{dg}{dx} = 0 \therefore \frac{1}{3\sqrt[3]{x^2}} + \frac{2}{3\sqrt[3]{x^5}} = -2\sqrt[3]{x^2} + \sqrt[3]{x^5} \therefore -8x^2 + x^5 \therefore x^5(x^3 - 8) = 0 \therefore x = \sqrt[3]{-8} = -2$$

$$\cancel{\frac{dg}{dx}} : 3\sqrt[3]{x^2} = 0 \therefore x = 0$$

$$\therefore 3\sqrt[3]{x^2} = 0 \therefore x = 0$$

$$N_c = \{-2, 0\}$$

$$39. F(x) = x^{1/5}(x-4)^2 = x^{1/5}(x^2 - 8x + 16) = x - 8x^{1/5} + 16x^{1/5} = \sqrt[5]{x^4} - 8\sqrt[5]{x^4} + 16\sqrt[5]{x^4}$$

$$\frac{dF}{dx} = \frac{14}{5}x^{-4/5} - \frac{72}{5}x^{-3/5} + \frac{64}{5}x^{-2/5} = \frac{14\sqrt[5]{x^4}}{5} - \frac{72\sqrt[5]{x^4}}{5} + \frac{64\sqrt[5]{x^4}}{5}$$

$$\frac{dF}{dx} = 0 \text{ au } \neq \frac{dF}{dx}$$

$$\frac{dF}{dx} = 0 \therefore x \in \left\{ \frac{8}{7}, 4 \right\}$$

$$\cancel{\frac{dF}{dx}} : x = 0$$

$$N_c = \left\{ 0, \frac{8}{7}, 4 \right\}$$

$$40. g(\theta) = 4\theta - \tan \theta$$

$$\frac{dg}{d\theta} = 4 - \sec^2 \theta$$

$$\frac{df}{d\theta} = 0 \text{ au } \nexists \frac{df}{d\theta}$$

$$\frac{df}{d\theta} = 0 : \sin^2 \theta = 4 \therefore \frac{1}{\cos^2 \theta} = 4 \therefore \cos \theta = \pm \frac{1}{2} \therefore \theta = \pi/3 + 2k\pi \quad k \in \mathbb{Z}.$$

$$\nexists \frac{df}{d\theta} : \nexists \sin^2 \theta : \cos^2 \theta = 0 \therefore \cos \theta = 0 \therefore \theta = \pi/2 + 2k\pi \quad k \in \mathbb{Z}.$$

$$N_c = \{\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi\}$$

$$41. f(\theta) = 2\cos \theta + \sin^2 \theta$$

$$\frac{df}{d\theta} = -2\sin \theta + 2\sin \theta \cos \theta$$

$$\frac{df}{d\theta} = 0 \text{ au } \nexists \frac{df}{d\theta}$$

$$\frac{df}{d\theta} = 0 : \sin \theta \cos \theta = \sin \theta \therefore \cos \theta = 1 \therefore \theta = 0 + 2k\pi = 2k\pi \quad k \in \mathbb{Z}.$$

$$\nexists \frac{df}{d\theta} : N/4$$

$$N_c = \{2k\pi\}$$

$$43. f(x) = x^2 e^{3x}$$

$$\frac{df}{dx} = 2x e^{3x} + x^2 \cdot (-3) e^{3x-1} \cdot e^x = 2x e^{3x} - 3x^2 e^{3x}$$

$$\frac{df}{dx} = 0 \text{ au } \nexists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 \therefore 2x e^{3x} - 3x^2 e^{3x} : 3x = 2 \therefore x = 2/3$$

$$\nexists \frac{df}{dx} : N/1$$

$$N_c = \{2/3\}$$

$$44. f(x) = x^2 \ln x$$

$$\frac{df}{dx} = -2x^3 \cdot \frac{1}{x} = -2x^{-4}$$

$$\frac{df}{dx} = 0 \text{ au } \nexists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 \therefore \frac{-2}{x^4} = 0 \therefore -2 = 0 : \text{imposibile}$$

$$\nexists \frac{df}{dx} \therefore x^4 = 0 \therefore x = 0$$

$$N_c = \{0\}$$

$$45. f(x) = 3x^2 - 12x + 5, [0, 3]$$

$$\frac{df}{dx} = 6x - 12$$

$$N_c : \frac{df}{dx} = 0 \text{ au } \nexists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 : 6x = 12 \therefore x = 2$$

$$N_c = \{2\}$$

$$f(0) = 5 \quad \text{Máximo}$$

$$f(2) = -7$$

$$f(3) = -22 \quad \text{Mínimo}$$

$$46. f(x) = x^3 - 3x + 1, [0, 3]$$

$$\frac{df}{dx} = 3x^2 - 3$$

$$Nc: \frac{df}{dx} = 0 \therefore x = 1 \quad \text{elimina-se a raiz negativa pois não pertence ao intervalo}$$

$$f(0) = 1$$

$$f(1) = -3 \quad \text{Mínimo}$$

$$f(3) = 19 \quad \text{Máximo}$$

$$49. f(x) = 2x^3 - 3x^2 - 12x + 1, [-2, 3]$$

$$\frac{df}{dx} = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$

$$Nc: \frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$$

$$f(-2) = -3$$

$$f(-1) = 8$$

$$f(1) = -19 \quad \text{Máximo}$$

$$f(2) = -8 \quad \text{Mínimo}$$

$$50. f(x) = x^3 - 6x^2 + 5, [-3, 5]$$

$$\frac{df}{dx} = 3x^2 - 12x$$

$$Nc: \frac{df}{dx} = 0 \therefore x \in \{0, 4\}$$

$$f(-3) = -76 \quad \text{Mínimo}$$

$$f(0) = 5 \quad \text{Máximo}$$

$$f(4) = -27$$

$$f(5) = -20$$

$$51. f(x) = 3x^4 - 4x^3 - 12x^2 + 1, [-2, 3]$$

$$\frac{df}{dx} = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$$

$$Nc: \frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$$

$$f(-2) = 33 \quad \text{Máximo}$$

$$f(-1) = -4$$

$$f(2) = -31 \quad \text{Mínimo}$$

$$f(3) = 28$$

$$52. f(x) = (x^2 - 1)^3, [-1, 1]$$

$$\frac{df}{dx} = 3(x^2 - 1)^2 \cdot 2x$$

$$Nc: \frac{df}{dx} = 0 \therefore x \in \{-1, 0, 1\}$$

$$f(-1) = 0$$

$$f(0) = -1 \quad \text{Mínimo}$$

$$f(1) = 0$$

$$f(2) = 23 \quad \text{Máximo}$$

$$53- f(x) = x + \frac{1}{x}, [0, 2, 4]$$

$$\frac{df}{dx} = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

$$N_c: \frac{df}{dx} = 0 \text{ en } \nexists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 \therefore \frac{1}{x^2} + 1 \therefore x = \pm 1 \quad \sim \sim \sim 1 \notin [0, 4]$$

$$\nexists \frac{df}{dx} \therefore x = 0$$

$$N_c: x \in \{0\}$$

$$f(0) = \underline{\underline{2}}$$

$$f(0, 2) = 5,2 \quad \text{Máximo}$$

$$f(1) = 2 \quad \text{Mínimo}$$

$$f(4) = \underline{\underline{-\frac{15}{4}}}$$

$$54- f(x) = \frac{x}{x^2 - x + 1}, [0, 3]$$

$$\frac{df}{dx} = \frac{x^2 - x + 1 - x(2x-1)}{(x^2 - x + 1)^2}$$

$$N_c: \frac{df}{dx} = 0 \text{ en } \nexists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 \therefore x^2 - x + 1 - 2x^2 + x = 0 \therefore x^2 + 1 = 0 \therefore \nexists$$

$$\nexists \frac{df}{dx} \therefore x^2 - x + 1 = 0 \therefore \nexists$$

$$N_c = \emptyset$$

$$f(0) = 0 \quad \text{Mínimo}$$

$$f(3) = 3 \mid \underline{\underline{3}} \quad \text{Máximo}$$

$$55- f(t) = t\sqrt{4-t^2}, [-1, 2]$$

$$\frac{df}{dt} = \sqrt{4-t^2} + \frac{t}{2}(4-t^2) \cdot (-2t) = \sqrt{4-t^2} - \frac{t}{\sqrt{4-t^2}} = \frac{4-t^2-t}{\sqrt{4-t^2}}$$

$$N_c: \frac{df}{dt} = 0 \text{ en } \nexists \frac{df}{dt}$$

$$\frac{df}{dt} = 0 \therefore -t^2 - t + 4 = 0 \therefore t = \frac{-1 \pm \sqrt{17}}{2} \quad \sim \sim \sim \text{a raíz negativa es menor que -1; luego, es eliminada.}$$

$$\nexists \frac{df}{dt} \therefore 4-t^2 > 0 \therefore t < 2 \quad \sim \sim \sim -2 < -1, \text{ eliminada}$$

$$N_c = \left\{ -1 + \frac{\sqrt{17}}{2}, 2 \right\}$$

$$f(-1) = -\sqrt{3} \quad \text{Mínimo}$$

$$f(2) = 0$$

$$f\left(-1 + \frac{\sqrt{17}}{2}\right) \approx 1,95 \quad \text{Máximo}$$

(B) $a, b > 0 ; x \in [0,1]$

$$f(x) = x^a(1-x)^b$$

$$\frac{df}{dx} = ax^{a-1}(1-x)^b + x^a \cdot b(1-x)^{b-1} = ax^{a-1}(1-x)^b - x^a \cdot b(1-x)^{b-1}$$

$$\text{Nc: } \frac{df}{dx} = 0 \therefore ax^{a-1}(1-x)^b = x^a \cdot b(1-x)^{b-1} \rightarrow \text{geometricamente, } a:b=1$$

$$x^a(1-x) = x \cdot (1-x)^0 \therefore (1-x) = x \therefore 2x = 1 \therefore x = 1/2$$

$$f(0) = 0$$

$$f(1/2) = \frac{1}{4} \quad \text{Máximo}$$

$$f(1) = 0$$

67- $f(x) = x\sqrt{x-x^2} = \sqrt{x(x-x^2)} = \sqrt{x^3-x^4}$

$$\frac{df}{dx} = \frac{1}{2}(x-x^2)^{-1/2} \cdot (3x^2-4x^3) = \frac{3x^2-4x^3}{\sqrt{x^3-x^4}}$$

$$\text{Nc: } \frac{df}{dx} = 0 \text{ ou } \frac{df}{dx} \neq 0$$

$$\frac{df}{dx} = 0 \therefore x^2(3-4x) = 0 \therefore x = 3/4 \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Nc: } x \in \{3/4, 1\}$$

$$\frac{df}{dx} \neq 0 \therefore x^2-x = 0 \therefore x^2(1-x) = 0 \therefore x = 1$$

Note que o raiz $\sqrt{x-x^2} = \sqrt{x(1-x)} = \sqrt{x} \cdot \sqrt{1-x}$.

Para que exista temha solução em \mathbb{R} , $x \geq 0$ e $1-x \geq 0$.

Para rodar fazer $x \geq 0$ e $x \leq 1$, $x \in [0,1]$.

$$f(0) = 0 \quad \text{Mínimo}$$

$$f(3/4) = \frac{3\sqrt{3}}{16} \quad \text{Máximo}$$

$$f(1) = 0$$

68- $V(t) = 999,87 - 0,06426t + 0,0085043t^2 - 0,0000679t^3$, $[0,30]$

$\downarrow P = \frac{m}{V}$: Ponta mínima de volume

$$\text{Nc: } \frac{dV}{dt} = 0 \text{ ou } \frac{dV}{dt} \neq 0$$

$$\frac{dV}{dt} = -0,06426 + 2 \cdot 0,0085043t - 3 \cdot 0,0000679t^2$$

$$\frac{dV}{dt} = 0 \therefore t \in \{3,96, 7,53\}$$

$$\text{Nc} = \{3,96\}$$

$$V(0) = 999,87 \text{ cm}^3$$

$$V(3,96) = 999,7445 \text{ cm}^3$$

$$V(7,53) = 1003,76 \text{ cm}^3$$

A água tem P máxima na ponta de volume mínima, que é atingida em $t=3,96^\circ\text{C}$

70- $F(\theta) = \frac{\sin \theta}{\sin \theta + \cos \theta}$, $[0, \pi/2]$

$$\text{Nc: } \frac{dF}{d\theta} = 0 \text{ ou } \frac{dF}{d\theta} \neq 0$$

$$\frac{dF}{d\theta} = \frac{1}{(\sin \theta + \cos \theta)^2} \cdot (-\sin^2 \theta - \cos^2 \theta) = -\frac{1}{\sin \theta + \cos \theta}$$

$$\frac{dF}{d\theta} = -\mu mg(\cos\theta + \cos\theta) = \frac{-\mu mg}{(\sin\theta + \cos\theta)^2}$$

$$\frac{dF}{d\theta} = 0 \therefore -\mu mg = 0$$

$$\nexists \frac{dF}{d\theta} : (\cos\theta + \cos\theta)^2 = 0 \therefore \cos\theta = -\cos\theta \therefore u = -\cos\theta$$

$$F(0) = \mu mg$$

$$F(\pi/2) = mg$$

17- $2x + \cos x = 0$

Suponha que existam $a < b$, $a+b$, tal que $2a + \cos a = 0$ e $2b + \cos b = 0$.

Existiria também $c \in [a, b]$ de forma que $\frac{d}{dc}(2c + \cos c) = \frac{d}{dc}0 = 0$

$\frac{d}{dc}(2c + \cos c) = 2 - \sin c = 0$. Portanto, $\sin c = 2$, o que é impossível.

Assim, a premissa é falsa e a equação tem apenas 1 raiz.

18- $x^3 + e^x = 0$

Suponha que exista $a < b$, $a+b$, que satisfazem essa equação.

Se assim fizesse, haveria $c \in [a, b]$ de forma que $\frac{d}{dc}$ da equação fosse zero.

$\frac{d}{dc} = 3c^2 + e^c = 0$. Para tanto, $c^2 = -\frac{e^c}{3}$; o que é impossível, pois $c^2 \geq 0 \forall c \in \mathbb{R}$.

Assim, a equação tem apenas uma raiz.

19- $f(x) = x^3 - 15x + c$, $[2, 2]$

$f(2) = 8 - 30 + c = 22 + c$

$f(2) = 8 - 30 + c = -22 + c$

Pela Tvi, há ao menos um valor de x tal que $f'(x) = 0$

Suponha $x_1 \neq x_2$, de forma que $f(x_1) = f(x_2) = 0$.

Se assim fizesse, existiria $d \in [x_1, x_2]$ tal que $\frac{df}{dd} = 0$ (Teo. Rolle)

$$\frac{df}{dd} = 3d^2 - 15 = 0 \therefore d = \pm\sqrt{5}$$

9- $f(x) = 2x^3 + 3x^2 - 36x$

a) $\frac{df}{dx} = 6x^2 + 6x - 36 = 6(x^2 + x - 6)$

$\frac{df}{dx} = 0 : x = \frac{-1 \pm \sqrt{1+24}}{2} \leq -3$



b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$\frac{df}{dx} = 0 \therefore x \in \{-3, 2\}$

$f(-3) = 31$ Máximo

$f(2) = -44$ Mínimo

c) $\frac{d^2f}{dx^2} = 12x + 6$

$12x + 6 = 0 \therefore x = -\frac{1}{2}$



$x \in (-\infty, -\frac{1}{2})$: $f(x)$ tem concavidade para baixo

$x \in (-\frac{1}{2}, +\infty)$: $f(x)$ tem concavidade para cima

$x = -\frac{1}{2}$ é um ponto de inflexão

10- $f(x) = 4x^3 + 3x^2 - 6x + 1$

a) $\frac{df}{dx} = 12x^2 + 6x - 6 = 6(2x^2 + x - 1)$

$\frac{df}{dx} = 0 : x = \frac{-1 \pm \sqrt{1+8}}{2} \leq -\frac{5}{2}$



b) Nc: $x \in \{-5/2, 1/4\}$

$f(-5/2) = -96$ Mínimo

$f(1/4) = 5$ Máximo

c) $\frac{d^2f}{dx^2} = 24x + 6$

$24x + 6 = 0 \therefore x = -1/4$



$x \in (-\infty, -\frac{1}{4})$: $f(x)$ tem concavidade para baixo

$x \in (-\infty, +\infty) : f(x) \neq 0$ Cima

$x = -1/4$: Ponto de inflexão

II- $f(x) = x^4 - 2x^2 + 3$

a) $\frac{df}{dx} = 4x^3 - 2x = x(4x^2 - 2)$

$\frac{df}{dx} = 0 : x = 0 \text{ ou } x = \pm \sqrt{\frac{1}{2}}$



b) Nc: $\frac{df}{dx} = 0$

$f(-\sqrt{1/2}) \approx 7,91$ MÁXIMO

$f(0) = 3$ MÍNIMO

$f(\sqrt{1/2}) \approx 5,985$

c) $\frac{d^2f}{dx^2} = 12x^2 - 2 = 2(6x^2 - 1)$

$\frac{d^2f}{dx^2} = 0 : x = \frac{0 \pm \sqrt{10+25}}{12} \leftarrow \frac{5}{12}, -\frac{5}{12}$



$x \in (-\infty, -5/12) : f'' < 0$

$x \in (-5/12, 5/12) : f'' > 0$

$x \in (5/12, +\infty) : f'' < 0$

$x \in \{-5/12, 5/12\}$ - Ponto de inflexão

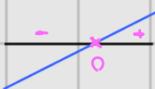
D) $f(x) = \frac{x^2}{x^2 + 3}$

a) $\frac{df}{dx} = \frac{2x(x^2 + 3) - x^2 \cdot 2x}{(x^2 + 3)^2} = \frac{6x}{(x^2 + 3)^2}$

$\frac{df}{dx} = 0$

$\frac{df}{dx} = 0 : x = 0$

Cima se denominador é estritamente positivo,



b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Sempre teremos $\frac{df}{dx}$, pois $x^2 + 3 > 0 \forall x \in \mathbb{R}$.

Nc: {0}

$$f(0) = 0 \quad \text{Mínimo}$$

c) $\frac{d^2f}{dx^2} = \frac{6(x^2+3)^2 - 12x(x^2+3) \cdot 2x}{(x^2+3)^4}$

$$\frac{d^2f}{dx^2} = 0 \therefore 6(x^2+3)^2 - 24x^2(x^2+3) = 0$$

$$\therefore (x^2+3)[6(x^2+3) - 24x^2] = 0$$

$$\therefore -12x^2 + 3 = 0 \therefore x = \pm \frac{\sqrt{6}}{6}$$



$x \in (-\infty, -\frac{\sqrt{6}}{6}) \cup (\frac{\sqrt{6}}{6}, +\infty)$ f é côncava para baixo

$x \in (-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6})$ f é côncava para cima

$x \in \{-\frac{\sqrt{6}}{6}\}$ são pontos de inflexão

13- $f(x) = x \sin x + \cos x, [0, 2\pi]$

a) $\frac{df}{dx} = \cos x - x \sin x$

$$\frac{df}{dx} = 0 \therefore \cos x = x \sin x$$

$$x \in \{\pi/4, 5\pi/4\}$$



b) Nc: $\frac{df}{dx} = 0 \therefore x \in \{\pi/4, 5\pi/4\}$

$$f(\pi/4) = 1 \quad \text{Máximo}$$

$$f(5\pi/4) = -2 \quad \text{Mínimo}$$

$$f(0) = 1 \quad \text{Máximo}$$

$$f(2\pi) = 1 \quad \text{Máximo}$$

c) $\frac{d^2f}{dx^2} = -x \sin x - \cos x$

$$\frac{d^2f}{dx^2} = 0 \therefore \cos x = -x \sin x \therefore \tan x = -1 \therefore x = -\frac{\pi}{4}$$



$x \in (-\infty, -\pi/4)$, f é ∪

$x \in (-\pi/4, +\infty)$, f é ∩

$x = -\pi/4$ é ponto de inflexão

14- $f(x) = \cos^2 x - 2x \sin x, [0, 2\pi]$

$$a) \frac{df}{dx} = -2\cos x \cdot \sin x - 2\cos x$$

$$\frac{df}{dx} > 0 \Leftrightarrow -2\cos x (\sin x + 1) > 0 \Leftrightarrow 2\cos x (\sin x + 1) < 0 \Leftrightarrow 2\cos x < 0 \text{ e } \sin x + 1 > 0 \text{ precisam ter raios trancados}$$



$$\begin{aligned} \sin x + 1 &> 0 \\ \sin x &> -1 \quad 0 \leq x \leq 2\pi, x \neq \frac{3\pi}{2} \end{aligned}$$

Quando $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$, f é crescente

Quando $x \leq \frac{\pi}{2}$ ou $x > \frac{3\pi}{2}$, f é decrescente.

$$b) Nc: \frac{df}{dx} = 0$$

$$Nc = \{x | 2, 3\pi/2\}$$

$$f(\pi/2) = -2 \text{ Mínimo}$$

$$f(3\pi/2) = 2 \text{ Máximo}$$

x	$\pi/2$	$3\pi/2$	2π
f'	-	0	+

↓ MIN ↑ MAX ↓

$$c) \frac{d^2f}{dx^2} = -2(\cos^2 x - \sin^2 x - \sin x) = -2(1 - 2\sin^2 x - \sin x) = 2(2\sin^2 x + \sin x - 1) > 0$$

$$2(2\sin x - 1)(\sin x + 1) > 0 \Leftrightarrow (2\sin x - 1) \text{ e } (\sin x + 1) \text{ precisam ter o mesmo sinal}$$

$$\sin x + 1 > 0 \Leftrightarrow \sin x > -1 \Leftrightarrow 0 \leq x \leq 2\pi \text{ e } x \neq \frac{3\pi}{2} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ Quando } \frac{\pi}{6} < x < \frac{5\pi}{6}, \text{ Cônica pra cima}$$

$$2\sin x - 1 > 0 \Leftrightarrow \sin x > \frac{1}{2} \Leftrightarrow \left. \begin{array}{l} \\ \end{array} \right\} \text{ Quando } 0 \leq x \leq \pi/6 \text{ ou } \pi/6 < x < \frac{3\pi}{2} \text{ ou } \frac{3\pi}{2} \leq x \leq 2\pi, \text{ Cônica pra baixo}$$

$$15- f(x) = e^{2x} + e^{-x}$$

$$a) \frac{df}{dx} = 2e^{2x} - e^{-x}$$

$$\frac{df}{dx} > 0 \Leftrightarrow 2e^{2x} - \frac{1}{e^x} > 0$$

$$\frac{2e^{3x}-1}{e^x} > 0. \text{ Como } e^x \text{ é sempre positiva,}$$

$$2e^{3x} - 1 > 0 \Leftrightarrow x > \frac{-\ln 2}{3}. \text{ Seja } a = e^{3x}, \text{ podemos}$$

encontrar a equação como $2a - b$.



$$b) x = -\frac{\ln 2}{3} \text{ é Mínimo.}$$

$$c) \frac{d^2f}{dx^2} = 4e^{2x} + e^{-x}$$

$$\frac{d^2f}{dx^2} > 0 \Leftrightarrow \frac{4e^{3x}+1}{e^x} > 0 \Leftrightarrow 4e^{3x} > -1 \Leftrightarrow e^{3x} > \frac{-1}{4} \Leftrightarrow x > \frac{\ln(-1/4)}{3}$$

$$\exists x \text{ tal que } \frac{d^2f}{dx^2} = 0.$$

$$x = -\frac{\ln 2}{3} \text{ é um ponto de inflexão.}$$

$$f \text{ é decrescente em } (-\infty, -\frac{\ln 2}{3}) \text{ e crescente em } (-\frac{\ln 2}{3}, +\infty)$$

$$16- f(x) = x^2 \ln x$$

$$a) \frac{df}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x = x(2 \ln x + 1)$$

$$x > e^{1/2}$$

$$\frac{df}{dx} > 0 \Leftrightarrow x(2 \ln x + 1) \text{ tem o mesmo sinal} \Leftrightarrow x > 0 \text{ e } (2 \ln x + 1) > 0 \Leftrightarrow x > 0$$

$$(1) \quad (2)$$

$$\therefore x > 0 \text{ e } (2 \ln x + 1) < 0 \Leftrightarrow x < e^{1/2}$$

$$\text{---} \quad 0 \quad \text{---} \quad e^{-1/2}$$

(1) $\frac{df}{dx} > 0$, f é crescente

(2) $\frac{df}{dx} < 0$, f é decrescente

b) $N_c = \{0, e^{-1/2}\}$

$$f(0) = 0$$

máximo

$$f(e^{-1/2}) = -\frac{1}{2} e^1 = -\frac{1}{2} \cdot \frac{1}{e} = -\frac{1}{2e}$$

Mínimo

c) $\frac{d^2f}{dx^2} = 2\ln x + 2 + 1 = 2\ln x + 3$

$$\frac{d^2f}{dx^2} > 0 \Leftrightarrow x > e^{-3/2}$$

$$\begin{array}{c} - \\ + \\ \hline e^{-3/2} \end{array}$$

$x \in (-\infty, e^{-3/2})$, f é côncava para baixo

$x \in (e^{-3/2}, +\infty)$, f é côncava para cima

$x = e^{-3/2}$ é ponto de inflexão

17- $f(x) = x^2 - x - \ln x$

a) $\frac{df}{dx} = 2x - 1 - \frac{1}{x} = \frac{2x^2 - x - 1}{x}$

$$\frac{df}{dx} > 0 \Leftrightarrow \frac{2x^2 - x - 1}{x} > 0$$

Para que ira ocorrer:

$$2x^2 - x - 1 > 0 \Leftrightarrow x > 0 \Leftrightarrow \left(x < -\frac{1}{2} \text{ ou } x > 1\right) \text{ e } (x > 0) \Leftrightarrow x > 1$$

$$2x^2 - x - 1 < 0 \Leftrightarrow x < 0 \Leftrightarrow \left(-\frac{1}{2} < x < 1\right) \Leftrightarrow (x < 0) \Leftrightarrow \frac{-1}{2} < x < 0 \quad \text{Como } \ln x \text{ não existe para } x > 0, \text{ descartar esse caso}$$

Portanto, em $x > 1$, $f(x)$ é crescente; em $x < 1$, $f(x)$ é decrescente

b) $N_c = \{1\}$

$$f(1) = 0 \quad \text{Mínimo}$$

$$\text{---} \quad \text{---}$$

c) $\frac{d^2f}{dx^2} = 2 + x^{-2} = 2 + \frac{1}{x^2} = \frac{2x^2 + 1}{x^2}$

$\frac{d^2f}{dx^2} > 0$: Como $x^2 > 0 \forall x \in \mathbb{R}$, então analisar o numerador:

$$2x^2 + 1 > 0 \Leftrightarrow x^2 > -\frac{1}{2} \Leftrightarrow x^2 > \frac{1}{2} \quad \forall x \in \mathbb{R}$$

Assim, $\frac{d^2f}{dx^2}$ é sempre positiva. Portanto, f tem concavidade para cima $\forall x \in \mathbb{R}$.

18- $f(x) = \sqrt{x} e^x = \frac{\sqrt{x}}{e^x}$

$$a) \frac{df}{dx} = \frac{1}{2} \times e^{-x} - \sqrt{x} e^{-x} = e^{-x} \left[\frac{1}{2\sqrt{x}} - \sqrt{x} \right]$$

$$\frac{df}{dx} > 0 \therefore \frac{1-2x}{2\sqrt{x}} > 0 \therefore 1-2x > 0 \therefore x < \frac{1}{2}$$

$$b) N_c: \frac{df}{dx} = 0 \text{ em } \exists \frac{df}{dx}$$

$$\frac{df}{dx} = 0 \therefore x = \frac{1}{2}$$

$$\exists \frac{df}{dx}: x = 0$$

$$f(1/2) = (2e)^{-1/2} \quad \text{Máximo}$$

$$f(0) = 0 \quad \text{Mínimo}$$

$$c) \frac{d^2f}{dx^2} = -e^{-x} \left[\frac{1}{2\sqrt{x}} - \sqrt{x} \right] + e^{-x} \left[-\sqrt{x} \cdot \frac{1}{2} - \frac{1}{2} \times \frac{1}{2\sqrt{x}} \right] = e^{-x} \left(\frac{-1}{2\sqrt{x}} + \sqrt{x} - 1 - \frac{1}{2\sqrt{x}} \right) = e^{-x} (\sqrt{x} - 1)$$

$$\frac{d^2f}{dx^2} > 0 \therefore \sqrt{x} - 1 > 0 \therefore \sqrt{x} > 1 \therefore x > 1$$

$x \in (1, +\infty)$: f é côncava para cima

$x \in (0, 1)$: f é côncava para baixo

$x = 1$ é ponto de inflexão

$$19- f(x) = x^5 - 5x + 3$$

$$\frac{df}{dx} = 5x^4 - 5$$

$$\frac{df}{dx} > 0 \therefore x^4 > 1 \therefore x > \pm 1$$

$$N_c: \frac{df}{dx} = 0 : \{-1, 1\}$$

$$f(-1) = -1 + 5 + 3 = 7 \quad \text{Máximo}$$

$$f(1) = 1 - 5 + 3 = -1 \quad \text{Mínimo}$$

$$20- f(x) = \frac{x^2}{x-1}$$

$$N_c: \frac{df}{dx} = 0 \text{ em } \exists \frac{df}{dx}$$

$$\frac{df}{dx} = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

$$\frac{df}{dx} = 0 \therefore x(x-2) = 0 \therefore x=0 \vee x=2$$

$$\exists \frac{df}{dx} \therefore x=1$$

$$f(0) = 0 \quad \text{Mínimo}$$

$$f(1) = \frac{1}{2}$$

$$f(2) = 4 \quad \text{Máximo}$$

$$21- f(x) = \sqrt{x} - \frac{4}{\sqrt{x}}$$

$$\frac{df}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{4}x^{-3/4} = \frac{1}{2\sqrt{x}} - \frac{1}{4\cdot 4\sqrt[4]{x^3}}$$

$$Nc: \frac{df}{dx} = 0$$

$$2\sqrt{x} = 4\sqrt[3]{x^3} \therefore x^2 = 16x^3 \therefore x^2(16x - 1) = 0$$

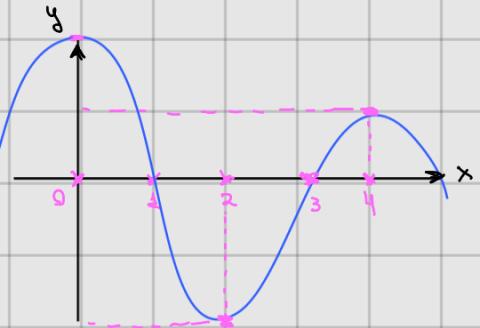
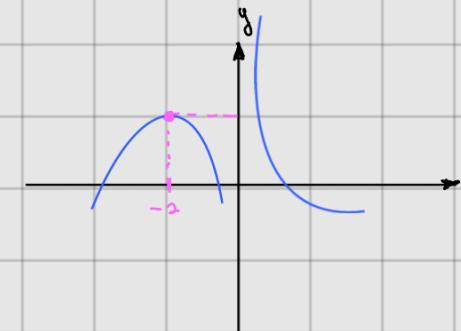
$$N_C = \{0, 1\}_{16}\}$$

$$f(0) = 0$$

MÁXIMO

$$f(1/16) = \frac{1}{4} - \frac{1}{2} = \frac{1-2}{4} = \frac{-1}{4}$$

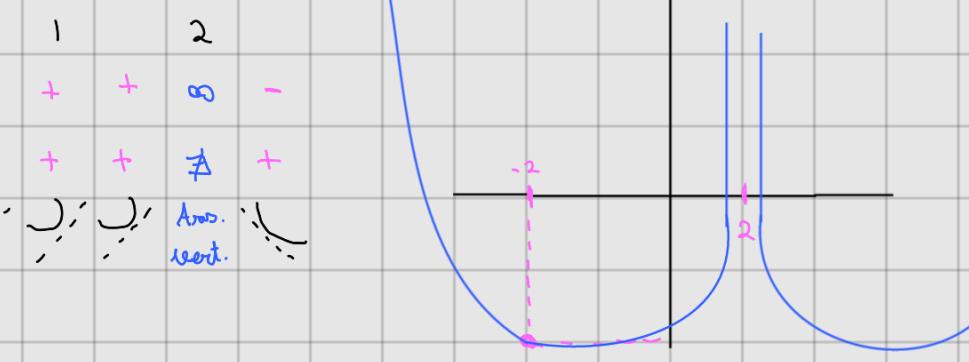
Minima

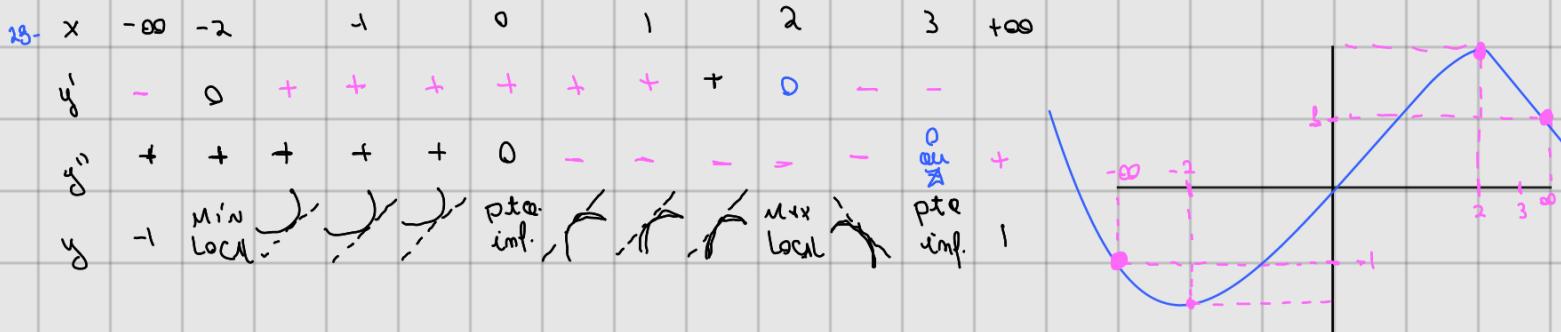


D: embora y' seja igual a zero, não é um ponto de inflexão. Pois o "zero"

na concavidade é explicada pela inclinação (y') constante.

•: saincos orleitivamente excolhidos





33- $f(x) = 2x^3 - 3x^2 - 12x$

a) $\frac{df}{dx} = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$\frac{df}{dx} = 0 \therefore x \in \{-1, 2\}$



b) Nc: $\frac{df}{dx} = 0 \text{ au } \nexists \frac{df}{dx}$

Nc: $\{-1, 2\}$

$f(-1) = -2 - 3 + 12 = 7 \quad \text{MAX}$

$f(2) = 16 - 12 - 24 = -28 \quad \text{MIN}$

c) $\frac{d^2f}{dx^2} = 12x - 6$



$x \in (-\infty, \frac{1}{2}) \quad f \in \wedge$

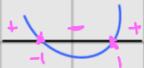
$x \in (\frac{1}{2}, +\infty) \quad f \in \cup$

$x = \frac{1}{2} \in \text{ptc. inf.}$

34- $f(x) = -x^3 + 3x + 2$

a) $\frac{df}{dx} = -3x^2 + 3$

$\frac{df}{dx} = 0 \therefore x^2 = 1 \therefore x = \pm 1$



b) Nc: $\frac{df}{dx} = 0 \text{ au } \nexists \frac{df}{dx}$

Nc: $\{-1, 1\}$

$f(-1) = 1 - 3 + 2 = 0 \quad \text{MIN}$

$f(1) = -1 + 3 + 2 = 4 \quad \text{MAX}$

c) $\frac{d^2f}{dx^2} = -6x$

d²f/dx²

$$\frac{df}{dx} = 0 \Leftrightarrow x = 0$$



$x \in (-\infty, 0)$ f ist ↗

$x \in (0, +\infty)$ f ist ↘

$x=0$ s. pta. inf.

35. $f(x) = -x^4 + 2x^2 + 2$

a) $\frac{df}{dx} = -4x^3 + 4x = 4x(-x+1)$

$\frac{df}{dx} > 0 \Leftrightarrow 4x(-x+1) > 0 \rightarrow$ Rahmen signiert

$$\begin{cases} 4x > 0 \Leftrightarrow x > 0 \\ (-x+1) > 0 \Leftrightarrow x < 1 \end{cases}$$

$$\begin{cases} 4x < 0 \Leftrightarrow x < 0 \\ (-x+1) < 0 \Leftrightarrow x > 1 \end{cases} \quad \text{impossibile}$$

$x \in (0, 1)$ f ist wachsend

$x \in (-\infty, 0) \cup (1, +\infty)$ f ist abnehmend

$x \in \{0, 1\}$ f ist konstante

b) Nc: $\frac{df}{dx} = 0$ an $\nexists \frac{df}{dx}$

Nc: $\{0, 1\}$

$f(0) = 2 \quad \text{min}$

$f(1) = -1 + 2 + 2 = 3 \quad \text{max}$

c) $\frac{d^2f}{dx^2} = -12x^2 + 4$

$\frac{df}{dx} = 0 \Leftrightarrow x = \pm \frac{\sqrt{3}}{3}$



$x \in (-\infty, -\sqrt{3}/3) \cup (\sqrt{3}/3, +\infty)$ f ist ↗

$x \in (-\sqrt{3}/3, \sqrt{3}/3)$ f ist ↘

$x \in \{-\sqrt{3}/3, \sqrt{3}/3\}$ s. pta. inf.

$4x^2 > 0 \quad \forall x \in \mathbb{R}$

26. $g(x) = x^4 + 8x^3 + 200$

a) $\frac{df}{dx} = 4x^3 + 24x^2 = 4x^2(x+6)$

$\frac{df}{dx} > 0 \Leftrightarrow x+6 > 0 \Leftrightarrow x > -6$



$x \in (-\infty, -6)$ f é decrescente

$x \in (-6, 0) \cup (0, +\infty)$ f é crescente

$x \in \{-6, 0\}$ f é constante

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

$$\frac{df}{dx} = 0 \therefore x \in \{-6, 0\}$$

$$f(-6) = 1296 - 1278 + 209 = 218 \quad \text{Max}$$

$$f(0) = 209 \quad \text{Min}$$

c) $\frac{d^2f}{dx^2} = 12x^2 + 48x = 12(x^2 + 4x)$



$x \in (-\infty, -4) \cup (0, +\infty)$ f é U

$x \in (-4, 0)$ f é ∩

$x \in \{-4, 0\}$ ponto p.c. inf.

37- $f(x) = (x+1)^5 - 5x - 2$

a) $\frac{df}{dx} = 5(x+1)^4 - 5 = 5[(x+1)^4 - 1]$

$\frac{df}{dx}$ é sempre maior ou igual a zero.

$$\frac{df}{dx} = 0 \therefore x = -1$$

$x \in (-\infty, -1) \cup (-1, +\infty)$ f é crescente

$x = -1$ f é constante

b) Nc: $\frac{df}{dx} = 0$ ou $\nexists \frac{df}{dx}$

Nc: $\{-1\}$

$$f(-1) = 5 - 2 = 3$$

c) $\frac{d^2f}{dx^2} = 20(x+1)^3 = 20(x+1) \cdot (x+1)^2$

$$\frac{d^2f}{dx^2} > 0 \therefore 20(x+1) > 0 \therefore x > -1$$

$$\frac{d^2f}{dx^2} = 0 \therefore x = -1$$

$x \in (-\infty, -1)$ f é ∩

$x \in (-1, +\infty)$ f é U

$x = -1$ è pta. inf.

$$38- h(x) = 5x^3 - 3x^5 = x^3(-x^2 + 5)$$

$$\text{a)} \frac{dh}{dx} = 15x^2 - 15x^4 = 15x^2(-x^2 + 1)$$

$$\frac{dh}{dx} > 0 \therefore x^2 > 1 \therefore x > \pm 1$$



$x \in (-\infty, -1) \cup (1, +\infty)$ h è decresc.

$x \in (-1, 0) \cup (0, 1)$ h è cresce.

$x \in \{-1, 0, 1\}$ h è costat.

$$\text{b)} \text{Nc: } \frac{dh}{dx} = 0 \text{ con } \exists \frac{dh}{dx}$$

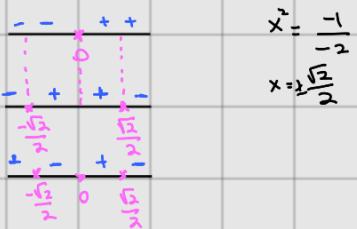
$$\frac{dh}{dx} = 0 \therefore x \in \{-1, 0, 1\}$$

$$h(-1) = 4$$

$$h(0) = 0 \quad \text{Min}$$

$$h(1) = 6 \quad \text{Max}$$

$$\text{c)} \frac{d^2h}{dx^2} = 30x - 60x^3 = 30x(-2x^2 + 1)$$



$$\begin{aligned} &x = -1 \\ &x = \pm \frac{\sqrt{2}}{2} \\ &x = 1 \end{aligned}$$

$x \in (-\infty, -\frac{\sqrt{2}}{2}) \cup (0, \frac{\sqrt{2}}{2})$ h è \cup

$x \in (-\frac{\sqrt{2}}{2}, 0) \cup (\frac{\sqrt{2}}{2}, +\infty)$ h è \cap

$x \in \{-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\}$ sono ptes. inf.

$$39- F(x) = x\sqrt{6-x}$$

$$\text{a)} \frac{dF}{dx} = \sqrt{6-x} + \frac{x}{2}(6-x)^{-1/2}(-1) = \sqrt{6-x} - \frac{x}{2\sqrt{6-x}} = \frac{12-3x}{2\sqrt{6-x}}$$

$$\frac{dF}{dx} > 0 \therefore 12-3x > 0 \therefore 3x < 12 \therefore x < 4$$

$x \in (-\infty, 4)$ f è cresce.

$x = 4$ f è costat.

$x \in (4, 6)$ f è decresc.

$$\text{b)} \text{Nc: } \frac{dF}{dx} = 0 \text{ con } \exists \frac{dF}{dx}$$

$$\frac{dF}{dx} = 0 \therefore x = 4$$

dx

$$\not\exists \frac{dF}{dx} : x = 6$$

$$Nc: \{4, 6\}$$

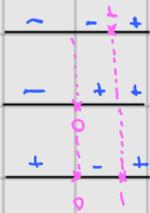
$$f(u) = 4\sqrt{2} \quad \text{MAX}$$

$$f(6) = 0 \quad \text{MIN}$$

u₁₀- G(x) = $5x^{\frac{2}{3}} - 2x^{\frac{5}{3}}$

$$\text{a)} \frac{dG}{dx} = \frac{10}{3}x^{-\frac{1}{3}} - \frac{10}{3}x^{\frac{2}{3}} = \frac{10}{3\sqrt[3]{x}} - \frac{10\sqrt[3]{x^2}}{3} = \frac{10 - 10x}{3\sqrt[3]{x}} = \frac{10 \cdot (-x+1)}{3\sqrt[3]{x}}$$

$$\frac{dG}{dx} > 0$$



$x \in (-\infty, 0) \cup (1, +\infty)$ G is cresc.

$x \in (0, 1)$ G is decreas.

b) Nc: $\frac{dG}{dx} = 0$ un $\not\exists \frac{dG}{dx}$

$$\frac{dG}{dx} = 0 \therefore x = 6$$

$$\not\exists \frac{dG}{dx} : x = 0$$

$$Nc: \{0, 6\}$$

$$G(0) = 0 \quad \text{MIN}$$

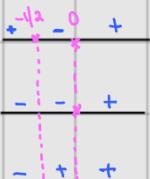
$$G(1) = 3 \quad \text{MAX}$$

$$\text{c)} \frac{d^2G}{dx^2} = \frac{-10}{9}x^{-\frac{4}{3}} - \frac{20}{9}x^{-\frac{1}{3}} = \frac{-10}{9x\sqrt[3]{x}} - \frac{20}{9\sqrt[3]{x}} = \frac{-90\sqrt[3]{x} - 180x\sqrt[3]{x}}{81x^2\sqrt[3]{x^2}} = \frac{-90}{81} \cdot \frac{\sqrt[3]{x} + 2x\sqrt[3]{x}}{x^2\sqrt[3]{x^2}}$$

$$2x\sqrt[3]{x} + \sqrt[3]{x} > 0 \therefore \sqrt[3]{x}(2x+1) > 0 \implies x < -\frac{1}{2} \vee x > 0$$

→ domain > 0 .

$$x^2\sqrt[3]{x^2} > 0 \therefore x^2 > 0 \therefore x > 0$$



$x \in (-\infty, -\frac{1}{2}) \cup (0, +\infty)$ G is \cap

$x \in (-\frac{1}{2}, 0) \cup (0, +\infty)$ G is \cup

$x \in \{-\frac{1}{2}, 0\}$ sso p.i.

u₁₁ $C(x) = x^{\frac{1}{3}}(x+4) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$

$$a) \frac{dc}{dx} = \frac{4}{3}x^{1/3} + \frac{1}{3}x^{-2/3} = \frac{4\sqrt[3]{x}}{3} + \frac{1}{3\sqrt[3]{x^2}} = \frac{4x+1}{3\sqrt[3]{x^2}}$$

$\frac{dc}{dx} > 0 \therefore$ depende apenas da numerador.

$$\frac{dc}{dx} > 0 \therefore 4x+1 > 0 \therefore x > -1/4$$

$x \in (-\infty, -1/4)$ C é decresc.

$x \in (-1/4, +\infty)$ C é cresc.

$x = -1/4$ C é extrema.

$$b) Nc: \frac{dc}{dx} = 0 \text{ eur } \nexists \frac{dc}{dx}$$

$$\frac{dc}{dx} = 0 \therefore x = -1/4$$

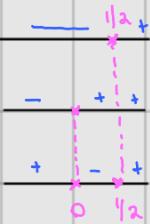
$$\nexists \frac{dc}{dx} \therefore x = 0$$

$$f(-1/4) \approx -2,3 \quad \text{min}$$

$$f(0) = 0 \quad \text{max}$$

$$c) \frac{d^2c}{dx^2} = \frac{4}{9}x^{-2/3} - \frac{2}{9}x^{-5/3} = \frac{4}{9\sqrt[3]{x^2}} - \frac{2}{9x\sqrt[3]{x^2}} = \frac{36x\sqrt[3]{x^2} - 18\sqrt[3]{x^2}}{9x^2\sqrt[3]{x}} = \frac{2\sqrt[3]{x^2}}{x^2} \cdot \frac{2x-1}{3\sqrt[3]{x}}$$

$$\frac{d^2c}{dx^2} > 0$$



$x \in (-\infty, 0) \cup (\sqrt[3]{2}, +\infty)$ C é v

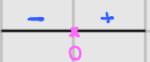
$x \in (0, \sqrt[3]{2})$ C é n

$x \in \{0, \sqrt[3]{2}\}$ são pontos de inflexão

$$42- f(x) = \ln(x^4+27)$$

$$a) \frac{df}{dx} = \frac{1}{x^4+27} \cdot 4x^3 = \frac{4x^3}{x^4+27}$$

O denominador é sempre positivo.



$x \in (-\infty, 0)$ f é decresc.

$x \in (0, +\infty)$ f é cresc.

$x = 0$ f é const.

$$b) Nc: \frac{df}{dx} = 0 \text{ eur } \nexists \frac{df}{dx}$$

Nc: ?

$$f(0) = \ln 27$$

$$c) \frac{d^2f}{dx^2} = \frac{12x^2(x^4+27)}{(x^4+27)^2} - 12x^6$$

$$y \frac{dx}{dx} = \frac{1}{(x^4 + 27)^2}$$

O denominador é sempre positivo.

$$\frac{d^2f}{dx^2} > 0 \dots 12x^2(x^4 + 27) > 16x^6 \dots 3(x^4 + 27) > 4x^4 \dots 3x^4 + 3^4 - 4x^4 > 0 \dots -x^4 > -3 \dots x^4 < 3 \dots x < \sqrt[4]{3}$$

$$\begin{array}{c} - \\ \text{x} \\ + \\ 3 \end{array}$$

$$x \in (-\infty, 3) f \in \cap$$

$$x \in (3, +\infty) f \in \cup$$

$x = 3$ é pta. infl.

43) $f(\theta) = 2\cos\theta + \cos^2\theta, [0, 2\pi]$

a) $\frac{df}{d\theta} = -2\sin\theta - 2\cos\theta \cdot \sin\theta$

$$\frac{df}{d\theta} > 0 \dots \cos\theta < -1$$

$\cos\theta$ é menor que -1 $\forall \theta \neq \pi$

$x = \pi$ f é cresc.

$x = \pi$ f é const.

~~$-2\sin\theta < 2\cos\theta \cdot \sin\theta$~~

$$-1 < \cos\theta$$

$$\cos\theta > -1 \Rightarrow \frac{df}{d\theta} < 0 \quad ???$$

b)

Pg 279 (4.4) JOÃO VAZ

1- a) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \neq$

b) $\lim_{x \rightarrow a} \frac{f(x)}{p(x)} = \frac{0}{\infty} = 0$

c) $\lim_{x \rightarrow a} \frac{f(x)}{p(x)} = \frac{1}{\infty} = 0$

d) $\lim_{x \rightarrow a} \frac{p(x)}{f(x)} = \frac{\infty}{0} \neq$

e) $\lim_{x \rightarrow a} \frac{p(x)}{g(x)} = \frac{\infty}{\infty} \neq$

2- a) $\lim_{x \rightarrow a} f(x)p(x) = 0 \cdot \infty \neq$

b) $\lim_{x \rightarrow a} f(x)p(x) = \infty \cdot \infty = \infty$

c) $\lim_{x \rightarrow a} p(x)g(x) = \infty \cdot \infty \neq$

3- a) $\lim_{x \rightarrow a} f(x) - p(x) = 0 - \infty = -\infty$

b) $\lim_{x \rightarrow a} p(x) - q(x) = \infty - \infty = \infty$

c) $\lim_{x \rightarrow a} p(x) + q(x) = \infty + \infty = \infty$

4- a) $\lim_{x \rightarrow a} f(x)^{g(x)} = 0^0 \neq$

b) $\lim_{x \rightarrow a} f(x)^{p(x)} = 0^\infty = 0$

c) $\lim_{x \rightarrow a} f(x)^{p(x)} = \infty^\infty \neq$

d) $\lim_{x \rightarrow a} p(x)^{f(x)} = \infty^0 = 1$

e) $\lim_{x \rightarrow a} p(x)^{g(x)} = \infty^\infty = \infty$

f) $\lim_{x \rightarrow a} \sqrt[p]{p(x)} = p(x)^{\frac{1}{p}} = p(x)^0 = 0^0 \neq$

5- $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 2} \frac{f'(x)}{g'(x)} = \frac{-\frac{18}{10}}{-\frac{4}{5}} = \frac{18}{10} \cdot \frac{5}{4} = \frac{9}{4}$

6- $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 2} \frac{f'(x)}{g'(x)} = \frac{1.5}{\frac{5}{2}} = 1.5$

7- $\lim_{x \rightarrow 1} \frac{x^2-1}{x+1} = \lim_{x \rightarrow 1} x-1 = 0$

8- $\lim_{x \rightarrow 1} \frac{x^a-1}{x^b-1} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 1} \frac{ax^{a-1}}{bx^{b-1}} = \frac{a}{b}$

9- $\lim_{x \rightarrow 1} \frac{x^3-2x^2+1}{x^3-1} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 1} \frac{3x^2-4x}{3x^2} = -1$

10- $\lim_{x \rightarrow 1/2} \frac{6x^2+5x-4}{4x^2+16x-9} = \frac{1.5+2.5-4}{1+8-9} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 1/2} \frac{12x+5}{8x+16} = \frac{11}{20}$

11- $\lim_{x \rightarrow \pi/2^+} \frac{\cos x}{1-\sin x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \pi/2^+} \frac{-\sin x}{-\cos x} = \frac{1}{0^+} = +\infty$

12- $\lim_{x \rightarrow 0} \frac{\tan 4x}{\tan 5x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\sec^2 4x \cdot 4}{\sec^2 5x \cdot 5} = \frac{4}{5}$

13- $\lim_{t \rightarrow 0} \frac{e^{2t}-1}{\ln e^t} = \frac{0}{0} \stackrel{LH}{=} \lim_{t \rightarrow 0} \frac{2e^{2t} \cdot 2}{1} = \frac{2}{1} = 2$

14- $\lim_{x \rightarrow 0} \frac{x^2}{1-\cos x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{2x}{\sin x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{2}{\cos x} = 2$

15- $\lim_{\theta \rightarrow \pi/2} \frac{1-\tan \theta}{1+\cos 2\theta} = \frac{0}{0} \stackrel{LH}{=} \lim_{\theta \rightarrow \pi/2} \frac{-\cos \theta}{0} = \frac{0}{2} = 0$

$$16. \lim_{\theta \rightarrow \pi/2} \frac{1-\cos\theta}{\cos\theta} = \lim_{\theta \rightarrow \pi/2} \frac{\sin\theta(1-\cos\theta)}{\cos\theta} = 1 \cdot 0 = 0$$

$$17. \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2x^{1/2}}} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{2}{x^{1/2}} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{2\sqrt{x}}{x} = \lim_{x \rightarrow \infty} \frac{2}{x} = \lim_{x \rightarrow \infty} \frac{x^{1/2}}{x} = \lim_{x \rightarrow \infty} \frac{1}{x^{1/2}} = 0$$

$$18. \lim_{x \rightarrow 1} \frac{\ln x}{\sin \pi x} = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{\pi \cos \pi x} = \frac{1}{1} = 1$$

$$19. \lim_{x \rightarrow 0^+} \frac{\ln x}{x} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{1}{1}} = \frac{1}{0^+} = +\infty$$

$$20. \lim_{x \rightarrow \infty} \frac{\ln \ln x}{x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\ln x} \cdot \frac{1}{x} = \lim_{x \rightarrow \infty} \frac{1}{\infty} \cdot \frac{1}{\infty} = 0 \cdot 0 = 0$$

$$21. \lim_{t \rightarrow 1} \frac{t^8 - 1}{t^5 - 1} = \lim_{t \rightarrow 1} \frac{\frac{8t^7}{t^5}}{\frac{5t^4}{t^5}} = \lim_{t \rightarrow 1} \frac{8t^3}{5} = \frac{8}{5}$$

$$22. \lim_{t \rightarrow 0} \frac{8^t - 5^t}{t} = \lim_{t \rightarrow 0} \frac{8^t \ln 8 - 5^t \ln 5}{t} = \ln 8 - \ln 5 = \ln(5/8)$$

$$23. \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}(1+2x) - \frac{1}{2}(1-4x)}{x} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+2x}} - \frac{2}{\sqrt{1-4x}} = \frac{-1}{1} = -1$$

$$24. \lim_{u \rightarrow \infty} \frac{e^{u/10}}{u^3} = \lim_{u \rightarrow \infty} \frac{\frac{1}{10}e^{u/10}}{3u^2} = \lim_{u \rightarrow \infty} \frac{1}{30}e^{u/10} = \lim_{u \rightarrow \infty} \frac{1}{60u} = \lim_{u \rightarrow \infty} \frac{1}{600u} = \infty$$

$$25. \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \rightarrow 0} \frac{\frac{e^x - 1}{x}}{2x} = \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$$

$$26. \lim_{x \rightarrow 0} \frac{\cosh x - x}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{e^x + e^{-x}}{2} - x}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{e^x - e^{-x}}{2} - x}{3x^2} = \lim_{x \rightarrow 0} \frac{\frac{e^x - e^{-x}}{2} - x}{3x^2} = \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2x}{6x^2} = \frac{2}{0} = \infty$$

* 27.

$$28. \lim_{x \rightarrow 0} \frac{x - \tan x}{x + \tan x} = \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{1}{x}} = \frac{1}{1} = 1$$

$$29. \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{2} = \infty$$

$$30. \lim_{x \rightarrow 0} \frac{3^x \cdot x}{3^x - 1} = \lim_{x \rightarrow 0} \frac{3^x \ln 3 \cdot x}{3^x \ln 3} = \frac{1}{\ln 3}$$

$$31. \lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2} = \lim_{x \rightarrow 0} \frac{-m^2 \sin mx + n^2 \sin nx}{2x} = \lim_{x \rightarrow 0} \frac{-m^2 \sin mx + n^2 \sin nx}{2} = \frac{-m^2 + n^2}{2}$$

$$32. \lim_{x \rightarrow 0} \frac{x + \tan x}{x + \cot x} = \lim_{x \rightarrow 0} \frac{x^2 - \tan^2 x}{(x + \cot x)(x - \tan x)} = \lim_{x \rightarrow 0} \frac{x^2 + \cot^2 x}{(x + \cot x)(x - \tan x)} = \lim_{x \rightarrow 0} \frac{2x - 2\tan^2 x}{(1 - \tan x)(x - \tan x) + (x + \cot x)(1 - \cot x)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{2 - 4x \tan x \cot x}{- \cot(x - \tan x) + 2(1 - \tan x)(1 - \cot x) + \tan x(x + \cot x)} = \frac{2}{0} = \infty$$

$$34. \lim_{x \rightarrow 0} \frac{x}{\cot 4x} = \lim_{x \rightarrow 0} x \cdot \tan 4x = 0$$

$$35. \lim_{x \rightarrow 1} \frac{1 - x + \ln x}{1 + \cot \pi x} = \lim_{x \rightarrow 1} \frac{-1 + \frac{1}{x}}{0} = \lim_{x \rightarrow 1} \frac{-x^2}{0} = \frac{-1}{\pi^2} = \frac{1}{\pi^2}$$

$$36. \lim_{x \rightarrow 0^+} \frac{x^2 - 1}{\tan x + x - 1}$$

$$37. \lim_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x-1)^2} = \lim_{x \rightarrow 1} \frac{ax^{a-1} - a}{2(x-1)} = \lim_{x \rightarrow 1} \frac{a(a-1)x^{a-2}}{2} = \frac{a(a-1)}{2}$$

$$38. \lim_{x \rightarrow \infty} \frac{e^x - e^{-x} - 2x}{x - \tan x} = \lim_{x \rightarrow \infty} \frac{\frac{e^x + e^{-x}}{2} - 2}{1 - \tan x} = \lim_{x \rightarrow \infty} \frac{\frac{e^x - e^{-x}}{2} - 2}{\tan x} = \lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{\tan x} = \frac{0}{1} = 0$$

$$39. \lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x \rightarrow 0} \frac{-\tan x + x}{4x^3} = \lim_{x \rightarrow 0} \frac{-\cos x + 1}{12x^2} = \lim_{x \rightarrow 0} \frac{\tan x}{12x} = \lim_{x \rightarrow 0} \frac{\sec^2 x}{24} = \frac{1}{24}$$

$$40. \lim_{x \rightarrow \alpha^+} \frac{\cos x \ln(x-\alpha)}{\ln(e^x - e^\alpha)} = \lim_{x \rightarrow \alpha^+} \frac{-\tan x \cdot \frac{1}{x-\alpha} \cdot 1}{\frac{1}{(e^x - e^\alpha)^2}} = \lim_{x \rightarrow \alpha^+} \frac{-\tan x \cdot (e^x - e^\alpha)}{(e^x - e^\alpha)^2} = \lim_{x \rightarrow \alpha^+} (e^x - e^\alpha)(-\tan x) - \tan x(e^x - e^\alpha) = 0 - 0 = 0$$

$$41. \lim_{x \rightarrow \infty} x \tan(\pi/x) = \infty = \lim_{x \rightarrow \infty} \frac{x \tan(\pi/x)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\cos(\pi/x)}{1 - \pi^2 x^2} = \lim_{x \rightarrow \infty} \frac{-\pi^2 \cos(\pi/x)}{-x^2} = \lim_{x \rightarrow \infty} \frac{\pi \cos(\pi/x)}{x} = \pi$$

$$42. \lim_{x \rightarrow \infty} \frac{x^{1/2}}{\sqrt{e^x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{2}x^{-1/2}}{\frac{1}{2}(e^x)^{1/2}} = \lim_{x \rightarrow \infty} \frac{x^{-1/2}}{e^{1/2}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x} \cdot \sqrt{e}} = 0$$

$$43. \lim_{x \rightarrow 0} \cot 2x \cdot \tan 6x = \lim_{x \rightarrow 0} \frac{\cot 2x \cdot 6}{\tan 2x} = \lim_{x \rightarrow 0} 3 \cot 6x \cdot \cot^2 2x = 3$$

$$44. \lim_{x \rightarrow 0^+} \tan x \ln x = 0 \cdot (-\infty) = \lim_{x \rightarrow 0^+} \frac{\tan x}{\frac{1}{\ln x}} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \rightarrow 0^+} \frac{x}{-1} = \lim_{x \rightarrow 0^+} -\frac{1}{x} = \infty$$

$$45. \lim_{x \rightarrow \infty} x^3 \cdot e^{-x^2} = \lim_{x \rightarrow \infty} \frac{x^3}{e^{x^2}} = \lim_{x \rightarrow \infty} \frac{3x^2}{2x^2} = \lim_{x \rightarrow \infty} \frac{3}{2} = \frac{3}{2} = 0$$

$$46 \lim_{x \rightarrow \infty} x \operatorname{tg}(1/x) = \infty \stackrel{P^I}{=} \lim_{x \rightarrow \infty} \frac{\operatorname{tg}(1/x)}{\frac{1}{x}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{-\operatorname{sec}^2(1/x) \cdot x^2}{-x^{-2}} = \lim_{x \rightarrow \infty} \frac{1}{(\operatorname{cos}^2(\frac{1}{x}))} = 1$$

$$47 \lim_{x \rightarrow \pm\infty} \ln x \cdot \operatorname{tg}(mx/2) = 0 \cdot \infty \stackrel{P^I}{=} \lim_{x \rightarrow \pm\infty} \frac{\ln x}{\operatorname{cotg}(mx/2)} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{x}}{-\operatorname{cosec}^2(mx/2) \cdot \frac{m}{2}} = \lim_{x \rightarrow \pm\infty} \frac{2}{-\pi \cdot \operatorname{cosec}^2(mx/2)} = \frac{-2}{\pi}$$

$$48 \lim_{x \rightarrow \frac{\pi}{2}^-} \operatorname{cos}x \cdot \operatorname{sec}5x = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\operatorname{cos}x}{\frac{1}{\operatorname{sec}5x}} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\operatorname{cos}x}{\operatorname{cos}5x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{-\operatorname{sen}x}{-5\operatorname{sen}5x} = \frac{1}{5}$$

$$49 \lim_{x \rightarrow \pm\infty} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \lim_{x \rightarrow \pm\infty} \frac{x \ln x - x + 1}{\ln x(x-1)} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \pm\infty} \frac{\ln x}{\frac{1}{x}(x-1) + \ln x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{x}}{-\frac{1}{x^2}(x-1) + \frac{1}{x} + \frac{1}{x}} = \frac{1}{0+1+1} = \frac{1}{2}$$

$$50 \lim_{x \rightarrow 0} (\operatorname{cos}x - \operatorname{cosec}x) = \lim_{x \rightarrow 0} \frac{1}{\operatorname{sen}x} - \frac{\operatorname{cos}x}{\operatorname{sen}x} = \lim_{x \rightarrow 0} \frac{1 - \operatorname{cos}x}{\operatorname{sen}x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1 + \operatorname{sen}x}{\operatorname{cos}x} = 1$$

$$51 \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) = \lim_{x \rightarrow 0^+} \frac{e^x - x}{x(e^x - 1)} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0^+} \frac{e^x - 1}{e^x - 1 + x e^x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0^+} \frac{e^x}{e^x + e^x + x e^x} = \frac{1}{2}$$

$$52 \lim_{x \rightarrow 0} \left(\operatorname{cosec}x - \frac{1}{x} \right) = \lim_{x \rightarrow 0} \left(\frac{\operatorname{cos}x}{\operatorname{sen}x} - \frac{1}{x} \right) = \lim_{x \rightarrow 0} \frac{x \operatorname{cos}x - \operatorname{sen}x}{x \operatorname{sen}x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\operatorname{cos}x - x \operatorname{sen}x - \operatorname{cos}x}{\operatorname{sen}x + x \operatorname{cos}x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{-\operatorname{sen}x - (\operatorname{sen}x + x \operatorname{cos}x) + x \operatorname{sen}x}{\operatorname{cos}x + \operatorname{cos}x - x \operatorname{sen}x} = \frac{0}{2} = 0$$

$$53 \lim_{x \rightarrow \infty} (x - \ln x) = \lim_{x \rightarrow \infty} x(1 - \frac{\ln x}{x}) \text{ Note que } \lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{LH}{=} \frac{1}{1} = 0. \text{ Portanto, } \lim_{x \rightarrow \infty} x(1 - \frac{\ln x}{x}) = \infty(1-0) = \infty$$

$$54 \lim_{x \rightarrow \infty} [\ln(x^2 - 1) - \ln(x^5 - 1)] = \lim_{x \rightarrow \infty} \ln \left(\frac{x^2 - 1}{x^5 - 1} \right) = \lim_{x \rightarrow \infty} \ln \left(\frac{(x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)}{(x-1)(x^4 + x^3 + x^2 + x + 1)} \right) = \ln \left(\frac{1}{5} \right) = \ln 5 - \ln 1$$

$$55 \lim_{x \rightarrow 0^+} \sqrt[x]{x} = a$$

$$\ln a = \lim_{x \rightarrow 0^+} [\sqrt[x]{x} \cdot \ln x] = \lim_{x \rightarrow 0^+} \frac{\sqrt[x]{x}}{\frac{1}{\ln x}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{2}x^{-1/2}}{-(\ln x)^{-2}} = \lim_{x \rightarrow 0^+} \frac{-\ln^2 x}{2\sqrt{x}} = \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\frac{2x}{\ln^2 x}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{2}x^{-1/2}}{\frac{2\ln^2 x - 2x \cdot 2 \ln x \cdot \frac{1}{x}}{\ln^4 x}} = \lim_{x \rightarrow 0^+} \frac{1}{2\ln^2 x - 2x \ln x} = \lim_{x \rightarrow 0^+} \frac{1}{2\ln x} \cdot \frac{\ln x}{x}$$

$$56 \lim_{x \rightarrow 0^+} (\operatorname{tg}2x)^x = a$$

$$\ln a = \lim_{x \rightarrow 0^+} x \ln(\operatorname{tg}2x) = 0 \cdot (-\infty) \stackrel{P^I}{=} \lim_{x \rightarrow 0^+} \frac{x}{\frac{1}{\ln(\operatorname{tg}2x)}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0^+} \frac{1}{-\ln^2(\operatorname{tg}2x) \cdot \operatorname{sec}^2 2x \cdot 2} = \lim_{x \rightarrow 0^+} \frac{-\ln(\operatorname{tg}2x)^2}{2 \cdot 2 \cdot 2x \cdot 2} = \frac{+\infty}{2} = +\infty$$

$$\ln a = \infty; a = e^\infty = \infty$$

$$\lim_{x \rightarrow \infty} (\operatorname{tg}2x)^x = \infty$$

$$57 \lim_{x \rightarrow 0} (1-2x)^{1/x} = a$$

$$\ln a = \lim_{x \rightarrow 0} \frac{1}{x} \ln(1-2x) = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} -\frac{2}{1-2x} \cdot \frac{1}{x} \cdot \frac{1}{1-2x} = \lim_{x \rightarrow 0} \frac{-\ln(1-2x)}{x^2} - \frac{2}{x(1-2x)} = \lim_{x \rightarrow 0} \frac{-x+2x^2 \ln(1-2x) - 2x^2}{x^2 \cdot 2x^2} = \frac{0}{0} \stackrel{LH}{=}$$

$$58 \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^{bx} = c$$

$$\ln c = \lim_{x \rightarrow \infty} [bx \cdot \ln \left(1 + \frac{a}{x} \right)] = \infty \cdot 0 \stackrel{P^I}{=} \lim_{x \rightarrow \infty} \left[b \ln \left(1 + \frac{a}{x} \right) \right] = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \infty} \frac{b \cdot \frac{1}{1+a/x} \cdot (-ax^{-2})}{-x^{-2}} = \lim_{x \rightarrow \infty} \frac{b \cdot a}{1+a/x} = ab$$

$$\ln c = ab; c = e^{ab}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} \right)^{bx} = e^{ab}$$

$$59 \lim_{x \rightarrow \pm\infty} x^{\frac{1}{1-x}} = a$$

$$\ln a = \lim_{x \rightarrow \pm\infty} \frac{1}{x-1} \ln x = \lim_{x \rightarrow \pm\infty} \frac{\ln x}{1-x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{x}}{-1} = -1$$

$$\ln a = -1; a = e^{-1}$$

$$\lim_{x \rightarrow 1^\pm} x^{\frac{1}{x-1}} = e^1$$

$$e^{\pm 1/(1+\ln x)} = a$$

$$60 - \lim_{x \rightarrow \infty} \frac{x}{e^{x+1/(x+\ln x)}} = \infty = a$$

$$\ln a = \lim_{x \rightarrow \infty} \frac{\ln 2}{1 + \ln x} \ln x = 0 \cdot \infty \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{\ln 2}{1 + \ln x}}{\frac{1}{\ln x}} = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{(1 + \ln x)^2} \cdot \frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\frac{x}{2}}{x} = \lim_{x \rightarrow \infty} \frac{x}{2} \cdot \frac{1}{x} = \frac{1}{2}$$

$$\ln a = \frac{1}{2}, a = \sqrt{e}$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \sqrt{e}$$

$$61 - \lim_{x \rightarrow \infty} \frac{1}{x^x} = \infty^0 = a$$

$$\ln a = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln x = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \frac{\infty}{\infty} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

$$\ln a = 0, a = e^0 = 1$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^x} = 1$$

$$62 - \lim_{x \rightarrow \infty} (e^x + x) = \infty^0 = a$$

$$\ln a = \lim_{x \rightarrow \infty} \frac{1}{x} \ln(e^x + x) = \lim_{x \rightarrow \infty} \frac{\ln(e^x + x)}{x} = \frac{\infty}{\infty} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{e^x + x} \cdot (e^x + 1)}{1} = \lim_{x \rightarrow \infty} \frac{e^x + 1}{e^x + x} = \frac{\infty}{\infty} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{e^x + 1} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x \left(1 + \frac{1}{e^x}\right)} = \frac{1}{1} = 1$$

$$\ln a = 1, a = e^1 = e$$

$$\lim_{x \rightarrow \infty} (e^x + x) = e^1$$

$$63 - \lim_{x \rightarrow 0^+} (4x+1)^{\cot x} = 1^0 = a$$

$$\ln a = \lim_{x \rightarrow 0^+} \cot x \cdot \ln(4x+1) = \lim_{x \rightarrow 0^+} \frac{\cot x \cdot \ln(4x+1)}{\tan x} = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{-\csc^2 x \ln(4x+1) + \cot x \cdot \frac{1}{4x+1} \cdot 4}{\sec^2 x} = \lim_{x \rightarrow 0^+} \frac{4}{1} = 4$$

$$\ln a = 4, a = e^4$$

$$\lim_{x \rightarrow 0^+} (4x+1)^{\cot x} = e^4$$

$$64 - \lim_{x \rightarrow \pi^-} (2-x)^{\tan(\frac{\pi x}{2})} = 1^\infty = a$$

$$\ln a = \lim_{x \rightarrow \pi^-} \tan(\frac{\pi x}{2}) \cdot \ln(2-x) = \infty \cdot 0 \stackrel{P}{=} \lim_{x \rightarrow \pi^-} \frac{\ln(2-x)}{\tan(\frac{\pi x}{2})} = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow \pi^-} \frac{\frac{1}{2-x}}{-\csc^2(\frac{\pi x}{2}) \cdot \frac{\pi}{2}} = \lim_{x \rightarrow \pi^-} \frac{-2 \csc^2(\frac{\pi x}{2})}{\pi(2-x)} = \frac{-2}{\pi}$$

$$\ln a = \frac{-2}{\pi}, a = e^{\frac{-2}{\pi}} = \sqrt[e^{-2/\pi}]$$

$$\lim_{x \rightarrow \pi^-} (2-x)^{\tan(\frac{\pi x}{2})} = e^{-2/\pi}$$

$$65 - \lim_{x \rightarrow 0^+} (\cos x)^{\frac{1}{x^2}} = 1^0 = a$$

$$\ln a = \lim_{x \rightarrow 0^+} \frac{1}{x^2} \ln(\cos x) = \lim_{x \rightarrow 0^+} \frac{\ln(\cos x)}{x^2} = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{\cos x} \cdot (-\sin x)}{2x} = \lim_{x \rightarrow 0^+} \frac{-\sin x}{2x \cos x} = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{-\cos x}{2 \cos x - 2x \sin x} = \frac{-1}{2}$$

$$\ln a = \frac{-1}{2}, a = e^{-1/2}$$

$$\lim_{x \rightarrow \pi^+} (\cos x)^{\frac{1}{x^2}} = e^{-1/2}$$

$$66 - \lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+5} \right)^{2x+1} = \infty^0 = a$$

$$\ln a = \lim_{x \rightarrow \infty} \left[(2x+1) \ln \left(\frac{2x-3}{2x+5} \right) \right] = \lim_{x \rightarrow \infty} \left[(2x+1) \ln \left(\frac{x(2-3/x)}{x(2+5/x)} \right) \right] = \lim_{x \rightarrow \infty} \left[(2x+1) \cdot \ln 1 \right] = \lim_{x \rightarrow \infty} \left[\frac{0}{2x+1} \right] = \frac{0}{0} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{0}{1 + (2x+1)^{-1} \cdot 2} = \lim_{x \rightarrow \infty} \frac{0}{-2} = 0$$

$$\ln a = 0, a = e^0 = 1$$

$$\lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+5} \right)^{2x+1} = 1$$

$$77 - \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} = \infty \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{1}{\frac{1}{2}(x^2+1)^{1/2} \cdot 2x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{x} = \frac{\infty}{\infty} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x}{1} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+1}} \text{ Loop}$$

$$\lim_{x \rightarrow \infty} \frac{x(\sqrt{x^2+1})'}{x^2+1} = \lim_{x \rightarrow \infty} \frac{x(\sqrt{x^2+1})'}{x(x+1/x)} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{x+1/x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2(1+1/x^2)}}{x+1/x} = \lim_{x \rightarrow \infty} \sqrt{1} = 1$$

$$78 - \lim_{x \rightarrow \infty} \frac{\cos x}{x} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \cos x = \lim_{x \rightarrow \infty} \cos x = 1$$

$$x \rightarrow (\eta_1)_0^+ \quad t \rightarrow x \quad x \rightarrow (\eta_1)_0^- \quad \text{Caso x} \quad \text{se x} \rightarrow -(\eta_1)_0^+$$

73- $\lim_{m \rightarrow \infty} A_0 \cdot \left(1 + \frac{n}{m}\right)^{nt} = A_0 \cdot 1^\infty = a$

$$\ln a = \lim_{m \rightarrow \infty} \left[\ln A_0 + nt \ln \left(1 + \frac{n}{m}\right) \right] = \lim_{m \rightarrow \infty} \left[nt \ln \left(1 + \frac{n}{m}\right) \right] + \ln A_0 = \lim_{m \rightarrow \infty} (\infty \cdot 0) + \ln A_0 \stackrel{H}{=} \lim_{m \rightarrow \infty} \left[\frac{\ln(1 + n/m)}{\frac{1}{nt}} \right] + \ln A_0 = \lim_{m \rightarrow \infty} \left(\frac{0}{0} \right) + \ln A_0 \stackrel{H}{=} \ln A_0 + \lim_{m \rightarrow \infty} \left[\frac{(1 + \frac{n}{m})^{\frac{1}{m}} \cdot (\frac{n}{m})^{-2}}{-\frac{1}{n^2 m^2}} \right]$$

$$\ln a = \ln A_0 + \lim_{m \rightarrow \infty} \frac{nt}{1 + \frac{n}{m}} = \ln A_0 + nt$$

$$a = e^{\ln A_0 + nt} = e^{\ln A_0} \cdot e^{nt} = A_0 e^{nt}$$

$$\lim_{m \rightarrow \infty} A_0 \left(1 + \frac{n}{m}\right)^{nt} = A_0 e^{nt}$$

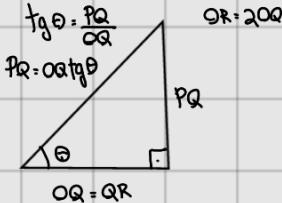
Área do setor circular com r = OR e d = θ

82- $A(\theta) = C(\theta) - \Delta POR = \frac{\theta}{2\pi} \cdot \pi \cdot (OR)^2 - 2B(\theta) = \frac{\theta \pi (OR)^2 - 4\pi B(\theta)}{2\pi}$

$$B(\theta) = \frac{QR \cdot PQ}{2} = \frac{OR \cdot PQ}{2} = \frac{OR^2 \operatorname{tg} \theta}{2}$$

$$A(\theta) = \theta \cdot 2OR^2 - OR^2 \operatorname{tg} \theta = OR^2 (\theta - \operatorname{tg} \theta)$$

$$\lim_{\theta \rightarrow 0^+} \frac{A(\theta)}{B(\theta)} = \lim_{\theta \rightarrow 0^+} \frac{2OR^2 (\theta - \operatorname{tg} \theta)}{OR^2 \operatorname{tg} \theta} = \lim_{\theta \rightarrow 0^+} \frac{4\theta - 2\operatorname{tg} \theta}{\operatorname{tg} \theta} = \frac{4 \cdot 0 - 2 \cdot 1}{1} = -2$$



$$1- y = x^3 + x = x(x^2 + 1)$$

domínio: $D = \mathbb{R}$

intersecções: $(0,0)$

assintotas vert.: \emptyset

assintotas horiz.: \emptyset

crescimento:

$$\frac{dy}{dx} = 3x^2 + 1$$

$x \in \mathbb{R}, y$ é crescente

máximos e mínimos: $N_c = \emptyset$

concavidade:

$$\frac{d^2y}{dx^2} = 6x$$

$x \in (-\infty, 0) y$ é \cap

$x=0$ é pta. infl.

$x \in (0, +\infty) y$ é \cup

$$2- y = x^3 + 6x^2 + 9x = x(x^2 + 6x + 9)$$

domínio: $D = \mathbb{R}$

intersecções: $(0,0), (-3,0)$

assintotas vert.: \emptyset

assintotas horiz.: \emptyset

crescimento:

$$\frac{dy}{dx} = 3x^2 + 12x + 9 = 3(x^2 + 4x + 3)$$



$x \in (-\infty, -3) \cup (1, +\infty) y$ é cresc.

$x \in (-3, -1) y$ é decresc.

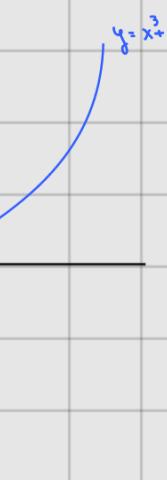
$x \in \{-3, -1\} y$ é cte.

máx. e mín.: $N_c = \{-3, -1\}$

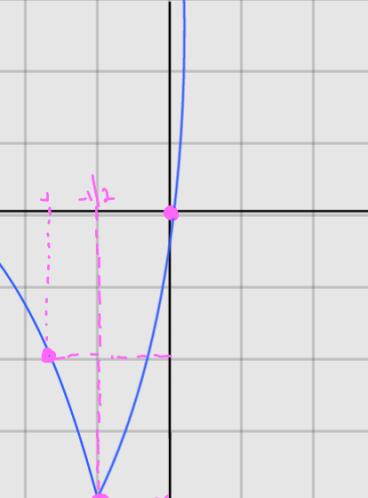
$$f(-3) = 0 \quad \text{Máx}$$

$$f(-1) = -4 \quad \text{Mín}$$

conc.:



$$y = x^3 + 6x^2 + 9x$$



$$\frac{dy}{dx} = 3(2x+4) = 6x+12$$



$x \in (-\infty, -1/2)$ y é ∩

$x \in (-1/2, +\infty)$ y é ∪

$x = -1/2$ é pta. infl.

3. $y = 2 - 15x + 9x^2 - x^3 = -x^3 + 9x^2 - 15x + 2$

dom.: $D = \mathbb{R}$

intervos: $(0, 2), (2, 0), (\frac{7-3\sqrt{5}}{2}, 0), (\frac{7+3\sqrt{5}}{2}, 0)$

assintotas: \emptyset

convergência:

$$\frac{dy}{dx} = -3x^2 + 18x - 15$$



$x \in (-\infty, 1) \cup (5, +\infty)$ y é decresc.

$x \in (1, 5)$ y é cresc.

$x \in (1, 5)$ y é dca.

max e min.: $N_c = \{1, 5\}$

$$f(1) = -5 \quad \text{MIN}$$

$$f(5) = 27 \quad \text{MAX}$$

concavidade

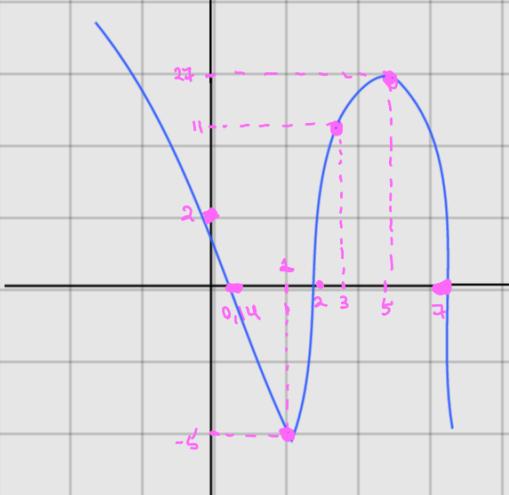
$$\frac{d^2y}{dx^2} = -6x + 18$$



$x \in (-\infty, 3)$ y é ∪

$x \in (3, +\infty)$ y é ∩

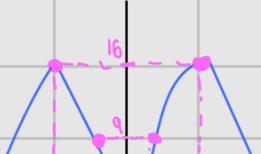
$x = 3$ é pta. infl.



4. $y = -x^4 + 8x^2$

dom.: \mathbb{R}

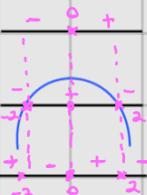
intervos: $(0, 0), (-2\sqrt{2}, 0), (2\sqrt{2}, 0)$



assintotas: \emptyset

crescimento:

$$\frac{dy}{dx} = -4x^3 + 16x = x(-4x^2 + 16)$$



$x \in (-\infty, -2) \cup (0, 2)$ y é cresc.

$x \in (-2, 0) \cup (2, +\infty)$ y é decresc.

$x \in \{-2, 0, 2\}$ y é cst.

máx. e min.: Nc = {-2, 0, 2}

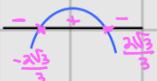
$$f(-2) = 16 \quad \text{MAX}$$

$$f(0) = 0 \quad \text{MIN}$$

$$f(2) = 16 \quad \text{MAX}$$

conc.

$$\frac{d^2y}{dx^2} = -12x^2 + 16$$



$x \in (-\infty, -\frac{2\sqrt{3}}{3}) \cup (\frac{2\sqrt{3}}{3}, +\infty)$ y é ^

$x \in (-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3})$ y é v

$x \in \{\pm \frac{2\sqrt{3}}{3}\}$ não pntos. impl.

5. $y = x(x-4)^3$

dom.: \mathbb{R}

interc.: $(0, 0), (4, 0)$

assintotas: \emptyset

crescimento:

$$\frac{dy}{dx} = (x-4)^3 + 3x(x-4)^2 = (x-4)^2(x-4+3x) = (x-4)^2(4x-4) = 4(x-4)^2(x-1)$$

$$(x-4)^2 = 0 \therefore x = 4$$

$$\exists 0 < x < 4$$

$$x-1 = 0 \therefore x = 1$$

$x \in (-\infty, 1)$ y é decresc.

$x \in (-\infty, 1) \cup (4, +\infty)$ y es crec.

$x \in [1, 4]$ y es cte.

max. e min.: Nc = {1, 4}

$$f(1) = -27 \quad \text{min}$$

$$f(4) = 0 \quad \text{max}$$

conc.

$$\frac{d^2y}{dx^2} = 3(x-4)^2 + 3(x-4) + 6x(x-4) = 3(x-4)(x-4+x+2x) = 3(x-4)(4x-8) = 12(x-4)(x-2) = 12(x^2-6x+8)$$



$x \in (-\infty, 2) \cup (4, +\infty)$ y es \cup

$x \in (2, 4)$ y es \cap

$x \in [2, 4]$ son ptos. infl.

✓ Ano ate o William
da essa matéria

