Logical Inner–Form Pattern Map

Cheat–Sheet for Choosing Proof Techniques

Strip off the quantifiers first, then match the logical "shape" of the remaining statement to a row below. The **preferred tactic** is usually the shortest route; if it stalls, switch to one of the alternatives in the "Why" column.

#	Canonical Inner Form	Typical Surface Wording	Preferred Proof Tactic	Why It's Natural
1	$R(x)$	Plain predicate (*e.g.* "x is prime")	Direct algebra / definition chase	No logical connectors to rearrange – just verify the property.
2	$P(x) \Rightarrow Q(x)$	"If x … then …"	Direct **or** Contrapositive	Choose the side (assume P vs. assume $\neg Q$) whose algebra is simpler.
3	$P(x) \Leftrightarrow Q(x)$	"… iff …"	Prove each direction as an implication	Each half may need a different trick.
4	$P(x) \Rightarrow [Q(x) \vee R(x)]$	"then Q *or* R"	Contrapositive	Negating the conclusion gives $\neg Q \wedge \neg R$, which is usually simpler than an inclusive OR.
5	$[P(x) \wedge Q(x)] \Rightarrow R(x)$	"P *and* Q then R"	Direct	You already have two facts to leverage.
6	$P(x) \Rightarrow [Q(x) \wedge R(x)]$	"then both Q *and* R"	Direct (prove each part)	Must establish two conclusions explicitly.
7	$\forall x\ \exists y\ P(x, y)$	"For every x there exists y such that …"	Construction	Reader expects a formula or algorithm that produces y from x.
8	$\exists x\ P(x)$	"There exists x such that …"	Construction *or* non–constructive (pigeonhole, contradiction)	One explicit example suffices; if hard, argue existence indirectly.
9	$\forall n \geq n_0\ P(n)$ where P references $n-1$, $n-2$, …	Recursive/iterative integer claims	Mathematical induction (ordinary or strong)	The truth "propagates" along n.

10	Parity / Rationality / Divisibility patterns	even/odd, rational/irrational, divisible by k	Contrapositive *or* contradiction	Negations have compact algebraic forms (*e.g.* "not even" $= 2k+1$).
11	Piecewise or absolute–value conditions	$\lvert x\rvert < 1$, max, min	Proof by cases	The definition itself already splits the domain.
12	Counting identity $A(n) = B(n)$	Binomial identities, combinatorial sums	Combinatorial (double–count) proof	Count the same set two different ways to avoid messy algebra.

Tip: When unsure, scribble one minute of scratch work using both the direct and contrapositive options; the cleaner path usually reveals itself quickly.