Key Formulas in Engineering Costs

1. Cost Behavior

$$
\begin{aligned}
\text{Fixed Cost (FC)}: &\ \text{Costs that do not change with output.} \\
\text{Variable Cost (VC)}: &\ \text{Costs that vary proportionally with output.} \\
\text{Total Cost (TC)} &= FC + VC \qquad (1) \\
\text{Average Cost (AC)} &= \frac{TC}{x} \qquad (2) \\
\text{Marginal Cost (MC)} &= \frac{d\,VC}{d\,x} \qquad (3)
\end{aligned}
$$

where x is the output (e.g., number of units produced).

2. Break-Even Analysis

Suppose revenue and cost functions are:

$$R(x) = \rho\, x, \qquad TC(x) = a_0 + b_0\, x,$$

where

- ρ = unit selling price ($/unit),
- a_0 = total fixed cost ($),
- b_0 = variable cost per unit ($/unit).

Profit (or net benefit) is:

$$\Omega(x) \;=\; R(x) \;-\; TC(x) \;=\; \rho\, x \;-\; \bigl(a_0 + b_0\, x\bigr). \tag{4}$$

Break-even output x^* occurs when $\Omega(x^*) = 0$:

$$0 = \rho\, x^* \;-\; \bigl(a_0 + b_0\, x^*\bigr) \quad\Longrightarrow\quad x^* = \frac{a_0}{\rho - b_0}. \tag{5}$$

3. Cost Indexes

To update a historical cost C_0 from year 0 to year 1 using an index:

$$C_1 \;=\; C_0 \;\times\; \frac{\text{Index}_1}{\text{Index}_0}. \tag{6}$$

Examples:

- Operating & Maintenance Cost Index ($\text{CPI}_{\text{O\&M}}$)
- Plant Cost Composite Index (PCCI)

4. Power-Sizing Model (Economies of Scale)

If two pieces of equipment have known sizes and costs at the same time,

$$\frac{C_A}{C_B} = \left(\frac{S_A}{S_B}\right)^y,$$

where

- C_A, C_B = capital costs of equipment A and B,
- S_A, S_B = capacities (sizes) of equipment A and B,
- y = size-exponent (typically $0.6 \le y \le 1.0$).
  - If $y < 1$, there are *economies of scale.*

Thus,

$$C_A = C_B \left(S_A/S_B\right)^y.$$

5. Scaling and Inflation Example for a Coal Plant

Reference Data (Year 2010):

- Base plant (Project–B):
  - Size: 400 MW
  - Capital cost: \$3,636 /kW $\Rightarrow$ \$1,454.4 M
  - Fixed O&M: \$16.84 M/year
  - Variable O&M: \$4.60 \$/MWh
  - PCCI_{2010}=100, PCCI_{2020}=172
  - $\text{CPI}_{\text{O\&M},2010}$=1.9, $\text{CPI}_{\text{O\&M},2020}$=2.2

5.1. Step 1: Scale from 400 MW to 500 MW at 2010 Prices

$$\text{Capital cost (2010)}: \quad C_{\text{cap},A}^{2010} = C_{\text{cap},B}^{2010}\left(\frac{500}{400}\right)^{1.0} = 1{,}454.4\text{ M} \times \frac{500}{400} = 1{,}818\text{ M}. \tag{7}$$

$$\text{Fixed O\&M (2010)}: \quad \text{FOM}_A^{2010} = \text{FOM}_B^{2010}\left(\frac{500}{400}\right)^{0.75} \approx 16.84\text{ M} \times \left(\frac{500}{400}\right)^{0.75} \approx 19.908\text{ M/year}. \tag{8}$$

$$\text{Variable O\&M (2010)}: \quad u_{\text{VOM},A}^{2010} = u_{\text{VOM},B}^{2010} = 4.60\text{ \$/MWh}. \tag{9}$$

5.2. Step 2: Inflate to 2020 Values

$$\text{Capital cost (2020)}:$$
$$C^{2020}_{\text{cap},A} = C^{2010}_{\text{cap},A} \times \frac{\text{PCCI}_{2020}}{\text{PCCI}_{2010}} = 1{,}818 \text{ M} \times \frac{172}{100} = 3{,}126.96 \text{ M}. \tag{10}$$

$$\text{Fixed O\&M (2020)}:$$
$$\text{FOM}^{2020}_{A} = \text{FOM}^{2010}_{A} \times \frac{\text{CPI}_{\text{O\&M},2020}}{\text{CPI}_{\text{O\&M},2010}} = 19.908 \text{ M} \times \frac{2.2}{1.9} \approx 23.0512 \text{ M/year}. \tag{11}$$

$$\text{Variable O\&M (2020)}:$$
$$u^{2020}_{\text{VOM},A} = u^{2010}_{\text{VOM},A} \times \frac{\text{CPI}_{\text{O\&M},2020}}{\text{CPI}_{\text{O\&M},2010}} = 4.60 \times \frac{2.2}{1.9} \approx 5.3263 \text{ \$/MWh}. \tag{12}$$

5.3. Step 3: Annual Cost and Revenue Functions (No Discounting)

Assume

- Plant life $N = 50$ years,
- Capacity factor $= 0.70$,
- Plant size $= 500$ MW.

Annualized Capital Recovery:

$$\text{CapRec}_A = \frac{C^{2020}_{\text{cap},A}}{N} = \frac{3{,}126.96 \text{ M}}{50} = 62.5392 \text{ M/year}.$$

Total Annual Cost Function: Let x = annual energy production (MWh). Then

$$x = 500 \text{ MW} \times 24 \frac{\text{h}}{\text{day}} \times 365 \frac{\text{days}}{\text{year}} \times 0.70 = 3.066 \times 10^6 \text{ MWh/year}.$$

Fixed costs per year:

$$\text{FixedAnnualCost} = \text{CapRec}_A + \text{FOM}^{2020}_{A} = 62.5392 + 23.0512 = 85.5904 \text{ M/year}.$$

Variable cost per MWh:

$$u^{2020}_{\text{VOM},A} = 5.3263 \text{ \$/MWh}.$$

Therefore,

$$TC_A(x) = \Big(85.5904\Big) + \Big(5.3263 \times 10^{-6}\Big)\, x \quad \text{(in M\$ per year)}. \tag{13}$$

Revenue Function: If the selling price is ρ \$/MWh, then

$$R_A(x) = \rho\, x \quad \text{(in \$ per year)}. \tag{14}$$

Profit Function:

$$\Omega_A(x) = R_A(x) - TC_A(x) = \rho\, x - \big[85.5904 + (5.3263 \times 10^{-6})\, x\big]. \tag{15}$$

Break-Even Price

At break-even, $\Omega_A(x) = 0$. Using $x = 3.066 \times 10^6$ MWh/year,

$$\begin{aligned}
0 &= \rho^* \times (3.066 \times 10^6) \; - \; \Big[85.5904 + (5.3263 \times 10^{-6})\,(3.066 \times 10^6)\Big] \\
\rho^* &= \frac{85.5904 + \left(5.3263 \times 10^{-6}\right)(3.066 \times 10^6)}{3.066 \times 10^6 \times 10^{-6}} \\
&\approx \frac{85.5904 + 16.3283}{3.066} = \frac{101.9187}{3.066} \approx 32.26 \text{ \$/MWh.}
\end{aligned} \tag{16}$$

6. Additional Cost Concepts

- **Sunk Cost:** A past cash outlay that cannot be recovered; should be ignored in current decision making.

7. Cost Estimation Models

1. **Per-Unit Model**:
$$\text{Cost} = (\text{Cost per Unit}) \times \text{Number of Units.}$$