Choosing a Proof Method in Elementary Set Theory

The table below is a quick decision guide for common proof tasks in introductory set theory.

What the statement looks like	**Typical goal**	**First-choice proof style**	**Sketch of the move**
$X = Y$	Equality	*Double containment* (element chasing)	Show $X \subseteq Y$ and $Y \subseteq X$ by picking arbitrary x.
$X \subseteq Y$	Inclusion	*Direct element-wise*	Take $x \in X$; unpack definitions until $x \in Y$.
Pure $\cup, \cap, \overline{}$ identity	Simplify/equality	*Algebra-of-sets manipulation*	Use commutative, distributive, De Morgan, absorption, etc.
Lots of complements	Inclusion/equality	*Element-wise with De Morgan* or Boolean-algebra view	Translate $x \notin A$ type statements; or use $\wedge, \vee, \neg$ identities.
"Or" in the conclusion	Inclusion	*Cases* or *Contrapositive*	Decide which branch; or turn $\cup$ into $\cap$ in the contrapositive.
Counting finite sets	Cardinality equality	*Counting / inclusion–exclusion*	Compute $\|X\|$ and $\|Y\|$ directly.
Indexed family $\bigcup_i A_i$	Inclusion	*Element-wise with index witness*	Exhibit the index i that works for the x.
Power sets / injections	Structure level	*Function construction* or Schröder–Bernstein	Build explicit map or injections in both directions.
Property over n	For all n	*Induction*	Base case + assume k to prove $k+1$.
Suspicious claim	Likely false	*Counterexample*	Provide one element that breaks the statement.