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Modern Methods for Calculating Ground-Wave Field Strength Over A Smooth Spherical Earth

by
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ABSTRACT

This report makes available the computer program that produces the proposed new FCC ground-wave propagation prediction curves for the new band of standard broadcast frequencies between 1605 and 1705 kHz. The curves are included in recommendations to the U.S. Department of State in preparation for an International Telecommunication Union Radio Conference.

The history of the FCC curves is traced from the early 1930's, when the Federal Radio Commission and later the FCC faced an intensifying need for technical information concerning interference distances. A family of curves satisfactorily meeting this need was published in 1940. The FCC reexamined this matter recently in connection with the planned expansion of the AM broadcast band, and the resulting new curves are a precise representation of the mathematical theory.

Mathematical background is furnished so that the computer program can be critically evaluated. This will be particularly valuable to persons implementing the program on other computers or adapting it for special applications. Technical references are identified for each of the formulas used by the program, and the history of the development of mathematical methods is outlined.

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MODERN METHODS FOR CALCULATING GROUND-WAVE FIELD STRENGTH OVER A SMOOTH SPHERICAL EARTH

Introduction

This report makes available the computer program that was used to draw the new FCC curves proposed for AM broadcast propagation prediction in the band 1605-1705 kHz. The program produces the theoretical value of ground-wave radio field intensity at given frequencies and distances and for given electric ground constants. The program will reproduce the particular family of curves published by the FCC [1] for use in the new band and can be used as an alternative to reading values from the curves themselves. Answers can be obtained for a continuous range of possible ground constants including the particular set of constants that determine the published family of curves.

The field strength values computed by this program are precise representations of the theory over a broad range of frequencies including the existing AM band (535-1605 kHz). They are more accurate than those presented in the old FCC curves which were published in 1940 and republished in 1954 with additions for low conductivity. It was not unexpected that greater accuracy could be obtained. A Canadian report [2] in 1963 called attention to discrepancies between theoretical computations and the FCC curves. The Canadian report found that actual measurements favored the theoretical computations rather than the FCC curves.

The values computed by this program are also more accurate than the curves adopted in 1985 [3] for the existing AM band. The present report provides a history of FCC ground-wave curves and mathematical background so that the situation can be critically evaluated if refinements in accuracy become desirable.

FCC curves have been to some extent arbitrary because they cannot be reproduced by calculation from explicit formulas. It is difficult to obtain consistent and repeatable results from the curves when interpolation is necessary. However, any sufficiently precise way of calculating the theoretical ground-wave field strength can be expected to agree with the FCC computer program reported here. In particular, it has been verified that the values computed by the FCC program agree with results available through time-sharing services provided by the Institute for Telecommunication Sciences of the U. S. Department of Commerce (ITS). The ITS program was independently developed by Leslie A. Berry approximately 10 years ago.

Propagation Model at Standard Broadcast Frequencies

The theoretical calculations that are the principal subject of this report assume a smooth, spherical earth with uniform dielectric constant and conductivity. The dielectric constant and ground conductivity are to be assigned values representing local conditions. The earth's radius is assumed to be greater than its actual value by a factor of 4/3 to account for atmospheric refraction. Both the transmitting antenna and the receiver are assumed to be on the ground.

This model was the basis for the original FCC curves [4]. Those curves were accompanied by text briefly describing the methods used to produce them. Plane earth formulas published by Norton [5] were used for sufficiently short distances such that the curvature of the earth does not introduce additional attenuation. For larger distances, the additional attenuation due to the curvature of the earth was introduced by the methods outlined in papers by van der Pol and Bremmer.

Succeeding editions of the FCC rules have continued to cite the same set of van der Pol and Bremmer papers [6,7,8,10] as the basis for the ground-wave curves. Reference [9] is another in the sequence of papers by these authors and is added to the list of references here for completeness. After World War II, Bremmer produced a book that includes all the earlier material [11].

The CCIR has developed a refined model with a more general characterization of the atmosphere [12]. The FCC model uses an effective earth's radius, justified by assuming that the refractive index of the atmosphere decreases with height in an approximately linear manner. The more general CCIR approach introduces methods for calculating the effects of an exponentially varying atmosphere. CCIR results are somewhat different from those of the FCC model at distances of about 100 miles and beyond for standard broadcast frequencies. At shorter distances where the curvature of the earth has a smaller effect, the apparent modification of this curvature by atmospheric refraction is a less critical part of the calculations. For such distances, results of both models tend to be the same and do not depend on the exact values specified in characterizing the atmosphere.

History of the Standard Broadcast Curves

Satisfactory field intensity curves for daytime standard broadcast were originally produced after an intensive effort from about 1930 to 1940 by radio engineers, mathematicians, physicists, broadcasters, the original Federal Radio Commission and later the FCC. Extensive measurement programs were conducted and a variety of empirical formulas were devised and tested while exact solutions were being sought to the fundamental mathematical equations.

Curves prepared by Rolf [13] in 1930 were based on promising theory but were actually misleading. Rolf relied on theoretical results published in 1909 by the physicist Arnold Sommerfeld [14], and these early results contained an error. Correct expressions for the field of an antenna over a flat earth had been worked out before 1930 by Sommerfeld himself as well as others, but disagreement with the 1909 result had not been noticed. Rolf used the earlier result thereby creating confusion.

For a time, measurements like those in the Kirby and Norton report [15] were presented in relation to the Rolf curves although agreement was poor. It was five years before the error in the theory of Rolf's graphs was identified [16]. It was still later that an experiment was conducted conclusively showing the theory to be wrong [17].

The Radio Commission compiled empirical formulas and curves which were published with its annual report [18] in 1931. The figure below is reproduced from that annual report. It gives a good impression of the prediction accuracies achievable in the early 1930s. It appears from subsequent annual reports of the Radio Commission that the curves in the figure were used in hearings and allocation matters at least until 1933.

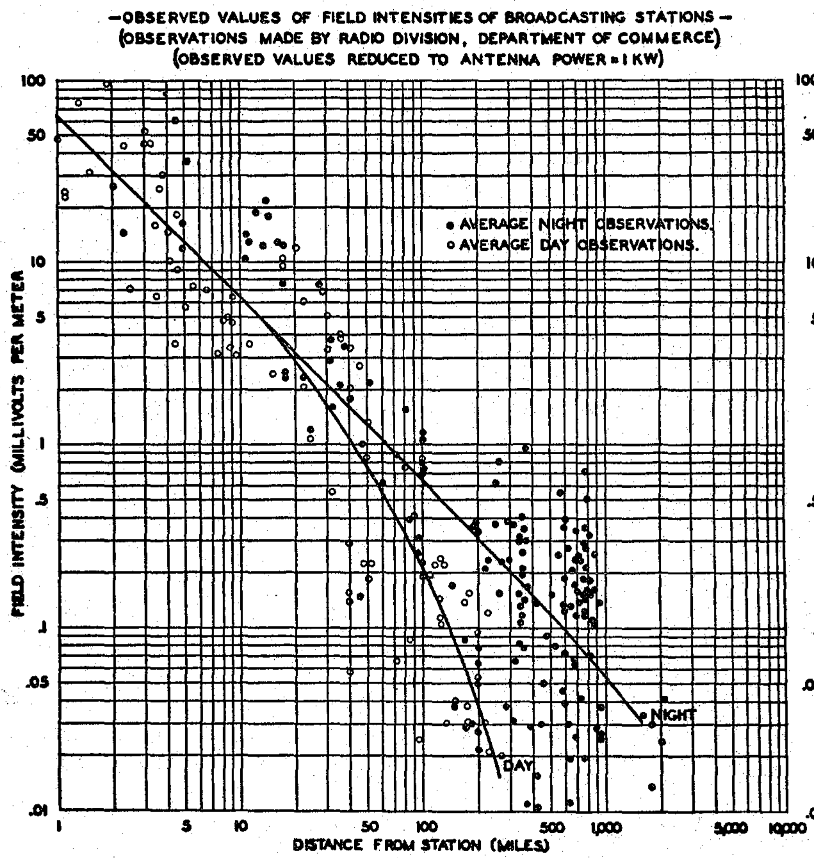


Figure reproduced from annual report of the Radio Commission in 1931. A single curve labeled "DAY" predicted ground-wave propagation at all broadcast frequencies and for all types of ground.

As soon as the correct solution was identified, Norton [5] in 1936 was able to construct a universal curve for prediction of field strength at relatively short distances. He was employed by the FCC at that time. This universal curve appears even today in Section 73.184 of the FCC rules as Graph 20, "Ground Wave Field Intensity versus Numerical Distance over a Plane Earth."

Norton's universal curve represented a very big step forward because it established a reliable theoretical basis for estimating ground conductivity in the vicinity of operating transmitters. Conductivity data, in turn, permitted better cataloguing of propagation measurements made at greater distances. The effects of the earth's curvature could thereafter be examined separately from those of the ground constants.

Also in 1936, Norton [5] provided curves for greater distances in what is called the diffraction region. These curves, however, were based on an incompletely developed theory. Mathematical solutions were being developed in Europe, and it took a few more years for that work to be consolidated into engineering graphs. Other attempts to provide curves prior to full development of the mathematics were those of Eckersley [19] in 1932, the van der Pol Committee [20] in 1932 (reported 1933), and the Dellinger Committee [21] in 1933.

The FCC announced [22] in its annual report for 1938 that it was beginning to prepare propagation prediction curves in conformity with a new theory of ground-wave propagation developed by several investigators in Europe. The necessary computations were made and drafting procedures were designed during the period 1938 through 1939. The curves, bearing the date January 1940, were published as an appendix to the Standards for Good Engineering Practice Concerning Standard Broadcast Stations [4]. They have been used with little change ever since.

The 1940 curves were drawn by methods developed by Norton. He described the calculation procedures at an FCC hearing in March 1940 [23] and a year later in February 1941 at the Fourth Annual Broadcast Engineering Conference [24]. The text and graphs originally presented at the March 1940 FCC hearing were eventually published in the Proceedings of the Institute of Radio Engineers [25].

At the time of the 1940 curves it was not possible to calculate field strengths over the entire range of distances of interest. Instead, calculations were made covering the range from the transmitter out to about 50 miles and resuming again at greater distances (typically about 200 miles) depending on the frequency and ground constants. The middle sections of the curves were estimates drawn to satisfy the overall requirement of smoothness and guided by those values near the transmitter and far away that could be calculated with greater accuracy.

It is interesting that the curves Norton presented to the 4th NAB convention at Columbus, Ohio a year later do not match those that were added to the FCC rules. Comparisons with exact calculations show that the curves put in the rules were slightly in error. The curve segments for great distances are systematically shifted upward. This may have been an expedient to overcome the difficulty of drafting smoothly fitting curve segments at intermediate distances where mathematical computations were not practical. The sea-water curves, for which the intermediate distance problem is minimal, are quite accurate. Inaccuracies in the 1940 curves are generally small in comparison to prediction errors expected due to many uncertain physical factors in real situations. However, the discrepancies were a concern of a Canadian report of measurements on the Great Lakes in 1962 [2].

New curves were added in 1954 for very low conductivity. These are quite accurate, which might be explained by the fact that the drafting job was not so huge as the earlier one and more care was exercised. When the low-conductivity curves were added, freehand drawing was still necessary to join the Sommerfeld curve segment to the curve segment calculated for the diffraction field at relatively great distances.

The curves in the rules were considered satisfactory for regulatory purposes until it became necessary to convert to metric units. In a 1979 FCC report, McMahon [26] described several methods for recalculating the curves in order to convert to metric units. He recommended that the method found in Bremmer's 1949 book [11] be used, and he provided a computer program. The McMahon program was subsequently used to produce new FCC curves in 1985 [3] which agree within about 1 or 2 decibels with the previous curves in the rules. However, the 1979 computer program is mathematically deficient in its ability to cover all the range of intermediate distances, and the great-distance values it computes are shifted upward to force a match in the middle.

FCC curves drawn for the band 1605-1705 kHz are the most recent. They are the result of precise calculations of field strength over the full range of distances of interest, including the previously troublesome intermediate distances. The theoretical model determining the values calculated is that of a smooth spherical earth with an effective radius 4/3 larger than actual. Transmitting and receiving antennas are assumed to be on the surface of the earth. The calculations are precise to at least 3 significant decimal digits and within at most a few hundredths of a decibel. This is substantially more precise than warranted by considerations of prediction accuracy alone, but it is useful for purposes of comparisons with calculation methods that may be developed independently.

History of Ground-Wave Prediction Mathematics

Some general background on the theory is provided here to contribute to an understanding of the computer program and make it easier to adapt the calculation procedure for special applications. A more complete review with derivations of the important equations can be found in the paper James R. Wait contributed to "Advances in Radio Research" [27].

The concept of numerical distance has been used in virtually every treatment of ground-wave propagation since it was first introduced by Prof. Arnold Sommerfeld in 1909 [14]. In his 1909 paper, Sommerfeld showed how to calculate the attenuation of radio waves spreading out from a vertical dipole mounted on a flat earth of given electric properties. The answer was conveniently expressed in terms of a dimensionless quantity, the numerical distance. This quantity is defined as the actual distance from the transmitting antenna measured in wavelengths and multiplied by a factor involving the ground conductivity and the dielectric constant of the ground relative to air.

Sommerfeld's results were for a flat earth, and they only apply relatively near the transmitter. The ground-wave propagates beyond such distances by diffraction, and there is a great amount of attenuation in addition to that predicted by flat earth theory alone. The Watson transformation, described in 1918 by G. N. Watson [28], was an essential step for solution of the problem of the diffraction of radio waves around the surface of the earth.

The Watson transformation made possible the solution of diffraction problems in the previously difficult case in which the radius of curvature of the obstacle is large in comparison to wavelength. This is the case for the earth, which is tens of thousands of wavelengths in diameter at standard broadcast frequencies. The scattering of radio waves by a raindrop is the opposite type of problem and is solved without the Watson transformation. In both cases the radio field is the sum of a series of functions tailored to fit the geometry of a sphere, but when that sphere is as large as the earth the number of terms that have to be included is very large. The Watson transformation converts the sum of this series to an integral for which there is a familiar evaluation technique.

The Watson integral is a kind that can be evaluated as a sum of <u>residues</u> of the integrand. The residues are associated with points in the complex plane at which the integrand becomes infinite. These are poles of the integrand, and the residues are easily calculated in the radio diffraction problem as soon as these critical points in the complex plane have been located. Watson found the poles and solved the radio diffraction problem for the special case in which the earth is a perfect conductor. This was useful in connection with maritime radio but gave only an upper bound on the fields that could be expected over land.

Solution of the radio diffraction problem still awaited discovery of practical procedures for finding the residue points that determine propagation over the real, imperfectly conducting earth.
T. L. Eckersley [29] applied an interesting method to evaluation of the Watson integral in about 1931. The Eckersley approach, however, did not directly evaluate the residues. It was an approximate method that gave the shape of curves representing field strength versus distance but still required shifting of those curves vertically by an undetermined amount. Eckersley suggested in 1934 [30] that his curves be shifted vertically until they are tangent or nearly tangent to Sommerfeld's curves. The results of making such a shift were examined in detail by Charles R. Burrows [31] and by K. A. Norton [5].

The necessary procedures for locating the residue points of the radio diffraction problem were developed between 1935 and 1938. B. Wwedensky [32] showed how to calculate the location of these points by what has been called the "tangent approximation." Balthazar van der Pol spoke before the IRE in New York in November 1936 describing how to find the residue points by the more accurate "Hankel approximation." Van der Pol was speaking of results obtained by himself and H. Bremmer. Part I of the van der Pol and Bremmer work was published in 1937 [6] and addressed the two extreme cases of perfect reflection and negligible conductivity at the surface of the earth. It appears from an FCC annual report [22] that the Bucharest meeting of the CCIR in 1937 was an important occasion for the exchange of information. The van der Pol and Bremmer paper published in 1938 [9] was a complete solution of the radio diffraction problem for propagation over electrically homogeneous spheres the size of the earth.

By 1940 the FCC, through the work of K. A. Norton [23], had developed a practical method for constructing curves approximately representing the theoretical predictions. The method used the flat-earth theory of Sommerfeld out to a distance of about 80 kilometers, and the diffraction-theory result was then used at those great distances where it was sufficient to include only a single term in computing the sum of the residues. Graphical techniques were used to complete the curves at intermediate distances.

George A. Hufford [33] in 1952 provided a basis for unifying the ground-wave prediction methods of Sommerfeld with those developed from the Watson transformation. The Hufford integral equation leads immediately to a representation of the ground-wave field as a Laplace transform, and the latter can be inverted by alternative methods appropriate respectively for distances near and far from the transmitter. Near the transmitter, the Laplace transform is inverted by examining its asymptotic form for large values of its argument. For distances relatively far from the transmitter, inversion of the Laplace transform is equivalent to the usual sum of residues. In fact, in the half-plane opposite the one in which the significant asymptotic behaviour occurs, the function to be inverted has poles corresponding to those that determine the residue series.

In deriving consequences of the unified representation arising from the Hufford integral equation, Bremmer [34] identified correction terms applicable to the Sommerfeld flat-earth formula to account for the effects of the earth′s curvature. Bremmer′s publication of these correction terms in 1958 completed the search for practical formulas engineers can use to calculate ground wave field strength. Previously there was no alternative to bold graphical interpolation in the intermediate range of distances which for standard broadcast frequencies may extend from about 50 to 300 kilometers (depending on the exact frequency and the ground constants).

The Bremmer correction terms are needed even though it is now possible with the help of a computer to include a large number of terms in summing the residue series. The number of terms included must still be limited to avoid an accumulation of errors. The formulas that use the Bremmer correction terms are complimentary to residue series computations. In the range where the same answer is obtained by both methods, the faster method can be used. This is the technique used by the FCC computer program described in this report.

The FCC Ground-wave Computer Program

The computer program improves on previous graphical methods by making exact computations in the intermediate range of distances. When the residue series is used, as many terms are included as necessary rather than just the one dominant at large distances. Closer to the transmitter when there are too many terms in the residue series, correction terms are added to the flat earth formula.

The program is mathematically justified by the details collected in Appendix A. Mathematical derivations are not repeated since they can be found in a number of places, particularly Wait [27]. Instead, Appendix A identifies sources of the formulas that have been translated into FORTRAN. The formulas are given in ordinary mathematical notation so that their FORTRAN implementation can be examined critically. See Gerks [35] for a detailed description of the development of a similar program.

Wherever possible the FORTRAN symbols chosen reflect the mathematical notation used by Norton [25]. These symbols will be familiar to radio engineers who prepare technical materials for filing with the FCC because formulas involving these symbols have been included in FCC rules since the Standards of Good Engineering Practice were first published in 1940.

Norton's symbols are supplemented by those used by Bremmer [11]. These supplementary symbols have the same meanings as were attached to them in the early van der Pol and Bremmer references [6,7,8,10] cited in FCC rules as the basis for the curves. Many of them are complex-valued and used in this form because the program can be written much more concisely this way.

The program architecture is described by flow diagrams in Appendix B, and the program itself is listed in Appendix C. The main program is elementary. It operates interactively and demonstrates use of the two major subroutines that actually determine ground-wave attenuation in the case of relatively short distances and long distances respectively.

The program has been used successfully on current-model microcomputers. At distances less than about 80 kilometers where it is not necessary to use the residue series, execution time is several seconds on the microcomputers that have been tried. On these computers it takes about 1 second to locate a residue point. Up to 30 residue points may be needed, but the computations necessary to locate these points are not repeated so long as there is no change in frequency or ground constants (that is, so long as the program is only being used to find the field strength for a range of distances).

Independently developed software should produce results that match those calculated by the FCC program to 3 significant decimal digits. This degree of precise match has been verified for the FCC program in comparison with ITSGW, a program independently developed by the Institute for Telecommunications Sciences of the Department of Commerce some years ago.

Applications

An important use of the FCC computer program is the prediction of field strength for ground constants other than those represented in the family of FCC curves. The ground dielectric constant, for example, may differ from the standard value of 15. Currently, FCC rules in Section 73.184 explicitly state that engineering showings may be based on computer program results in such circumstances.

The program or combinations of its subroutines can be used for analytical curve-fitting in place of older methods. When it is suspected that ground constants differ from the usual assumptions and thereby substantially affect radio service or interference, it is common practice to make field strength measurements. Subsequent estimates of the true ground constants have in the past been made by graphical techniques using, for example, the cumbersome method of matching semi-transparent tracings over a light table as described in the FCC rules. With a computer, analytical techniques using the FCC computer program can be substituted.

Acknowledgments

Confidence in the correctness of the FCC program has been reinforced by comparison with results of the ITS program and also with the results of a program representing the CCIR exponential atmosphere model. These comparisons were made with the help of the staff of ITS, and their help is acknowledged with thanks.

The author also thanks Mark Weissberger, of the Voice of America, and John Cavanagh, of the Dahlgren Naval Surface Weapons Center, for their help reviewing the work. One objective has been to make FCC propagation prediction methods fully understood in relation to methods that may be employed by other agencies, and their interest in these matters has been encouraging.

[24] K. A. Norton, "Ground Wave Propagation," presented at the Fourth Annual Broadcast Engineering Conference, February 10-21, 1941, sponsored by the Department of Electrical Engineering of the Ohio State University with the cooperation of the National Association of Broadcasters.

[25] K. A. Norton, "The Calculation of Ground-Wave Field Intensity over a Finitely Conducting Spherical Earth," Proceedings of the IRE, December 1941, pages 623-639.

[26] J. H. McMahon, "Investigation of Methods for Converting the FCC Ground Wave Field Intensity Curves to the Metric System," FCC/OCE Report RS79-01, January 1979

[27] J. R. Wait, "Electromagnetic Surface Waves," chapter contributed to "Advances in Radio Research," J. A. Saxton, Ed., Academic Press, N.Y., 1964, pages 157-217.

[28] G. N. Watson, "The Diffraction of Radio Waves by the Earth," Proceedings of the Royal Society, Series A, Vol.95, pages 83-99, 1918.

[29] T. L. Eckersley, "Radio Transmission Problems Treated by Phase Integral Methods," Proceedings of the Royal Society, Series A, Vol. 136, pages 499-526, June 1932.

[30] T. L. Eckersley, "Study of Curves of Propagation of Waves," Document A. G., No. 11, Communication II, International Scientific Radio Union, Fifth Assembly, London, September 1934. This information identifies the occasion on which the document was considered, although the document itself may be no longer be available.

[31] C. R. Burrows, "Radio Propagation over Spherical Earth," Proceedings of the IRE, Vol. 23, pages 470-480, May 1935. Also published in Bell System Technical Journal, Vol. XIV, pages 477-480, July 1935.

[32] B. Wwedensky, "The Diffractive Propagation of Radio Waves" (in English), Technical Physics of the U.S.S.R., Vol. II, No. 6, pages 624-639, 1935.

[33] Geo. A. Hufford, "An Integral Equation Approach to the Problem of Wave Propagation over an Irregular Surface," Quarterly Applied Mathematics, Vol. 9, No. 4, pages 391-404, 1952.

[34] H. Bremmer, "Applications of Operational Calculus to Ground-wave Propagation, Particularly for Long Waves," IRE Transactions on Antennas and Propagation, Vol. AP-6, pages 267-272, July 1958.

[35] I. H. Gerks, "Use of a High-speed Computer for Ground-wave Calculations," IRE Transactions on Antennas and Propagation, Vol. AP-10, pages 292-299, May 1962.

APPENDIX A

MATHEMATICAL BASIS

The major variables are identified in this appendix, and important formulas are reproduced so that their translation into FORTRAN can be examined critically.

This appendix also links the variables and formulas of the FCC ground-wave computer program to technical publications where definitions and mathematical derivations can be found. Four references are needed:

K. A. Norton, "The Calculation of Ground-Wave Field Intensity over a Finitely Conducting Spherical Earth", Proceedings of the IRE, December 1941, pages 623-639.

H. Bremmer, Terrestrial Radio Waves, Elsevier Publishing Co., 1949.

H. Bremmer, "Applications of Operational Calculus to Ground-wave Propagation, Particularly for Long Waves", IRE Transactions on Antennas and Propagation, Vol. AP-6, pages 267-272, July 1958.

M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions", National Bureau of Standards Mathematics Series 55, U.S. Gov. Printing Office, Orig. printing 1964, ninth printing 1970 (also Dover Publications, N.Y., 1968).

VARIABLE	USUAL MATH SYMBOL	NAME USED IN FORTRAN PROGRAM	SIGNIFICANCE
Ground Conductivity	σ	SIGMA	Input variable. Units of millisiemens/meter.
Relative Dielectric Constant	ϵ	EPSILON	Input variable. Measured relative to air and therefore dimensionless.
Frequency	f	FREQ	Input variable. Megahertz.
Distance	d	DIST	Input variable. Kilometers.
Magnitude of Numerical Distance	p	P	Numerical distance completely determines the field attenuation for a flat earth.
Phase Angle of Numerical Distance	b	B	Magnitude and angle of the numerical distance given by Norton 1941 equations 4 to 9.
Norton's "K"	K	K	Dependent on ratio of wavelength to earth radius. Used in both short and long distance calculations. Given by Norton 1941 equation 13.
Bremmer's "χ"	χ	CHI	Used in long distance calculations. Proportional to distance. Defined by Bremmer 1949 equation III-31.
Ground-wave Attenuation	f(p,b) etc.	A	Magnitude of the ratio of the ground-wave field to the field produced by the same antenna over a perfectly conducting flat earth.

Used by the main program in calls to first-level subroutines.

PRINCIPAL VARIABLES USED IN FCC GROUND-WAVE PROGRAM

Table A-1

VARIABLE	MATH SYMBOL	NAME USED IN FORTRAN PROGRAM	DEFINITION / SIGNIFICANCE
Complex Numerical Distance	ρ	RHO	Magnitude p; phase angle b. $\rho = p \exp(jb)$
Square Root of Numerical Distance	$\sqrt{\rho}$	RHOROOT	Magnitude $\sqrt{p}$; phase b/2. $\sqrt{\rho} = \sqrt{p} \exp(jb/2)$
Bremmer´s "δ"	δ	DEL	Magnitude K; phase angle 135 degrees minus b/2. $\delta = K \exp[j(3\pi/4 - b/2)]$ Defined by Bremmer 1949, equation III-22.
Residue Points (numbered by s = 1, 2, ..)	τ_s	TAU(S)	These are the points in the complex plane at which residues of the diffraction field integrand are to be evaluated. Determined from and either $\tau_{s,0}$ or $\tau_{s,\infty}$ (below).
Residue points for strongly absorbing earth	$\tau_{s,0}$	TAU0	Locations in the complex plane at which to evaluate residues in special cases of very low conductivity or very high permittivity ($\delta \longrightarrow 0$).
Residue points for perfectly conducting earth	$\tau_{s,\infty}$	TAU1	Locations in the complex plane at which to evaluate residues in the special case of very high conductivity ($\delta \longrightarrow \infty$).

Modern versions of FORTRAN will evaluate expressions involving complex variables, and this capability allows the ground-wave calculation procedure to be written much more concisely.

PRINCIPAL COMPLEX VARIABLES

Table A-2

QUANTITY	FORMULA AND SOURCE
Numerical Distance (RHO)	When the conductivity, σ, is in millisiemens/meter, f is in Mhz, and the distance, d, is in the same units as the wavelength, λ, define $x = 17.97\ \sigma/f$, $b'' = \arctan(\epsilon/x)$, $b' = \arctan[(\epsilon - 1)/x]$. Then the complex numerical distance, ρ, is given by $\rho = p\exp(jb)$ where $b = 2b'' - b'$ and $p = \pi d\cos^2 b''/(\lambda x\cos b')$. SOURCE: Norton 1941, vertical polarization.
δ (DEL)	When wavelength, λ, and the effective earth radius, a, are in the same units, define b, b′ and b′′ as above and let $K = [\lambda/(2\pi a)]^{1/3}(x\cos b')^{1/2}/\cos b''$ $\psi = b/2$. Then $\delta = K\exp[j(3\pi/4 - \psi)]$ SOURCE: Norton 1941 equation 13a, Bremmer 1949 equation III-29.
Flat-earth Attenuation (ZA)	Let ρ represent numerical distance as above and denote the complex attenuation by Z_a. Then $Z_a = 1 + j\sqrt{\pi\rho}\,e^{-\rho}\operatorname{erfc}(-j\sqrt{\rho})$ $= 1 + j\sqrt{\pi\rho}\,e^{-\rho} - 2\rho + (2\rho)^2/(1\cdot 3) - (2\rho)^3/(1\cdot 3\cdot 5) + \cdots$ $= 1 + j\sqrt{\pi\rho}\,w(\sqrt{\rho})$. SOURCE: Norton 1941 equations 44 and 47, Abramowitz equation 7.1.3
Adjusted Flat-earth Attenuation (ZADJ)	Denote the adjusted value by Z_{adj}, and let Z_a and ρ respectively be flat-earth attenuation and numerical distance as above. Then $Z_{adj} =$ $Z_a + [(1 + 2\rho)Z_a - 1 - j\sqrt{\pi\rho}]\delta^3/2$ $+ [(\rho^2/2 - 1)Z_a + j\sqrt{\pi\rho}(1 - \rho) + 1 - 2\rho + 5\rho^2/6]\delta^6$. SOURCE: Bremmer 1958 equation 24.

The effects of the curvature of the earth enter through the quantity δ.

FORMULAS FOR GROUND-WAVE ATTENUATION AT RELATIVELY SHORT DISTANCES

Table A-3

QUANTITY	FORMULA AND SOURCE
Residue points (TAU0) for strongly absorbing earth	The residue points $\tau_{s,0}$ are equal to a constant factor times the zeroes of the Airy function. Let a_s denote these zeroes as found in Table 10.13 and equation 10.4.94 of Abramowitz. Then $\tau_{s,0} = -a_s \exp(j\pi/3)/(2)^{1/3}$ SOURCE: Bremmer 1949, equations III-25a and 25b.
Residue points (TAU1) for perfectly conducting earth	The residue points $\tau_{s,\infty}$ are found in terms of the zeroes of the derivative of the Airy function. Let a'_s denote these zeroes as found in Table 10.13 and equation 10.4.95 of Abramowitz. Then $\tau_{s,\infty} = -a'_s \exp(j\pi/3)/(2)^{1/3}$ SOURCE: Bremmer 1949, equations III-24a and 24b.
Residue Points (TAU)	When δ is small, the points τ_s are found by power series in δ with coefficients that are polynomials in TAU0. Similar power series in $q = 1/\delta$, with coefficients determined by TAU1, are used when δ is large. In between, the computer program finds τ_s by numerical integration. See Table A-5. SOURCE: Bremmer 1949, equations III-27 and 28.
χ (CHI)	When wavelength, λ, and the effective earth radius, a, are in the same units as the distance, d, from the transmitter $\chi = (2\pi a/\lambda)^{1/3} d/a$ SOURCE: Bremmer 1949, equation III-31.
Attenuation in Diffraction Region	Denote the complex attenuation by Z_a, and let τ_s for s = 1, 2, ... denote the residue points as above. Let δ be as in table A-3. Then $Z_a = \sqrt{2\pi j\chi}\sum_{s=1}^{\infty}\exp(j\tau_s\chi) / [2\tau_s - (1/\delta)^2]$ SOURCE: Bremmer 1941 equation III-33.

Bremmer's formulation numbers the residue points starting with s = 0, but this convention is altered in the computer program in order to associate TAU0 and TAU1 with the accurate tabulations of Abramowitz which start with s = 1.

FORMULAS FOR GROUND-WAVE ATTENUATION AT RELATIVELY LONG DISTANCES

Table A-4

QUANTITY	FORMULA
Coefficients of Power Series in δ, for determination of r_s when δ is small	Let $c_0 = r_{s,0}$. Then $r_s = c_0 + c_1\delta + c_2\delta^2 + c_3\delta^3 + \cdots$, where the next 10 coefficients following c_0 are $c_1 = -1$ $\quad$ $c_6 = 14c_0/9$ $c_2 = 0$ $\quad$ $c_7 = -(5 + 8c_0^3)/7$ $c_3 = -2c_0/3$ $\quad$ $c_8 = 58c_0^2/15$ $c_4 = 1/2$ $\quad$ $c_9 = -8[(287/63) + 2c_0^3]c_0/9$ $c_5 = -4c_0^2/5$ $\quad$ $c_{10} = 8[(47/8) + (582/15)c_0^3]/35$.
Coefficients of Power Series in $q = 1/\delta$, determining r_s when δ is large	Let $d = r_{s,\infty}$. Then $r_s = d_0 + d_1 q + d_2 q^2 + d_3 q^3 + \cdots$, where $q = 1/\delta$ and coefficients after d_0 are $d_1 = -1/(2d_0)$ $d_2 = -1/(8d_0^3)$ $d_3 = -[1 + 3/(4d_0^3)]/(12d_0^2)$ $d_4 = -[(7/3) + (5/4d_0^3)]/(32d_0^4)$ $d_5 = -[(1/5) + 21/(40d_0^3) + 7/(32d_0^6)]/(8d_0^3)$ $d_6 = -[(29/45) + 77/(80d_0^3) + 21/(64d_0^6)]/(16d_0^5)$ $d_7 = -[(2/7) + 76/(45d_0^3) + 143/(80d_0^6) + 33/(64d_0^9)]/(32d_0^4)$ $d_8 = -[(97/70) + 163/(40d_0^3) + 429/(128d_0^6) + 429/(512d_0^9)]/(64d_0^6)$.

Bremmer's results (Bremmer 1949, equations III-27 and 28) must be extended in order to locate the residue points with a precision of 3 decimal digits. When δ is either very small or very large in magnitude, Bremmer's power series technique may be used except that additional terms should be included in the respective power series. In the intermediate region it is necessary to use numerical integration.

The differential equation relating r_s to $r_{s,0}$ is: $dr_s/d\delta = 1/(2\delta^2 r_s - 1)$. When δ is such that neither power series is strongly convergent, $r_s(\delta)$ may be found by numerical integration of this equation starting with $r_s(0) = r_{s,0}$.

The coefficients c_1, c_2, etc. are determined in terms of $c_0 = r_{s,0}$ by applying this differential equation to the power series $r_s = c_0 + c_1\delta + c_2\delta^2 + \cdots$. The coefficients d_1, d_2, etc. are similarly determined in terms of $d_0 = r_{s,\infty}$ by this equation. Bremmer 1949 gives only c_1 through c_5 and d_1 through d_4, and the additional coefficients tabulated above give improved accuracy and reduce the number of situations in which numerical integration will be required.

EXTENSION OF RESULTS RELATING TO DETERMINATION OF r_s BY POWER SERIES

Table A-5

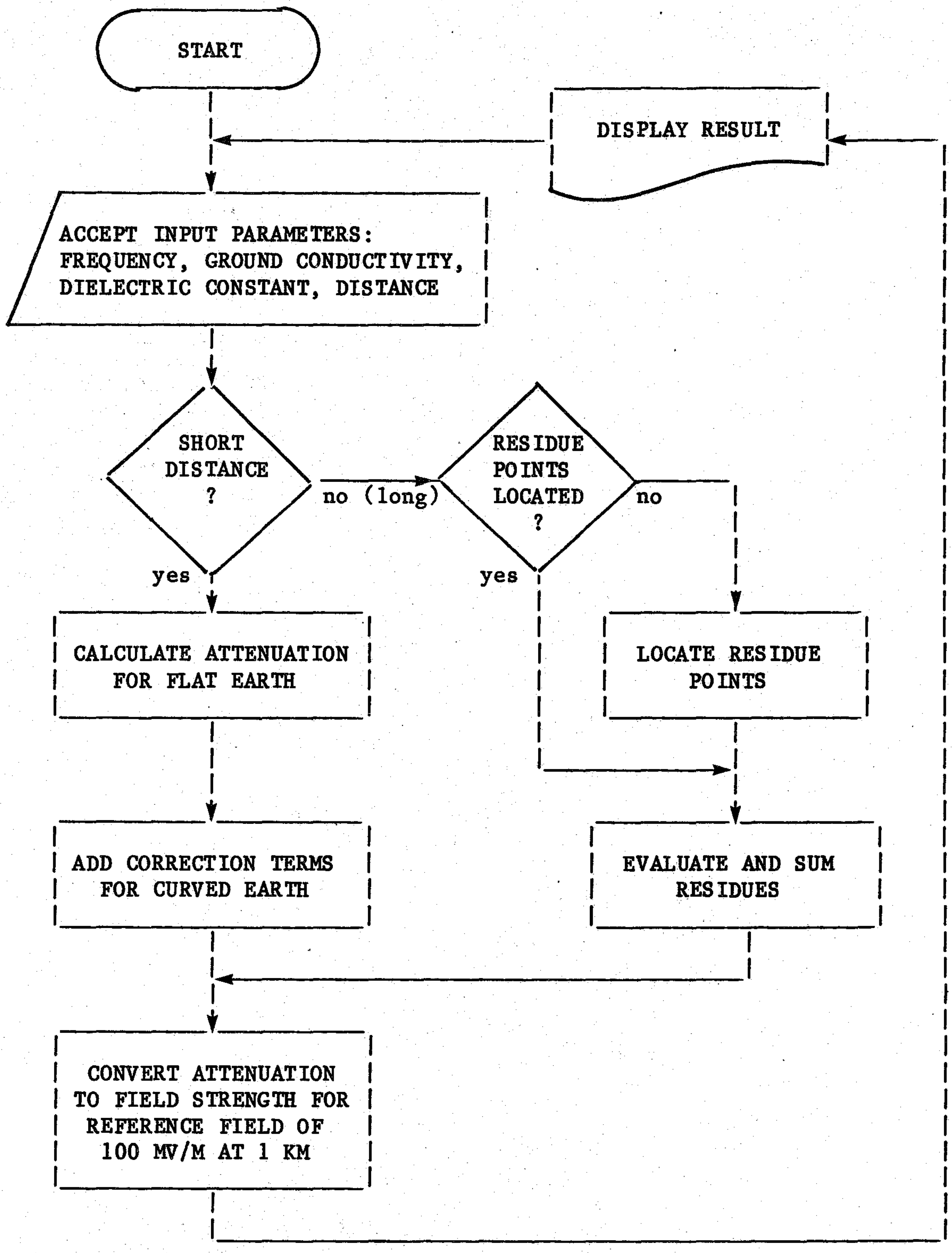
APPENDIX B

COMPUTER PROGRAM FLOW DIAGRAMS

Important features of the overall procedure are (1) the choice between short- and long-distance subroutines and (2) the conversion of attenuation to field strength.

The main program must choose whether to use the short-distance subroutine, which computes the Sommerfeld flat-earth attenuation and then corrects for earth curvature, or the subroutine for long distances which involves an evaluation of the residue series. At very short and very long distances there is a relatively little calculation to be done and the subroutines work very fast. Either routine will give an accurate answer in the intermediate range of distances, and the best choice is the one that is fastest.

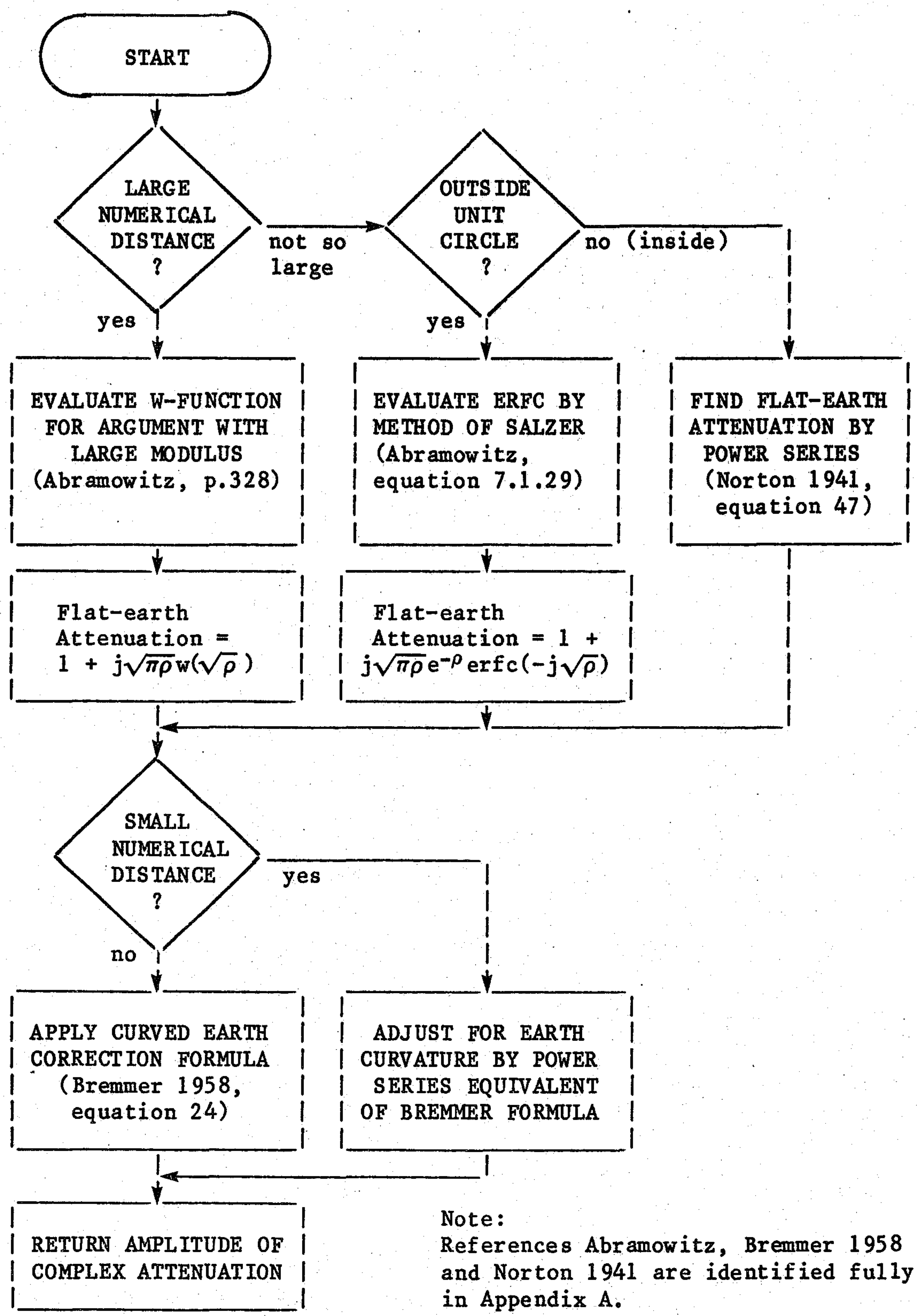
The output of both routines is the attenuation relative to an inverse-distance field. This is converted to field strength by multiplying by the reference field (100 mv/m) and dividing by the given distance.



Output of both the long and short distance calculations is the attenuation, A, the magnitude of the ratio of the ground-wave field to the field of the same antenna over a perfectly conducting earth.

FLOW DIAGRAM FOR CALCULATION OF GROUND-WAVE FIELD STRENGTH

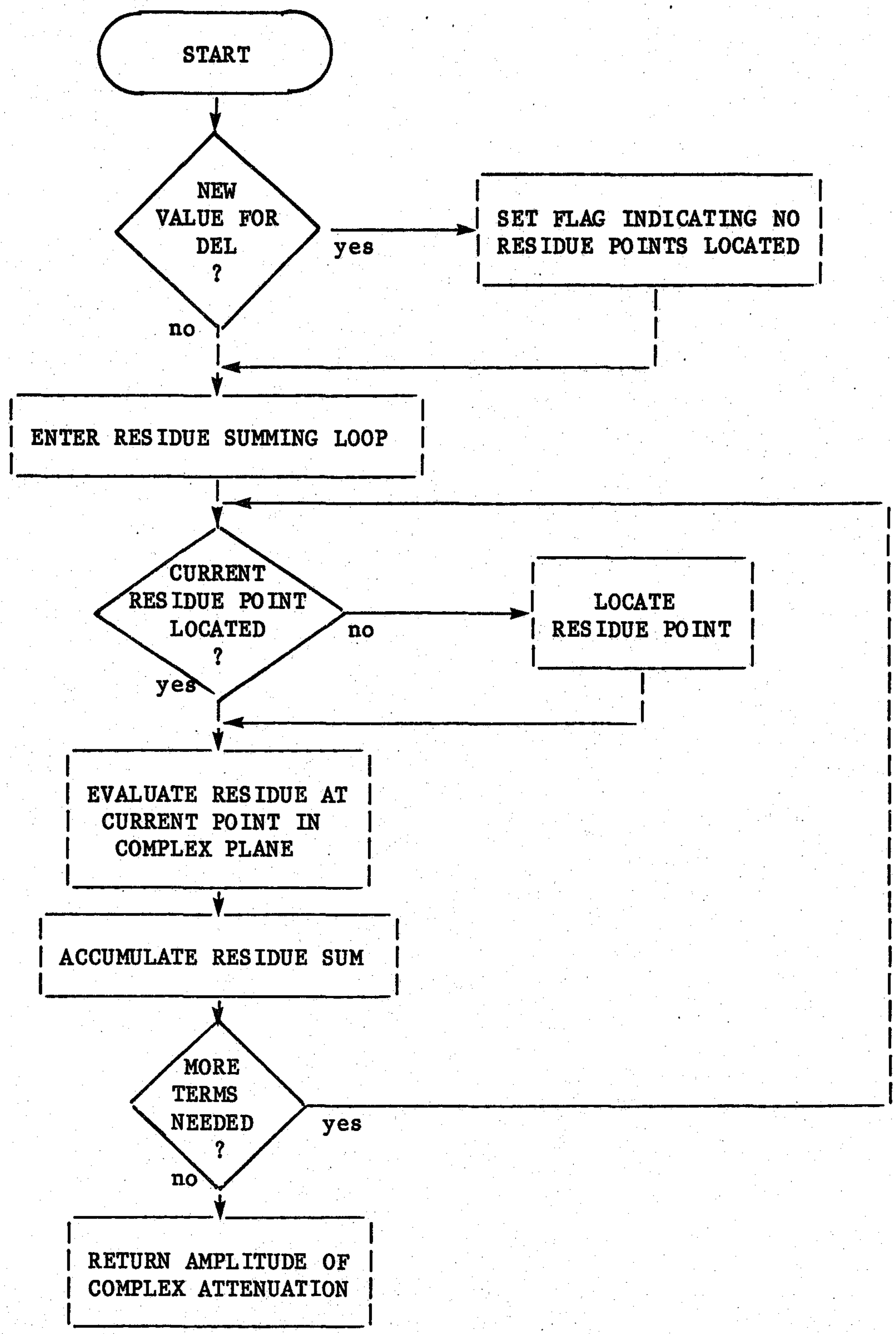
Figure B-1



Find flat-earth attenuation and then correct for earth curvature.

CALCULATION OF FIELD STRENGTH FOR RELATIVELY SHORT DISTANCES

Figure B-2



Residues of the diffraction field integrand are evaluated starting with the pole nearest the origin and working outward until the residue series converges.

FIELD STRENGTH CALCULATION FOR RELATIVELY LONG DISTANCES

Figure B-3

APPENDIX C

FORTRAN COMPUTER PROGRAM

The program and subprograms are identified below in groups. In the first group are the main program and a module named QUERY. These are elementary programs for entering input and controlling the major subroutines to obtain field-strength answers interactively.

The subroutine GWCONST converts input values of frequency, distance, and ground constants to dimensionless quantities, e.g. the numerical distance. All remaining calculations will be carried out in relation to these dimensionless quantities.

SURFACE and the other subprograms of its group are used for calculating ground-wave attenuation for relatively short distances. These calculations begin by constructing complex-valued quantities from the real values supplied.

RESIDUES is the lead subprogram of the group that makes the appropriate calculations for greater distances. The task of locating the residue points requires a relatively large number of FORTRAN statements. Complex quantities are used, and many more statements would be needed without the efficiency of complex notation.

Program/Subprogram	Description
FCCGW.FOR	Main program
QUERY.FOR	Prompt for and accept inputs
GWCONST.FOR	Establish constants
SURFACE.FOR	Calculations by extended flat-earth theory
SOMMERFLD.FOR	Find basic flat-earth field
SALZER.FOR	Evaluate complex error function
SRS1.FOR	First power series adjusting for curved earth
SRS2.FOR	Power series to make second adjustment
RESIDUES.FOR	Sum residues to evaluate diffraction field
AIRY0.FOR	Locate residue points of type 0
AIRY1.FOR	Locate residue points of type 1

```
      PROGRAM FCCGW
C
C  Calculate ground-wave field strength for vertically
C  polarized MF waves.
C
      REAL K
      CHARACTER*12 NAME( 2 )
      DATA NAME / 'ADJ. SOMRFLD',
     &            'RESIDUE SUM ' /
C
      DATA SIGMA, EPSILON, FREQ, DIST / 5., 15., .55, 80. /
      DATA PI / 3.1415927 /
C
C  Define input and output units.
C
      COMMON / INOUT / IN, IO
      DATA IN, IO / 5, 6 /
C
C  Get input parameters.
C
 10   CONTINUE
      IQUIT = 0
      WRITE ( IO, * )
      CALL QUERY( 'Ground conductivity in mS/m', 27,
     &  SIGMA, IQUIT )
      IF ( IQUIT .GT. 0 ) STOP 'Stopped on CNTRL-Z'
      CALL QUERY( 'Dielectric constant relative to air ', 35,
     &  EPSILON, IQUIT )
      IF ( IQUIT .GT. 0 ) GOTO 10
      CALL QUERY( 'Frequency in MHz', 16, FREQ, IQUIT )
      IF ( IQUIT .GT. 0 ) GOTO 10
      CALL QUERY( 'Distance in km', 14, DIST, IQUIT )
      IF ( IQUIT .GT. 0 ) GOTO 10
C
C  Show parameters accepted.
C
      WRITE ( IO, 1000 )
     &  'At ', DIST, ' km',
     &  'for ground conductivity = ', SIGMA, ' mS/m,',
     &  'dielectric constant = ', EPSILON,
     &  'and frequency = ', FREQ, ' MHz:'
C
C  Evaluate constants required by major subroutines.
C
      CALL GWCONST(
     &  SIGMA, EPSILON, FREQ, DIST,                      !Given
     &  P, B, K, CHI )                                   !Find
C
C  Set distance beyond which to use residue series.
C
      FAR = 80 / FREQ ** .3333
```

```
C
C  Calculate ground-wave attenuation by approriate method.
C
       IF ( DIST .LE. FAR ) THEN
           METHOD = 1
           CALL SURFACE( P, B, K, A )
       ELSE
           METHOD = 2
           PSI = 0.5 * B
           CALL RESIDUES( CHI, K, PSI, A )
       END IF
C
C  Multiply inverse-distance field (100 mv/m at 1 km) by
C  the attenuation.
C
       FIELD = A * 100 / DIST
C
C  Show answer and loop for another set of inputs.
C
       WRITE ( IO, 1001 )
       WRITE ( IO, 1002 ) NAME( METHOD ), FIELD
       GOTO 10
C
 1000 FORMAT( /1X, A, F6.1, A /1X, A, F7.2, A /1X, A, F5.2,
     &  /1X, A, F8.4, A )
 1001 FORMAT( /
     &  4X, 'METHOD', 9X, 'FIELD (mV/m)'/
     &  4X, '------', 9X, '------------' )
 1002 FORMAT( 1X, A, 5X, 1PG9.3 )
C
       END
```

```
C
C**********************************************************
C
C  QUERY - Prompt for input and accept real number.  Up to
C  50 characters are allowed for description of the parameter
C  to be entered.  The description is provided in the string
C  PROMPT in which characters 1 through LENGTH are significant.
C
C**********************************************************
C
      SUBROUTINE QUERY( PROMPT, LENGTH, VALUE, IQUIT )
C
      CHARACTER PROMPT*50, ANSWER*8
      CHARACTER STRING*80
C
C  Provide character-by-character access to STRING and ANSWER by
C  making them equivalent to arrays LINE and WORD respectively.
C
      CHARACTER*1 LINE(80), WORD(8)
      EQUIVALENCE ( LINE, STRING )
      EQUIVALENCE ( WORD, ANSWER )
C
C  Calling program must identify input and output devices by
C  filling in common block INOUT.
C
      COMMON / INOUT / IN, IO
C
C  Compose the prompting message.  Erase characters that
C  came from the string PROMPT but are beyond its actual
C  length.
C
      WRITE ( STRING, '(A50, A, 1PG12.6, A)' ) PROMPT,
     &     ' = [', VALUE, ']? '
      DO 10 I = LENGTH + 1, 50
         LINE( I ) = ' '
   10 CONTINUE
C
C  Remove extra spaces and find last non-blank character.
C
      L = 80
      DO 100 I = LENGTH + 1, 79
         IF ( I .GT. L - 1 ) GOTO 200
   20    IF ( LINE( I ) .EQ. ' ' .AND. LINE( I+1 ) .EQ. ' ' ) THEN
            L = L - 1
            IF ( I .GE. L ) GOTO 200
            DO 30 J = I, L
               LINE( J ) = LINE( J+1 )
   30       CONTINUE
            GOTO 20
         END IF
  100 CONTINUE
```

```
C
C  Issue prompt.
C
  200   WRITE ( IO, * ) ( LINE( I ), I = 1, L )
C
C  Accept answer in character form.
C
        READ ( IN, '( A )', ERR = 410, END = 400 ) ANSWER
        IF ( ANSWER .NE. ' ' ) GOTO 300
C
C  Nothing typed except ENTER key.  Return without changing the
C  argument VALUE so that calling routine may use default.
C
        RETURN
C
C  Something typed in.  Right justify it.
C
  300   IF ( WORD( 8 ) .NE. ' ' ) GO TO 320
        DO 310 I = 7, 1, -1
           WORD( I+1 ) = WORD( I )
  310   CONTINUE
        WORD( 1 ) = ' '
        GO TO 300
C
C  Decode and exit.
C
  320   READ ( ANSWER, '(F8.0)', ERR = 410 ) VALUE
C
        RETURN
C
C  Exit on Control-Z or read error.
C
  400   IQUIT = 1
        RETURN
C
  410   STOP 'Error in subroutine QUERY'
C
        END
```

```
C
C*************************************************************
C
C  GWCONST - Set groundwave constants.  The independent variables
C  passed as arguments of this subroutine are:
C
C      SIGMA, the ground conductivity in millisiemens/meter
C      EPSILON, the relative dielectric constant (1.0 for air)
C      FREQ, the frequency in MHz
C      DIST, radio path length in kilometers
C
C  Quantities calculated in this subroutine and returned are:
C
C      P, amplitude of the complex numerical distance introduced
C      for the solution of radio propagation problems by Arnold
C      Sommerfeld in 1909
C
C      B, phase angle of the numerical distance
C
C      K, a dimensionless parameter proportional to the cube-
C      root of the ratio of wavelength to the effective earth
C      radius, and dependent also upon the ground constants
C
C      CHI, a dimensionless parameter proportional to the cube-
C      root of the effective earth radius measured in wavelengths,
C      and proportional also to the radio path distance measured
C      as the angle subtended from the center of the earth.
C
C  Use of the symbols P and B for the amplitude and phase of
C  the numerical distance follows Norton, Proc. IRE 1941.
C
C  The symbol K is used to denote exactly the same quantity in
C  NBS Tech Note 101, in Norton's 1941 IRE paper and in the
C  1949 book by Bremmer.  See references below.
C
C  CHI is used in evaluating the residue series.  The Greek letter
C  of that name was used for this quantity by Bremmer in his 1949
C  book.
C
C  References:
C
C      K. A. Norton, "The Calculation of Ground-Wave Field
C      Intensity over a Finitely Conducting Spherical Earth",
C      Proc. IRE, Dec 1941, pages 623-639.
C
C      H. Bremmer, Terrestrial Radio Waves, Elsevier
C      Publishing Co., 1949.
C
C*************************************************************
C
        SUBROUTINE GWCONST(
     &     SIGMA, EPSILON, FREQ, DIST,                   !Given
     &     P, B, K, CHI )                                !Find
```

```
C
      REAL K
C
      DATA PI / 3.1415927 /
      DATA EARTHRAD / 6370. /          !Radius of the earth, km
      DATA FACTOR / 1.333333 /         !Assumed effective radius factor
C
C  Speed of light in air (refractive index 1.00031)
C
      DATA SPEED / 299700. /           !km/second
C
C  Begin execution.  Determine effective earth radius from
C  given earth radius factor.
C
      EFFRAD = FACTOR * EARTHRAD
C
C  Find wavelength.  Distance and the effective earth radius will
C  be divided by wavelength to produce dimensionless quantities.
C
      WAVELENGTH = SPEED / ( 1E6 * FREQ )     !km
C
C  Intermediate variables X, B1 and B2 are derived from the
C  ground constants by formulas that appear in Norton, Proc.
C  IRE, 1941.
C
      X  = 17.97 * SIGMA / FREQ
      B1 = ATAN2( EPSILON - 1, X )
      B2 = ATAN2( EPSILON, X )
C
C  Calculation of numerical distance, P, and its phase angle, B
C
      P = PI * ( DIST / WAVELENGTH ) * COS( B2 ) ** 2
    &     / ( X * COS( B1 ) )
      B = 2 * B2 - B1
C
C  Calculation of K.  See Norton, Proc. IRE, Dec 1941, page 628.
C
      K = ( WAVELENGTH /
    &     ( 2 * PI * EFFRAD ) ) ** ( 1./3 )
    &     * SQRT( X * COS( B1 ) ) / COS( B2 )
C
C  Calculation of CHI.  Confer Bremmer, Terrestrial Radio Waves,
C  page 49, equation (III, 31).
C
      CHI = DIST / EFFRAD *
    &     ( 2 * PI * EFFRAD / WAVELENGTH ) ** (1./3)
C
      RETURN
      END
```

```
C
C*************************************************************
C
C  SURFACE - Calculation of surface wave attenuation.  The
C  flat earth value is found by the usual formula due to A.
C  Sommerfeld.  Corrections for curved earth are then applied.
C  The curved earth corrections are from H. Bremmer, "Applications
C  of Operational Calculus to Ground-Wave Propagation,
C  Particularly for Long Waves", IRE Transactions on Antennas
C  and Propagation, July 1958.
C
C  Inputs are the numerical distance, P, its phase, B, and
C  the parameter K.  The parameter K (so denoted by Norton
C  and in NBS Tech Note 101) carries information concerning
C  the effective earth radius so that spherical earth
C  corrections can be applied.
C
C  Output is the attenuation factor, A, the magnitude of the
C  ratio of the ground-wave field to the field produced by
C  the same antenna over a perfectly conducting flat earth.
C
C*************************************************************
C
      SUBROUTINE SURFACE( P, B, K, A )
C
      IMPLICIT COMPLEX ( Z )
      COMPLEX DEL, RHO, ERFC, SRS1, SRS2
      REAL K
C
      DATA PI / 3.1415927 /
C
C  Spherical earth correction formulas.  Use when numerical
C  distance is not close to 0.  Notice formulas are in cascade
C  so that the last automatically uses those previous.  They
C  are written this way so that they can be examined separately,
C  but direct reference in the program is made only to
C  the last formula.
C
C  ZA denotes the Sommerfeld flat-earth attenuation which must
C  be calculated separately.
C
      ZADJ1( DEL, RHO, ZA ) = ZA
     &   + DEL ** 3 *
     &   1./2 * ( ( 1 + 2 * RHO ) * ZA
     &   - 1 - (0,1) * SQRT( PI * RHO ) )
C
      ZADJ2( DEL, RHO, ZA ) = ZADJ1( DEL, RHO, ZA )
     &   + DEL ** 6 *
     &   ( ( 1./2 * RHO ** 2 - 1 ) * ZA
     &   + (0,1) * SQRT( PI * RHO ) * ( 1 - RHO )
     &   + 1 - 2 * RHO + 5./6 * RHO ** 2 )
```

```
C
C  Begin execution.  Convert input variables P, B to complex form.
C  The resulting complex variable, RHO,  will always be in the
C  upper half-plane.
C
      RHO = P * ( COS( B ) + (0,1) * SIN( B ) )
C
C  The complex parameter DEL is determined by K and the angle B.
C
      DEL = K *
     &    ( COS( 3*PI/4 - B/2 ) + (0,1) * SIN( 3*PI/4 - B/2 ) )
C
C  Find complex attenuation for flat earth
C
      CALL SOMMERFLD( RHO, ZA )
C
C  Adjust for spherical earth. The functions SRS1 and SR2 are
C  power series corresponding to ZADJ1 and ZADJ2 respectively.
C  When RHO is small and consequently ZA is near unity, the power
C  series must be used.  ZADJ1 and ZADJ2 are not accurate under
C  these conditions because the formulas involve the difference
C  between ZA and a number near unity.
C
      IF ( ABS( RHO ) .GT. 0.5 ) THEN
         ZADJ = ZADJ2( DEL, RHO, ZA )
      ELSE
         Z1 = (0,1) * SQRT( RHO )
         Z3 = ( DEL * Z1 ) ** 3
         ZADJ = ZA - Z3 * ( SRS1( Z1 )
     &                 - Z3 * ( SRS2( Z1 ) ))
      END IF
C
C  Return the amplitude
C
      A = ABS( ZADJ )
      RETURN
C
      END
```

```
C
C*************************************************************
C
C  SOMMERFLD - Calculation of the surface wave attenuation
C  factor for given "numerical distance".  Numerical
C  distance is the parameter introduced by A. Sommerfeld
C  in 1909 when he showed how to find the field of a short
C  dipole radiating over a finitely conducting plane earth.
C
C  Input is the numerical distance in complex form, RHO.
C
C  Output ZA is the complex surface wave attenuation factor
C  for a flat earth.
C
C*************************************************************
C
      SUBROUTINE SOMMERFLD( RHO, ZA )
C
      IMPLICIT COMPLEX ( S, T, W, Z )
      COMPLEX ERFC, RHO, RHOROOT
C
      DATA PI / 3.1415927 /
C
C  Coefficients C1, D1 etc. to approximate w-function of
C  large modulus (M. Abramowitz and I. Stegun, Handbook of
C  Mathematical Functions, National Bureau of Standards,
C  1964, page 328).
C
      DATA C1, C2, C3 / 0.4613135, 0.09999216, 0.002883894 /
      DATA D1, D2, D3 / 0.1901635, 1.7844927,  5.5253437 /
C
C  Approximation formula for w-function of large modulus.
C  Error less than 2 parts in 1E6 provided the absolute
C  value of X exceeds 3.9, or Y > 3.
C
      W( Z ) = (0,1) * Z *
     &    ( C1 / ( Z ** 2 - D1 )
     &    + C2 / ( Z ** 2 - D2 )
     &    + C3 / ( Z ** 2 - D3 ) )
C
C  Begin execution.  The numerical distance, as a complex
C  variable, should always be found in the upper half-plane.
C  Calculate its square root, RHOROOT, which will be located in the
C  first quadrant.
C
      IF ( AIMAG( RHO ) .LT. 0 ) STOP
     &    'ERROR: Complex numerical distance in lower half-plane'
      RHOROOT = SQRT( RHO )
```

```
C
C  Determine most appropriate method.
C
         IF ( REAL ( RHOROOT ) .GT. 3.9
     &      .OR. AIMAG( RHOROOT ) .GT. 3.0 ) GO TO 300
         IF ( ABS( RHOROOT ) .GT. 1 ) GO TO 200
C
C  Power series for small RHO.  For ABS( RHO ) = 1 or less,
C  the I-th term will be less than 1E-35 in magnitude after
C  33 terms.
C
  100    CONTINUE
         TERM = 1
         SUM = 1
         DO 110 I = 1, 33
            TERM = - 2 * RHO * TERM / ( 2 * I - 1 )
            SUM = SUM + TERM
            IF ( ABS( TERM ) .LT. ABS( SUM ) / 1E5 )
     &         GO TO 120
  110    CONTINUE
  120    ZA = SUM + (0,1) * SQRT( PI * RHO ) * EXP( -RHO )
         RETURN
C
C  When RHOROOT is outside the unit circle but below the horizontal
C  line Y = 3.0 and left of X = 3.9, calculate the complex surface
C  attenuation in terms of the complementary error function
C  The latter is to be evaluated at -(0,1) * RHOROOT, and the series
C  of Salzer provides a close approximation.
C
  200    CONTINUE
         ERFC = SALZER( -(0,1) * RHOROOT )
         ZA = 1 + (0,1) * SQRT( PI * RHO ) * EXP( -RHO ) * ERFC
         RETURN
C
C  For arguments with large absolute magnitude, compute the
C  complex attenuation function in terms of the w-function
C  described in Abramowitz.
C
  300    CONTINUE
         ZA = 1 + (0,1) * SQRT( PI * RHO ) * W( RHOROOT )
         RETURN
C
         END
```

```
C
C*******************************************************
C
C  SALZER - Compute the complementary error function of
C  the complex argument, Z, using the method described by
C  H. E. Salzer, Formulas for Calculating the Error Function
C  of a Complex Variable, Math. Tables and Other Aids to
C  Computation (journal published by National Research Council),
C  Vol. V, 1951.  The formulas also appear in Abramowitz and
C  Stegun, page 299.
C
C*******************************************************
C
      FUNCTION SALZER( Z )
C
      IMPLICIT COMPLEX ( S, Z )
      COMPLEX TEST
C
      COMMON / INOUT / IN, IO
      DATA PI / 3.1415927 /
C
C  Coefficients P, and A1, A2, etc. to approximate error
C  function of real argument (C. Hastings, Approximations
C  for Digital Computers, Princeton Univ. Press, 1955 )
C
      DATA P, A1, A2, A3, A4, A5 / 0.3275911, 0.2548296,
     &      -0.2844967, 1.4214137, -1.4531520, 1.0614054 /
C
C  Approximation of complementary error function for real,
C  non-negative arguments
C
      T( X ) = 1 / ( 1 + P * X )
      REALERFC( X ) =
     &          T( X ) *
     &    ( A1 + T( X ) *
     &    ( A2 + T( X ) *
     &    ( A3 + T( X ) *
     &    ( A4 + T( X ) *
     &    ( A5 )))))
     &    * EXP( - X ** 2 )
C
C  Functions for approximating ERF of complex arguments.
C
      F( X, Y, N ) = 2 * X
     &     - 2 * X * COSH( N * Y ) * COS( 2 * X * Y )
     &        + N * SINH( N * Y ) * SIN( 2 * X * Y )
      G( X, Y, N ) =
     &      2 * X * COSH( N * Y ) * SIN( 2 * X * Y )
     &        + N * SINH( N * Y ) * COS( 2 * X * Y )
C
C  Begin execution.
C
      X = REAL ( Z )
```

```
      Y = AIMAG( Z )
      SUM = 0
      IF ( Y .EQ. 0 ) GO TO 20
C
      N = MIN( ABS( 80/Y ), 50. )
      DO 10 I = 1, N
         SUM = SUM + EXP( - .25 * ( I ** 2 ) )
     &      * ( F( X, Y, I ) + (0,1) * G( X, Y, I ) )
     &      / ( I ** 2 + 4.0 * X ** 2 )
         IF ( I .GT. 1 .AND.
     &      ABS( SUM - TEST ) .LT. ABS( TEST ) / 1E5 )
     &      GO TO 20
         TEST = SUM
   10 CONTINUE
      WRITE ( IO, * )
     &   'Unexpected error: Salzer series failed to converge'
C
   20 CONTINUE
      SALZER = - 2 * EXP( - X ** 2 ) * SUM / PI
      IF ( X .NE. 0. )
     &   SALZER = SALZER - EXP( - X ** 2 ) *
     &  ( 1 - COS( 2 * X * Y ) + (0,1) * SIN( 2 * X * Y ) ) /
     &  ( 2 * PI * X )
C
C  When called to aid evaluation of the Sommerfeld complex
C  surface wave attenuation, the variable Z will be in the
C  4th quadrant.  Additional provision is made below to
C  return the value of the complementary error function
C  independent of what quadrant Z is in.
C
      IF ( X .GE. 0. ) THEN
         SALZER = SALZER + REALERFC( X )
      ELSE
         SALZER = SALZER + 2 - REALERFC( -X )
      END IF
      RETURN
C
      END
```

```
C
C*****************************************************
C
C  SRS1 - Compute the first of two power series associated
C  with correction terms to the complex surface wave
C  attenuation.  An effective earth radius factor is needed
C  to apply the correction, and the appropriate factor is
C  applied to the sum of the power series addressed by this
C  subprogram.
C
C  The input is a single complex value, and the output is
C  also complex.
C
C*****************************************************
C
      FUNCTION SRS1( Z )
C
      IMPLICIT COMPLEX ( S, T, Z )
      COMPLEX ODDTERM, EVENTERM
C
      COMMON / INOUT / IN, IO
      DATA PI / 3.1415927 /
C
      ODDTERM = 4 * Z / ( 3 * SQRT( PI ) )
      EVENTERM = 1
      SUM = 1 + 2 * ODDTERM
      DO 100 I = 2, 50
         IF ( MOD( I, 2 ) .EQ. 0 ) THEN
            EVENTERM = 2 * EVENTERM * Z ** 2 / ( I + 2 )
            TERM = EVENTERM
         ELSE
            ODDTERM = 2 * ODDTERM * Z ** 2 / ( I + 2 )
            TERM = ODDTERM
         END IF
         SUM = SUM + ( I + 1 ) * TERM
         IF ( I .GT. 2 .AND.
     &        ABS( SUM - TEST ) .LT. ABS( TEST ) / 1E6 )
     &        GO TO 110
         TEST = SUM
  100 CONTINUE
      WRITE ( IO, * ) 'Slow convergence in series 1'
C
  110 SRS1 = SUM * SQRT( PI ) / 2
      RETURN
C
      END
```

```
C
C*********************************************************
C
C  SRS2 - Compute the second of two power series associated
C  with correction terms to the complex surface wave
C  attenuation.  An effective earth radius factor is needed
C  to apply the correction, and the appropriate factor is
C  applied to the sum of the power series addressed by this
C  subprogram.
C
C  The input is a single complex value, and the output is
C  also complex.
C
C*********************************************************
C
      FUNCTION SRS2( Z )
C
      IMPLICIT COMPLEX ( S, T, Z )
      COMPLEX ODDTERM, EVENTERM
C
      COMMON / INOUT / IN, IO
      DATA PI / 3.1415927 /
C
      EVENTERM = 8 / ( 15 * SQRT( PI ) )
      ODDTERM = Z / 6
      SUM = 7 * EVENTERM + 2 * 8 * ODDTERM
      DO 200 I = 2, 50
         IF ( MOD( I, 2 ) .EQ. 0 ) THEN
            EVENTERM = 2 * EVENTERM * Z ** 2 / ( I + 5 )
            TERM = EVENTERM
         ELSE
            ODDTERM = 2 * ODDTERM * Z ** 2 / ( I + 5 )
            TERM = ODDTERM
         END IF
         SUM = SUM + ( I + 1 ) * ( I + 7 ) * TERM
         IF ( I .GT. 2 .AND.
     &        ABS( SUM - TEST ) .LT. ABS( TEST ) / 1E6 )
     &        GO TO 210
         TEST = SUM
  200 CONTINUE
      WRITE ( IO, * ) 'Slow convergence in series 2'
C
  210 SRS2 = SUM * SQRT( PI ) / 8
      RETURN
C
      END
```

```
C
C*************************************************************
C
C  RESIDUES - Calculation of diffraction loss over smooth
C  finitely conducting earth using residue series
C
C  The computations follow the method outlined by H. Bremmer,
C  Terrestial Radio Waves, Elsevier Publishing Co., 1949.
C
C*************************************************************
C
      SUBROUTINE RESIDUES( CHI, K, PSI,                    !Given
     &      ATTENUATION )                                  !Find
C
      IMPLICIT COMPLEX ( C, D, Q, T, Z )
      REAL K, CHI
      INTEGER S
      PARAMETER ( MAXTERMS = 30 )
C
C  TAU(S) denotes the points in the complex plane at which
C  residues of the diffraction field integrand are to be
C  evaluated.
C
      DIMENSION TAU( MAXTERMS )
C
C  No need to recalaulate residue points if no change in DEL.
C
      COMMON / SAVE / DEL, QSQR, TAU, NUMPOINTS
      SAVE / SAVE /
C
C  MAXTERMS refers to how many terms may be included in the
C  residue series.  The number actually included will be
C  determined by a convergence criterion represented by the
C  parameter PRECISION.
C
      DATA PRECISION / 1E-4 /
C
C  FINENESS determines the size of the element of integration
C  used to locate residue points.
C
      DATA FINENESS / 0.05 /
      DATA PI / 3.1415927 /
C
C  TAU0 and TAU1 denote reference points where residues
C  would be evaluated in certain limiting cases.  These
C  reference points are all in the first quadrant on the line
C  of slope 60 degrees.  The amplitudes of the complex numbers
C  representing reference points TAU0 and TAU1 are determined
C  from the amplitudes of corresponding roots of the Airy
C  function and its derivative.
C
C  The required roots of the Airy function and its derivative
C  are found by FUNCTION AIRY0 and FUNCTION AIRY1 respectively.
```

```
C
C  Function producing TAU0 or TAU1 from the amplitude
C  AIRY0 or AIRY1:
C
       TFN( AIRY ) = AIRY / 2 ** ( 1./3) * EXP( (0,1) * PI/3 )
C
C  Functions for finding points TAU, at which residues
C  are evaluated, from the reference points TAU0.  These
C  formulas are used when DEL is small.
C
       C3 ( TAU ) = -2./3 * TAU
       C5 ( TAU ) = -4./5 * TAU ** 2
       C6 ( TAU ) = 14./9 * TAU
       C7 ( TAU ) = - ( 5 + 8 * TAU ** 3 ) / 7
       C8 ( TAU ) = 58./15 * TAU ** 2
       C9 ( TAU ) = - TAU * ( 2296./567 + 16/9 * TAU ** 3 )
       C10( TAU ) = 47./35 + 4656/525 * TAU ** 3
C
       TAUFN0( TAU, DEL ) =
     &               TAU + DEL *
     &    (          - 1 + DEL *
     &    (            0 + DEL *
     &    ( C3 ( TAU ) + DEL *
     &    (         1./2 + DEL *
     &    ( C5 ( TAU ) + DEL *
     &    ( C6 ( TAU ) + DEL *
     &    ( C7 ( TAU ) + DEL *
     &    ( C8 ( TAU ) + DEL *
     &    ( C9 ( TAU ) + DEL *
     &    ( C10( TAU ) ))))))))))
C
C  Functions for finding TAU from TAU1.  Used when DEL
C  is large after setting Q = 1/DEL.
C
       D1( TAU ) =  -   1 / ( 2 * TAU )
       D2( TAU ) =  -   1 / ( 8 * TAU ** 3 )
       D3( TAU ) =  -   1 / ( TAU ** 2 ) *
     &  (   1./12   +   1 / ( 16 * TAU ** 3 ) )
       D4( TAU ) =  -   1 / ( TAU ** 4 ) *
     &  (   7./ 96  +   5 / ( 128 * TAU ** 3 ) )
       D5( TAU ) =  -   1 / ( TAU ** 3 ) *
     &  (   1./ 40  +   1 / ( TAU ** 3 ) *
     &  (  21./320  +   7 / ( 256 * TAU ** 3 ) ))
       D6( TAU ) =  -   1 / ( TAU ** 5 ) *
     &  (  29./ 720 +   1 / ( TAU ** 3 ) *
     &  (  77./1280 +  21 / ( 1024 * TAU ** 3 ) ))
       D7( TAU ) =  -   1 / ( TAU ** 4 ) *
     &  (   1./ 112 +   1 / ( TAU ** 3 ) *
     &  (  19./ 360 +   1 / ( TAU ** 3 ) *
     &  ( 143./2560 +  33 / ( 2048 * TAU ** 3 ) )))
       D8( TAU ) =  -   1 / ( TAU ** 6 ) *
     &  (  97./4480 +   1 / ( TAU ** 3 ) *
     &  ( 163./2560 +   1 / ( TAU ** 3 ) *
```

```
     &   ( 429./8192 + 429 / ( 32768 * TAU ** 3 ) )))
C
        TAUFN1( TAU, Q ) =
     &              TAU + Q *
     &      ( D1( TAU ) + Q *
     &      ( D2( TAU ) + Q *
     &      ( D3( TAU ) + Q *
     &      ( D4( TAU ) + Q *
     &      ( D5( TAU ) + Q *
     &      ( D6( TAU ) + Q *
     &      ( D7( TAU ) + Q *
     &      ( D8( TAU ) ))))))))
C
C  Formula for finding TAU by integration
C
        DELTAU( TAU, DEL, DELDEL ) =
     &      DELDEL / ( 2 * TAU * DEL ** 2 - 1 )
C
C  Begin execution.  Define quantities that will be used
C  repeatedly in the residue summing loop.
C
        DELNEW = K * EXP( (0,1) * ( 3*PI/4 - PSI ) )
        IF ( DELNEW .NE. DEL ) THEN
           DEL = DELNEW
           QSQR = ( 1 / DEL ) ** 2
           NUMPOINTS = 0
        END IF
C
C  Clear the variable used to accumulate the residue sum, and
C  initialize variable used to test convergence.
C
        ZS = 0
        TEST = 0
C
C  Begin calculation of sum of residues.  If the S-th residue
C  point been located by a previous call to this subroutine,
C  jump past the calculation of TAU(S).
C
        DO 100 S = 1, MAXTERMS
           IF ( S .LE. NUMPOINTS ) GO TO 90
C
C  Find TAU(S) from TAU0 if K is small, or from TAU1 if K is
C  large.  For intermediate values of K, accuracy requires an
C  integration procedure.
C
           TAU0 = TFN( AIRY0( S ) )
           TAU1 = TFN( AIRY1( S ) )
           IF ( ABS( TAU0 * K ** 2 ) .LT. 0.25 ) THEN
              TAU( S ) = TAUFN0( TAU0, DEL )
           ELSE IF ( ABS( TAU1 * K ** 2 ) .GT. 1.0 ) THEN
              TAU( S ) = TAUFN1( TAU1, 1/DEL )
           ELSE
              T = TAU0
```

```
               N = MAX( ABS( DEL ) / FINENESS, 2.0 )
               DELDEL = DEL / N
C
C  Integrate along a diagonal path in the complex DEL-plane
C  using a fourth-order Runge-Kutta method.  The variable of
C  integration, DEL1, runs from 0 to DEL.
C
               DEL1 = 0
               DO 80 I = 1, N
                  TK1 = DELTAU( T, DEL1, DELDEL )
                  DEL1 = DEL1 + DELDEL/2
                  TK2 = DELTAU( T + TK1/2, DEL1, DELDEL )
                  TK3 = DELTAU( T + TK2/2, DEL1, DELDEL )
                  DEL1 = DEL1 + DELDEL/2
                  TK4 = DELTAU( T + TK3, DEL1, DELDEL )
                  T = T + ( TK1 + 2 * TK2 + 2 * TK3 + TK4 ) / 6
 80            CONTINUE
               TAU(S) = T
            END IF
C
C  Remember how many residue points have been located.  This
C  permits reuse of this subroutine without recalculation of
C  TAU(S) so long as the value of DEL remains unchanged, that
C  is for changes in distance only.
C
            NUMPOINTS = S
C
C  Evaluate residue at TAU(S).
C
  90     CONTINUE
            Z = EXP( (0,1) * TAU(S) * CHI ) /
     &          ( 2 * TAU(S) - QSQR )
C
C  Accumulate and loop to calculate next residue until
C  test indicates convergence is satisfactory.
C
            ZS = ZS + Z
            IF ( ABS( ZS - TEST ) .LT. PRECISION * ABS( TEST ) )
     &         GO TO 200
            TEST = ZS
 100     CONTINUE
C
C  Exit with diffraction loss determined from sum of residues.
C
 200     CONTINUE
         ATTENUATION = ABS( ZS ) * SQRT( 2 * PI * CHI )
         RETURN
C
         END
```

```
C
C**********************************************************
C
C  AIRY0 - Locate the zeroes of the Airy function.
C
C  The zeroes of interest are located on the negative real
C  axis, and this subprogram returns positive values equal
C  to minus the x-coordinates of these zeroes.
C
C  The first 10 values of AIRY0 are those tabulated in
C  Abramowitz and Stegun, Handbook of Functions, page 478;
C  values for indices larger than 10 are calculated using
C  equations (10.4.94) etc. on page 450 of the same reference.
C
C**********************************************************
C
      FUNCTION AIRY0( S )
C
      DIMENSION A0( 10 )
      INTEGER S
      DATA PI / 3.1415927 /
C
      DATA A0 /
     &    2.3381074,  4.0879494,  5.5205598,  6.7867081,
     &    7.9441336,  9.0226508, 10.0401743, 11.0085243,
     &   11.9360156, 12.8287767 /
C
C  Formulas used to calculate the amplitudes A0( S ) for
C  indices greater than 10.
C
      X( S ) = 3 * PI * ( 4 * S - 1 ) / 8
      F( X ) = X ** (2./3) * ( 1 + 5./48 * (1/X) ** 2 )
C
      IF ( S .LE. 10 ) THEN
         AIRY0 = A0( S )
      ELSE
         AIRY0 = F( X( S ) )
      END IF
      RETURN
C
      END
```

```
C
C***********************************************************
C
C  AIRY1 - Locate the zeroes of the derivative of the Airy
C  function.
C
C  The zeroes of interest are located on the negative real
C  axis, and this subprogram returns positive values equal
C  to minus the x-coordinates of these zeroes.
C
C  The first 10 values of AIRY1 are those tabulated in
C  Abramowitz and Stegun, Handbook of Functions, page 478;
C  values for indices larger than 10 are calculated using
C  equations (10.4.94) etc. on page 450 of the same reference.
C
C***********************************************************
C
      FUNCTION AIRY1( S )
C
      DIMENSION A1( 10 )
      INTEGER S
      DATA PI / 3.1415927 /
C
      DATA A1 /
     &     1.0187930,  3.2481976, 4.8200992,  6.1633074,
     &     7.3721773,  8.4884867, 9.5354491, 10.5276604,
     &    11.4750566, 12.3847884 /
C
C  Formulas used to calculate the amplitudes A1( S ) for
C  indices greater than 10.
C
      Y( S ) = 3 * PI * ( 4 * S - 3 ) / 8
      G( Y ) = Y ** (2./3) * ( 1 - 7./48 * (1/Y) ** 2 )
C
      IF ( S .LE. 10 ) THEN
         AIRY1 = A1( S )
      ELSE
         AIRY1 = G( Y( S ) )
      END IF
      RETURN
C
      END
```