Stan

Software Ecosystem for Modern Bayesian Inference

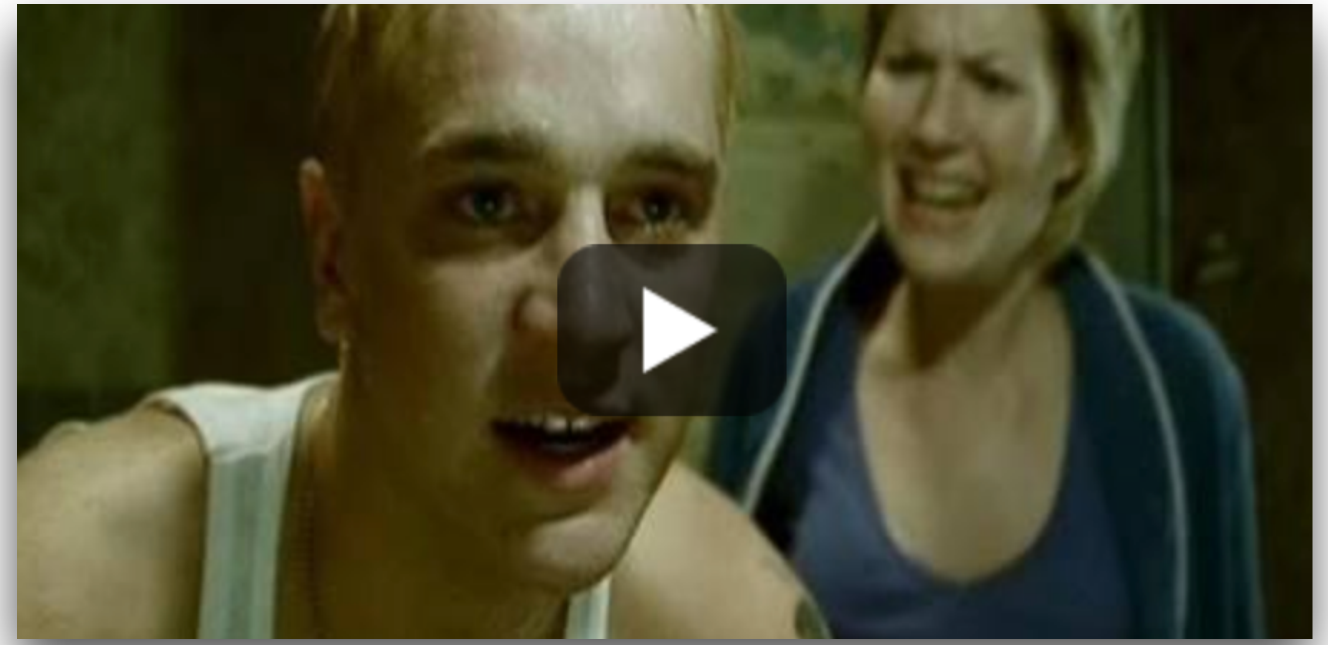
StanCon 2018 Helsinki

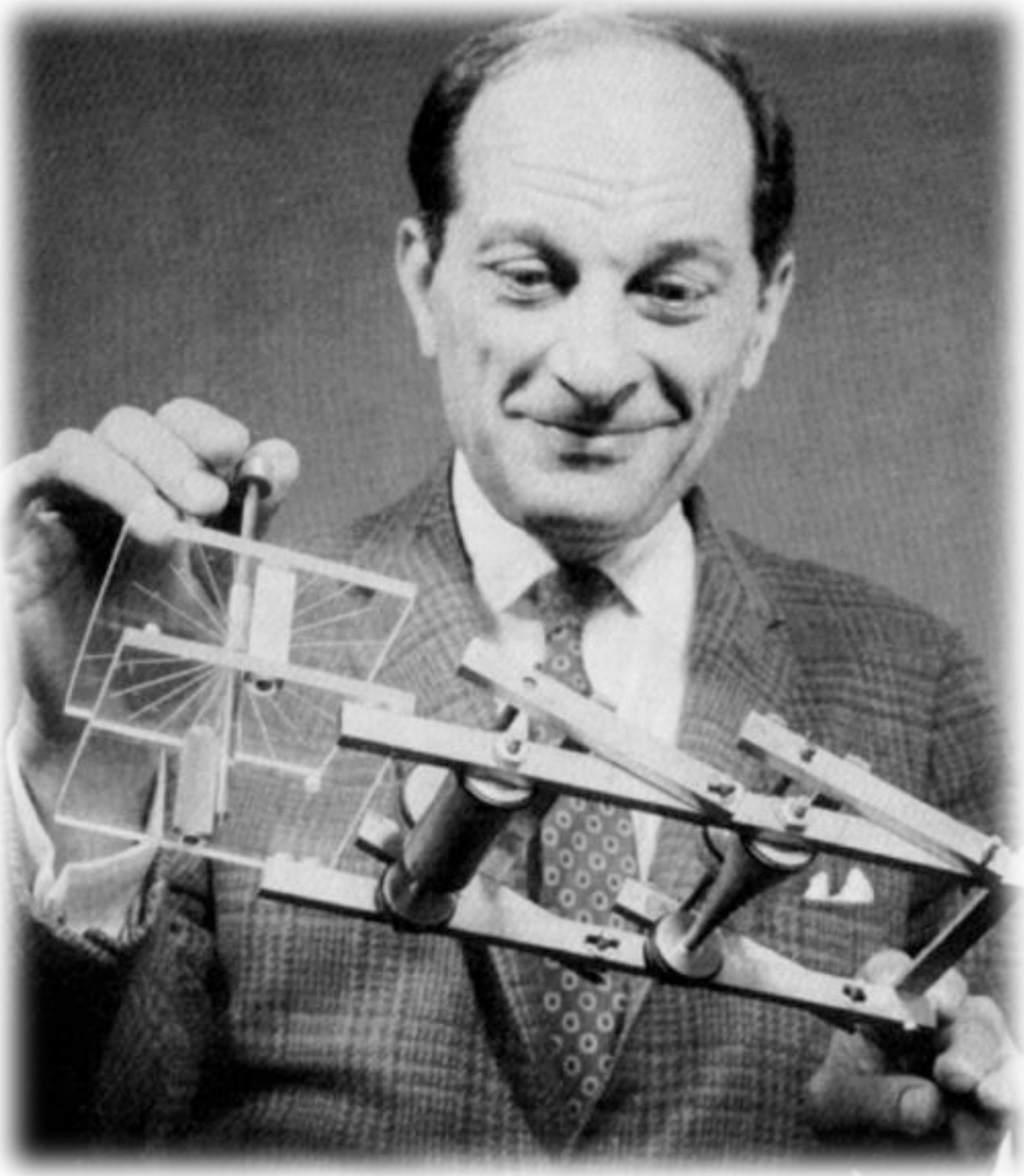
Jonah Gabry & Lauren Kennedy
Columbia University

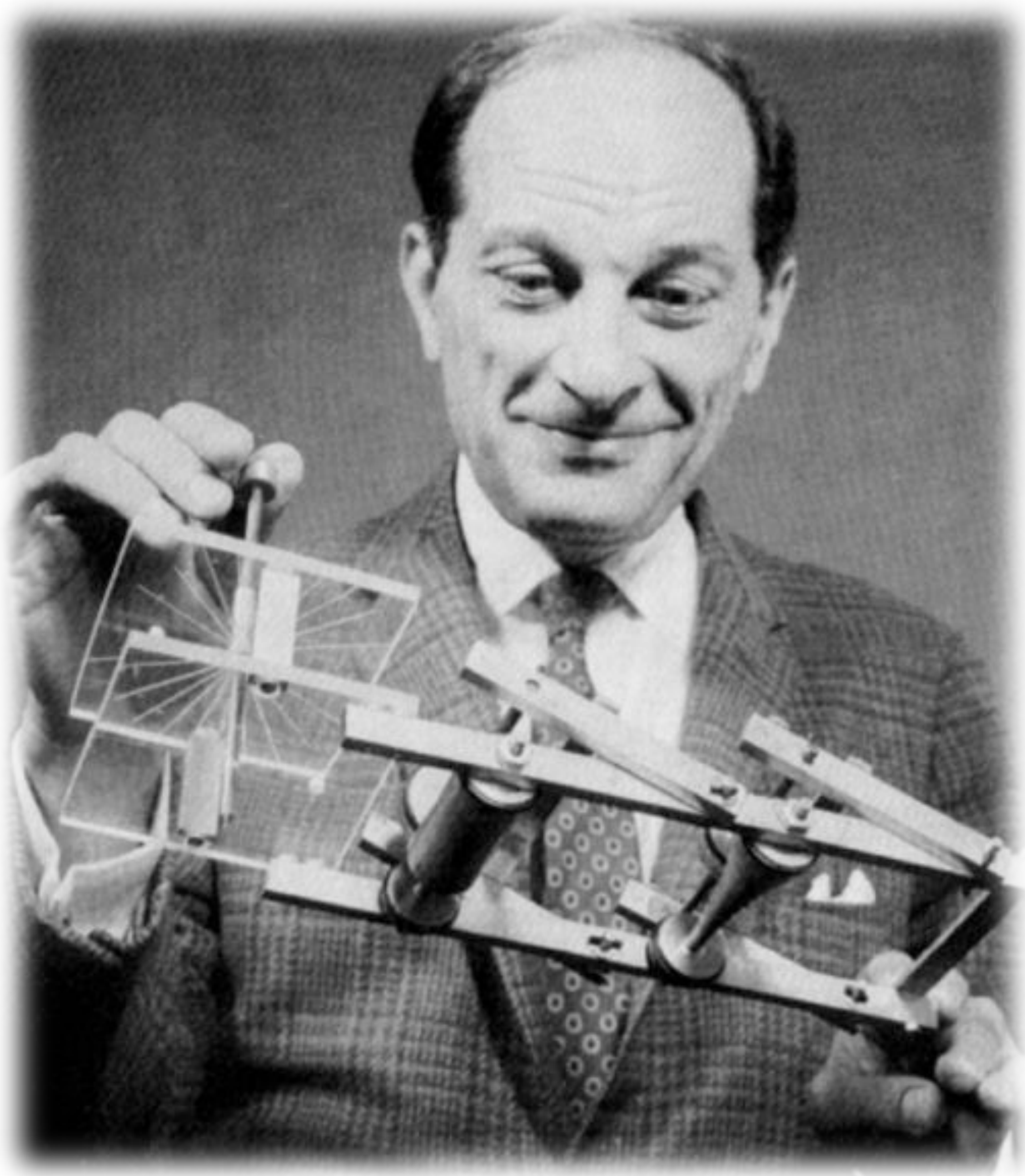
github.com/jgabry/stancon2018helsinki_intro

Why “Stan”?

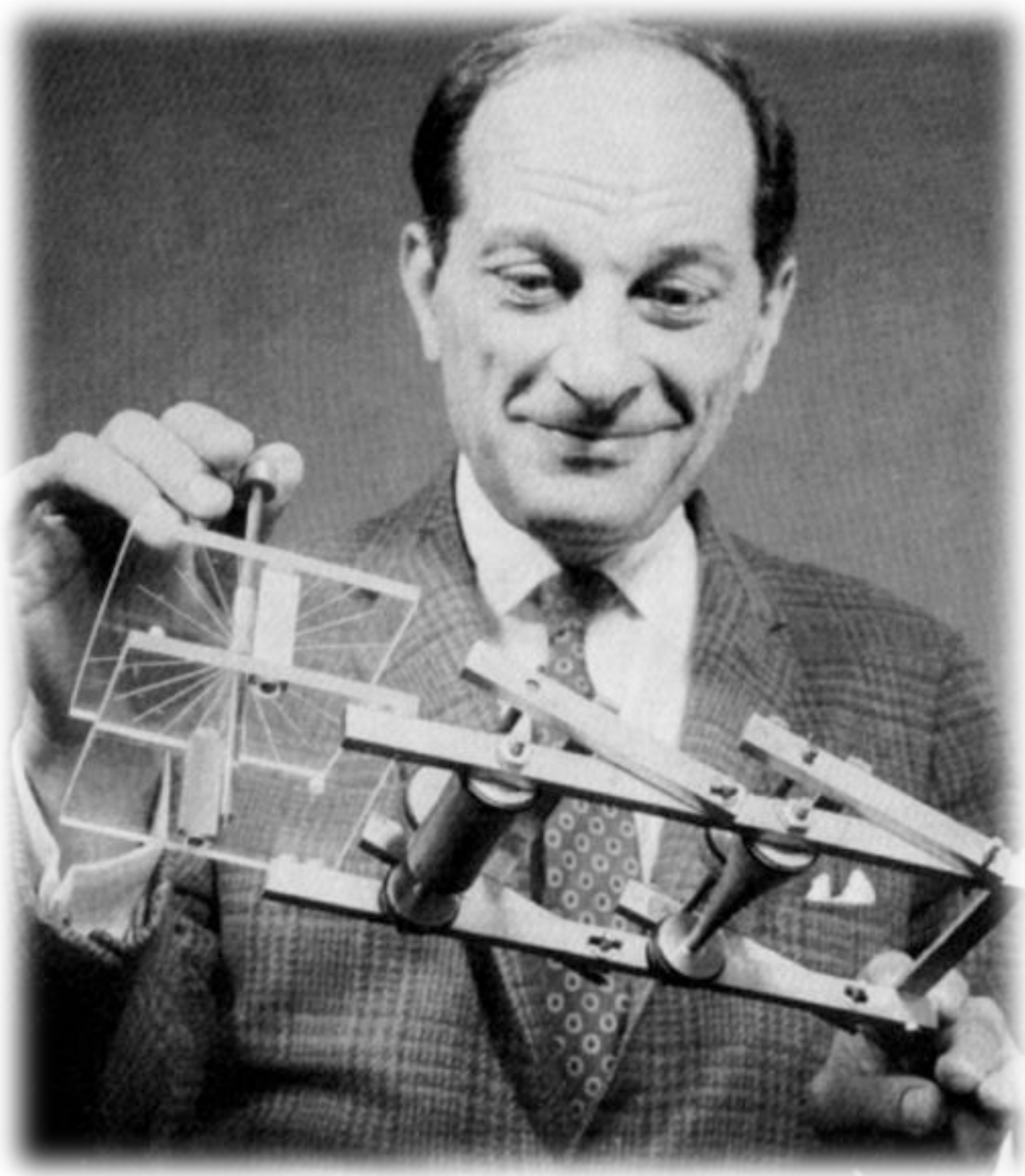
suboptimal SEO







Stanislaw Ulam
(1909–1984)



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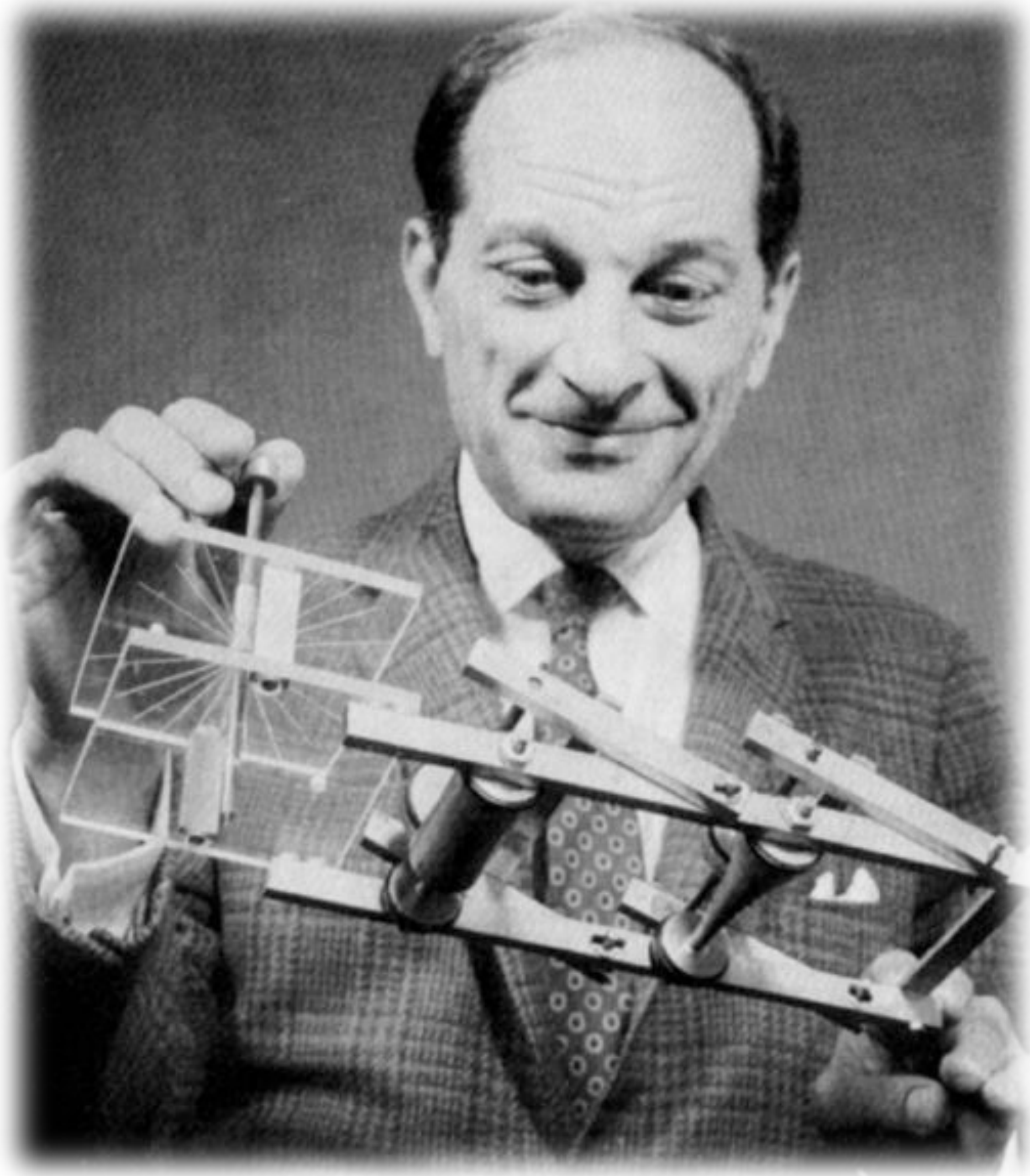
Monte Carlo
Method



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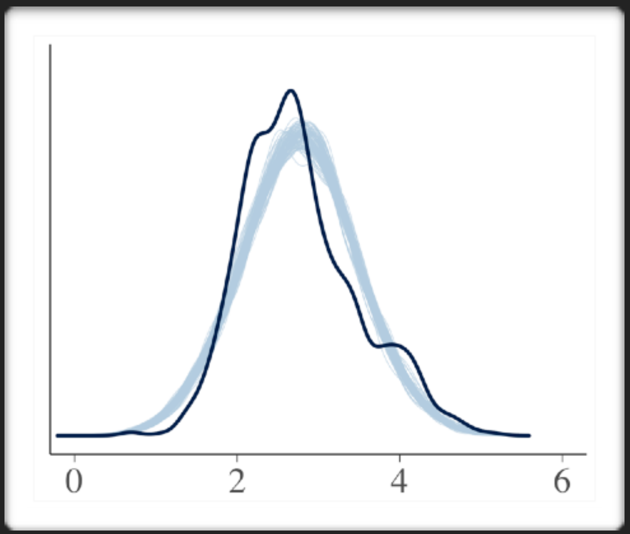
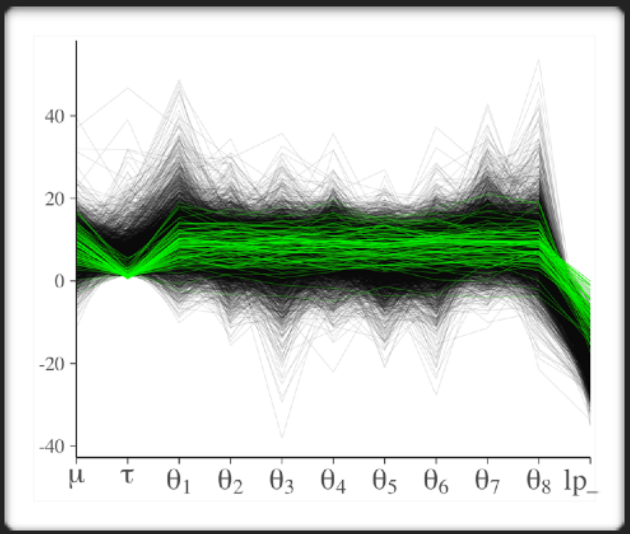
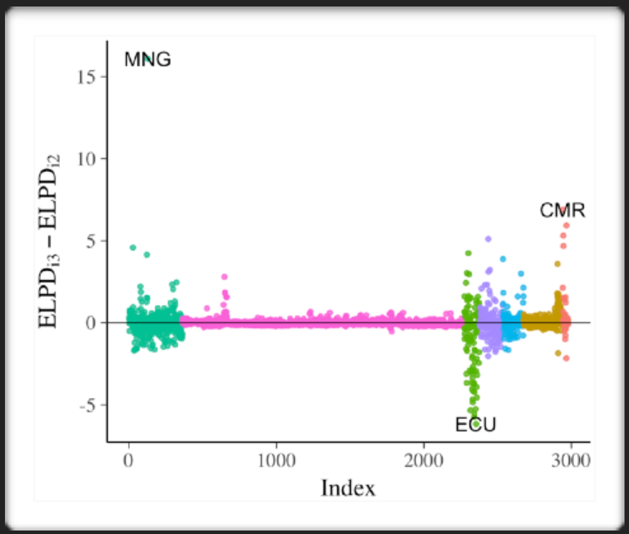
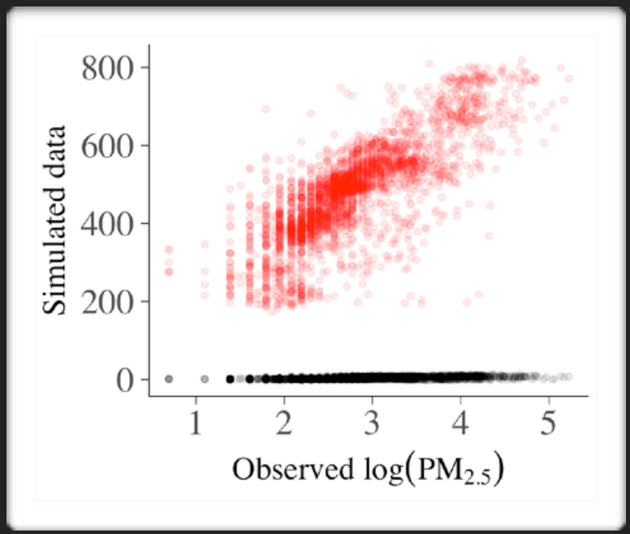
- Stan **ecosystem**
  - lang, math library (C++)
  - interfaces and tools (R, Python, many more)
  - documentation (example model repo, user guide & reference manual, case studies, R package vignettes)
  - online community (Stan Forums on Discourse)

Visualization in Bayesian workflow









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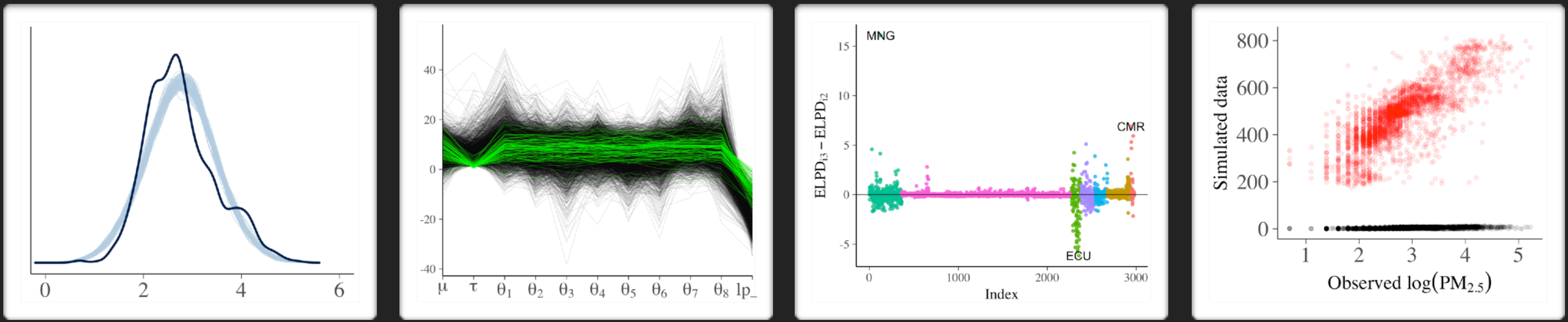

Gabry, J., Simpson, D., Vehtari, A., Betancourt, M., and Gelman, A. (2018).
Visualization in Bayesian workflow.
Journal of the Royal Statistical Society Series A, accepted for publication.
arXiv preprint: arxiv.org/abs/1709.01449
Code: github.com/jgabry/bayes-vis-paper

Workflow

Bayesian data analysis

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Bayesian data analysis

- Exploratory data analysis

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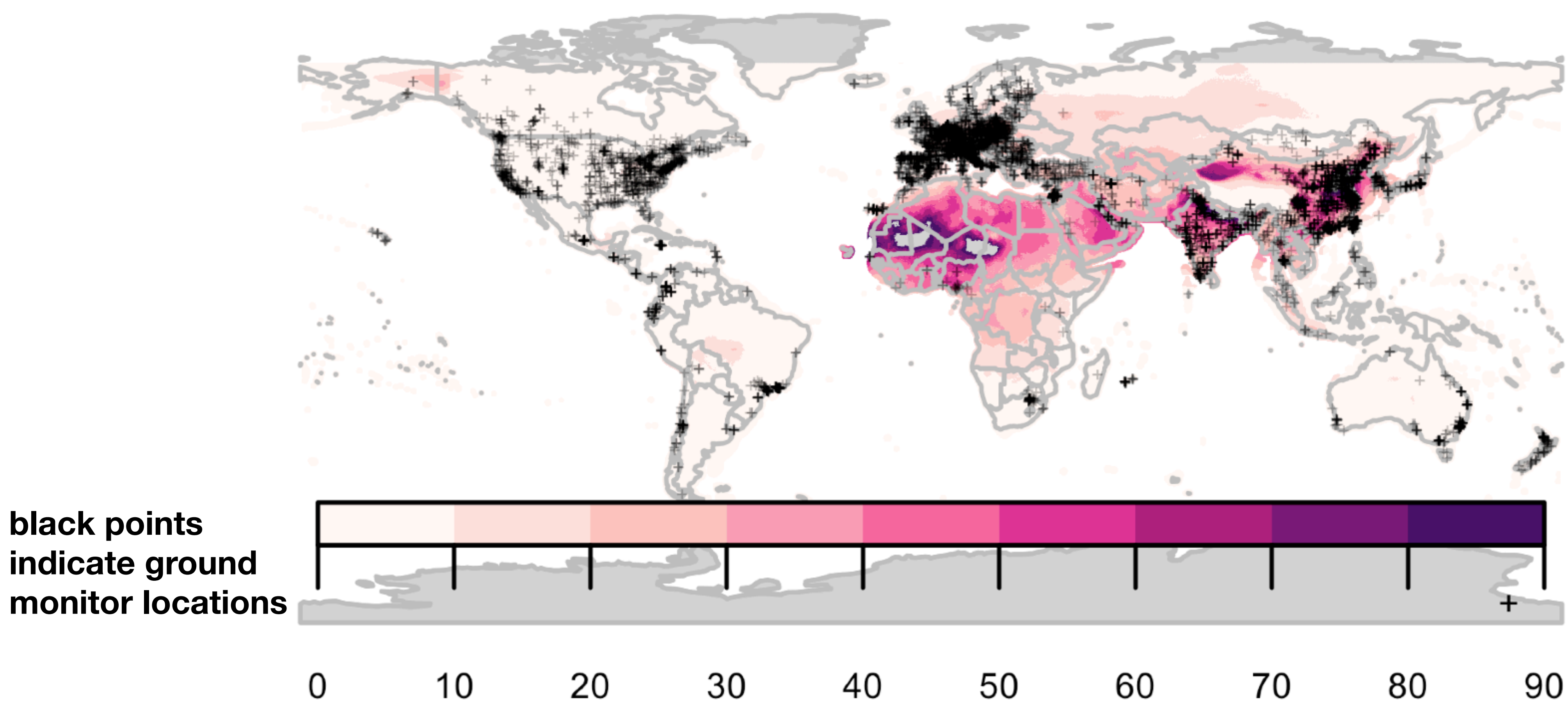

Satellite estimates of PM2.5 and ground monitor locations

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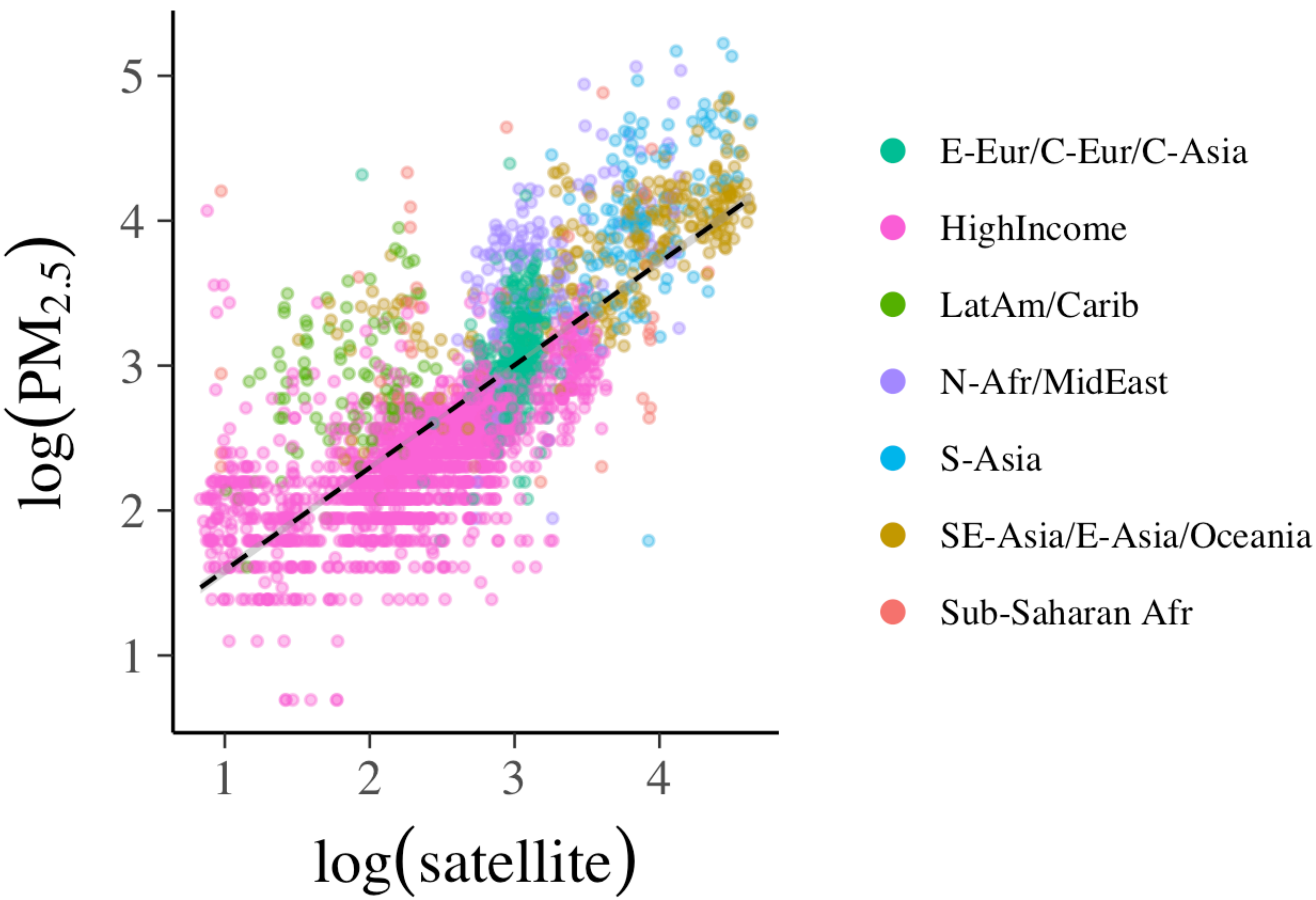
Exploratory data analysis
building a network of models



Exploratory data analysis
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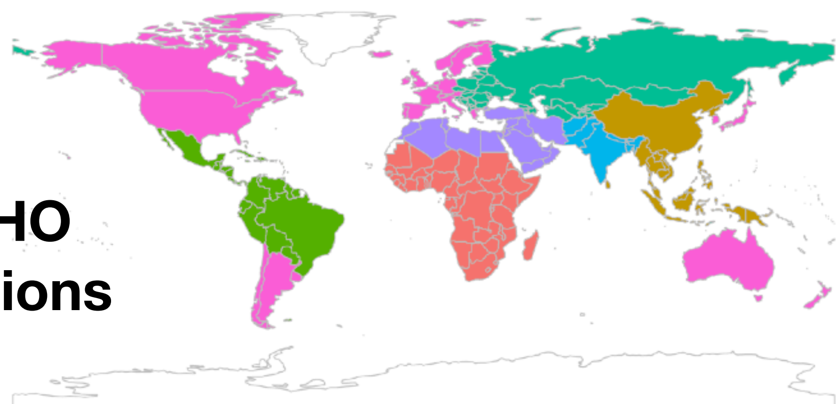
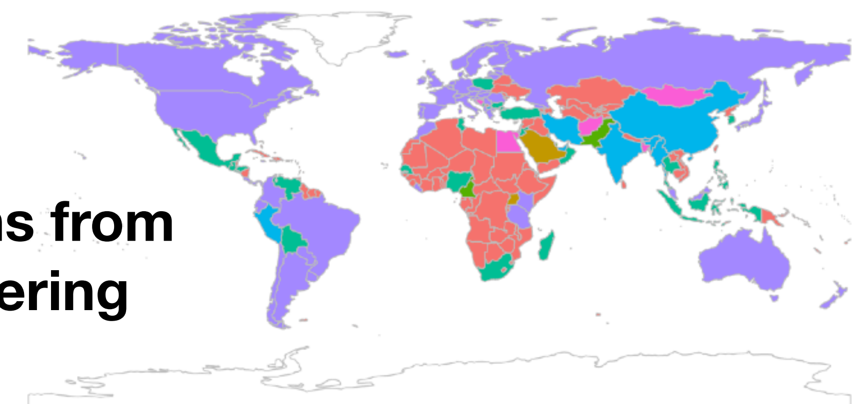
WHO Regions



Regions from clustering



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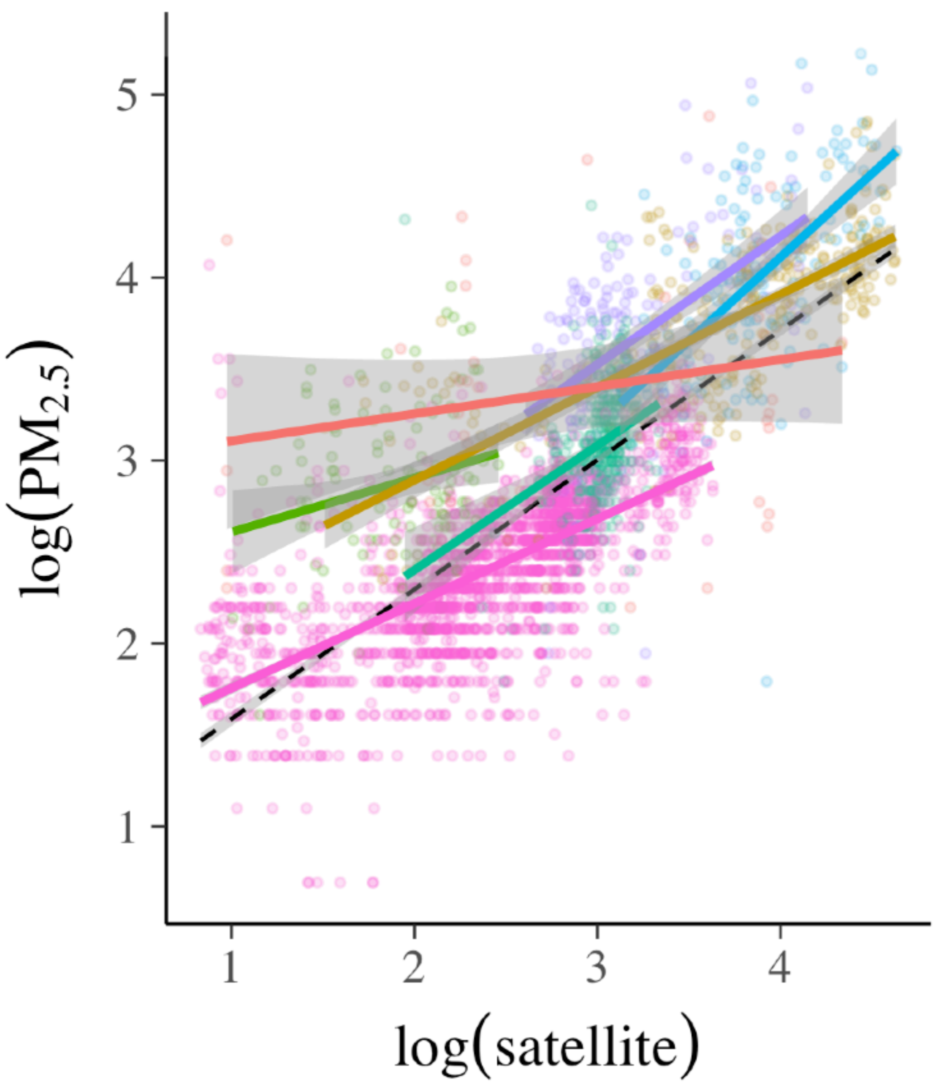

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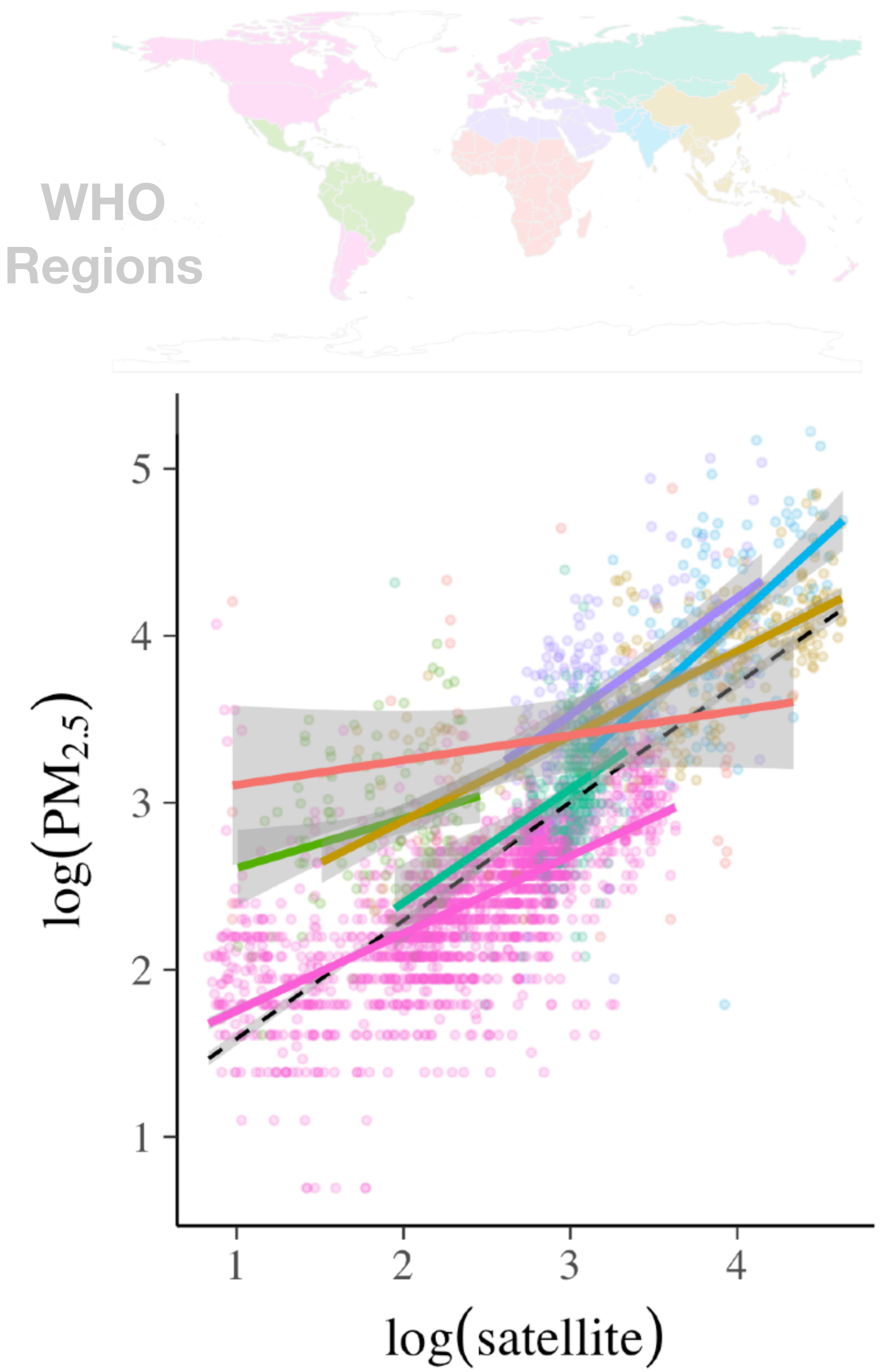

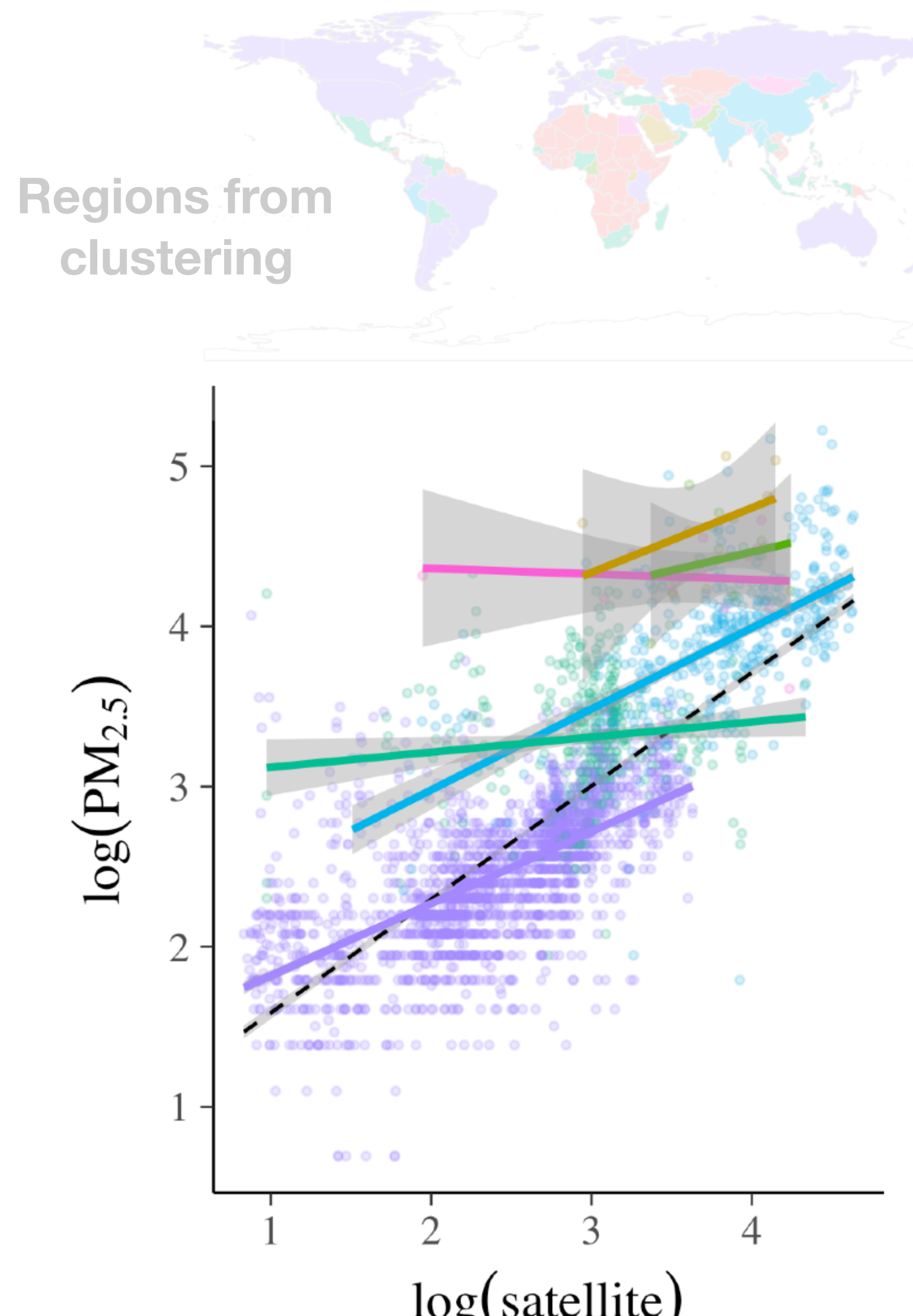

Exploratory data analysis

building a network of models

For measurements $n = 1, \ldots, N$

and regions $j = 1, \ldots, J$

Model 1

$$\log(\mathbf{PM}_{2.5})_{n,j} = \alpha + \beta \log(\mathbf{sat})_{n,j} + \epsilon_{n,j}, \quad \epsilon_{n,j} \sim N(0,\sigma)$$

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Gelman, A., Simpson, D., and Betancourt, M. (2017).
The prior can often only be understood in the context of the likelihood.
arXiv preprint: arxiv.org/abs/1708.07487

A Bayesian modeler commits to an a priori *joint distribution*

Likelihood x Prior

$$p(\mathbf{y}, \boldsymbol{\theta}) = p(\mathbf{y} \mid \boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\boldsymbol{\theta} \mid \mathbf{y})p(\mathbf{y})$$

Posterior x Marginal Likelihood

Data
(observed)

Parameters
(unobserved)

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- Proper but diffuse is better than improper but is still often problematic

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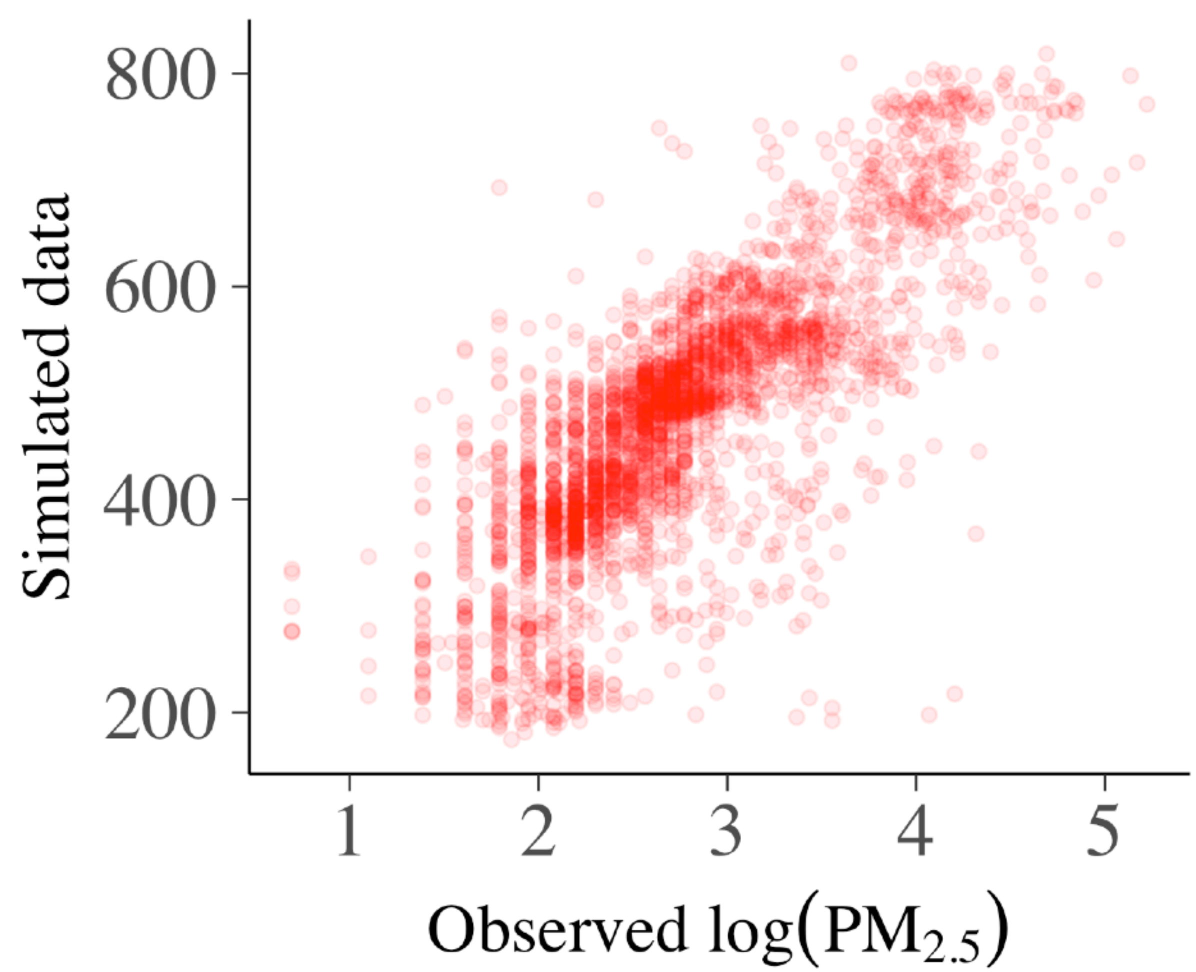

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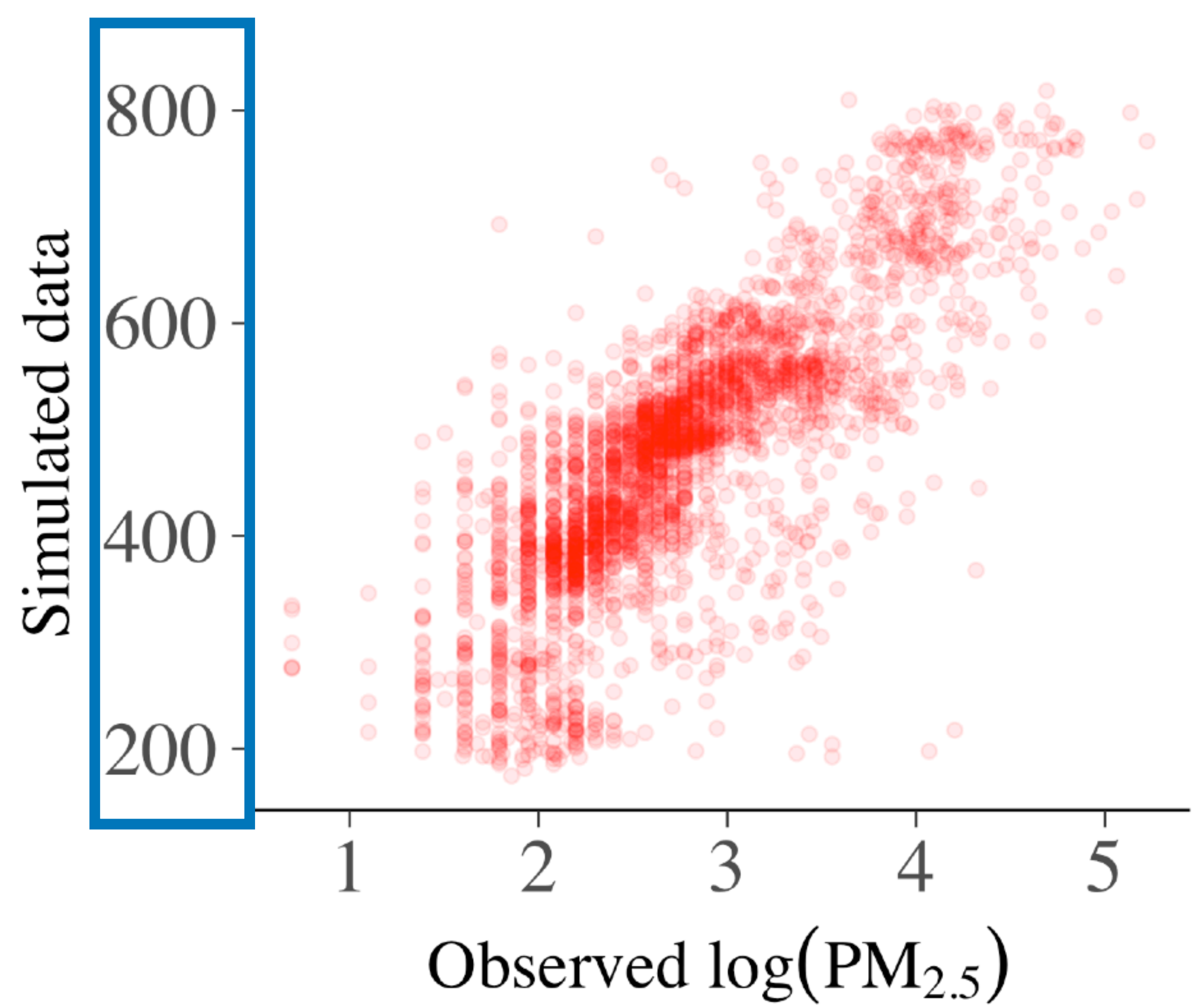

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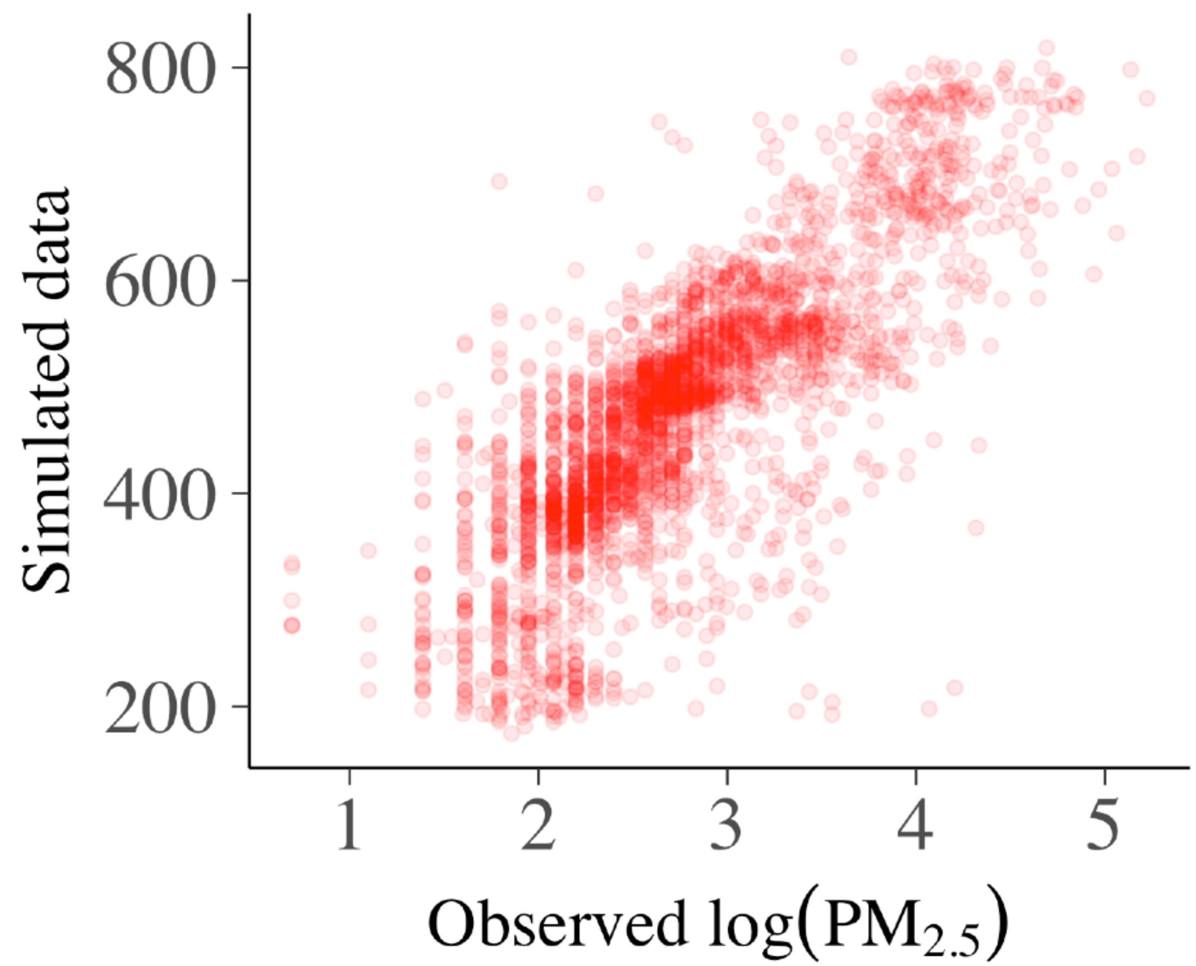

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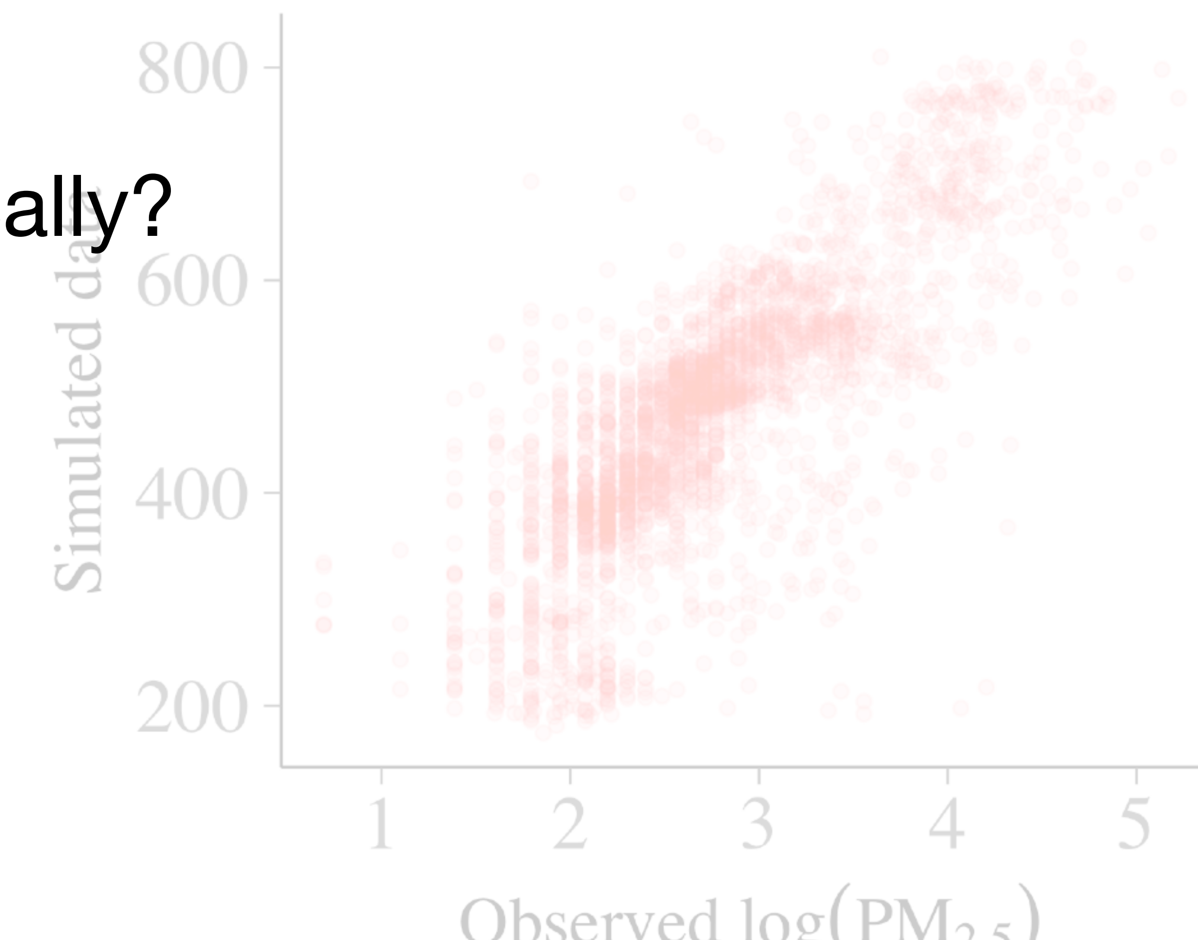

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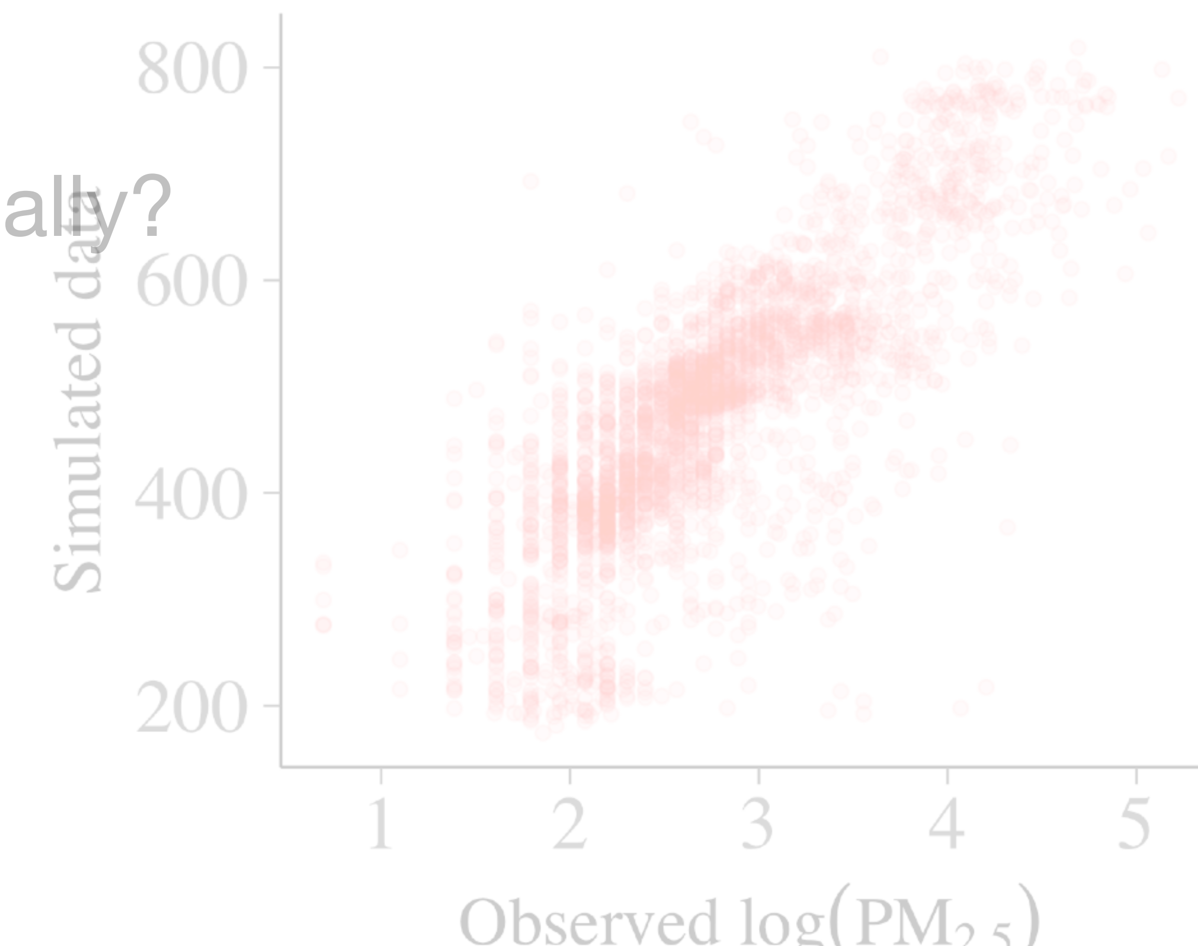

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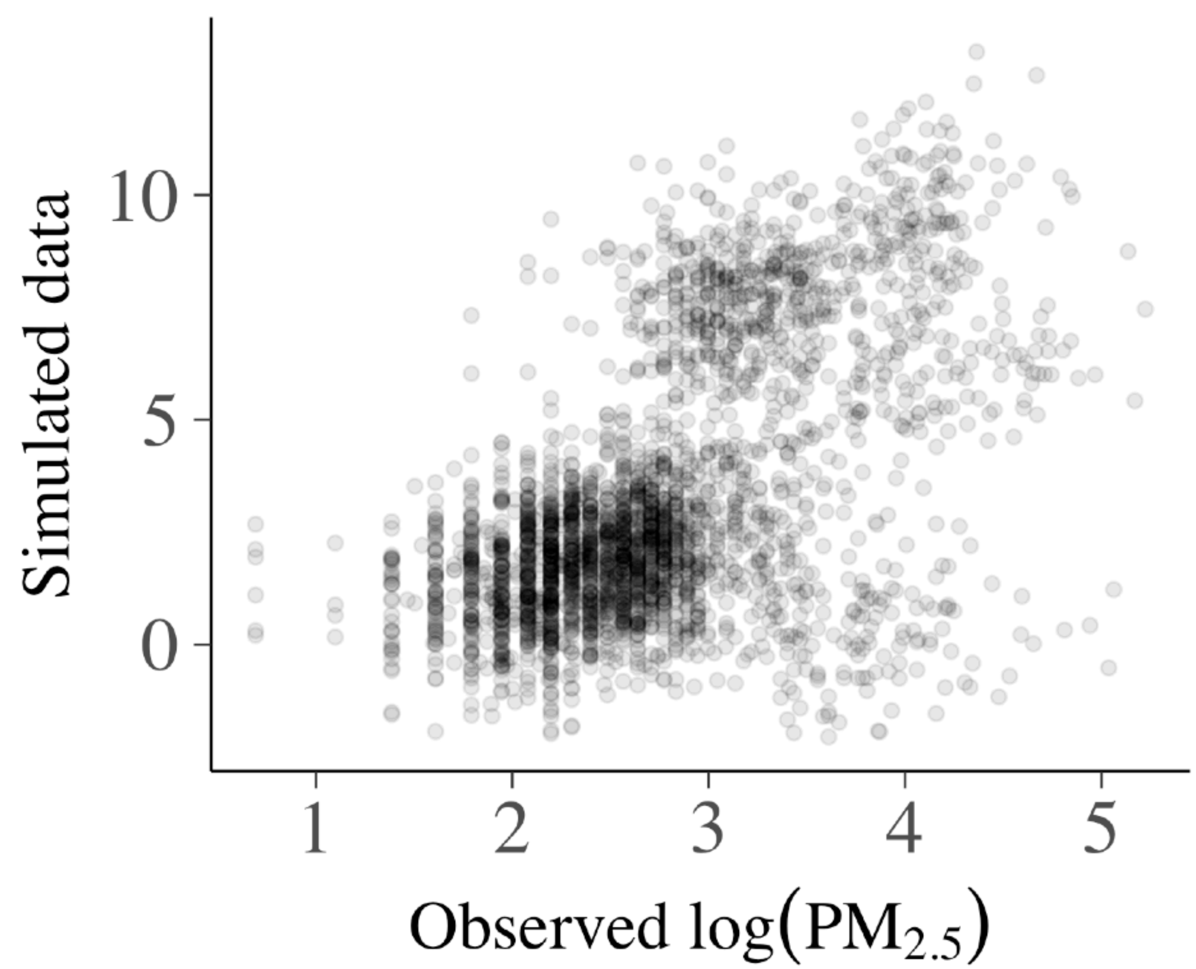

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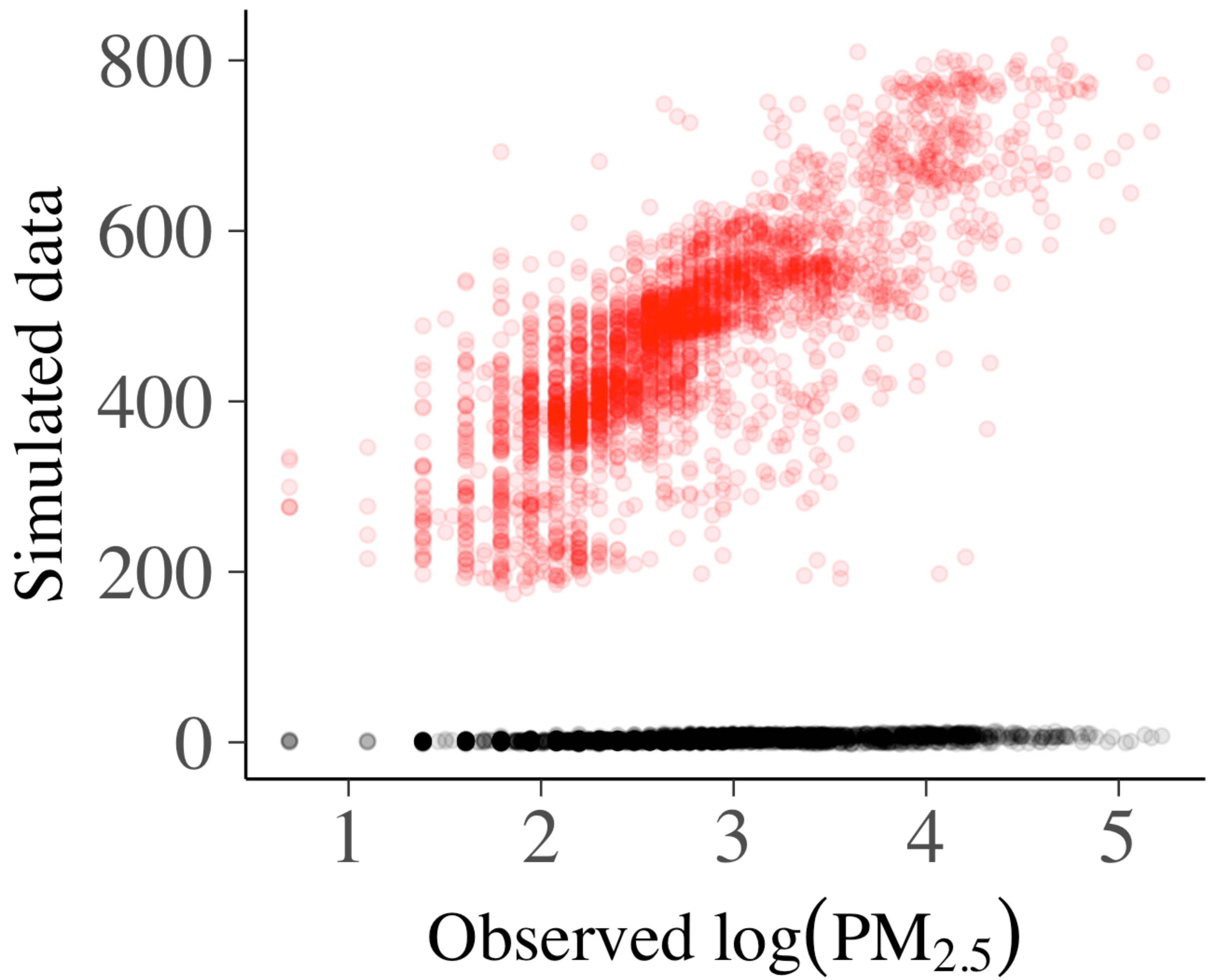

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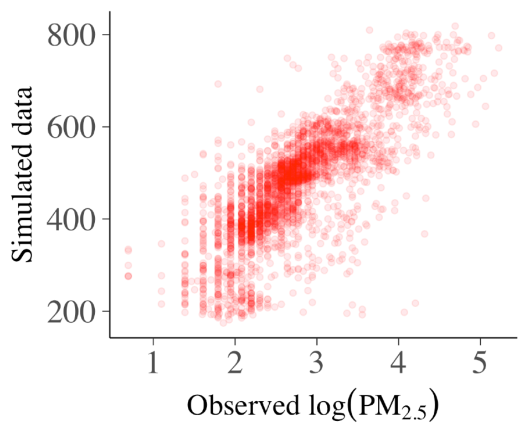

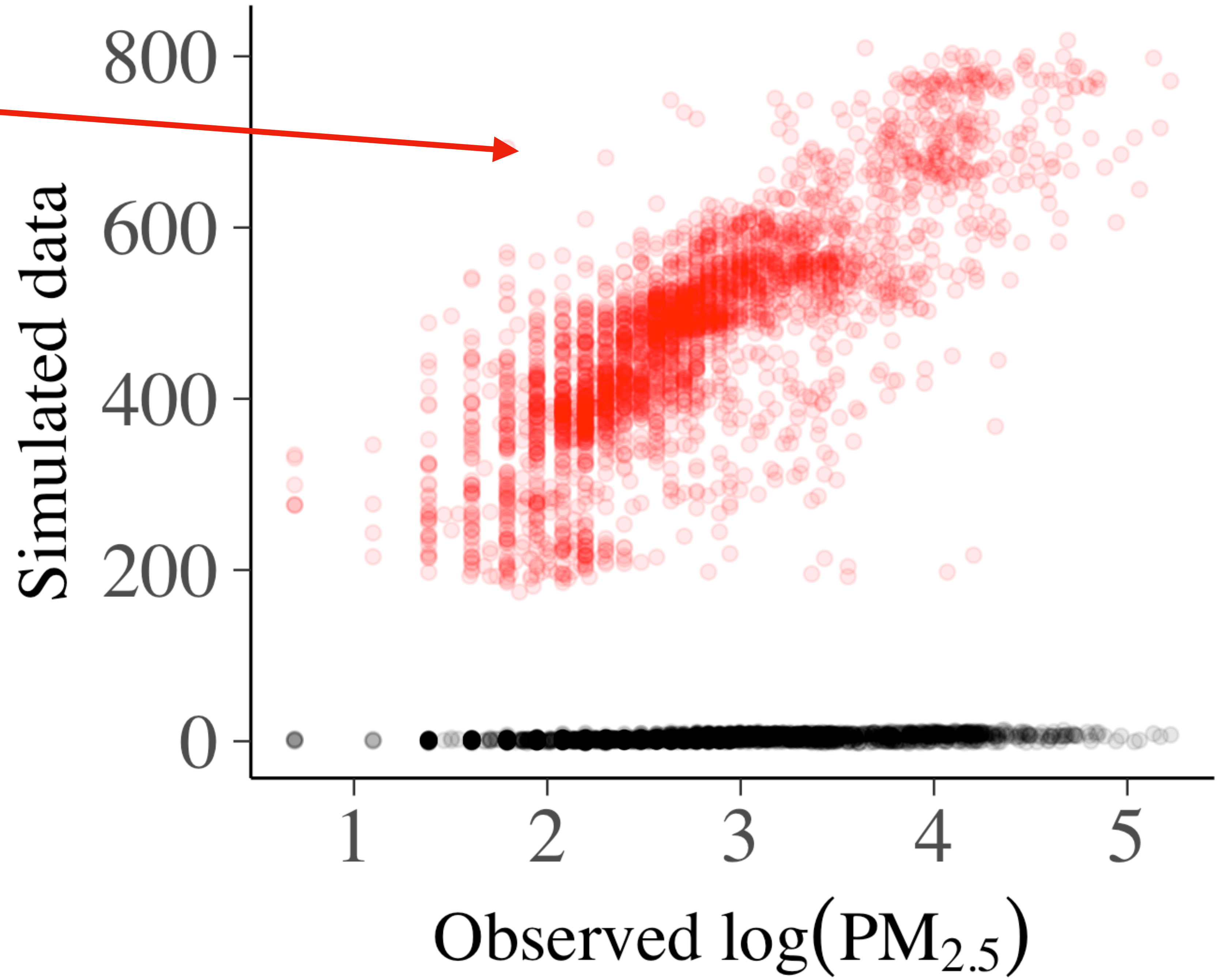

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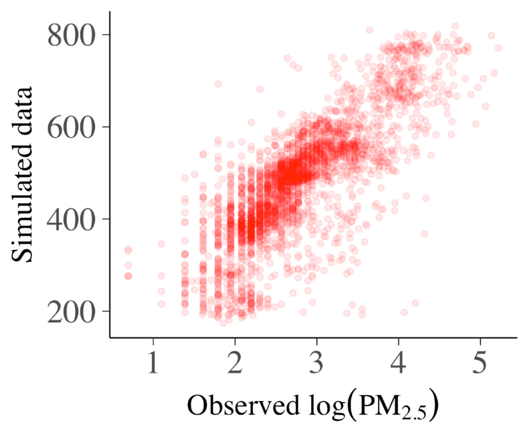
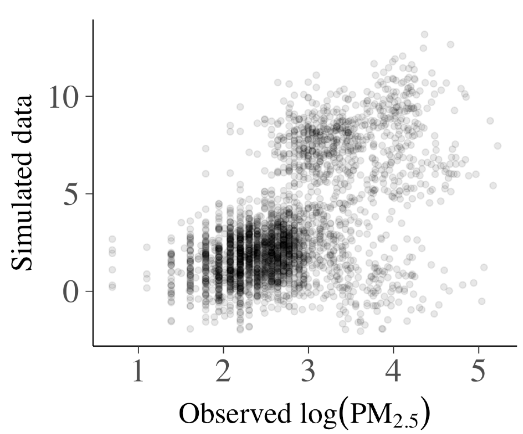
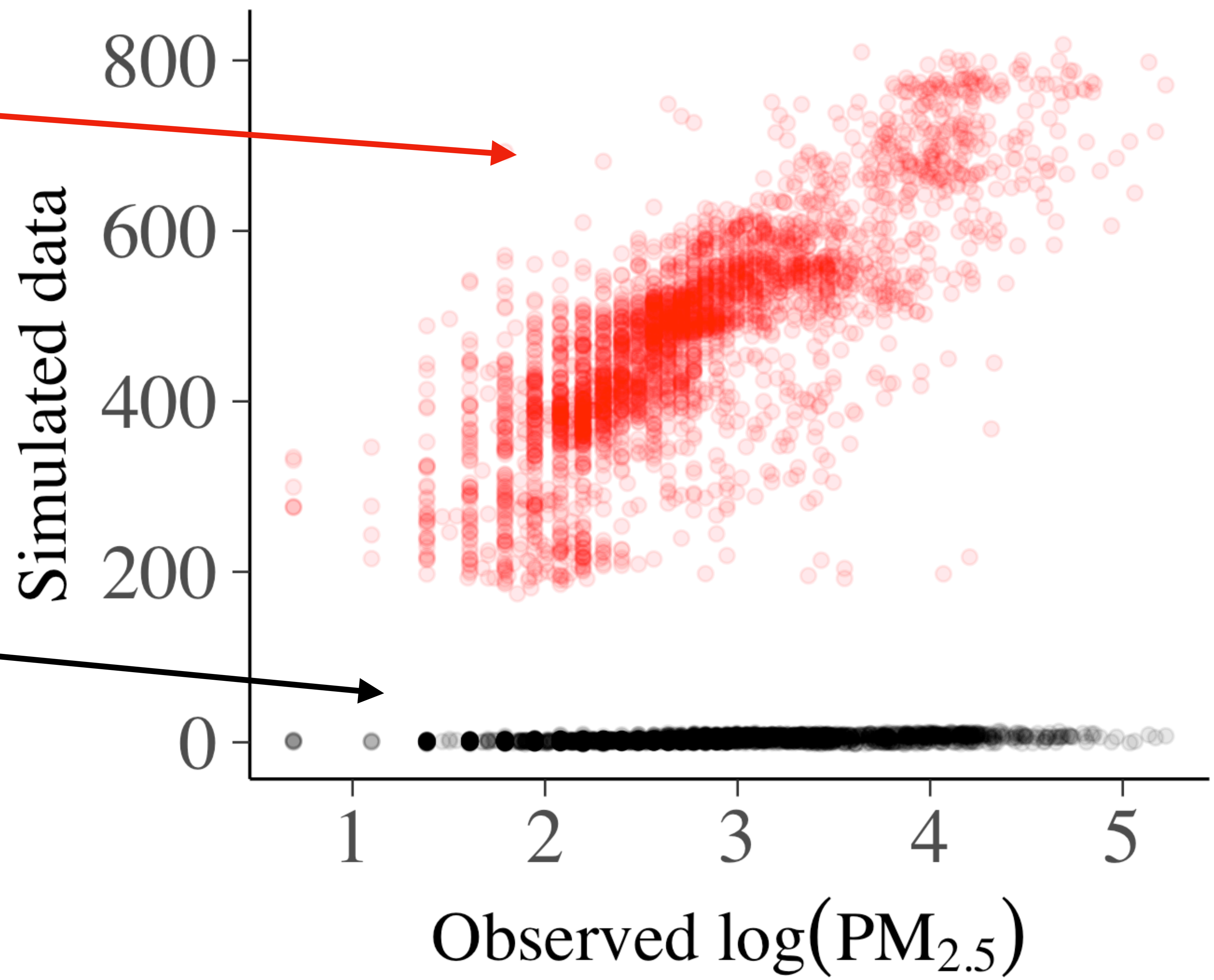


Weakly informative





Markov Chain Monte Carlo

MCMC, HMC, NUTS, other fancy acronyms…

Chi Feng's MCMC demos: http://chi-feng.github.io/mcmc-demo/

Betancourt, M. (2017).
A conceptual introduction to Hamiltonian Monte Carlo.
arXiv preprint:
arxiv.org/abs/1701.02434

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visual model evaluation

The *posterior predictive distribution* is the average data generation process over the entire model

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$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)\, p(\theta|y)\, d\theta$$

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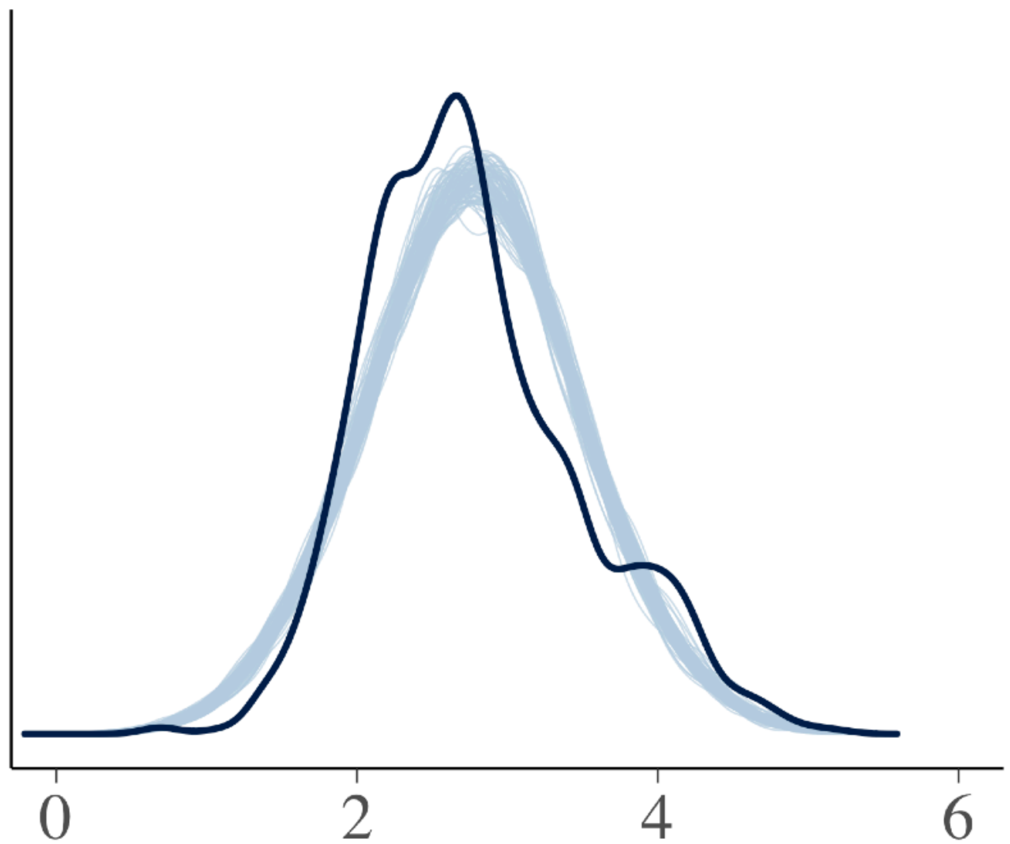
Posterior predictive checking

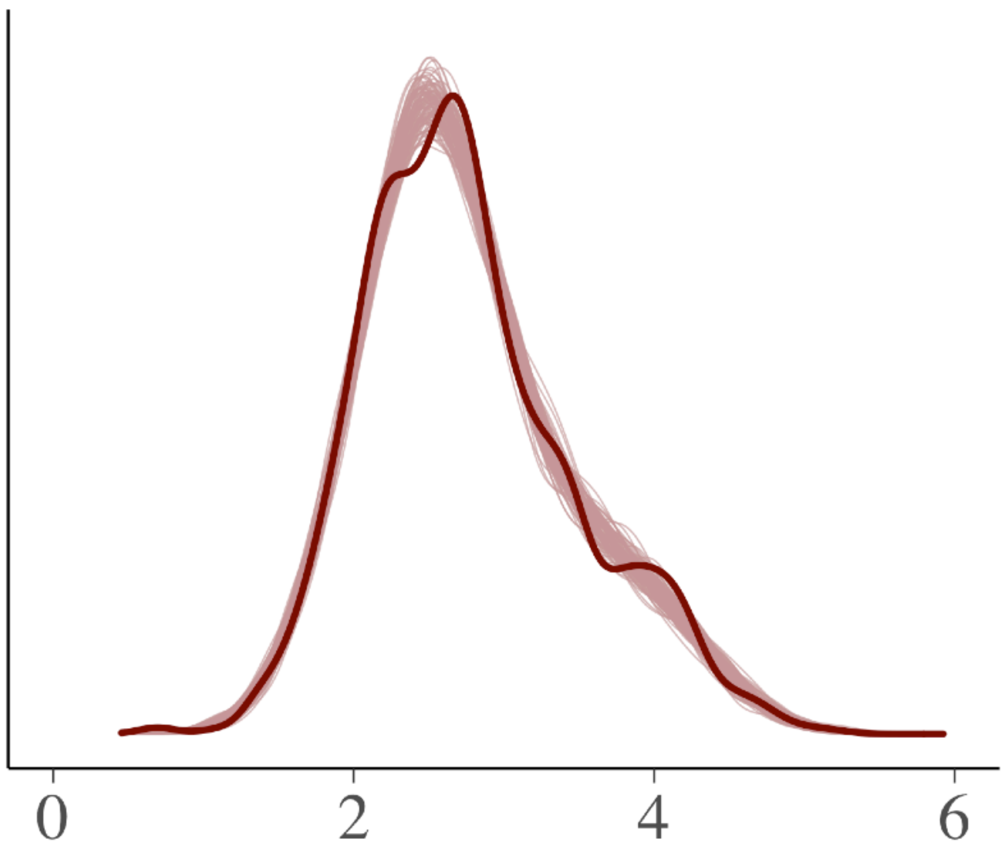
visual model evaluation

Observed data vs posterior predictive simulations

Model 1 (single level)



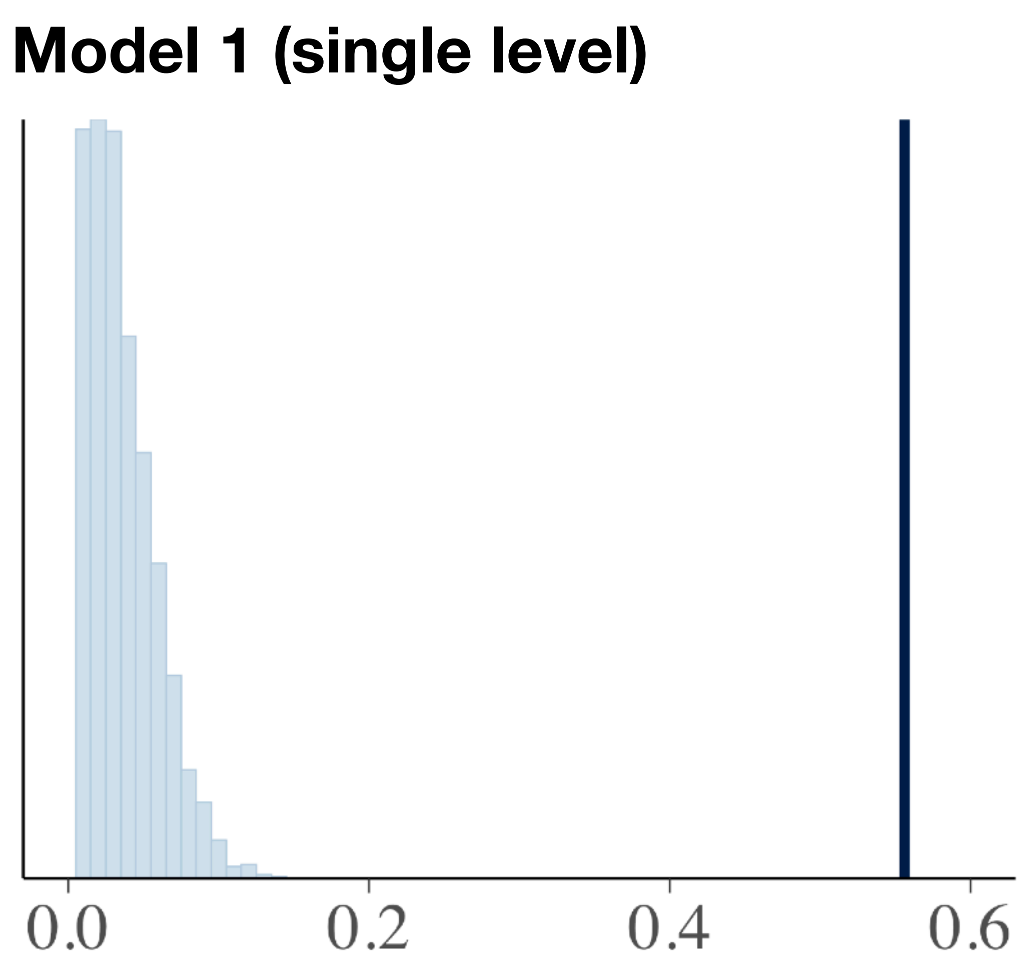
Model 3 (multilevel)

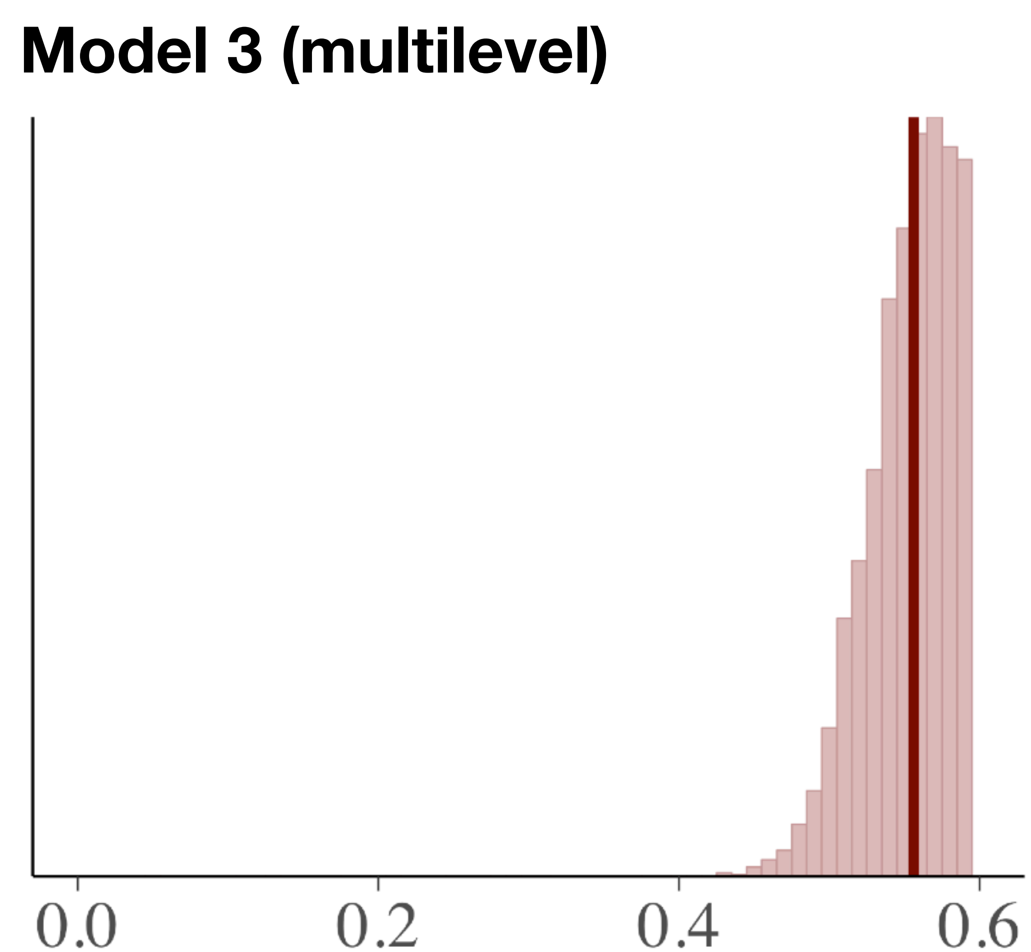


Posterior predictive checking

visual model evaluation

Observed statistics vs posterior predictive statistics

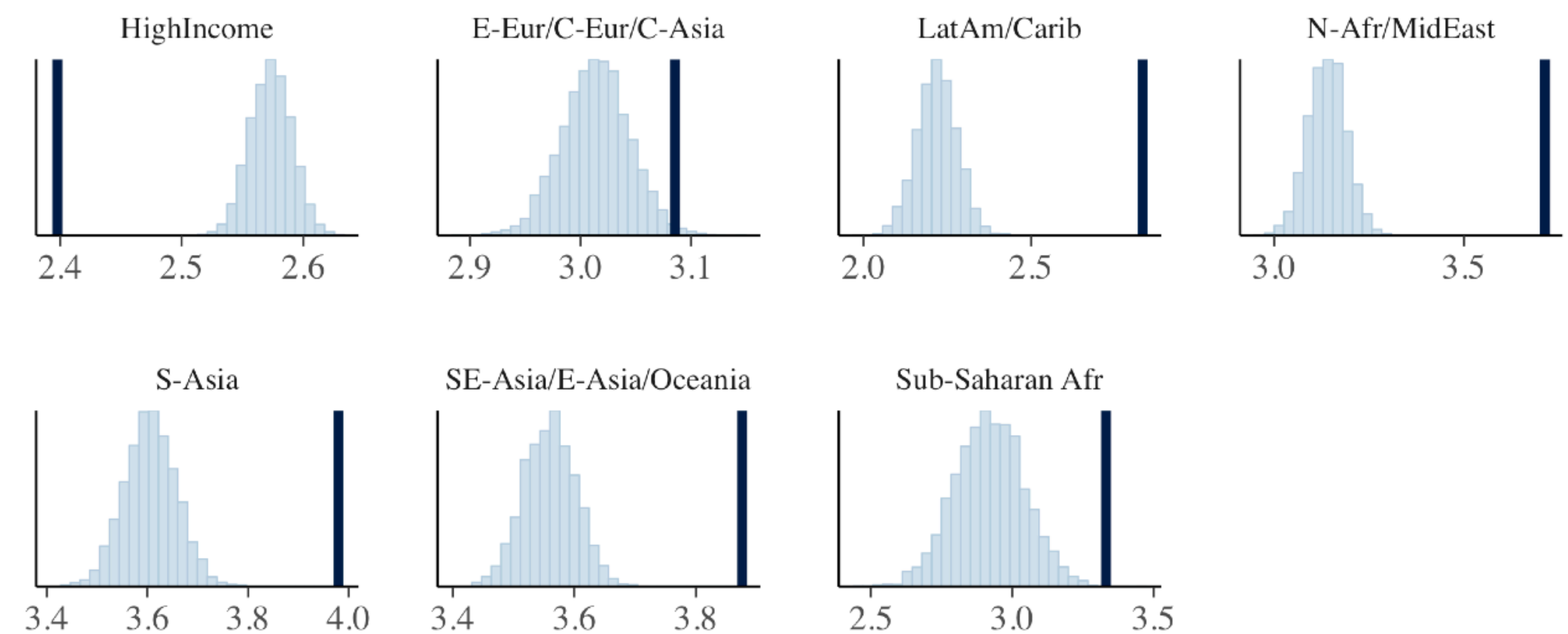




$$T(y) = \mathrm{skew}(y)$$

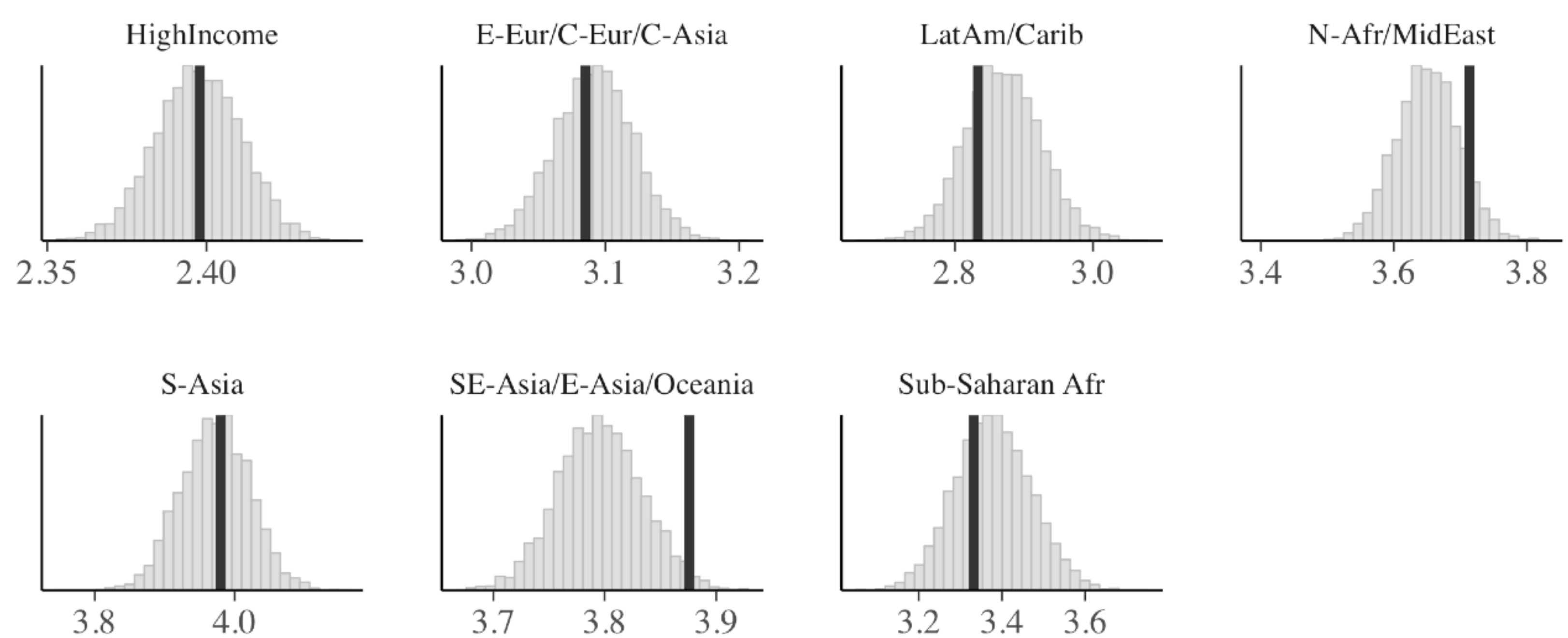
Posterior predictive checking:
visual model evaluation

Model 1 (single level)



$$T(y) = \mathrm{med}(y|\mathrm{region})$$

Model 2 (multilevel)



Workflow

Bayesian data analysis

- Exploratory data analysis
- *Prior* predictive checking
- Model fitting and algorithm diagnostics
- *Posterior* predictive checking
- **Model comparison (e.g., via cross-validation)**

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16:30 - 17:30 **Model assessment and selection** Aki Vehtari