Modeling the Overall Energy in Music Tracks: Linear Regression Approach

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STAT 510: REGRESSION ANALYSIS

Introduction

The data for this project were scraped from Spotify's web application programming interface (API). The dataset consists of audio features of 694 tracks from various artist's across multiple genres. The variables with a short description are:

Variable	Data Type	Description
Energy (response)	Numeric	Perceptual measure of intensity and activity
Genre (1)	Categorical	Genre of music
Mode (2)	Categorical	Modality of track, $1 = \text{major}, 0 = \text{minor}$
Tempo (1)	Numeric	Overall beats per minute (BPMs)
Danceability (2)	Numeric	Measures how suitable the track is for dancing
Loudness (3)	Numeric	Overall loudness of a track in decibels (dB)
Speechiness (4)	Numeric	Detects the presence of spoken works in a track
Acousticness (5)	Numeric	Measures the acoustic sound of a track
Liveness (6)	Numeric	Detects presence of an audience in the recording
Instrumentalness (7)	Numeric	Predicts presence of vocals in the track
Valence (8)	Numeric	Measure of musical positivity conveyed by a track
Duration (9)	Numeric	Length of track in milliseconds

The purpose of this study is to implement regression analysis techniques to find insights on what determines the overall energy in a track based on the overall audio features. Since the audio features are describing the entire track, linear regression is appropriate since there will be no dependence on time.

Research Questions

This project will be guided by two research questions that will be answered directly, along with a third research question for exploratory purposes of potential future research. These questions are:

1. How can linear regression be implemented in order to understand what determines the overall energy of a track in the simplest manner?
2. What variable has the largest effect on the overall energy in a track?
3. Disregarding simplicity, are there any interactions in the data that contribute to understanding the energy in a track? If so, are they intuitive?

Regression Method

In an attempt to find the desired insights from this data, a variety of regression methods can be used. To address the first research question, building a linear regression model using variable selection techniques can be used to find a starting point. The model will need to be assessed to ensure the proper assumptions are met before reliable insights can be drawn from it. If assumptions are not fulfilled, procedures such as model reduction, transformation, and influential point detection are options to aid in building a correctly specified model with

proper assumptions. Answering the first research question would come from building a first-order linear regression model where all estimated coefficients are significant.

The second question follows from the first since it entails finding the variable that has the largest effect on energy. In attempt to finding this answer, a potential solution is to use the first-order model built and for each predictor in the model, take it out and conduct a general F-test against the first-order model. Since the model was built to have all significant predictors, the F-test will result in the variable being significant. Instead what can be observed is the increase in residual sum of squares to determine which of the predictors can explain the most variation in energy.

The third question can be addressed by starting at the initial starting model found from variable selection procedures. A screening can be done to determine which interaction is the most significant. Only the most significant interaction effect will be added, and the screening process will be repeated. This can be continued until there are no significant interaction terms in the scope of the model. Then the model can be reduced if necessary and assessed for linear regression model assumptions.

Regression Analysis, Results, and Interpretation

Before any research questions can be addressed, an exploratory look at the data should take place to learn about the data. From observing a scatterplot matrix, there are two variables that have notable correlations with the response, being x_3 = loudness and x_5 = acousticness. Those are the only two correlations where the magnitude is above 0.5 in the entirety of the data.

Procedures for Research Question 1:

To find an initial starting model, the variable selection procedure of choice is best subsets regression because it gives an optimal model option for each number of predictors in the data. It also provides various criteria for a decision to be made including R_a^2, Bayesian Information Criterion, and Mallows' C_p statistic. The MSE can also be calculated since it provides the residual sums of squares. The results are organized in a tibble for easier comparison of all available models. The selected subset is that of 8 predictors, since it provides the ideal results for R_a^2, MSE, and BIC. Mallows' C_p statistic is not too far from the number of predictors either. The resulting model is:

```
# A tibble: 11 x 7
   nvars    R2 Ajd_R2   RSS     MSE   BIC    Cp
   <int> <dbl>  <dbl> <dbl>   <dbl> <dbl> <dbl>
 1     1 0.524  0.523  8.62 0.0125  -502. 409.
 2     2 0.608  0.607  7.10 0.0103  -630. 217.
 3     3 0.634  0.633  6.63 0.00960 -672. 158.
 4     4 0.679  0.677  5.81 0.00844 -756.  56.6
 5     5 0.698  0.696  5.47 0.00795 -792.  14.6
 6     6 0.701  0.699  5.41 0.00788 -792.   9.87
 7     7 0.703  0.700  5.38 0.00784 -790.   7.62
 8     8 0.704  0.701  5.36 0.00782 -786.   6.92
 9     9 0.704  0.701  5.36 0.00783 -780.   8.36
10    10 0.705  0.700  5.35 0.00784 -774.  10.1
11    11 0.705  0.700  5.35 0.00785 -768.  12
```

$$Y_i = \beta_0 + \beta_1 c_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8 + \epsilon$$

This will serve as a starting model for now, but linear regression model assumptions need to be met before it is sufficient.

Identify Potential Influential Points:

Before assessing model assumptions, influential points should be considered because they could potentially interfere with the model assumptions by heavily swaying predictions. Methods of identification will include leverages to account for predictor variables and studentized residuals to observe potential outliers in the response. An observation with a studentized residual where $|r_i| > 3$ will be considered an outlier and leverages will be considered at both twice and three times the mean. Since there are only a total of 29 potential influential points out of 694 observations, all will be dropped. After influential data points have been dropped, best subsets regression is repeated and the subset with 8 predictors was chosen again yielding the model:

```
Leverage > 3*mean : 14
Leverage > 2*mean : 28
Outliers          : 1
```

$$Y_i = \beta_0 + \beta_1 c_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_4 + \beta_6 x_5 + \beta_7 x_6 + \beta_8 x_8 + \epsilon$$

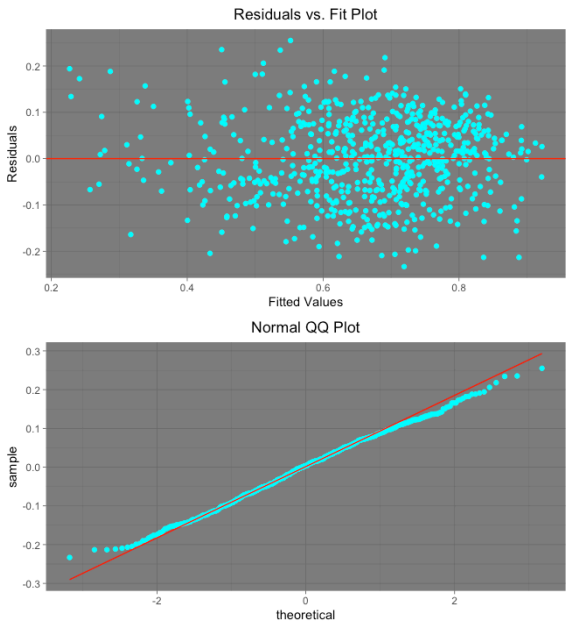
This model has an adjust coefficient of determination of $R_a^2 = 0.6828$, meaning that it can explain about 68.28% of the variation in energy. The summary table provides t-tests for the coefficient estimates for the hypotheses:

$$H_0: \beta_k = 0 \ \ vs. \ \ H_1: \beta_k \neq 0$$

Results deem the estimates for x_4 and x_6 being insignificant, meaning H_0 is accepted. The partial F-test is beneficial here, testing the hypothesis:

$$H_0: \beta_5 = \beta_7 = 0 \ \ vs. \ \ H_1: \beta_k \neq 0 \ (k = 5, 7)$$

The p-value from this test is $0.3606 > 0.05 = \alpha$ so the null hypothesis is not rejected and both variables are safely taken out of the model. The summary table of the reduced model indicates all numeric predictors are significant having a $R_a^2 = 0.6827$ thus, about 68.27% of the variation in energy can be explained by this model. Before this is deemed reliable, the assumptions must be assessed. The residuals vs. fit plot does not indicate any obvious issues with variance or linearity and the normal QQ plot seems to show a normal distribution for the residuals. This can be confirmed with the Shapiro-Wilk test, testing $H_0: e_i's \sim N(0, \sigma^2)$. The p-value is $0.1186 > 0.05 = \alpha$, so it can be concluded that the residuals are normally distributed, and the model is safe to use for inference and prediction.




```
Shapiro-Wilk Test:

Null: The variable is normally distributed
Alt : The variable is not normally distributed

        Shapiro-Wilk normality test

data:  augment(model)$.resid
W = 0.99627, p-value = 0.1186
```

Answering Research Question 1:

The first-order linear regression model with all significant predictors is the simplest way to try and understand what determines the energy in a track. This model is found to be:

$$Y_i = \beta_0 + \beta_1 c_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_5 + \beta_6 x_8 + \epsilon$$

95% Confidence Interval for Beta Coefficients:

	Pred	Low	Est	Up
	<chr>	<dbl>	<dbl>	<dbl>
1	(Intercept)	0.886	0.945	1.00
2	c12	-0.0110	0.0107	0.0325
3	c13	-0.00905	0.0157	0.0405
4	c14	-0.0428	-0.0183	0.00622
5	c15	-0.0407	-0.0187	0.00337
6	x1	0.0000262	0.000275	0.000524
7	x2	-0.239	-0.173	-0.107
8	x3	0.0373	0.0410	0.0448
9	x5	-0.276	-0.240	-0.205
10	x8	0.130	0.168	0.206

Observing the confidence intervals for the estimated coefficients, we can see that for all the numeric predictors the interval does not cover zero. This indicates that the estimates will be reliable and within these intervals 95% of the time. One thing to note is the genres, all corresponding intervals do include zero. This may be something to pay attention to for interaction terms.

Procedures for Research Question 2:

The proposed solution to determining which variable has the largest effect on the energy in a track is to conduct F-tests for a single parameter, but to observe the difference in residual sums of squares as a measure of the magnitude of effect. The difference in residual sums of squares for those that did not have the largest effect are shown in the table below:

c_1	x_1	x_2	x_5	x_8
0.0973	0.0358	0.2028	1.3354	0.5694

Answering the Question:

When x_3 is left out of the model, the difference in residual sums of squares is:

$$8.4306 - 4.9851 = 3.4455$$

Analysis of Variance Table

Model 1: y ~ c1 + x1 + x2 + x5 + x8
Model 2: y ~ c1 + x1 + x2 + x3 + x5 + x8

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	657	8.4306				
2	656	4.9851	1	3.4455	453.41	< 2.2e-16 ***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Thus, by a high margin, x_3 (loudness) is the variable that has the largest effect on energy.

Procedures for Research Question 3:

To answer the third research question using the proposed solution, this is a slightly rigorous task. The starting point is the initial starting model from best subsets regression after influential points were taken out. Recall the model:

$$Y_i = \beta_0 + \beta_1 c_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_4 + \beta_6 x_5 + \beta_7 x_6 + \beta_8 x_8 + \epsilon$$

Model:
y ~ c1 + x1 + x2 + x3 + x4 + x5 + x6 + x8

	Df	Sum of Sq	RSS	AIC	F value	Pr(>F)
<none>			4.9695	-3238.0		
c1:x4	4	0.154410	4.8151	-3251.1	5.2110	0.0003891 ***
x1:x4	1	0.006418	4.9631	-3236.9	0.8444	0.3584896
x2:x4	1	0.062740	4.9068	-3244.5	8.3494	0.0039861 **
x3:x4	1	0.001218	4.9683	-3236.2	0.1601	0.6891963
x4:x5	1	0.159414	4.8101	-3257.8	21.6412	3.981e-06 ***
x4:x6	1	0.002491	4.9671	-3236.4	0.3275	0.5673561
x4:x8	1	0.087004	4.8825	-3247.8	11.6361	0.0006867 ***

Since there are so many interactions to observe, the set of them for x_4 will be displayed. The most significant interaction term is that of $x_4 x_5$, so that is the first update.

The model from above is updated and the process repeats.

The model from the third update includes the interaction effects $(x_4, x_5), (x_2, x_8)$, and (x_2, x_3). The summary table for this update is shown to the right. Recall that the initial model had of $R_a^2 = 0.6828$. This model with interaction terms has $R_a^2 = 0.6982$ and all interaction effects are significant.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.9553183  0.0682683  13.994  < 2e-16 ***
c12          0.0076810  0.0109000   0.705  0.48126
c13          0.0175970  0.0126021   1.396  0.16308
c14         -0.0138000  0.0140158  -0.985  0.32518
c15         -0.0183631  0.0111329  -1.649  0.09954 .
x1           0.0002720  0.0001253   2.171  0.03028 *
x2          -0.1820846  0.0996317  -1.828  0.06807 .
x3           0.0591375  0.0084363   7.010 5.98e-12 ***
x4          -0.1231630  0.0563418  -2.186  0.02917 *
x5          -0.2943701  0.0226782 -12.980  < 2e-16 ***
x6           0.0241019  0.0327036   0.737  0.46140
x8           0.3881869  0.0792594   4.898 1.22e-06 ***
x4:x5        0.9447581  0.2060741   4.585 5.46e-06 ***
x2:x8       -0.3400200  0.1149546  -2.958  0.00321 **
x2:x3       -0.0282154  0.0125989  -2.240  0.02546 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08503 on 651 degrees of freedom
Multiple R-squared:  0.7045,    Adjusted R-squared:  0.6982
F-statistic: 110.9 on 14 and 651 DF,  p-value: < 2.2e-16
```

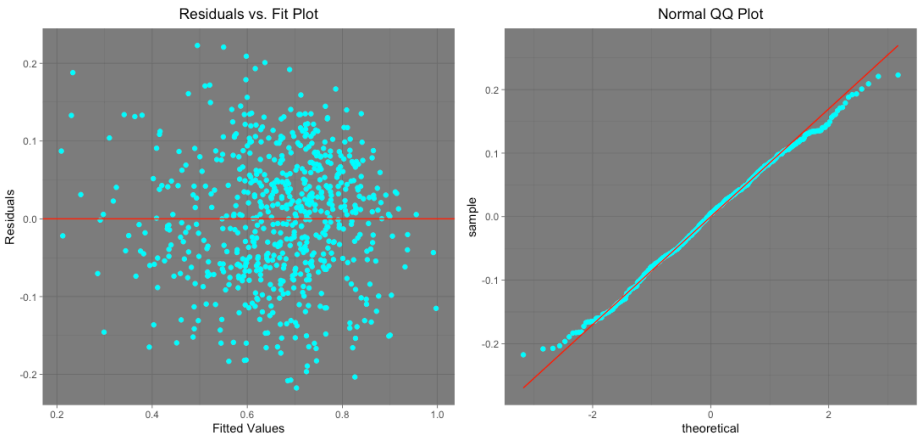
After being repeated numerous times, the screening indicated no significant interactions after the model was updated a total of 8 times. The 8 interaction effects present in the final model include:

$$(x_4, x_5), (x_2, x_8), (x_2, x_3), (x_3, x_4), (x_5, x_8), (c_1, x_8), (x_4, x_8), (c_1, x_4)$$

The variables x_1 and x_6 were tested for significance for significance via the partial F-test. The model was reduced because they did not test to be significant. The final linear regression model with interaction effects has $R_a^2 = 0.7121$, meaning that it can explain about 71.21% of the variation in energy.

Residual Analysis:

The residuals vs. fit plot and the normal QQ plot both appear to be well-behaved.



The Shapiro-Wilk test does not look needed, but to be safe, the test is done.

The p-value $= 0.08209 > 0.05 = \alpha$ thus, the null hypothesis is accepted, and the residuals are normally distributed.

```
Shapiro-Wilk Test:

Null: The variable is normally distributed
Alt : The variable is not normally distributed

        Shapiro-Wilk normality test

data:  augment(model)$.resid
W = 0.99594, p-value = 0.08209
```

Answer to Research Question:

A simple answer to this question is yes, there are interactions in the data that help understand the energy in a track. This is also intuitive if you think about the interactions. For

x_4x_5, this makes sense because music that has more acoustic sounds come more from bands that perform together. For x_2x_8 most people are likely more inclined to dance when they are in a positive environment. For x_2x_3, when it is thought of to go out dancing, the music will typically be louder. For x_3x_4, this is not as intuitive, this could potentially just be because if words are detected they have to be loud enough to be heard over the music. For x_4x_8 and x_5x_8, these kind of go hand in hand since it should be easier to convey a happy message with words, and lyrics tend to be more meaningful and present in general in acoustic music. The last two have to do with interactions with genres, meaning different genres will have different values for these features.

Thinking deeper than the question itself, note that all the interactions between two numeric variables have at least two interactions out of five variables interactions. The table below indicates that there could be some deeper more complex interactions in the data at a higher order or potentially hierarchical level.

(The numbers are the subscripts for the x_i)

2	**3**	**4**	**5**	**8**
3	2	3	4	2
8	4	5	8	4
		8		5

Notice how x_4 interacts with all others except for x_2, but they both interact with x_8. This likely indicates that there indeed more complexity to the interaction effects found between these variables.

Conclusion

The energy in music is a feature that has immense power, but why is that? This study was aimed at trying to understand what determines energy and how it varies. If it was possible to determine energy in music with a single feature, it would be the volume it is played at. The loudness was a huge factor in explaining energy, but it is far too complex to be described by one feature. It was found that the danceability, loudness, speechiness, acousticness, and valence all interacted with more than just one of each other to the point where they essentially all interact with each other, whether it be directly or indirectly. Further, the genre of music also plays a role in energy as well, because these features are a lot more or less likely to be as strong in some genres rather than others. Acousticness for example, would not likely be very strongly heard, if heard at all, in hip hop, pop, or electronic music.

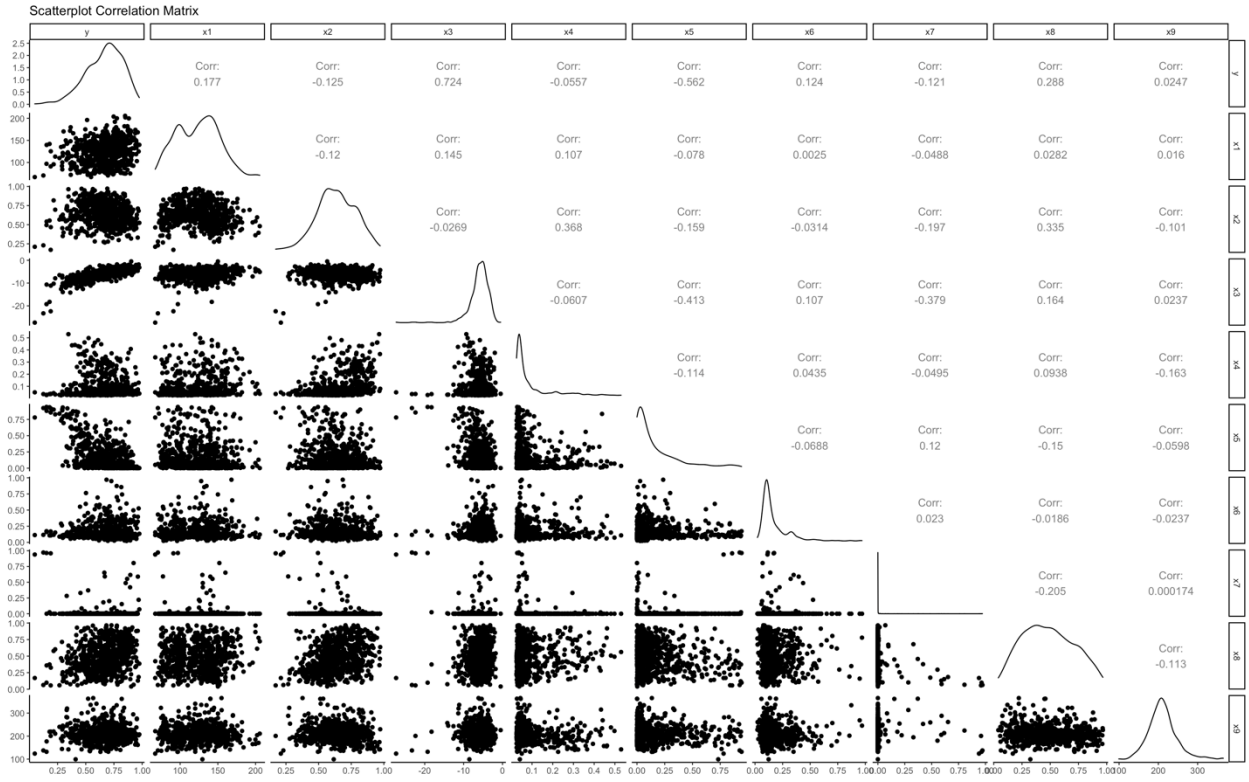
While there were many interesting observations made, it is very likely that there is far more exploration that can be done in regard to the energy in music. Interesting insights may come from exploring higher order interactions, or hierarchical relationships. Energy could potentially differ significantly when music is heard from an instrument rather than from an audio source.

Appendix

- **Section 1:** Extra Output
- **Section 2:** R Codes

Section 1:

Scatterplot Correlation Matrix



Genre Variable Means

```
# A tibble: 5 x 11
  c1       mY   mX1   mX2   mX3   mX4   mX5   mX6       mX7   mX8   mX9
  <fct> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>     <dbl> <dbl> <dbl>
1 1     0.669  119. 0.654 -6.46 0.0613 0.150 0.164 0.00675   0.615  215.
2 2     0.722  126. 0.563 -5.29 0.0452 0.190 0.154 0.0000113 0.547  207.
3 3     0.692  126. 0.550 -5.79 0.0683 0.170 0.201 0.0649    0.308  220.
4 4     0.632  126. 0.781 -6.08 0.197  0.102 0.181 0.00339   0.468  206.
5 5     0.606  119. 0.658 -6.21 0.0790 0.255 0.159 0.00693   0.469  206.
```

Summary of Final Model

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.958490   0.072737  13.177  < 2e-16 ***
c12         -0.017063   0.033618  -0.508 0.611943
c13          0.078185   0.031427   2.488 0.013107 *
c14         -0.053838   0.033864  -1.590 0.112361
c15         -0.037936   0.030827  -1.231 0.218916
x2          -0.251834   0.110477  -2.280 0.022964 *
x3           0.066195   0.008478   7.808 2.38e-14 ***
x4           1.088381   0.259865   4.188 3.20e-05 ***
x5          -0.188550   0.042976  -4.387 1.34e-05 ***
x8           0.484150   0.098287   4.926 1.07e-06 ***
x4:x5        0.939777   0.240455   3.908 0.000103 ***
x2:x8       -0.422737   0.138702  -3.048 0.002400 **
x2:x3       -0.046883   0.013473  -3.480 0.000536 ***
x3:x4        0.055888   0.019081   2.929 0.003522 **
x5:x8       -0.229084   0.086078  -2.661 0.007978 **
c12:x8       0.075004   0.053275   1.408 0.159653
c13:x8      -0.098761   0.067365  -1.466 0.143123
c14:x8       0.174753   0.059094   2.957 0.003218 **
c15:x8       0.068685   0.050571   1.358 0.174883
x4:x8       -0.748680   0.227716  -3.288 0.001065 **
c12:x4      -0.210310   0.269533  -0.780 0.435516
c13:x4      -0.528088   0.208494  -2.533 0.011550 *
c14:x4      -0.542693   0.169122  -3.209 0.001399 **
c15:x4      -0.326775   0.196197  -1.666 0.096292 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08304 on 642 degrees of freedom
Multiple R-squared:  0.7221,    Adjusted R-squared:  0.7121
F-statistic: 72.52 on 23 and 642 DF,  p-value: < 2.2e-16
```

Confidence Intervals for β Estimates of Final Model

```
95% Confidence Interval for Beta Coefficients:

# A tibble: 24 x 4
   Pred            Low     Est      Up
   <chr>         <dbl>   <dbl>   <dbl>
 1 (Intercept)  0.816   0.958   1.10
 2 c12         -0.0831 -0.0171  0.0490
 3 c13          0.0165  0.0782  0.140
 4 c14         -0.120  -0.0538  0.0127
 5 c15         -0.0985 -0.0379  0.0226
 6 x2          -0.469  -0.252  -0.0349
 7 x3           0.0495  0.0662  0.0828
 8 x4           0.578   1.09    1.60
 9 x5          -0.273  -0.189  -0.104
10 x8           0.291   0.484   0.677
11 x4:x5        0.468   0.940   1.41
12 x2:x8       -0.695  -0.423  -0.150
13 x2:x3       -0.0733 -0.0469 -0.0204
14 x3:x4        0.0184  0.0559  0.0934
15 x5:x8       -0.398  -0.229  -0.0601
16 c12:x8      -0.0296  0.0750  0.180
17 c13:x8      -0.231  -0.0988  0.0335
18 c14:x8       0.0587  0.175   0.291
19 c15:x8      -0.0306  0.0687  0.168
20 x4:x8       -1.20   -0.749  -0.302
21 c12:x4      -0.740  -0.210   0.319
22 c13:x4      -0.938  -0.528  -0.119
23 c14:x4      -0.875  -0.543  -0.211
24 c15:x4      -0.712  -0.327   0.0585
```

Observing Data for Largest Residuals

Average values for all data: *Genres:* 1: Alt/Reg | 2: Country 3: Electronic | 4: Hip Hop 5: Pop	# A tibble: 5 x 9 c1 n mY mPr mX2 mX3 mX4 mX5 mX8 <fct> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> 1 1 134 0.673 0.673 0.661 -6.37 0.0620 0.140 0.623 2 2 147 0.725 0.725 0.562 -5.23 0.0453 0.191 0.549 3 3 119 0.700 0.700 0.557 -5.26 0.0698 0.156 0.320 4 4 130 0.628 0.628 0.783 -6.13 0.196 0.101 0.465 5 5 136 0.607 0.607 0.660 -6.22 0.0797 0.257 0.471
Average values for data where the residuals are greater than 0.1:	# A tibble: 5 x 9 c1 n mY mPr mX2 mX3 mX4 mX5 mX8 <fct> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> 1 1 11 0.756 0.623 0.715 -7.18 0.063 0.173 0.631 2 2 15 0.798 0.663 0.591 -5.74 0.0475 0.307 0.511 3 3 12 0.822 0.693 0.576 -4.99 0.0825 0.219 0.368 4 4 15 0.746 0.611 0.751 -6.89 0.186 0.0866 0.507 5 5 22 0.722 0.591 0.684 -6.61 0.0730 0.273 0.501
Average values for data where the residuals are greater than 0.15:	# A tibble: 5 x 9 c1 n mY mPr mX2 mX3 mX4 mX5 mX8 <fct> <int> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> 1 1 2 0.744 0.555 0.845 -7.55 0.126 0.416 0.657 2 2 3 0.821 0.619 0.629 -5.89 0.0548 0.439 0.505 3 3 2 0.823 0.654 0.568 -5.50 0.0378 0.304 0.322 4 4 3 0.723 0.558 0.877 -7.35 0.258 0.0451 0.529 5 5 4 0.712 0.519 0.566 -7.24 0.0711 0.434 0.408

Section 2:

Functions:

```
load_libraries = function(){
  # loads necessary libraries
  library(tidyverse)
  library(GGally)
  library(ggpubr)
  library(broom)
  library(leaps)
}


load_and_process_data = function(path){
  # given a path, loads data and preprocesses variables
  # this is specific to this dataset
  data = read_csv(path)
```

```
  data = data %>% select(genre:duration)
  df = data %>% mutate(y = energy,
                       c1 = factor(genre), c2 = factor(mode),
                       x1 = tempo, x2 = danceability, x3 = loudness,
                       x4 = speechiness, x5 = acousticness,
                       x6 = liveness, x7 = instrumentalness,
                       x8 = valence, x9 = duration/1000)
  df = df %>% select(y:x9)
  df = df %>% mutate(c1 = if_else(c1=='Alternative/Reggae', '1',
                                  if_else(c1=='Country', '2',
                                          if_else(c1=='Electronic Dance', '3'
,
                                                  if_else(c1=='Hip Hop', '4',
'5')))))
  df = df %>% mutate(c1 = factor(c1)) %>% print()
  return(df)
}
var_map = function(){
  # reference map for what each variable is
  cat('Response:  Y  = Energy\n\nCategorical Predictors:',
      '\n\nc1 = Genre   (1: Alt/Reg, 2: Country, 3: Electronic, 4: Hip Hop, 5
: Pop)',
      '\nc2 = Mode\n\nNumeric Predictors:',
      '\n\nx1 = Tempo (BPMs)\nx2 = Danceability\nx3 = Loudness',
      '\nx4 = Speechiness\nx5 = Acousticness\nx6 = Liveness',
      '\nx7 = Instrumentalness\nx8 = Valence\nx9 = Duration (s)')
}

var_select_explore = function(data, reg_eq){
  # provides summary of linear regression
  # given a dataset and regression formula
  model = lm(reg_eq, data = data)
  summary(model)
}
var_select_bestsubsets_comps = function(X, y, df){
  # performs best subsets regression
  # creates dataset to easily compare models
  # based on the values extracted
  model_subsets = regsubsets(X, y, nvmax = ncol(df))
  mods = summary(model_subsets)
  rs = mods$rss
  n = nrow(df)
  nsubsets = length(rs)
  mses = c()
  for (i in 1:nsubsets){
    mse = rs[i]/(n-i-1)
    mses = append(mses, mse)
  }
  mod_comps = tibble(nvars = 1:nsubsets, R2 = mods$rsq, Ajd_R2 = mods$adjr2,
                     RSS = rs, MSE = mses, BIC = mods$bic, Cp = mods$cp)
```

```
  mod_comps %>% print()
}
var_select_bestsubsets_predictors = function(X, y, df){
  # shows which variables are in model subsets
  nv = ncol(df)
  model_subsets = regsubsets(X, y, nvmax = nv)
  mods = summary(model_subsets)
  mods$which
}

assess_res_fit = function(model){
  # Residuals vs. Fit Plot
  augment(model) %>% ggplot(., aes(x=.fitted, y=.resid)) +
    geom_point(color = 'cyan') + geom_hline(yintercept = 0, color = 'red') +
    labs(title = 'Residuals vs. Fit Plot', x = 'Fitted Values', y = 'Residual
s') +
    theme_dark() + theme(plot.title = element_text(hjust = 0.5))
}
assess_norm_qq = function(model){
  # Normal QQ Plot
  augment(model) %>% ggplot(., aes(sample = .resid)) +
    stat_qq(color = 'cyan') + stat_qq_line(color = 'red') + ggtitle('Normal Q
Q Plot') +
    theme_dark() + theme(plot.title = element_text(hjust = 0.5))
}
assess_outliers_leverage = function(model, df){
  # finds leverages and outliers
  # returns dataframe with values to determine both
  df$hv = hatvalues(model)
  df$rs = rstandard(model)
  cat('Leverage > 3*mean :', length(which(df$hv > (3*sum(df$hv)/nrow(df)))),
      '\nLeverage > 2*mean :', length(which(df$hv > (2*sum(df$hv)/nrow(df))))
,
      '\nOutliers          :', length(which(abs(df$rs) > 3)),
      '\n')
  return(df)
}

test_LRT = function(alpha, full, reduced){
  # conducts likelihood ratio test to determine better fitting model
  k = length(full$coefficients) - length(reduced$coefficients)
  lr= -2*(logLik(reduced) - logLik(full))
  p = pchisq(lr, df = k, lower.tail = F)
  cat('\nLikelihood Ratio Test:',
      '\n\nNull: Smaller model has a better fit',
      '\nAlt : Larger model has a better fit',
      '\n\nTest Statistic:', lr, '\nDegrees of Freedom:', k)
  if (p < alpha){
    cat('\nThe p-value =', p, '<', alpha, '= alpha',
        '\n\nReject Null, choose larger model\n')
```

```
  }else{
    cat('\nThe p-value =', p, '>', alpha, '= alpha',
        '\n\nAccept Null, choose smaller model\n')
  }
}
test_shapiro = function(model){
  # Shapiro-Wilk Test for Normality
  cat('\nShapiro-Wilk Test:\n\nNull: The variable is normally distributed',
      '\nAlt : The variable is not normally distributed\n')
  shapiro.test(augment(model)$.resid)
}
inf_confints = function(model, level = 0.95){
  coeffs = as.data.frame(model$coefficients)
  ci = tibble(Pred = rownames(coeffs),
              Low = confint(model, level = level)[,1],
              Est = model$coefficients,
              Up = confint(model, level = level)[,2])
  cat('\n\n95% Confidence Interval for Beta Coefficients:\n\n')
  ci %>% print(n = Inf)
}
```

Analysis:

```
load_libraries()
data = load_and_process_data('audio_features.csv')

var_map()

data %>%
  select(y, x1:x9) %>%
  ggpairs(title = 'Scatterplot Correlation Matrix', progress = F) +
  theme_classic()

data %>%
  group_by(c1) %>%
  summarise(mY = mean(y),
            mX1 = mean(x1), mX2 = mean(x2), mX3 = mean(x3),
            mX4 = mean(x4), mX5 = mean(x5), mX6 = mean(x6),
            mX7 = mean(x7), mX8 = mean(x8), mX9 = mean(x9))

attach(data)
X = cbind(c1, c2, x1, x2, x3, x4, x5, x6, x7, x8, x9)
var_select_bestsubsets_comps(X, y, data)

var_select_bestsubsets_predictors(X, y, data)
detach(data)

fit = lm(y ~ c1 + x1 + x2 + x3 + x5 + x6 + x7 + x8, data = data)
summary(fit)

df = assess_outliers_leverage(fit, data)
```

```r
df = df %>% filter(hv < 2*sum(hv)/nrow(df), abs(rs) < 3)
df = df %>% select(y, c1, x1:x8) %>% print()

attach(df)
X = cbind(c1, x1, x2, x3, x4, x5, x6, x7, x8)
var_select_bestsubsets_comps(X, y, df)

var_select_bestsubsets_predictors(X, y, df)
detach(df)

fit = lm(y ~ c1 + x1 + x2 + x3 + x4 + x5 + x6 + x8, data = df)
summary(fit)

assess_res_fit(fit)

assess_norm_qq(fit)

test_shapiro(fit)

assess_outliers_leverage(fit, df) %>% filter(hv > 3*sum(hv)/nrow(df))

df %>% ggplot(aes(x = x7, y = augment(fit)$.resid)) +
  geom_point(color = 'blue') + geom_hline(yintercept = 0, color = 'red') +
  labs(title='Residuals vs. x7', x='x7', y='Residuals') + theme_classic()

anova(fit)

rfit = lm(y ~ c1 + x1 + x2 + x3 + x5 + x8, data = df)
summary(rfit)

anova(rfit)

test_LRT(0.05, fit, rfit)

anova(rfit, fit)

p1 = assess_res_fit(rfit)
p2 = assess_norm_qq(rfit)
ggarrange(p1, p2, nrow = 1, ncol = 2)

test_shapiro(rfit)

model = lm(y ~ c1 + x1 + x2 + x3 + x5 + x8, data = df)
summary(model)

inf_confints(model, 0.95)

augment(model)

augment(model) %>% filter(.resid > 0.15)

augment(model) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPr = mean(.fitted),
```

```
            mX1 = mean(x1), mX2 = mean(x2), mX3 = mean(x3),
            mX5 = mean(x5), mX8 = mean(x8))

augment(model) %>%
  filter(.resid > 0.1) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPred = mean(.fitted),
            mX1 = mean(x1), mX2 = mean(x2), mX3 = mean(x3),
            mX5 = mean(x5), mX8 = mean(x8))

augment(model) %>%
  filter(.resid > 0.15) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPred = mean(.fitted),
            mX1 = mean(x1), mX2 = mean(x2), mX3 = mean(x3),
            mX5 = mean(x5), mX8 = mean(x8))

reg_form = y ~ x1 + x2 + x3 + x5 + x8
anova(lm(reg_form, data = df), model)

reg_form = y ~ c1 + x2 + x3 + x5 + x8
anova(lm(reg_form, data = df), model)

reg_form = y ~ c1 + x1 + x3 + x5 + x8
anova(lm(reg_form, data = df), model)

reg_form = y ~ c1 + x1 + x2 + x5 + x8
anova(lm(reg_form, data = df), model)

reg_form = y ~ c1 + x1 + x2 + x3 + x8
anova(lm(reg_form, data = df), model)

reg_form = y ~ c1 + x1 + x2 + x3 + x5
anova(lm(reg_form, data = df), model)

ifit = list()

add1(fit, ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, test = '
F')
add1(fit, ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, test = '
F')
add1(fit, ~.+ x2*c1 + x2*x1 + x2*x3 + x2*x4 + x2*x5 + x2*x6 + x2*x8, test = '
F')
add1(fit, ~.+ x3*c1 + x3*x1 + x3*x2 + x3*x4 + x3*x5 + x3*x6 + x3*x8, test = '
F')
add1(fit, ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x3 + x4*x5 + x4*x6 + x4*x8, test = '
F')
add1(fit, ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x4 + x5*x6 + x5*x8, test = '
F')
add1(fit, ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, test = '
F')
```

```
add1(fit, ~.+ x8*c1 + x8*x1 + x8*x2 + x8*x3 + x8*x4 + x8*x5 + x8*x6, test = '
F')

ifit = append(ifit, list(update(fit, ~.+ x4*x5)))
summary(ifit[[1]])

add1(ifit[[1]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, te
st = 'F')
add1(ifit[[1]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[1]], ~.+ x2*c1 + x2*x1 + x2*x3 + x2*x4 + x2*x5 + x2*x6 + x2*x8, te
st = 'F')
add1(ifit[[1]], ~.+ x3*c1 + x3*x1 + x3*x2 + x3*x4 + x3*x5 + x3*x6 + x3*x8, te
st = 'F')
add1(ifit[[1]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x3 + x4*x6 + x4*x8, test = 'F'
)
add1(ifit[[1]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6 + x5*x8, test = 'F'
)
add1(ifit[[1]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[1]], ~.+ x8*c1 + x8*x1 + x8*x2 + x8*x3 + x8*x4 + x8*x5 + x8*x6, te
st = 'F')

ifit = append(ifit, list(update(ifit[[1]], ~.+ x2*x8)))
summary(ifit[[2]])

add1(ifit[[2]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, te
st = 'F')
add1(ifit[[2]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[2]], ~.+ x2*c1 + x2*x1 + x2*x3 + x2*x4 + x2*x5 + x2*x6, test = 'F'
)
add1(ifit[[2]], ~.+ x3*c1 + x3*x1 + x3*x2 + x3*x4 + x3*x5 + x3*x6 + x3*x8, te
st = 'F')
add1(ifit[[2]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x3 + x4*x6 + x4*x8, test = 'F'
)
add1(ifit[[2]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6 + x5*x8, test = 'F'
)
add1(ifit[[2]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[2]], ~.+ x8*c1 + x8*x1 + x8*x3 + x8*x4 + x8*x5 + x8*x6, test = 'F'
)

ifit = append(ifit, list(update(ifit[[2]], ~.+ x2*x3)))
summary(ifit[[3]])

add1(ifit[[3]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, te
st = 'F')
add1(ifit[[3]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[3]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
```

```
add1(ifit[[3]], ~.+ x3*c1 + x3*x1 + x3*x4 + x3*x5 + x3*x6 + x3*x8, test = 'F'
)
add1(ifit[[3]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x3 + x4*x6 + x4*x8, test = 'F'
)
add1(ifit[[3]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6 + x5*x8, test = 'F'
)
add1(ifit[[3]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[3]], ~.+ x8*c1 + x8*x1 + x8*x3 + x8*x4 + x8*x5 + x8*x6, test = 'F'
)

ifit = append(ifit, list(update(ifit[[3]], ~.+ x3*x4)))
summary(ifit[[4]])

add1(ifit[[4]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, te
st = 'F')
add1(ifit[[4]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[4]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
add1(ifit[[4]], ~.+ x3*c1 + x3*x1 + x3*x5 + x3*x6 + x3*x8, test = 'F')
add1(ifit[[4]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x6 + x4*x8, test = 'F')
add1(ifit[[4]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6 + x5*x8, test = 'F'
)
add1(ifit[[4]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[4]], ~.+ x8*c1 + x8*x1 + x8*x3 + x8*x4 + x8*x5 + x8*x6, test = 'F'
)

ifit = append(ifit, list(update(ifit[[4]], ~.+ x5*x8)))
summary(ifit[[5]])

add1(ifit[[5]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6 + c1*x8, te
st = 'F')
add1(ifit[[5]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[5]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
add1(ifit[[5]], ~.+ x3*c1 + x3*x1 + x3*x5 + x3*x6 + x3*x8, test = 'F')
add1(ifit[[5]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x6 + x4*x8, test = 'F')
add1(ifit[[5]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6, test = 'F')
add1(ifit[[5]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[5]], ~.+ x8*c1 + x8*x1 + x8*x3 + x8*x4 + x8*x6, test = 'F')

ifit = append(ifit, list(update(ifit[[5]], ~.+ c1*x8)))
summary(ifit[[6]])

add1(ifit[[6]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6, test = 'F'
)
add1(ifit[[6]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[6]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
```

```
add1(ifit[[6]], ~.+ x3*c1 + x3*x1 + x3*x5 + x3*x6 + x3*x8, test = 'F')
add1(ifit[[6]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x6 + x4*x8, test = 'F')
add1(ifit[[6]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6, test = 'F')
add1(ifit[[6]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[6]], ~.+ x8*x1 + x8*x3 + x8*x4 + x8*x6, test = 'F')

ifit = append(ifit, list(update(ifit[[6]], ~.+ x4*x8)))
summary(ifit[[7]])

add1(ifit[[7]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x4 + c1*x5 + c1*x6, test = 'F'
)
add1(ifit[[7]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[7]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
add1(ifit[[7]], ~.+ x3*c1 + x3*x1 + x3*x5 + x3*x6 + x3*x8, test = 'F')
add1(ifit[[7]], ~.+ x4*c1 + x4*x1 + x4*x2 + x4*x6, test = 'F')
add1(ifit[[7]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6, test = 'F')
add1(ifit[[7]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[7]], ~.+ x8*x1 + x8*x3 + x8*x6, test = 'F')

ifit = append(ifit, list(update(ifit[[7]], ~.+ c1*x4)))
summary(ifit[[8]])

add1(ifit[[8]], ~.+ c1*x1 + c1*x2 + c1*x3 + c1*x5 + c1*x6, test = 'F')
add1(ifit[[8]], ~.+ x1*c1 + x1*x2 + x1*x3 + x1*x4 + x1*x5 + x1*x6 + x1*x8, te
st = 'F')
add1(ifit[[8]], ~.+ x2*c1 + x2*x1 + x2*x4 + x2*x5 + x2*x6, test = 'F')
add1(ifit[[8]], ~.+ x3*c1 + x3*x1 + x3*x5 + x3*x6 + x3*x8, test = 'F')
add1(ifit[[8]], ~.+ x4*x1 + x4*x2 + x4*x6, test = 'F')
add1(ifit[[8]], ~.+ x5*c1 + x5*x1 + x5*x2 + x5*x3 + x5*x6, test = 'F')
add1(ifit[[8]], ~.+ x6*c1 + x6*x1 + x6*x2 + x6*x3 + x6*x4 + x6*x5 + x6*x8, te
st = 'F')
add1(ifit[[8]], ~.+ x8*x1 + x8*x3 + x8*x6, test = 'F')

rmod = lm(y ~ c1 + x2 + x3 + x4 + x5 + x8 + x4:x5 + x2:x8 + x2:x3 +
              x3:x4 + x5:x8 + c1:x8 + x4:x8 + c1:x4, data = df)
anova(rmod, ifit[[8]])

summary(rmod)

model = lm(y ~ c1 + x2 + x3 + x4 + x5 + x8 + x4:x5 + x2:x8 + x2:x3 +
               x3:x4 + x5:x8 + c1:x8 + x4:x8 + c1:x4, data = df)
summary(model)

assess_res_fit(model)

assess_norm_qq(model)

test_shapiro(model)
```

```r
inf_confints(model, 0.95)

augment(model) %>% filter(.resid > 0.15)

augment(model) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPr = mean(.fitted),
            mX2 = mean(x2), mX3 = mean(x3),
            mX4 = mean(x4), mX5 = mean(x5), mX8 = mean(x8))

augment(model) %>%
  filter(.resid > 0.1) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPred = mean(.fitted),
            mX2 = mean(x2), mX3 = mean(x3),
            mX4 = mean(x4), mX5 = mean(x5), mX8 = mean(x8))

augment(model) %>%
  filter(.resid > 0.15) %>%
  group_by(c1) %>%
  summarise(n = n(), mY = mean(y), mPred = mean(.fitted),
            mX2 = mean(x2), mX3 = mean(x3),
            mX4 = mean(x4), mX5 = mean(x5), mX8 = mean(x8))
```