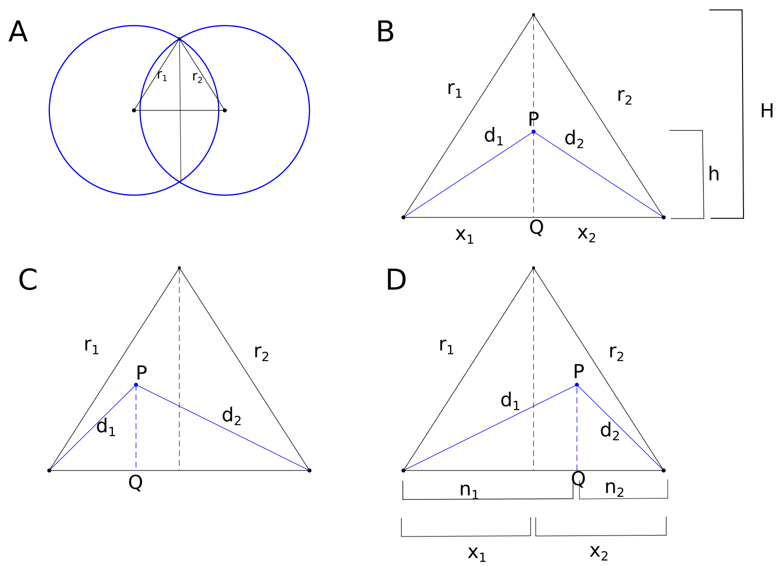

Figure 1: Examples of two intersecting circles and the division of the interect. A) The two circles, showing the triangle formed by the two centers and one intersection point. B) The triangle from A, with a random point chosen on the intersection line. C) The triangle from A, with a random point closer to circle 1. D) The triangle from A, with a random point closer to circle 2.

Definitions:

Circles 1 and 2 intersect with an overlapping region as shown in Figure 1A. A triangle can be formed from the two center points and one of the intersection points. If a random point is chosen in the intersection area, one wants to determine where it is relative to the center line. It can be on the line (Fig 1B), on the left of the line (Fig 1C) or on the right of the line (Fig 1D).

r_1 = the radius of circle 1.

r_2 = the radius of circle 2.

H = The the distance from the intersection point to the line between the center of the two circles.

h = The distance from the random point to the line between the center of the two circles.

d_1 = The distance from the point to the center of circle 1.

d_2 = The distance from the point to the center of circle 2.

n_1 = The distance from line PQ to the center of circle 1.

n_2 = The distance from line PQ to the center of circle 2.

In all cases

$$H^2 + x_1^2 = r_1^2 \tag{1}$$
$$H^2 + x_2^2 = r_2^2 \tag{2}$$
$$h^2 + n_1^2 = d_1^2 \tag{3}$$
$$h^2 + n_2^2 = d_2^2 \tag{4}$$

These can be simplified to :

$$n_1^2 - n_2^2 = d_1^2 - d_2^2 \tag{5}$$
$$x_1^2 - x_2^2 = r_1^2 - r_2^2 \tag{6}$$

In case B, where the random point falls on the line between the two circles, then $x_1 = n_1$ and $x_2 = n_2$ and Eq. 5 and 6 can be rearranged to:

$$d_1^2 - d_2^2 = r_1^2 - r_2^2 \tag{7}$$
$$d_1^2 - r_1^2 = d_2^2 - r_2^2 \tag{8}$$

In case C, the random point is in the left side of the bisected region, making the following true:

$$n_1 < x_1 \tag{9}$$
$$n_2 > x_2 \tag{10}$$

To simplify Eq 5 + 6 we need to find a relationship between the left sides of the equations. We also know that $x_1 + x_2 = n_1 + n_2$

$$n_1^2 - n_2^2 \stackrel{?}{=} x_1^2 - x_2^2 \tag{11}$$
$$(n_1 + n_2)(n_1 - n_2) \stackrel{?}{=} (x_1 + x_2)(x_1 - x_2) \tag{12}$$
$$(n_1 + n_2)(n_1 - n_2) \stackrel{?}{=} (n_1 + n_2)(x_1 - x_2) \tag{13}$$
$$(n_1 - n_2) \stackrel{?}{=} (x_1 - x_2) \tag{14}$$
$$n_1 - x_1 \stackrel{?}{=} n_2 - x_2 \tag{15}$$

From Eq 9 and 10, we know that $n_1 - x_1$ is negative and $n_2 - x_2$ is positive, therefore

$$n_1 - x_1 < n_2 - x_2 \tag{16}$$

If we substitute that back into Eq 11 then:

$$n_1^2 - n_2^2 < x_1^2 - x_2^2 \tag{17}$$

Therefore

$$d_1^2 - d_2^2 < r_1^2 - r_2^2 \tag{18}$$
$$d_1^2 - r_1^2 < d_2^2 - r_2^2 \tag{19}$$

Using the same logic, if the point is on the right side of the dividing line then

$$d_1^2 - r_1^2 > d_2^2 - r_2^2 \tag{20}$$

Conclusion: Using equations 8, 19 and 20, we can determine which side of the line a given point is on.