

# Galloway-Final Project 5131-Boiler Drum Modeling and Simulation

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December 1, 2019

## 1 Intro

### 1.1 Background

In operation of fired steam boilers, the water reservoir for production of steam is called the steam drum. Safety and reliability concerns require that the level of liquid in this reservoir be kept under tight control as too high a level will cause liquid phase water to flood the steam delivery network, and too low a level will result in the boiler tubes, which absorb the heat from the furnace/firebox section, to become exposed and fail due to temperatures exceeding design.

The level in the drum displays interesting dynamics under certain circumstances, adding water from the feed system to make up the mass lost through steam production can sometimes actually cause the level of the drum to decrease for a period of time before moving in the expected direction. Under the opposite conditions where steam is removed quicker than makeup water is added the liquid in the steam drum will increase prior to moving in the expected direction of decreasing. This phenomenon is called "boiler drum shrink and swell" and is the topic of interest.

### 1.2 Topic

During normal operations of industrial boilers, it is difficult if not impossible to examine the effects of varying inputs and outputs to the system individually. Here we examine the effects of mass and energy imbalances on the liquid and vapor phases of the steam drum for a boiler model tailored to a region of pressures typifying industrial applications. This includes:

- Positive and Negative System Mass Imbalances with the Energy Balance Held Equal
- Positive and Negative System Energy Imbalances with the Mass Balance Held Equal

The pressure region selected is 1250 psia to 1750 psia.

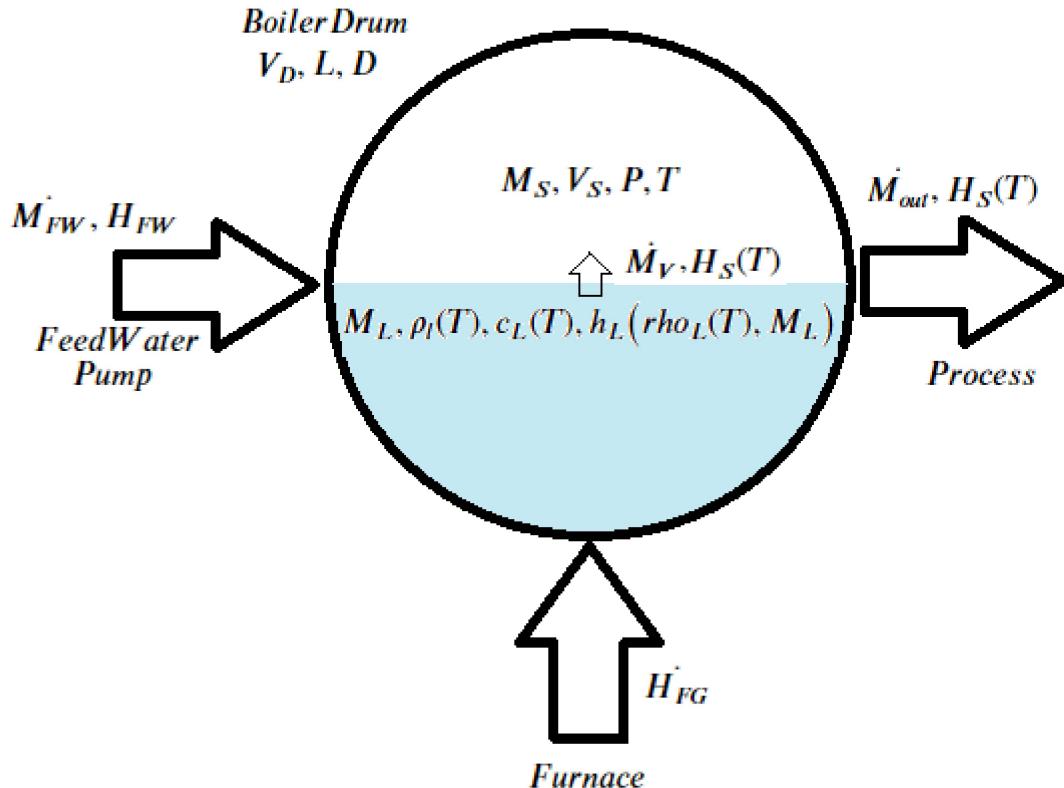
## 2 Model Development

### 2.1 Boiler Drum Level Physical System Definition

#### 2.1.1 Simplified System

To simplify the system we will omit the riser section (aka upcomers), downcomers and mud drum drawing a system boundary around just the drum portion of the process.

## Simplified Boiler Drum System



### 2.1.2 Variable Definitions

$\dot{H}_{FG}$  = Enthalpy Flow From Furnace Combustion [ $\frac{btu}{s}$ ]

$\dot{M}_{FW}$  = Mass Flow of Feed Water [psia]

$H_{FW}$  = Specific Enthalpy of Boiler Feed Water From Pump [ $\frac{btu}{lb}$ ]

$M_L$  = Mass of Water in Boiler Drum [lbs]

$\rho_L$  = Density of Liquid in Boiler Drum [ $\frac{lbs}{ft^3}$ ]

$c_L$  = Specific Heat Capacity of Water [ $\frac{btu}{lb - ^\circ F}$ ]

$\dot{M}_{out}$  = Mass Flow of Steam to Process [ $\frac{lbs}{s}$ ]

$H_S$  = Specific Enthalpy of Steam in the Boiler Drum [ $\frac{btu}{lb}$ ]

$V_D$  = Total Volume of Boiler Drum [ $ft^3$ ]

$L$  = Length of Boiler Drum [ft]

$D$  = Diameter of Boiler Drum [ft]

$M_S$  = Mass of Steam in Boiler Drum [lbs]

$V_S$  = Volume of Steam in Boiler Drum [ $ft^3$ ]

$P$  = Pressure in Boiler Drum [psia]

$T$  = Temperature of Contents of Boiler Drum [ $^\circ F$ ]

$\dot{M}_V$  = Mass Flow of Steam from Liquid Phase to Vapor Phase [psia]

### 2.1.3 Fitting Curves to Tabular Data

Using the `iapws` library for python, polynomial curves were fit to the **IAPWS97** table data as well as specific heat capacity from a table on Engineering Toolbox [2]. The results of those curve fits are here and of the form  $f(x) = p2 * x^2 + p1 * x + p0$ .

Table: Resulting Parameters from Curve Fitting

Curve	p2	p1	p0
$\rho_L$	0.000000	-0.080402	90.561163
$H_S$	0.000000	-0.624383	1540.647940
$c_L$	0.000052	-0.057233	16.979969
$T(P)$	0.000000	0.089223	462.004416

## 2.2 Boiler Drum First Principles Modeling

### 2.2.1 Assumptions

- Thermodynamic equilibrium exists between the liquid and vapor phases at all times. The consequence of which is the liquid and vapor temperature are equal and the energy balance for the vapor phase is not necessary. [1]
- Initial conditions will be such that the system is already at saturation temperature and there will be no heat-up stage required.

### 2.2.2 System Equations

Working from the mass and energy balances, other physical relationships and the curves fit to various table values for the region of interest we have,

#### Mass and Energy Balances

$$\frac{d(c_L M_L T)}{dt} = H_{FW} \dot{M}_{FW} + H_{FG} - H_S \dot{M}_V \quad (1)$$

$$\frac{dM_L}{dt} = \dot{M}_{FW} - \dot{M}_V \quad (2)$$

$$\frac{dM_S}{dt} = \dot{M}_V(T) - \dot{M}_{out} \quad (3)$$

## Curve Fit Equations

$$c_L(T) = \alpha_2 T^2 + \alpha_1 T + \alpha_0 \quad (4)$$

$$\rho_L(T) = \beta_1 T + \beta_0 \quad (5)$$

$$H_S(T) = \lambda_1 T + \lambda_0 \quad (6)$$

$$T(P) = \gamma_1 P + \gamma_0 \implies P(T) = \frac{T - \gamma_0}{\gamma_1} \quad (7)$$

## Physical and Thermodynamic Relationships

$$V_S = V_D - \frac{M_L}{\rho_L} \quad (8)$$

$$c_1 P V_S = M_S R_S (T + c_2) \implies M_L(M_S, T) = \rho_L(T) \left( V_D - \frac{M_S R_S (T + c_2)}{c_1 P(T)} \right) \quad (9)$$

$$\dot{M}_V(T) = \frac{\dot{H}_{FG}}{H_S(T) - H_{FW}} \implies \frac{\dot{H}_{FG}}{\lambda_1 T + \lambda_0 - H_{FW}} \quad (10)$$

Where  $\alpha_i$ ,  $\beta_i$ ,  $\lambda_i$  and  $\gamma_i$  are the constants from the curve polynomial fits to the tabular data,  $c_1 = 144 \left[ \frac{ft^2}{in^2} \right]$ ,  $c_2 = 459.67$  to convert to rankine, and  $R_S$  is the specific gas constant of water vapor  $85.556 \left[ \frac{lb f - ft}{lb m - R} \right]$

**Solving For a Final System in  $T, M_S$**  Substituting eq. 2, eq. 5, eq. 7 and eq. 9 into equation eq. 1 gives,

$$\frac{d(c_L M_L T)}{dt} = H_{FW} \dot{M}_{FW} + \dot{H}_{FG} - H_S \dot{M}_V \quad (1)$$

Let,  $f(T) = c_L(T) * T$ , then,

$$\begin{aligned} \implies \frac{d(M_L f(T))}{dt} &= M_L(M_S, T) \frac{df(T)}{dT} \frac{dT}{dt} + f(T) \frac{dM_L}{dt} \\ &= M_L(M_S, T) \frac{df(T)}{dT} \frac{dT}{dt} + f(T)(\dot{M}_{FW} - \dot{M}_V(T)) \end{aligned}$$

Now,

$$f(T) = c_L(T) * T = (\alpha_2 T^2 + \alpha_1 T + \alpha_0) T = \alpha_2 T^3 + \alpha_1 T^2 + \alpha_0 T \quad (11)$$

$$\text{and, } \frac{df(T)}{dT} = 3\alpha_2 T^2 + 2\alpha_1 T + \alpha_0 \quad (12)$$

And finally,

$$\frac{dT}{dt} = \frac{H_{FW} \dot{M}_{FW} + \dot{H}_{FG} - H_S(T) \dot{M}_V(T) - f(T)(\dot{M}_{FW} - \dot{M}_V(T))}{M_L(M_S, T) \frac{df(T)}{dT}} \quad (1a)$$

Thus from the synthesis of the system equations we arrive at a nonlinear, autonomous system of differential equations in  $M_S$  and  $T$ . All other variables of interest can be calculated after this system is solved.

## 2.3 Final System

$$\boxed{\frac{dT}{dt} = \frac{H_{FW} \dot{M}_{FW} + \dot{H}_{FG} - H_S(T) \dot{M}_V(T) - f(T)(\dot{M}_{FW} - \dot{M}_V(T))}{M_L(M_S, T) \frac{df(T)}{dT}}} \quad (1a)$$

$$\boxed{\frac{dM_S}{dt} = \dot{M}_V(T) - \dot{M}_{out}} \quad (3)$$

## 3 Simulation

### 3.1 Boiler Dimensions and Initial Conditions

First, to check the model, we fix initial conditions to match a boiler the student has worked on in the past. With the boiler at a quiescent point, the energy input from  $\dot{H}_{FG}$  will be equal to the amount of energy to heat the incoming feedwater to saturation temperature, plus the heat required to vaporize the liquid to replace the outgoing steam. This will allow us to compute a suitable  $\dot{H}_{FG}$  for balanced operation. The masses are trivial to balance.

Figure: Ammonia Plant: Enid, Oklahoma



Table: Parameters Taken From Example Boiler

Diameter	Length	Total Volume
6.0	42.0	1187.522

## 3.2 Simulation Case 1: Quiescent Operation

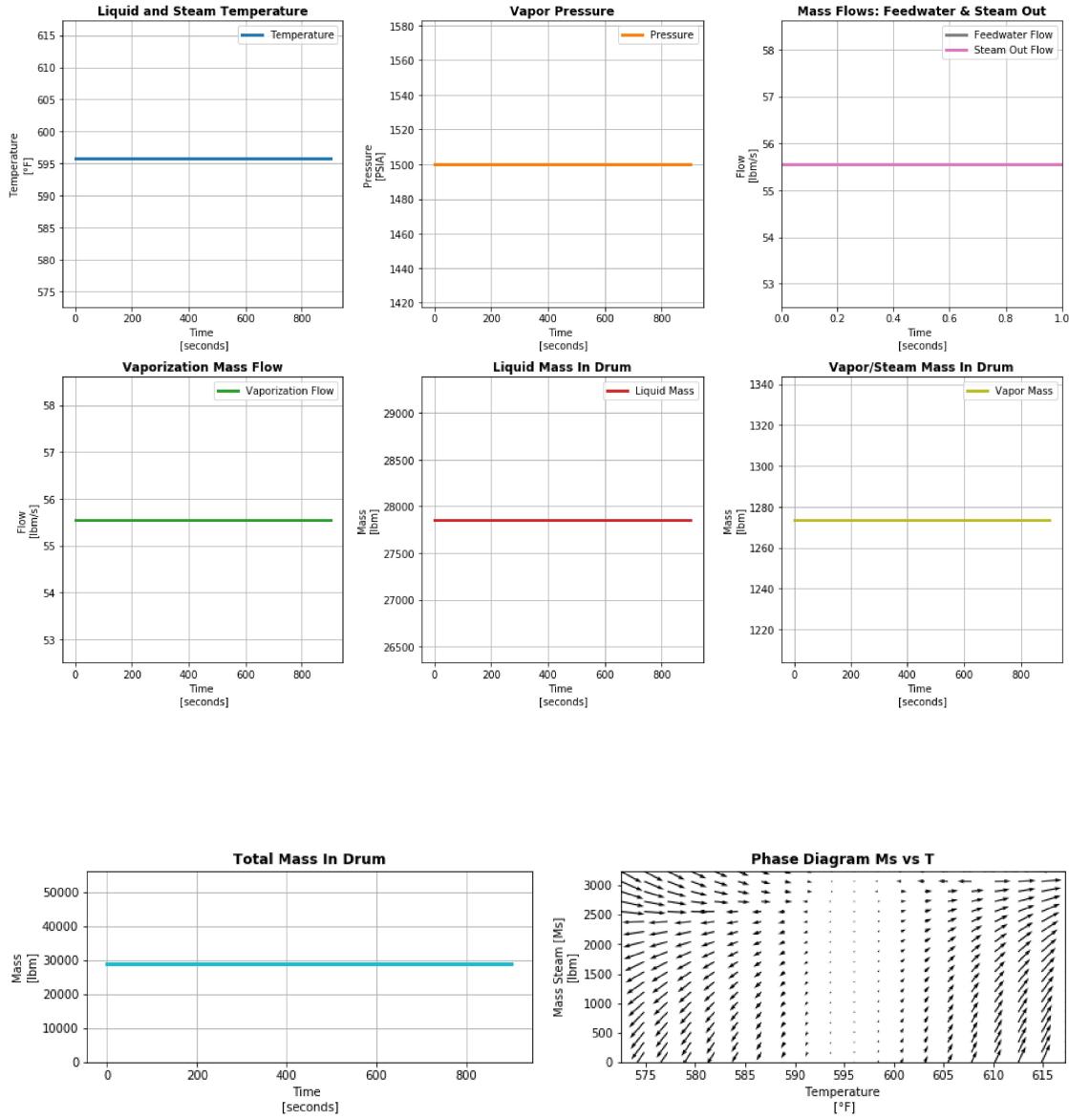
### 3.2.1 Initial Conditions for Case 1: Quiescent Operating Point

The energy and mass inputs/outputs were balanced such that the system should remain at initial conditions during the simulation.

Case 1: Quiescent Operating Point Parameters

Variable	Value	Units
$P_0$	1500.000000	PSIA
$T_0$	595.838660	°F
$M_{s0}$	1273.724450	lbm
$H_{FW}$	178.163918	btu/lbm
$\dot{M}_{FW}$	55.555556	lbm/s
$\dot{H}_{FG}$	55025.128314	btu/s
$\dot{M}_{out}$	55.555556	lbm/s

### 3.2.2 Results for Case 1: Quiescent Operating Point



As expected, with the system in a stable and balanced operational point the flows, masses, temperature and pressure all remain static. This verifies the model is working in the balanced case. There are nullclines at the expected current temperature of  $\approx 596F$  but unexpectedly, another nullcline starting around  $(570F, 2500lbm)$  and curving gently to the point around  $(610F, 3000lbm)$ . This makes sense, as for any given temperature, there would be a mass above which there is not enough energy to keep the vaporization flow high enough to replenish that which is lost via  $\dot{M}_{out}$ . Similarly, as the temperature increases the amount of energy in the system will be higher and simply convert all the mass to steam/vapor.

### 3.3 Simulation Case 2: Negative Mass Imbalance

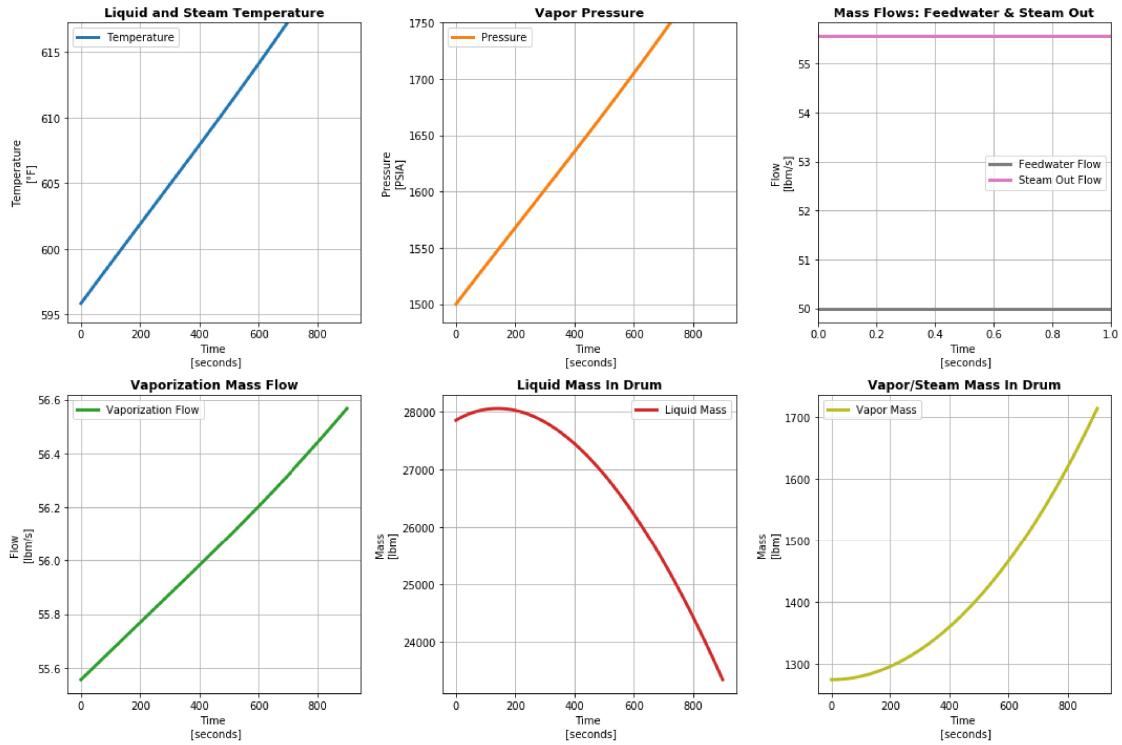
For this case, we examine the simulation with a mass imbalance induced by reducing the feedwater flow,  $\dot{M}_{FW}$ , to 90% of  $\dot{M}_{out}$ . This will create a negative change in total mass to the boiler drum over time.

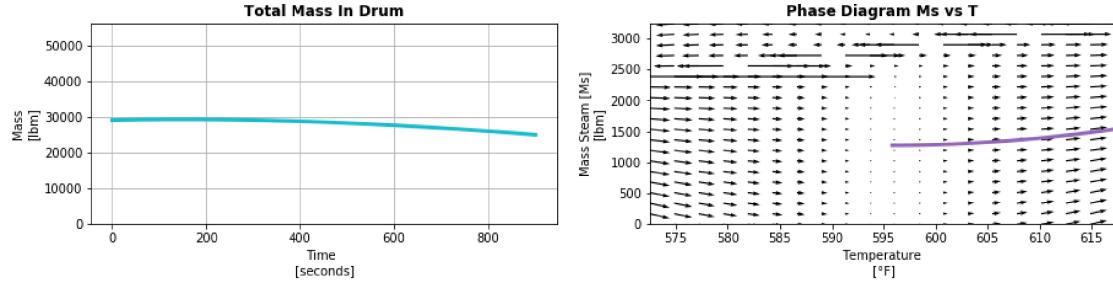
#### 3.3.1 Calculate and Tabulate Initial Conditions for Case 2: Negative Mass Imbalance

Case 2: Negative Mass Imbalance Parameters

Variable	Value	Units
$P_0$	1500.000000	PSIA
$T_0$	595.838660	°F
$M_{s0}$	1273.724450	lbm
$H_{FW}$	178.163918	btu/lbm
$\dot{M}_{FW}$	50.000000	lbm/s
$H_{FG}$	55025.128314	btu/s
$\dot{M}_{out}$	55.555556	lbm/s

#### 3.3.2 Results for Case 2: Negative Mass Imbalance





Here we see the classic "swell" phenomenon. Even though the mass balance is negative, which intuitively should make the mass in the drum decrease, the dynamic response is initially in the opposite direction and followed by the reduction in liquid and total mass expected. Under these conditions, the temperature, pressure and vaporization flows respond linearly without reaching equilibrium as expected, but the masses respond in a surprisingly nonlinear fashion. In a real boiler, the increase in drum pressure would cause the  $\dot{M}_{out}$  to increase as well the result of which helps to suppress the "swell" phenomenon. Since  $\dot{M}_{out}$  is held constant throughout the simulation, we can see the full effect in an isolated fashion.

### 3.4 Simulation Case 3: Positive Mass Imbalance

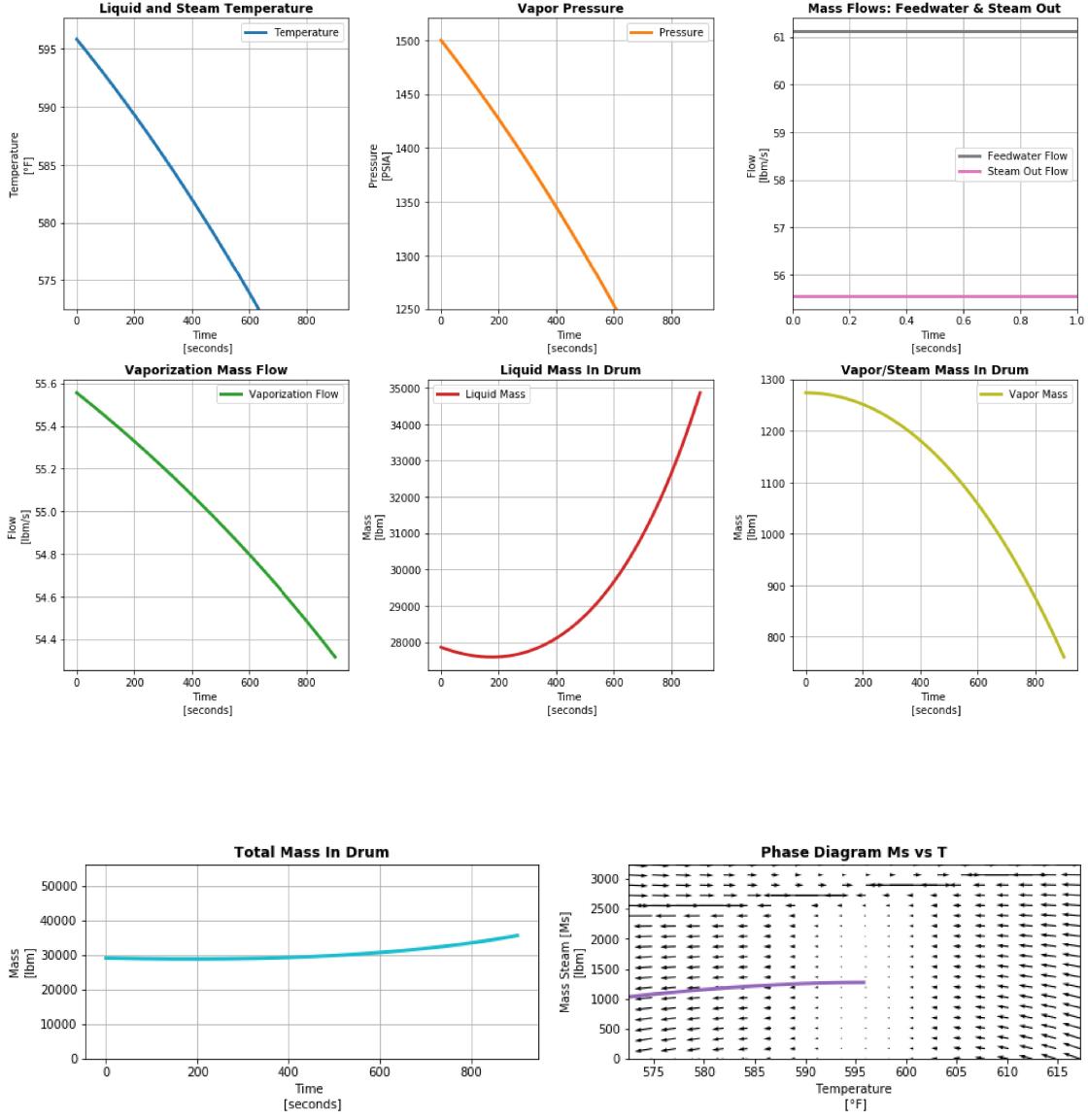
For this case, we examine the simulation with a positive mass imbalance induced by increasing the feedwater flow,  $\dot{M}_{FW}$ , to 110% of  $\dot{M}_{out}$ . This will create a positive change in total mass to the boiler drum over time.

#### 3.4.1 Calculate and Tabulate Initial Conditions for Case 3: Positive Mass Imbalance

Case 3: Positive Mass Imbalance Parameters

Variable	Value	Units
$P_0$	1500.000000	PSIA
$T_0$	595.838660	°F
$M_{s0}$	1273.724450	lbm
$H_{FW}$	178.163918	btu/lbm
$\dot{M}_{FW}$	61.111111	lbm/s
$\dot{H}_{FG}$	55025.128314	btu/s
$\dot{M}_{out}$	55.555556	lbm/s

### 3.4.2 Results for Case 3: Positive Mass Imbalance



Here we get another interesting result. The classic “shrink” phenomenon is evident as expected. The addition on extra mass to the system via the  $\dot{M}_{FW}$  imbalance causes a nonlinear response in the direction opposite to intuition. After roughly 200 seconds, the liquid phase and total mass in the drum begin to rise following an initial reduction. Surprisingly, the temperature, pressure and vaporization flow are now slightly nonlinear with their derivative becoming more negative as time moves forward. Since the process is integrating in nature and the flows and energy sources, the system doesn't reach an equilibrium again as expected.

### 3.5 Simulation Case 4: Negative Energy Imbalance

For this case, we examine the simulation with a positive energy imbalance induced by increasing the enthalpy flow from the furnace,  $\dot{H}_{FG}$ , to 95% of the calculated equilibrium value. This will

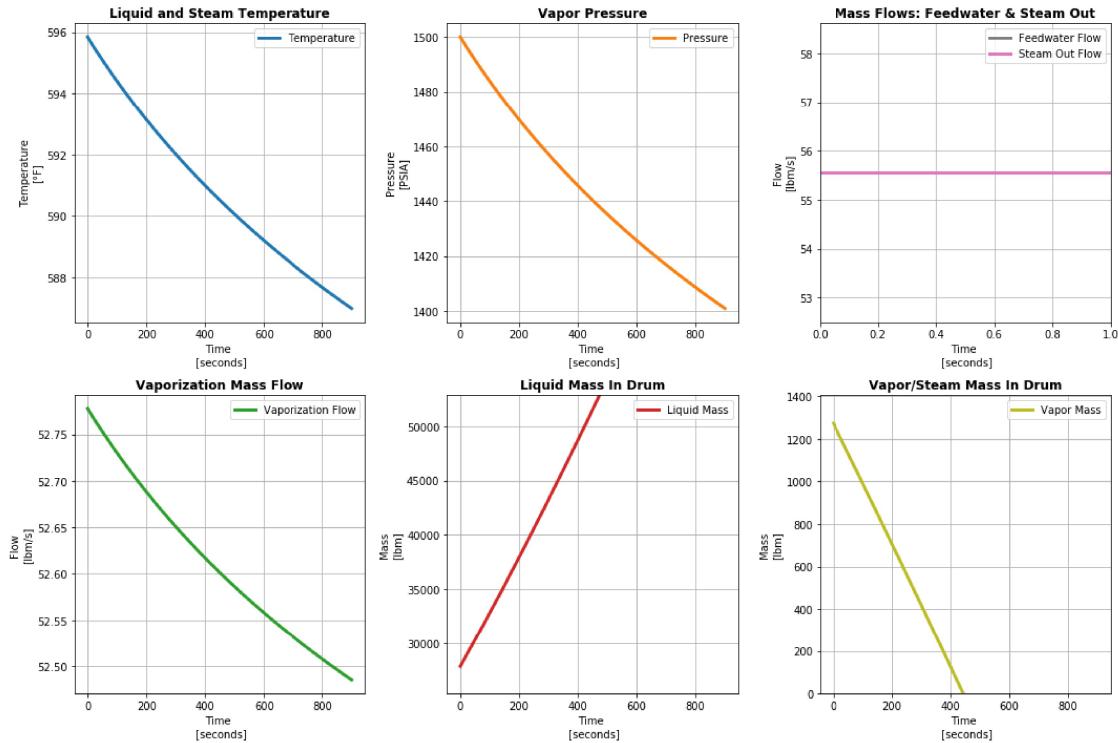
create a deficit in total energy flow to the boiler drum which should become increasingly negative over time.

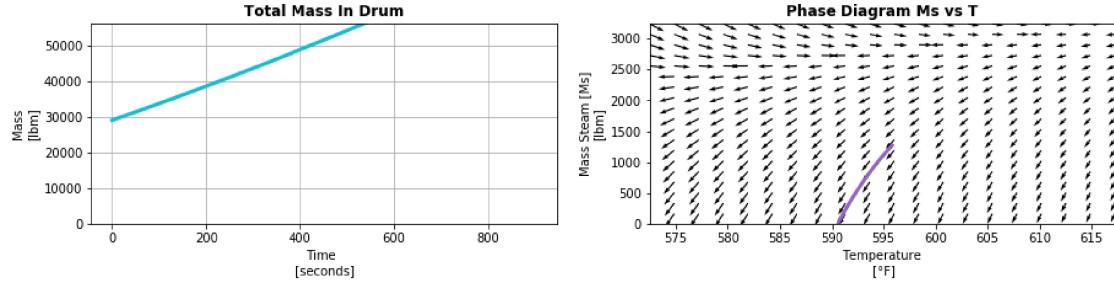
### 3.5.1 Calculate and Tabulate Initial Conditions for Case 4: Negative Energy Imbalance

#### Case 4: Negative Energy Imbalance Parameters

Variable	Value	Units
$P_0$	1500.000000	PSIA
$T_0$	595.838660	°F
$M_{s0}$	1273.724450	lbm
$H_{FW}$	178.163918	btu/lbm
$\dot{M}_{FW}$	55.555556	lbm/s
$\dot{H}_{FG}$	52273.871899	btu/s
$\dot{M}_{out}$	55.555556	lbm/s

### 3.5.2 Results for Case 4: Negative Energy Imbalance





In this simulation, the temperature, pressure and vaporization flows drop off in a slightly nonlinear fashion. We see that the results are again non-intuitive in the masses. A negative deficit in energy flow to the boiler drum produces a linear response with respect to the masses which would be expected for changes in mass balance, but instead occurs in response to energy balance. For the time frame selected, the vapor mass enters a negative region computationally. This is, of course, a loss of fidelity as the mass could not be negative. In this case, the boiler drum will eventually fill completely with liquid and carry over in to the steam delivery system.

### 3.6 Simulation Case 5: Positive Energy Imbalance

For this case, we examine the simulation with a positive energy imbalance induced by increasing the enthalpy flow from the furnace,  $\dot{H}_{FG}$ , to 105% of the calculated equilibrium value. This will create an increase in total energy flow to the boiler drum.

#### 3.6.1 Calculate and Tabulate Initial Conditions for Case 5: Positive Energy Imbalance

Case 5: Positive Energy Imbalance Parameters

Variable	Value	Units
$P_0$	1500.000000	PSIA
$T_0$	595.838660	°F
$M_{s0}$	1273.724450	lbm
$H_{FW}$	178.163918	btu/lbm
$\dot{M}_{FW}$	55.555556	lbm/s
$\dot{H}_{FG}$	57776.384730	btu/s
$\dot{M}_{out}$	55.555556	lbm/s

### 3.6.2 Results for Case 5: Positive Energy Imbalance

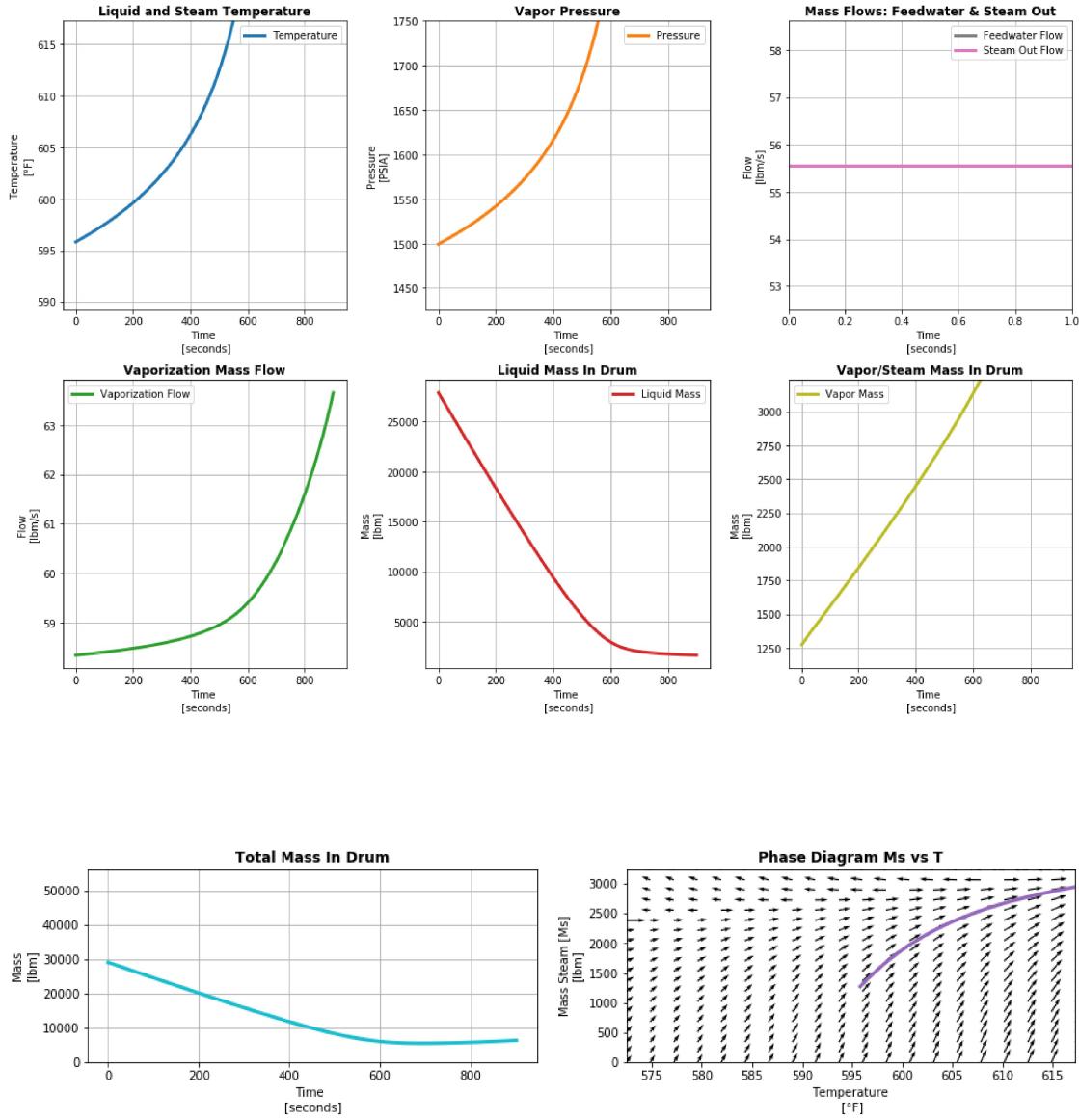


Table: Terminal Simulation Values

Variable	Final Values [lbm]
$M_L$	1657.784327
$M_S$	4787.473575
$M_L + M_S$	6445.2579020

In this simulation, we see that the increase in energy flow to the boiler drum causes the temperature, pressure and vaporization flow to grow exponentially. It appears that the liquid and

total mass reach an equilibrium point, but that point is just beyond the pressure region for which the model is valid. The phase diagram appears to show that, for the valid model region (approximately 570 to 620 °F), the system will reach an equilibrium. This seems to be due to the model's lack of limitation on steam drum pressure. The vapor can simply pressurize without limit accompanying an infinite mass of steam. This would not be the case in the real system, as a real drum would explode at certain pressure conditions. In practice, a safety relief valve would be designed to automatically open and the extra vapor would be purged to the atmosphere. This would empty the drum of liquid completely.

## 4 Conclusion

The model successfully shows the behavior of the “shrink and swell” phenomenon seen in practice. However, unlike real-world applications in which an increase or reduction in drum pressure would cause changes in the  $\dot{M}_{out}$  flow that would help to counteract the phenomenon, we are able to see the full effect isolated from interaction.

Over the course of the various cases, we induced positive and negative imbalances to the mass flow to the drum. Here we saw a nonlinear response in mass which is counter-intuitive, but expected from experience. Oddly though, we saw a roughly linear response in temperature, pressure and vaporization flow. Conversely, the induction of energy imbalances created a mostly linear response in the masses but nonlinear response in the temperature, pressure and vaporization flow. This would suggest that each system is coupled linearly, but individually has a nonlinear solution. Equilibriums do not appear to exist except at system boundaries which was expected due to the physical nature of the solution.

This model could be expanded to include other effects on the system such as the introduction of an economizing heat exchanger model which would vary the  $H_{FW}$  value with changes in mass flow and  $H_{FG}$  from the furnace section, and modeling the outlet of the steam drum as an orifice which would cause  $\dot{M}_{out}$  to respond dynamically to changes in vessel pressure. In these cases, it is likely that equilibrium solutions would show up with proper design of the parameters due to their counteracting forces. Uses could include inverting the model to convert it in to a control rule for  $\dot{M}_{FW}$  to better control difficult drum levels in a multivariate-nonlinear fashion. Additionally, creating multiple models linearizing various pressure regions could be synthesized to build a linear spline to further the model's application and validity.

## References

- [1] Franks, Roger G. E., "Modeling and Simulation In Chemical Engineering," *Wiley-Interscience*, pp. 105, 1972.
- [2] The Engineering Toolbox, "Specific heat for liquid water at temperatures from 32 to 675 °F"  
[www.engineeringtoolbox.com/specific-heat-capacity-water-d\\_660.html](http://www.engineeringtoolbox.com/specific-heat-capacity-water-d_660.html)