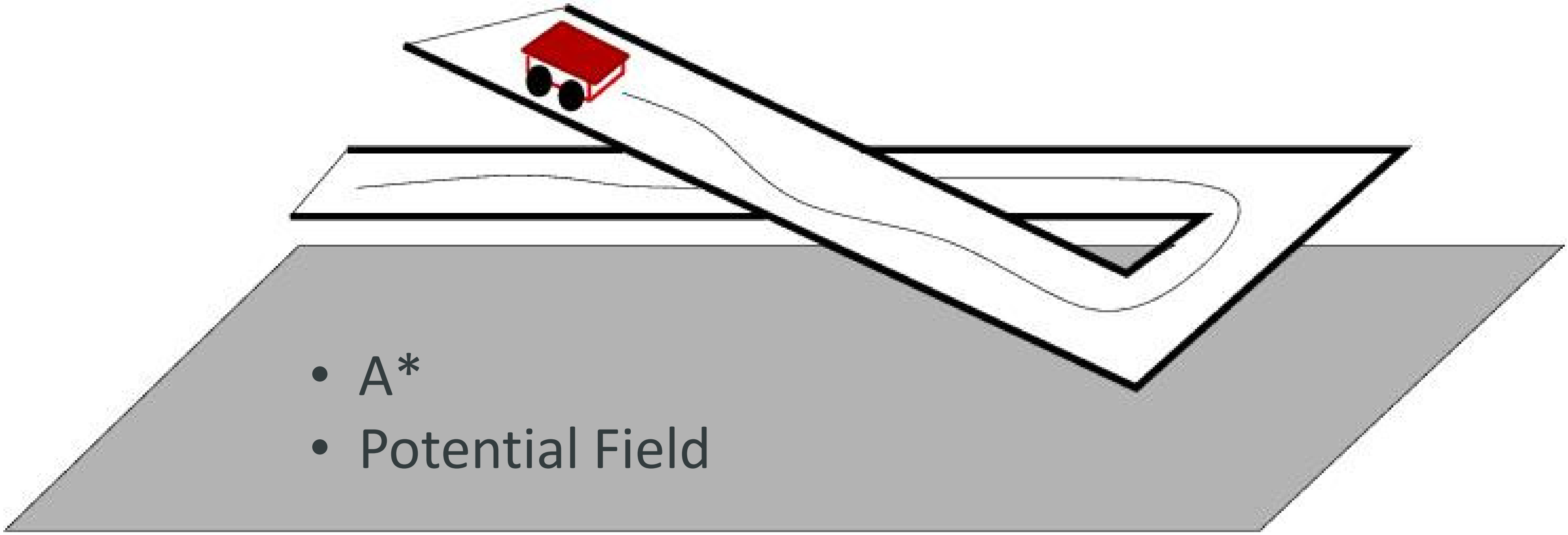
Lecture 8: ROBOT PATH PLANNING



- A*
- Potential Field

Robot Path Planning

GOALS

- collision-free trajectories
- robot should reach the goal location as fast as possible

requires an algorithm to find a suitable path between a start and target point whilst avoiding certain areas referred to as obstacles.

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Path planning algorithm - A*

- presented in 1968 by Hart *et al.*
- is admissible, i.e. guarantees to find the shortest path, if there is one
- is minimalistic, i.e. if compared to other admissible algorithms with the same knowledge of the search area, it will investigate the minimal amount of nodes necessary to find an optimal path.

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The A* Search

- A* is an algorithm that:
  – Uses heuristic to guide search
  – While ensuring that it will compute a path with minimum cost

"estimated cost"

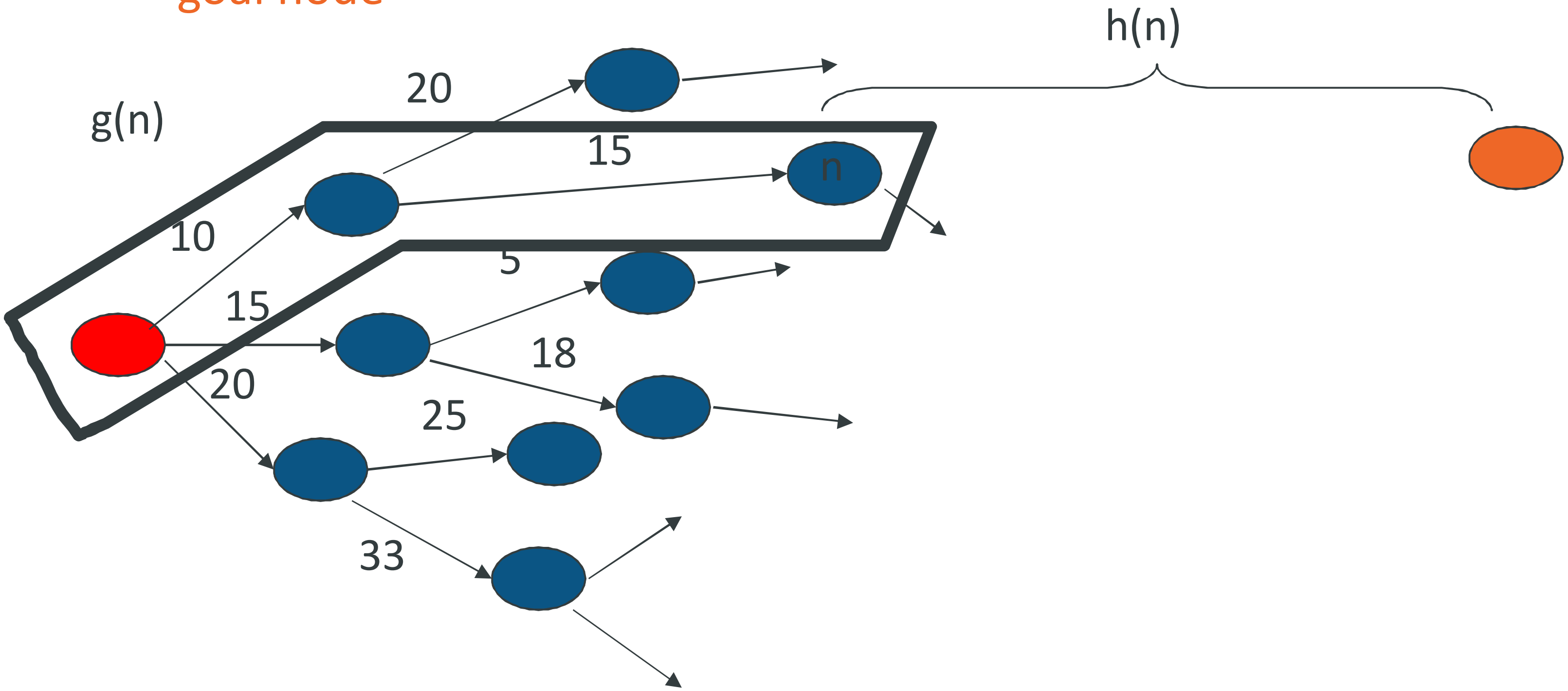
- A* computes the function $f(n) = g(n) + h(n)$

"actual cost"

➢ *g(n)* : cost of moving from the starting point to the node *n*
➢ *h(n)* : is an estimate of the cost of the distance between the node *n* and the goal point

The A* Search

- $f(n) = g(n) + h(n)$
  - $g(n)$ = “cost from the starting node to reach n”
  - $h(n)$ = “estimate of the cost of the cheapest path from n to the goal node”



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Heuristics

- technique designed for solving problem quickly when classic methods are too slow, or finding an approximate solution when classic methods fail to find an exact solution. It can be considered a shortcut.
- in A* to be admissible, the heuristic may never overestimate the cost of the optimal path from node n to the target node:

$$h(n) \leq \hat{h}(n) \text{ for all } n$$

where $h(n)$ is some estimate for the cost of the optimal path from node n to the target node and $\hat{h}(n)$ is the actual cost.

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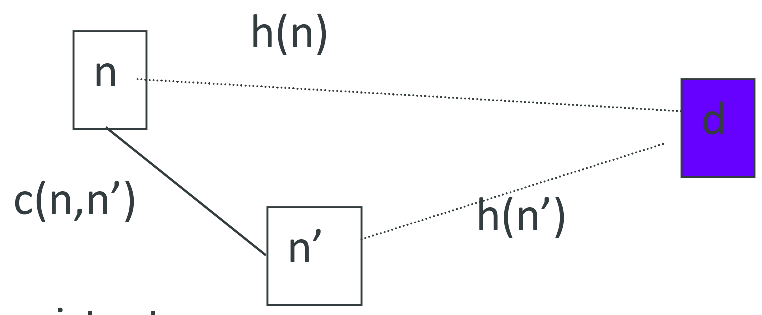
Heuristics (cont.)

- A* generates an optimal solution if h(n) is an admissible heuristic and the search space is a tree:
  – h(n) is **admissible** if it never overestimates the cost to reach the destination node

- A* generates an optimal solution if h(n) is a consistent heuristic and the search space is a graph:
  – h(n) is **consistent** if for every node n and for every successor node n' of n:

$$h(n) \leq c(n,n') + h(n')$$



- If h(n) is consistent then h(n) is admissible
- Frequently when h(n) is admissible, it is also consistent

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Path planning algorithm – A* implementation

Variables and functions:

- *n* : node in the search space
- *close list*: contains all the nodes that have already been explored and the obstacles(added before starting the search)
- *open list*: contains all the nodes that still need to be checked
- $f(n)$=g(n)+$h(n)$

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Path planning algorithm – A* implementation(cont.)

1. Set start node as current node and add it to the *closed list*
2. add all neighbours of the current node to *open list* along with *g(n)*, *h(n)*, *f(n)* and their parent node
3. if a neighbour is already in the open list, check if the *f(n)* score is lower and if so, update *g(n)*, *h(n)*, *f(n)* and its parent
4. choose the node with the smallest *f(n)* score in the *open list*, set it as the current node, remove it from the *open list* and add it to the *closed list*
5. repeat step 2 through 4, until the target node is added to the *closed list*, or until the *open list* is empty
6. if the target node is found, retrace the nodes by identifying their parents until the start node is found; this is the shortest path
7. if the *open list* is empty, no path to the target is possible

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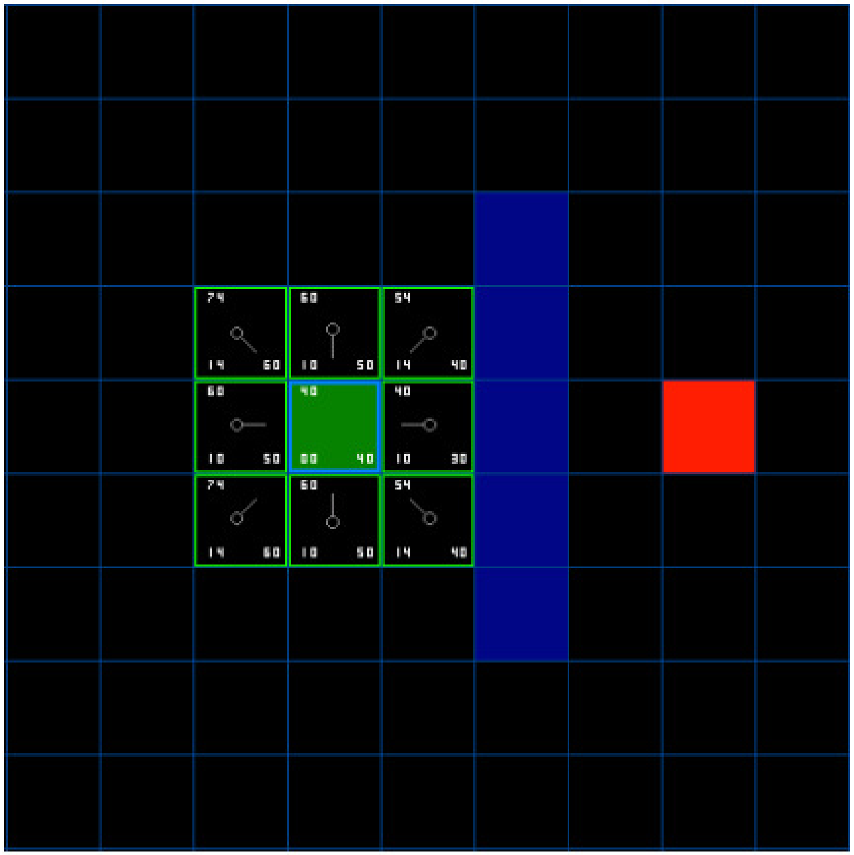
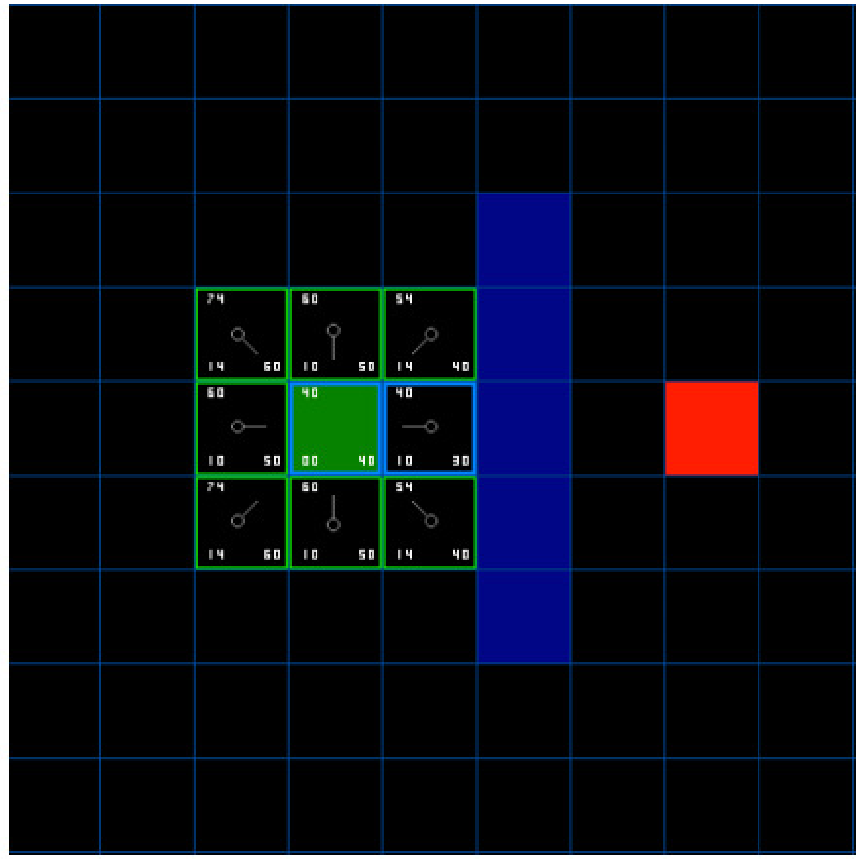
Path planning algorithm – A* implementation





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Heuristics(cont.)

MANHATTAN

$$h(n) = |x_n - x_{goal}| + |y_n - y_{goal}|$$

x_n, x_{goal} ,y_n and y_{goal} are the node x-position, goal x-position, node y-position and goal y-position.

DIAGONAL

$$h_{manhattan}(n) = |x_n - x_{goal}| + |y_n - y_{goal}|$$

$$h_{diagonal}(n) = \min(|x_n - x_{goal}|\ ,\ |y_n - y_{goal}|)$$

$$h(n) = \sqrt{2}\ h_{diagonal}(n) + (h_{manhattan}(n) - 2\ h_{diagonal}(n))$$

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Heuristics(cont.)

EUCLIDIAN

$$h(n) = \sqrt{(x_n - x_{goal})^2 + (y_n - y_{goal})^2}$$

Square root is relatively computational complex

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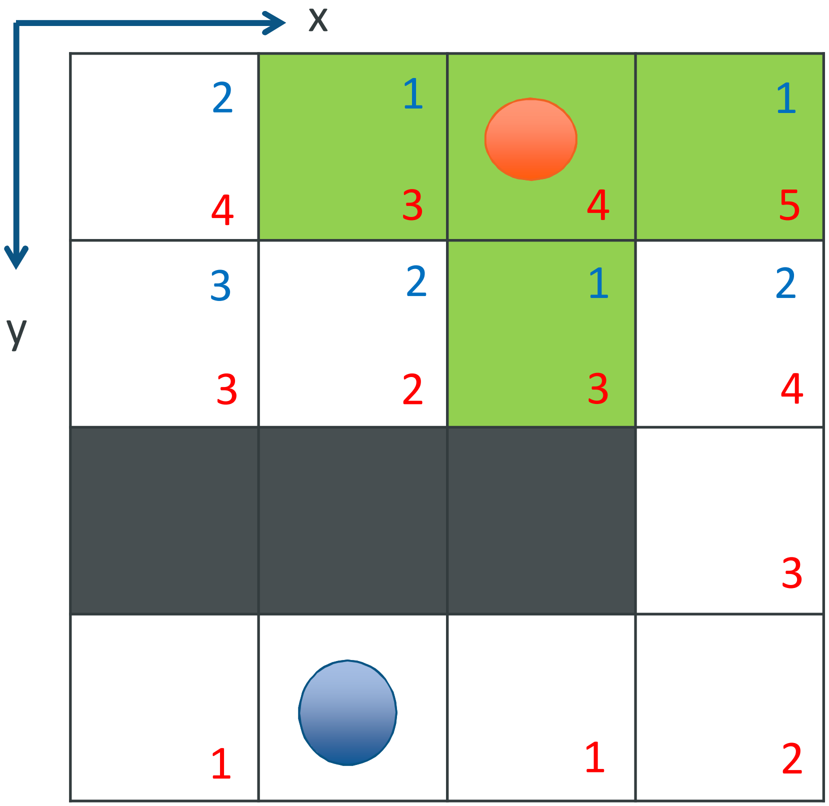
A*- Path Planning



(2,4)

H(n) g(n)=1

Close list
(1,3)
(2,3)
(3,3)
(3,1)

Starting node
(3,1)

Open list
(2,1)=4 (4,1)=6 (3,2)=4

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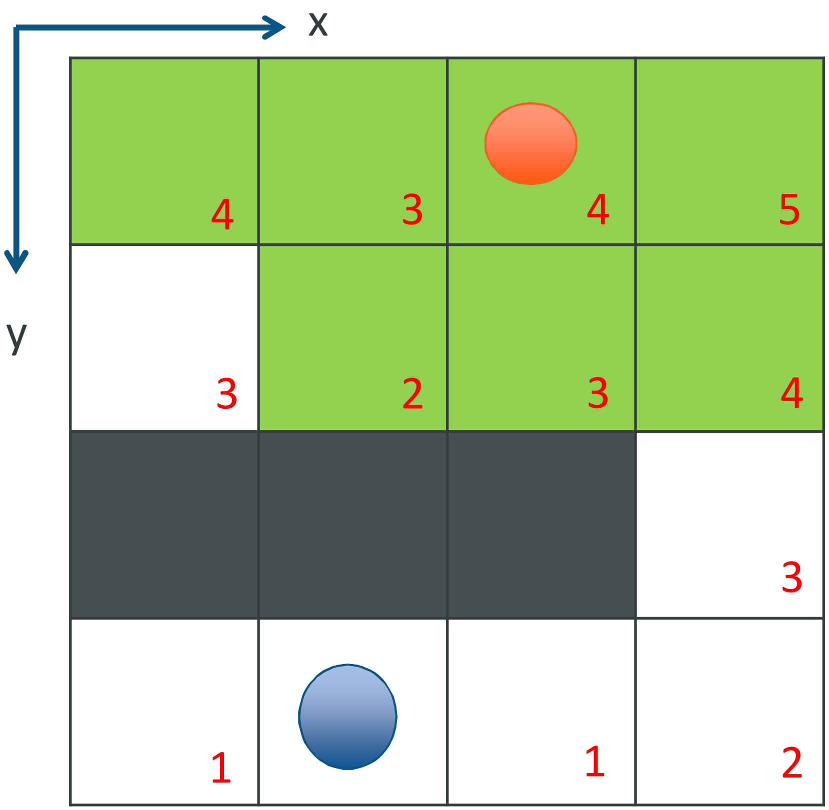
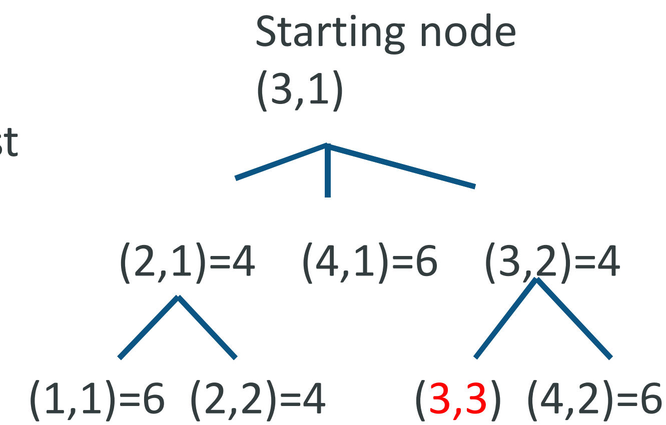
A*- Path Planning



(2,4)

Close list
(1,3)
(2,3)
(3,3)
(3,1)
(2,1)
(3,2)



H(n) (Manhattan) g(n)=1

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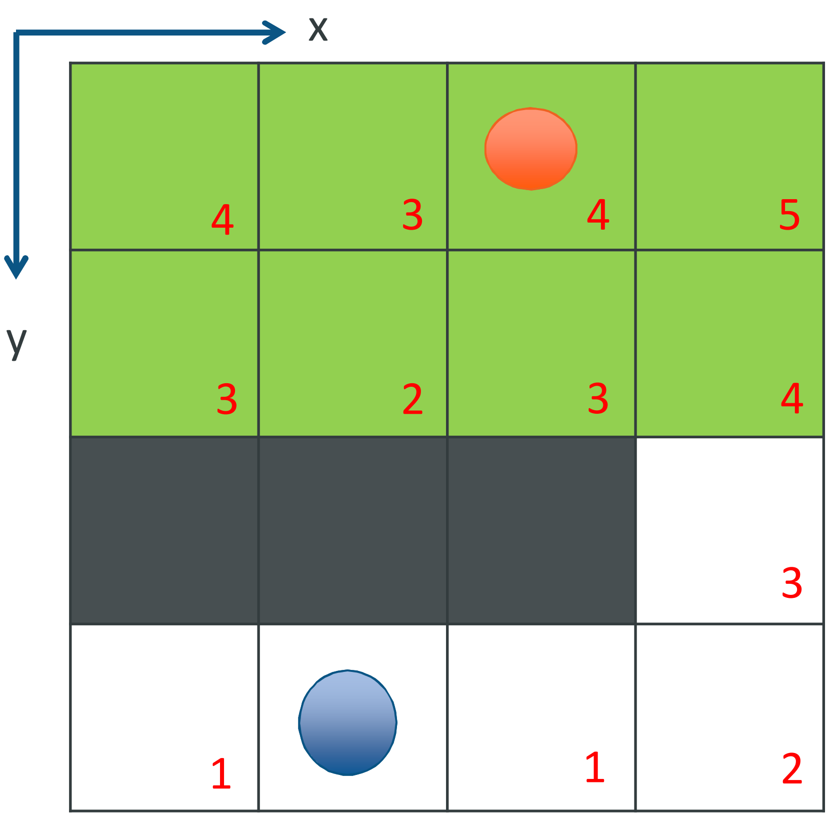
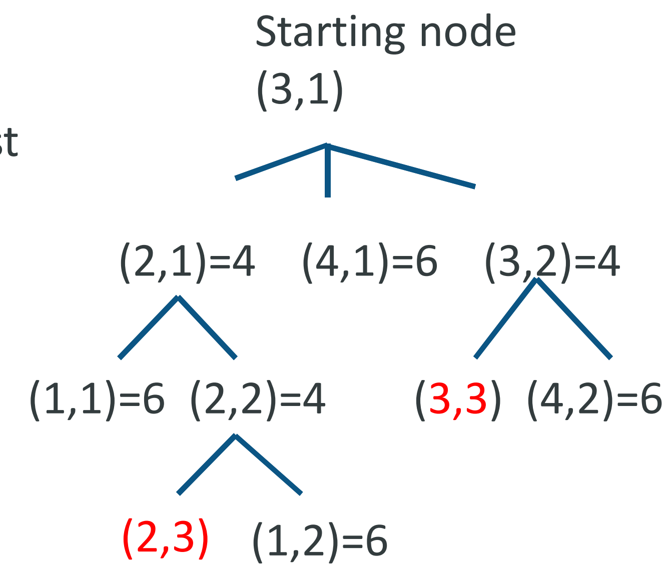


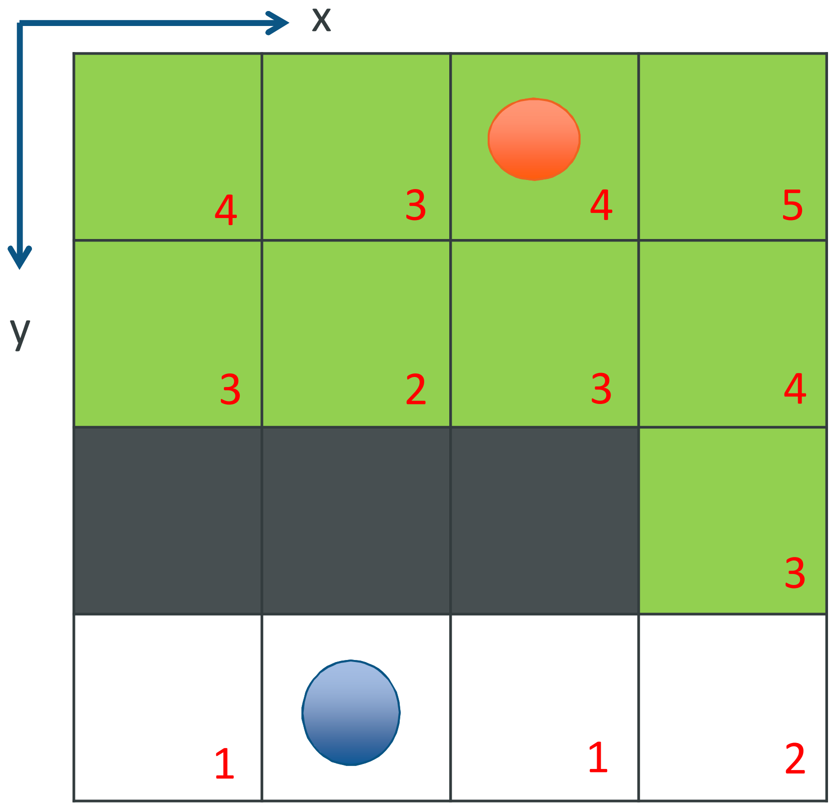
(2,4)

Close list
(1,3)
(2,3)
(3,3)
(3,1)
(2,1)
(3,2)
(2,2)



H(n) (Manhattan) g(n)=1

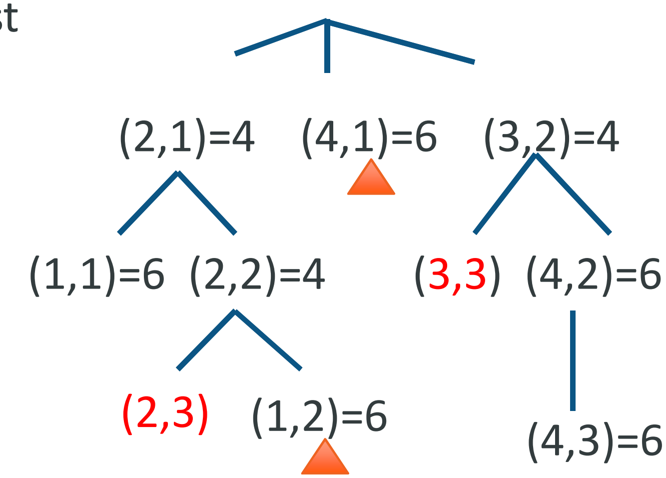
A*- Path Planning



(2,4)

Close list
(1,3)
(2,3)
(3,3)
(3,1)
(2,1)
(3,2)
(2,2)
(1,2)
(1,1)
(4,2)

Starting node
(3,1)



H(n) (Manhattan) + g(n)=1

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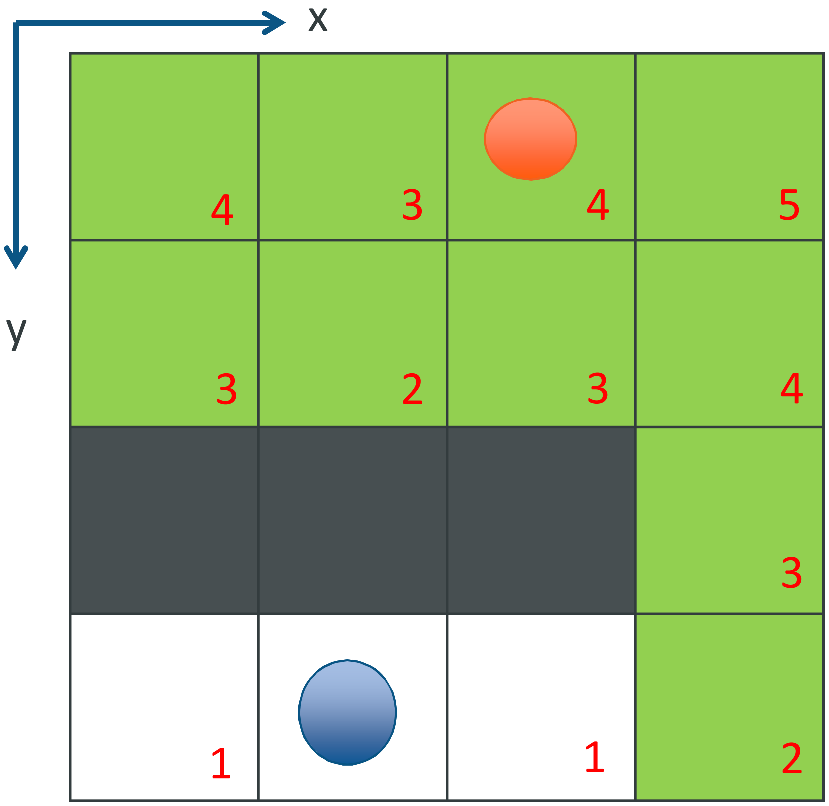
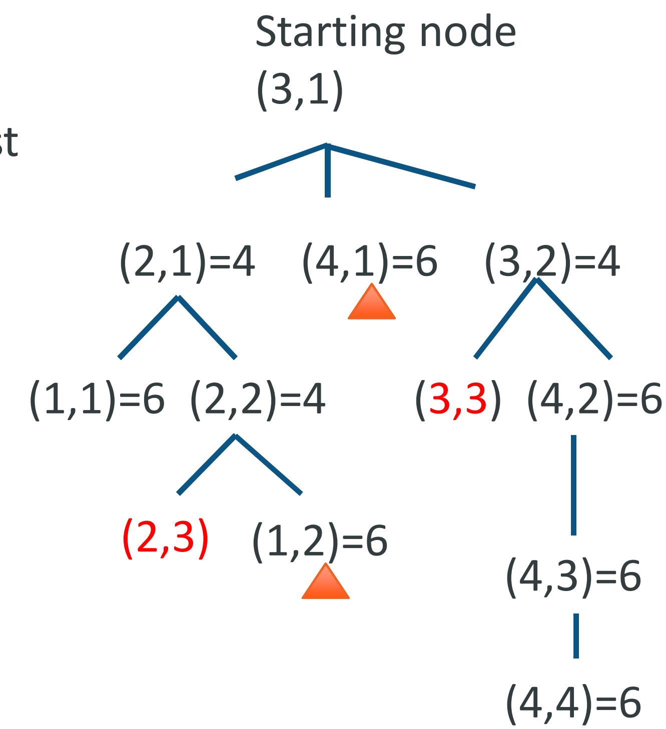


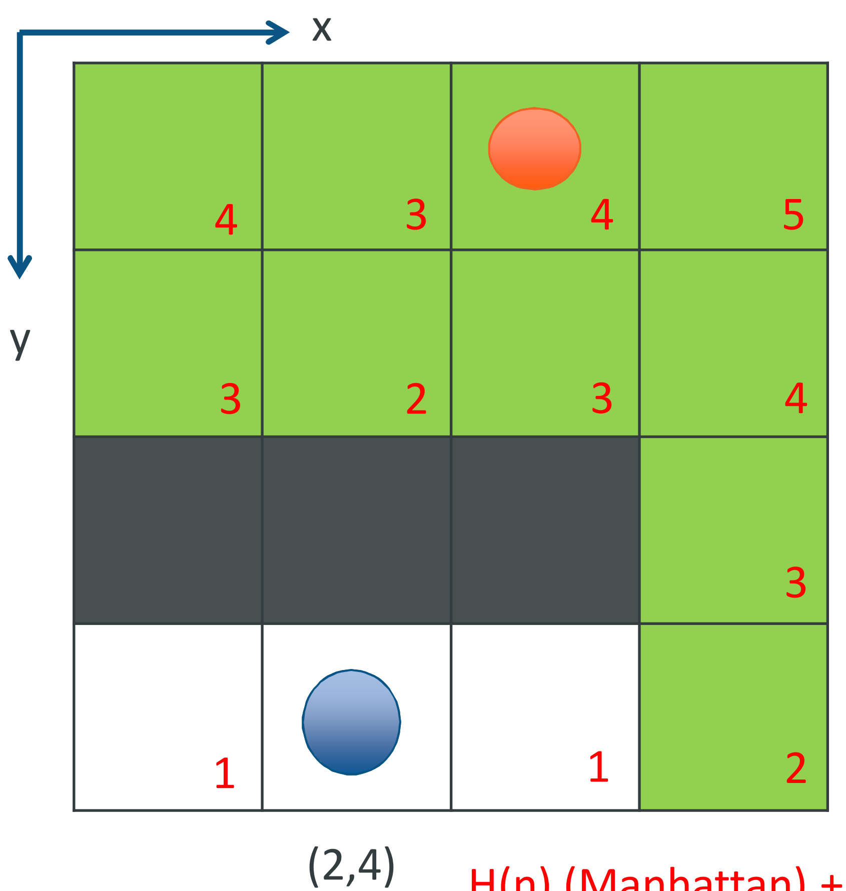
(2,4)

Close list
(1,3)
(2,3)
(3,3)
(3,1)
(2,1)
(3,2)
(2,2)
(1,2)
(1,1)
(4,2)
(4,3)

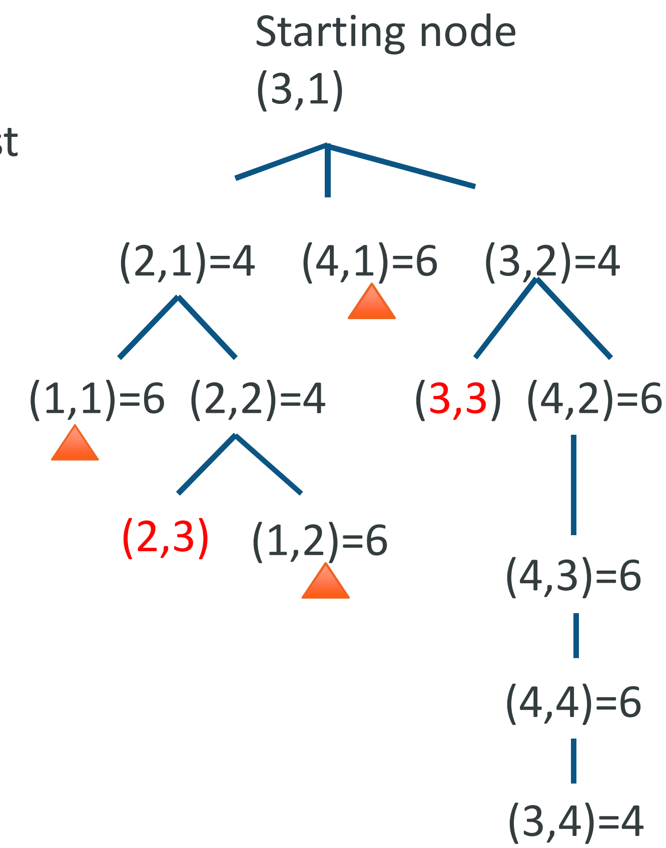


H(n) (Manhattan) + g(n)=1

A*- Path Planning



Close list
(1,3)
(2,3)
(3,3)
(3,1)
(2,1)
(3,2)
(2,2)
(1,2)
(1,1)
(4,2)
(4,3)
(4,4)



H(n) (Manhattan) + g(n)=1

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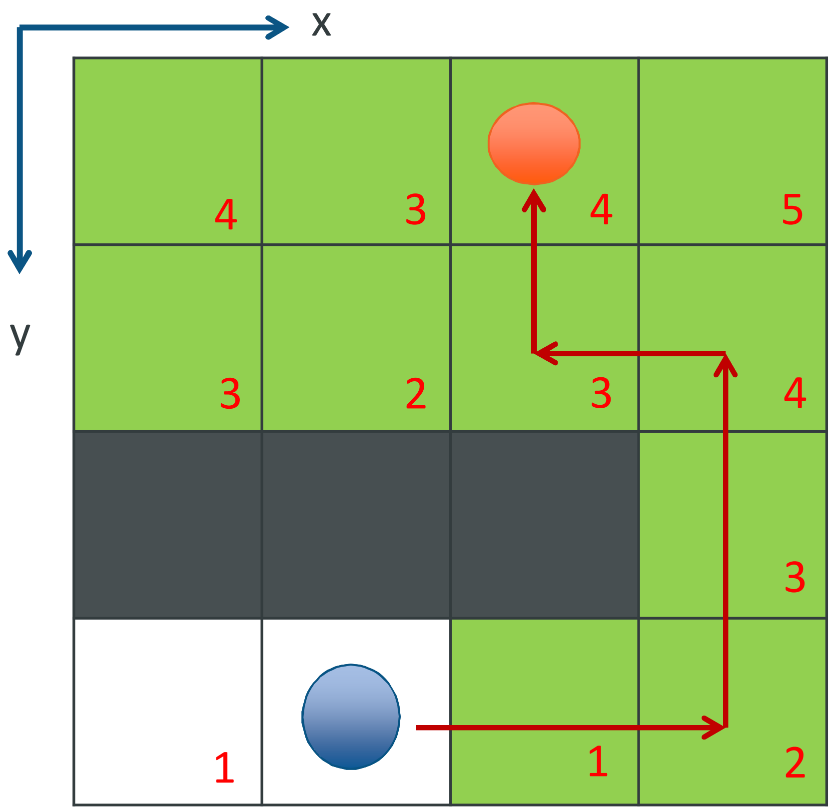
A*- Path Planning



(2,4)

Close list	Current node	Open list
(1,3)	(3,4)=2	(2,4)=goal
(2,3)		
(3,3)		
(3,1)		
(2,1)		
(3,2)		
(2,2)		
(1,2)		
(1,1)		
(4,2)		
(4,3)		
(4,4)		

H(n) (Manhattan) + g(n)=1

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Potential Fields

- **Initially proposed for real-time collision avoidance [Khatib 1986].**
- **A potential field is a scalar function over the free space.**
- **To navigate, the robot applies a force proportional to the negated gradient of the potential field.**
- **A** navigation function **is an ideal potential field that**
  - **has global minimum at the goal**
  - **has no local minima**
  - **grows to infinity near obstacles**
  - **is smooth**

Attractive & Repulsive Fields

$$F_{att} = -k_{att}(x - x_{goal})$$

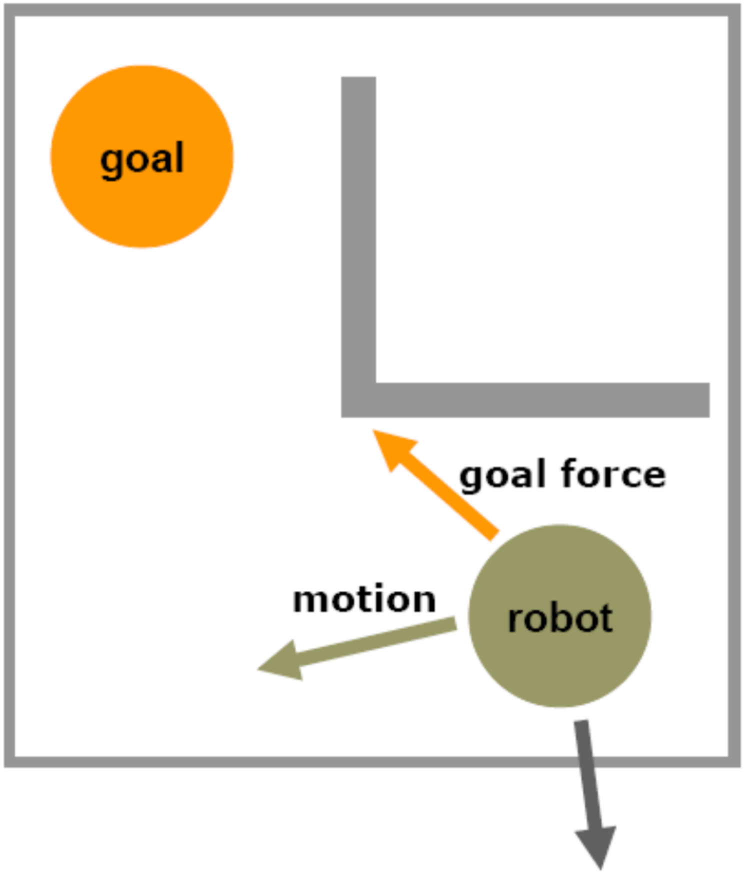
$$F_{rep} = \begin{cases} k_{rep}\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2}\frac{\partial\rho}{\partial x} & \text{if } \rho \le \rho_0, \\ 0 & \text{if } \rho > \rho_0 \end{cases}$$

k_{att}, k_{rep} : positive scaling factors

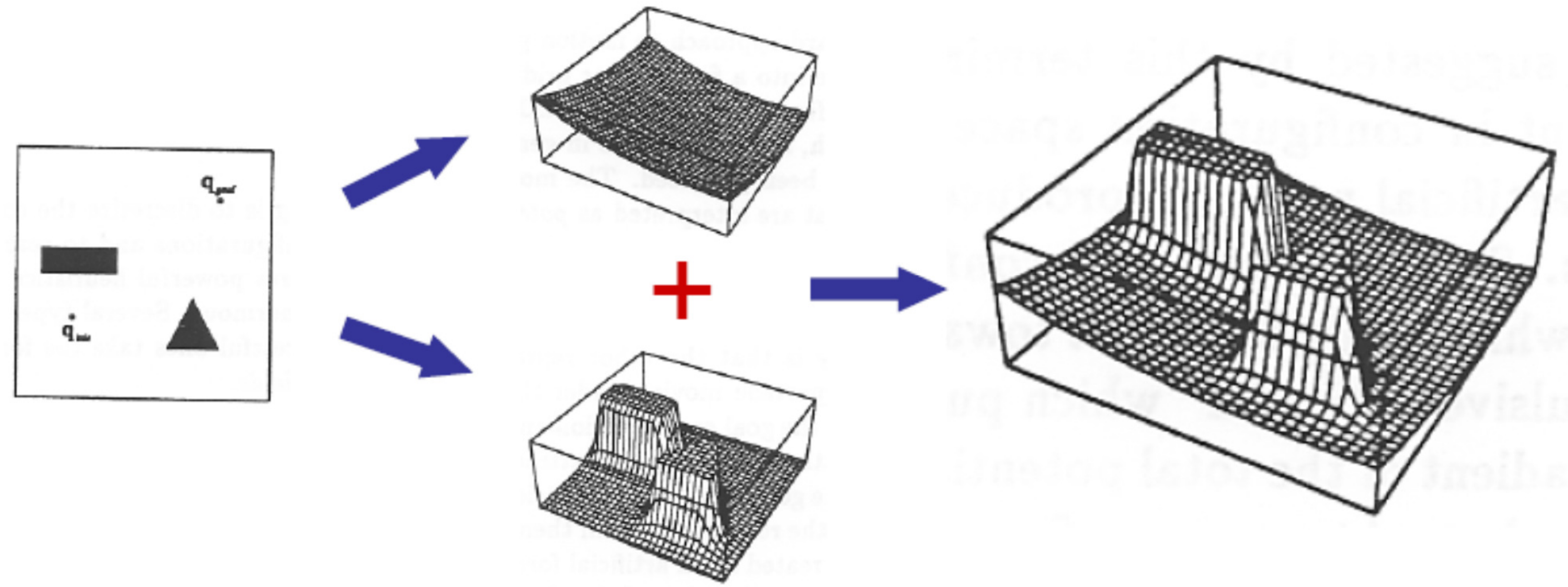
x : position of the robot

ρ : distance to the obstacle

ρ_0 : distance of influence



How Does It Work?

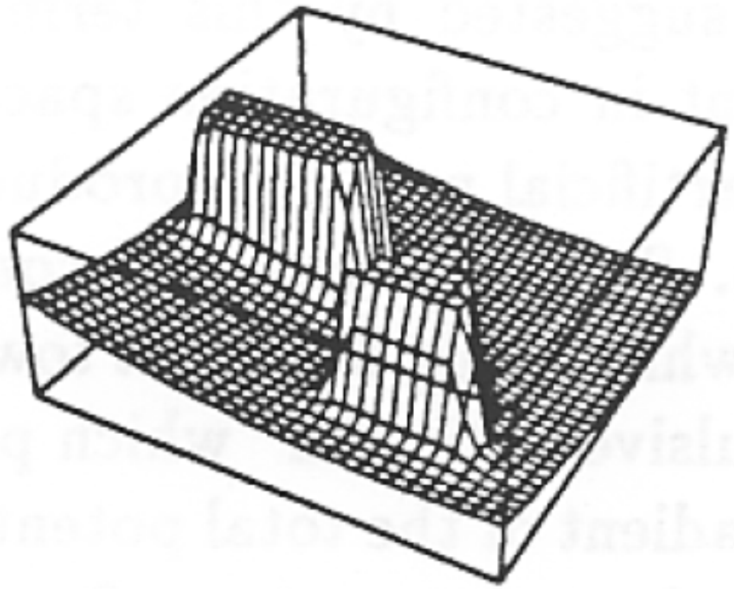


Algorithm Outline

- Place a regular grid G over the configuration space
- Compute the potential field over G
- Search G using a best-first algorithm-such as A* with potential field as the heuristic function

Local Minima

- **What can we do?**
  - **Escape from local minima by taking random walks**
  - **Build an ideal potential field – navigation function – that does not have local minima**



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