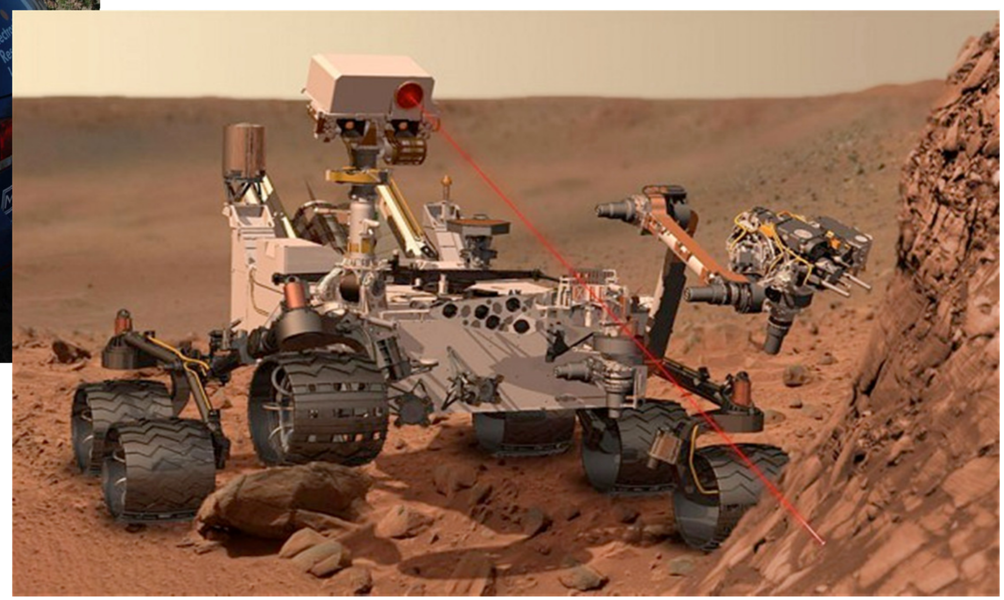
Robotics Group Project - 5CCS2RGP

Lecture 2: Kinematics of wheeled robot

Hongbin Liu, King's College London, 2014

Wheeled Robots for Autonomous Navigation

Self-driving car- Stanley



Mars Rover

Locomotion-Kinematics-Dynamics

- **Locomotion** is the process of causing an autonomous robot to move.

– In order to produce motion, forces must be applied to the vehicle

- **Kinematics** – study of the mathematics of motion without considering the forces that affect the motion.

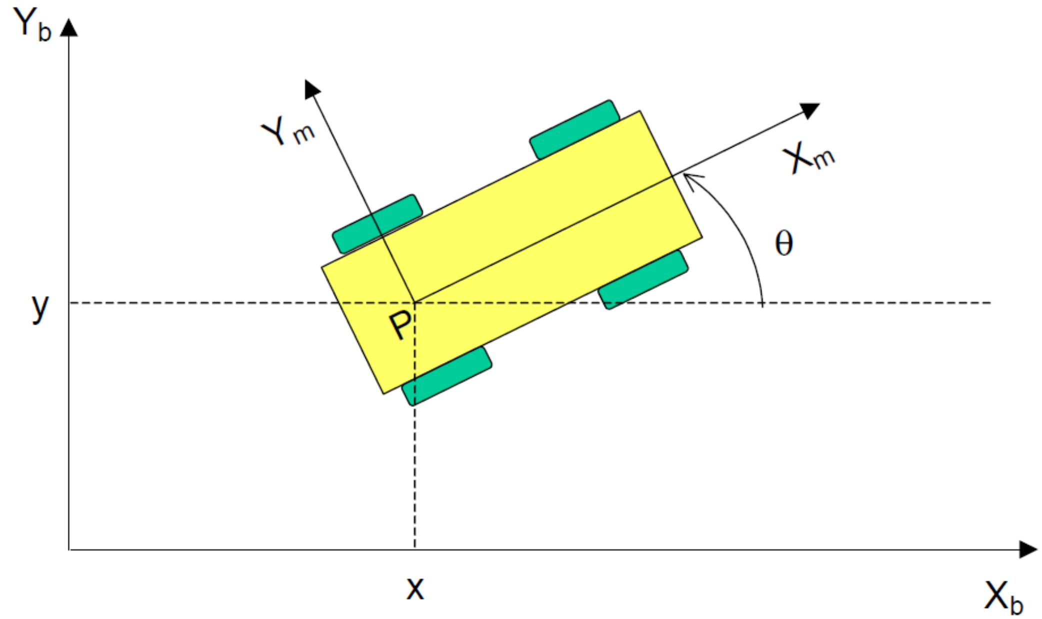
– Deals with the geometric relationships that govern the system

– Deals with the relationship between control parameters and the behaviour of a system in state space.

- **Dynamics** – the study of motion in which these forces are modeled

– Includes the energies and speeds associated with these motions

Notation

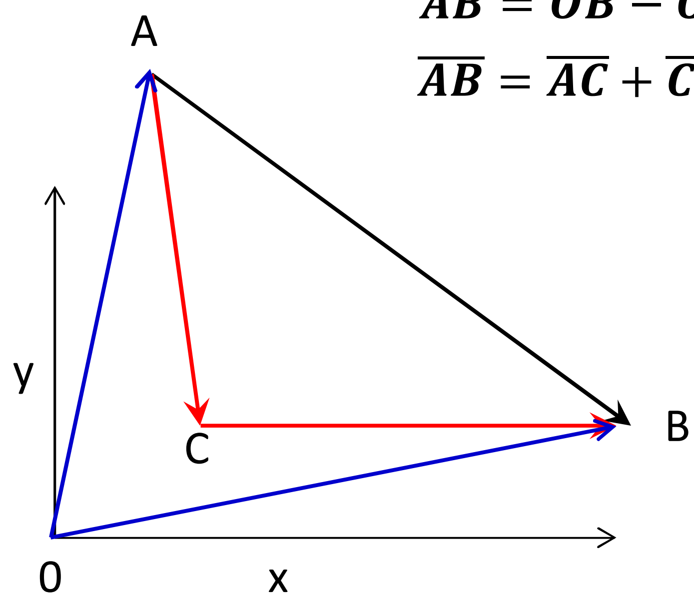


- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

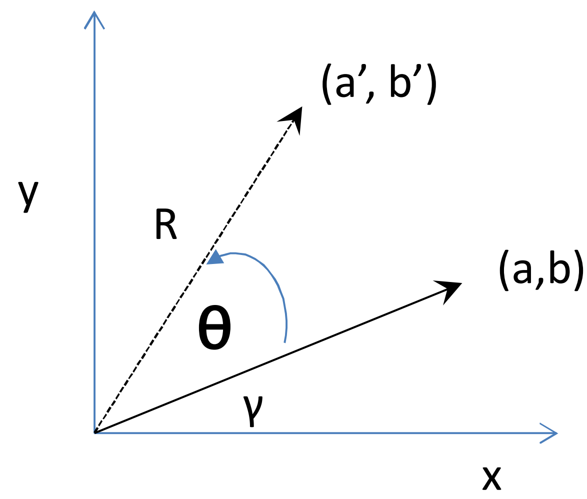
$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$ robot posture in base frame

Fundamental math-translation

$$\overline{\boldsymbol{AB}} = \overline{\boldsymbol{OB}} - \overline{\boldsymbol{OA}}$$
$$\overline{\boldsymbol{AB}} = \overline{\boldsymbol{AC}} + \overline{\boldsymbol{CB}}$$



Fundamental math-Rotation



$$\begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Proof using trigonometry :

a’ =R cos(θ+γ)

= R(cos θ cos γ– sin θ sin γ)

= R cos γ cos θ- R sinγ sin θ = a cos θ - b sin θ

b’ =R sin(θ+γ)

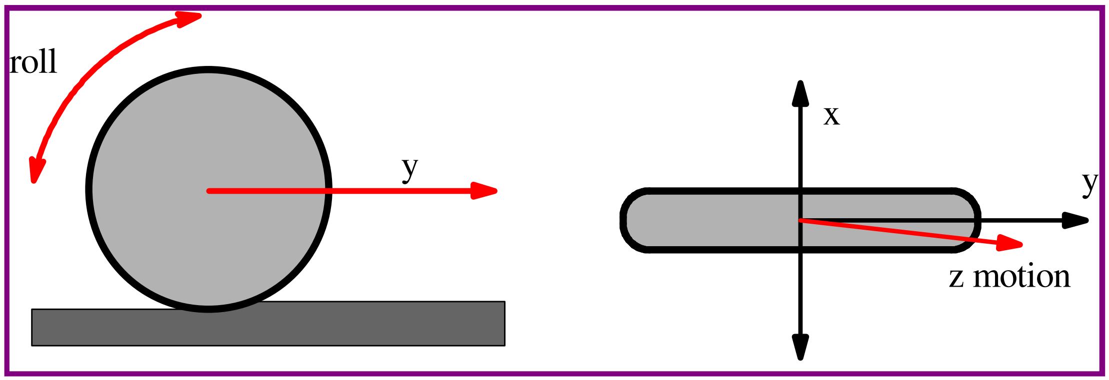
= R(sin θ cos γ + cos θ sin γ)

= R cos γ sin θ + R sinγ cos θ = a sin θ+ bcos θ

Locomotion of Wheeled Robots

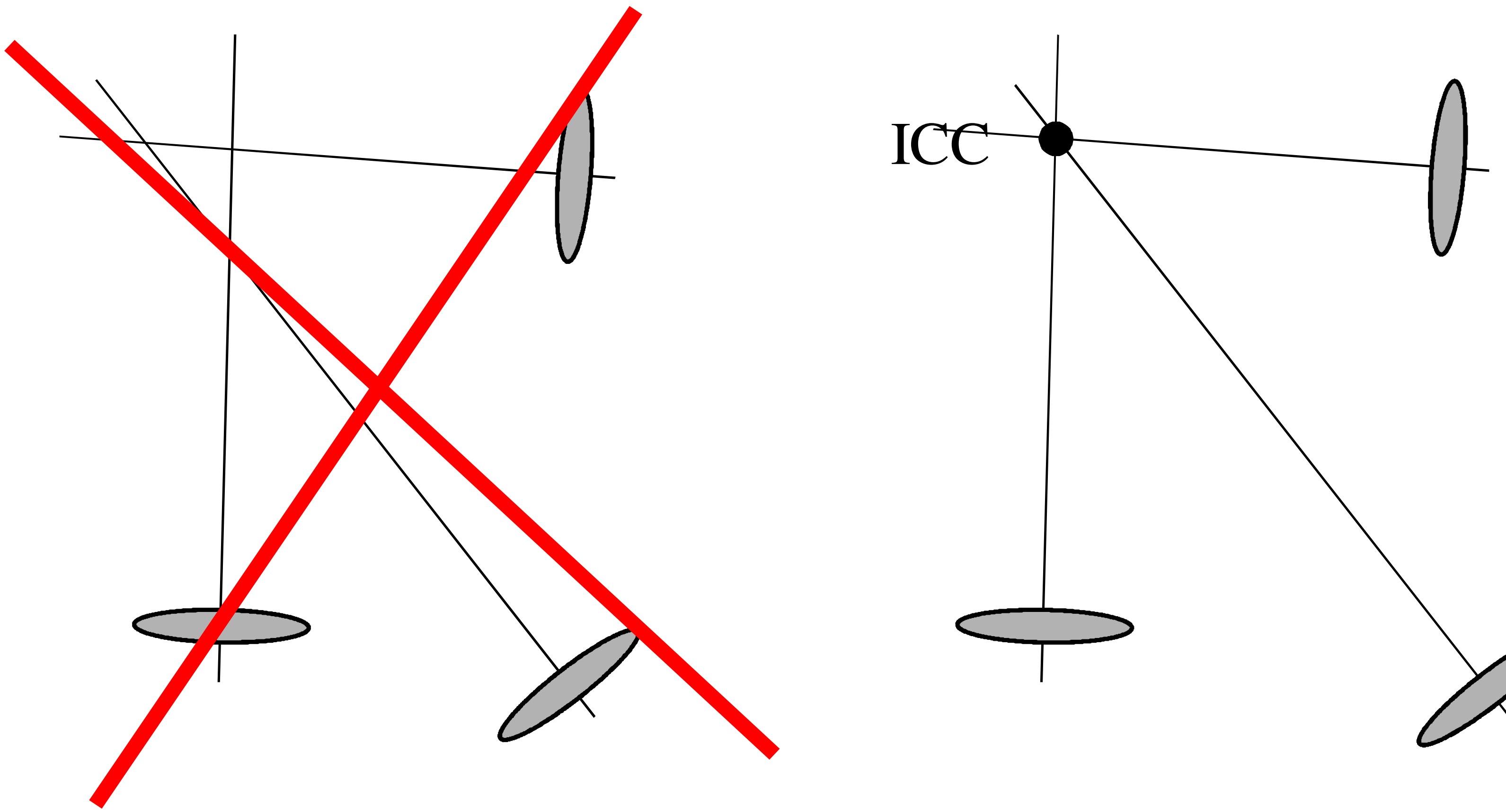
Locomotion (Oxford Dict.):

Power of motion from place to place



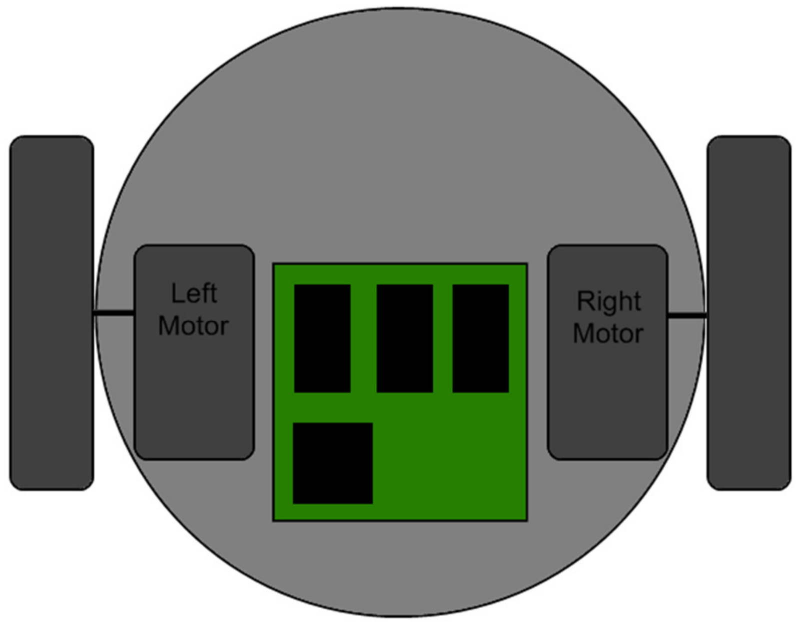
- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive
- Mecanum wheels

Instantaneous Center of Curvature



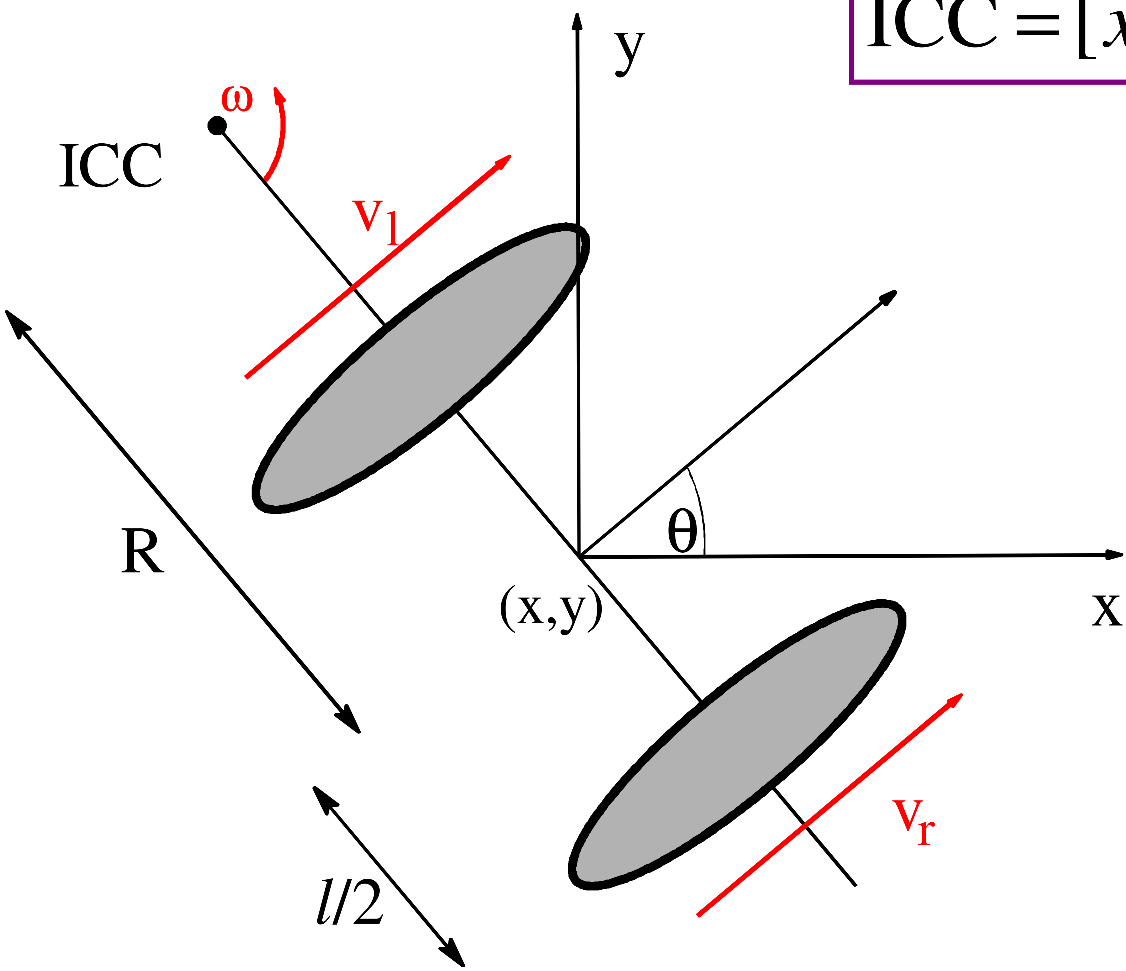
For rolling motion to occur, each wheel has to move along its y-axis

Differential Drive



Differential drive is the common drive mechanism used in mobile robots. It consists of 2 drive wheels mounted on a common axis, and each wheel can independently being driven either forward or back-ward.

Differential Drive



$$\boxed{\mathrm{ICC} = [x - R\sin\theta,\ y + R\cos\theta]}$$

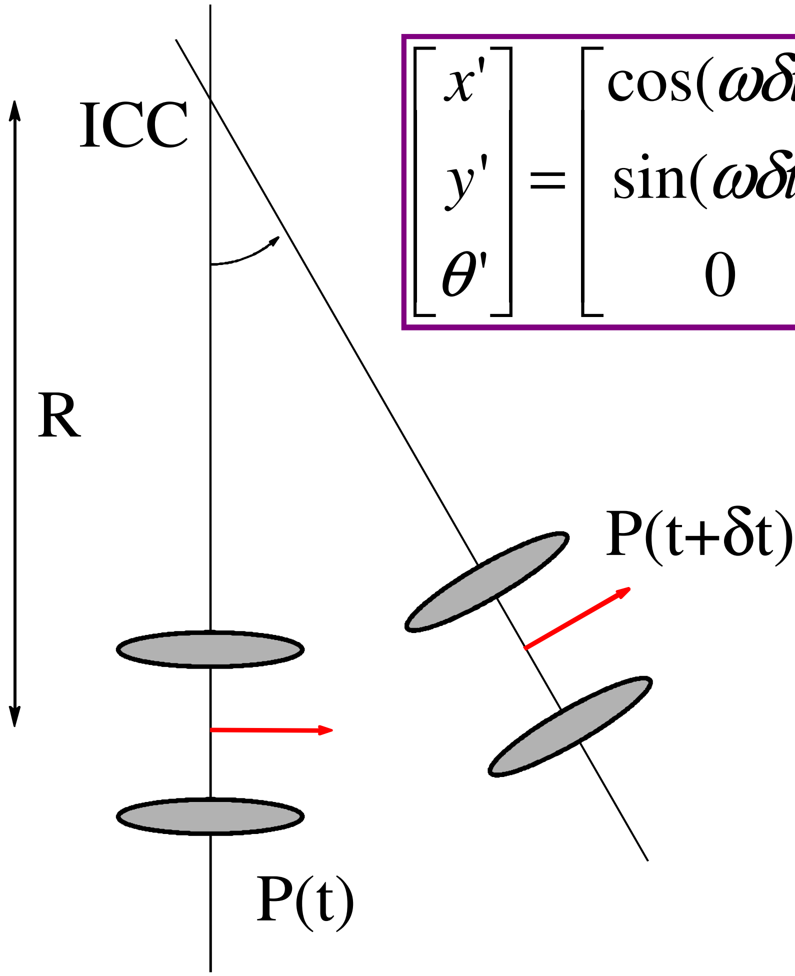
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

Differential Drive: Forward Kinematics



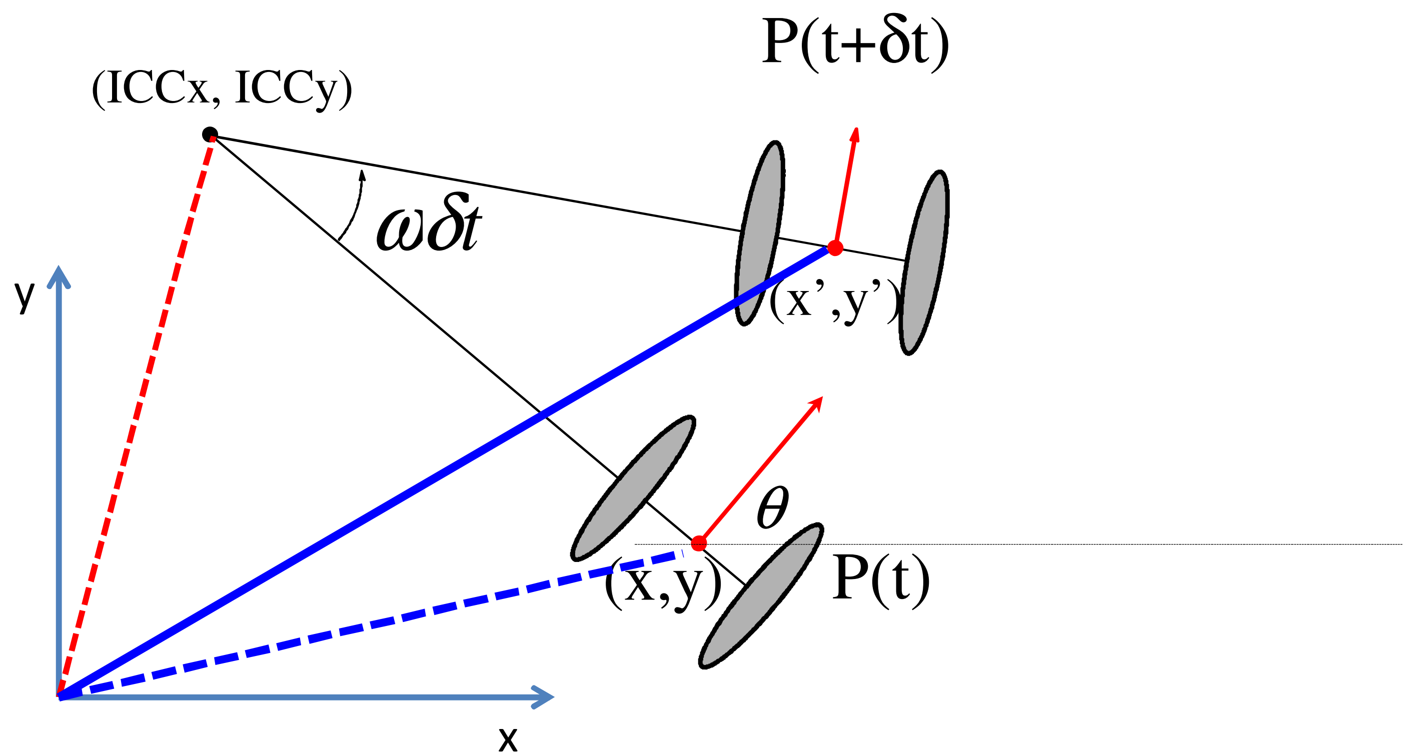
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \ \cos[\theta(t')]dt'$$

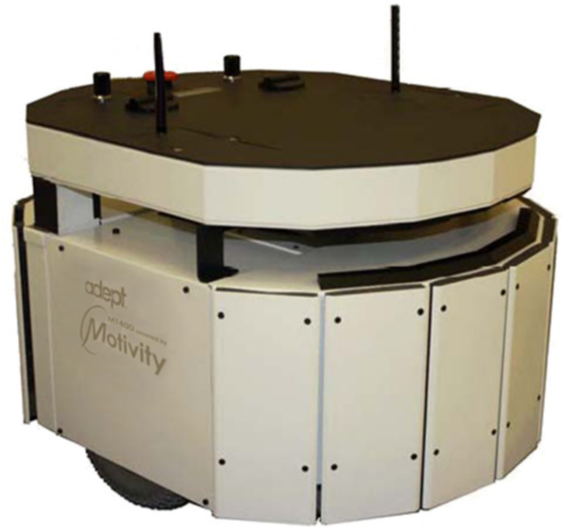
$$y(t) = \int_0^t v(t') \ \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t') \, dt'$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \mathrm{ICC}_x \\ y - \mathrm{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \mathrm{ICC}_x \\ \mathrm{ICC}_y \\ \omega\delta t \end{bmatrix}$$



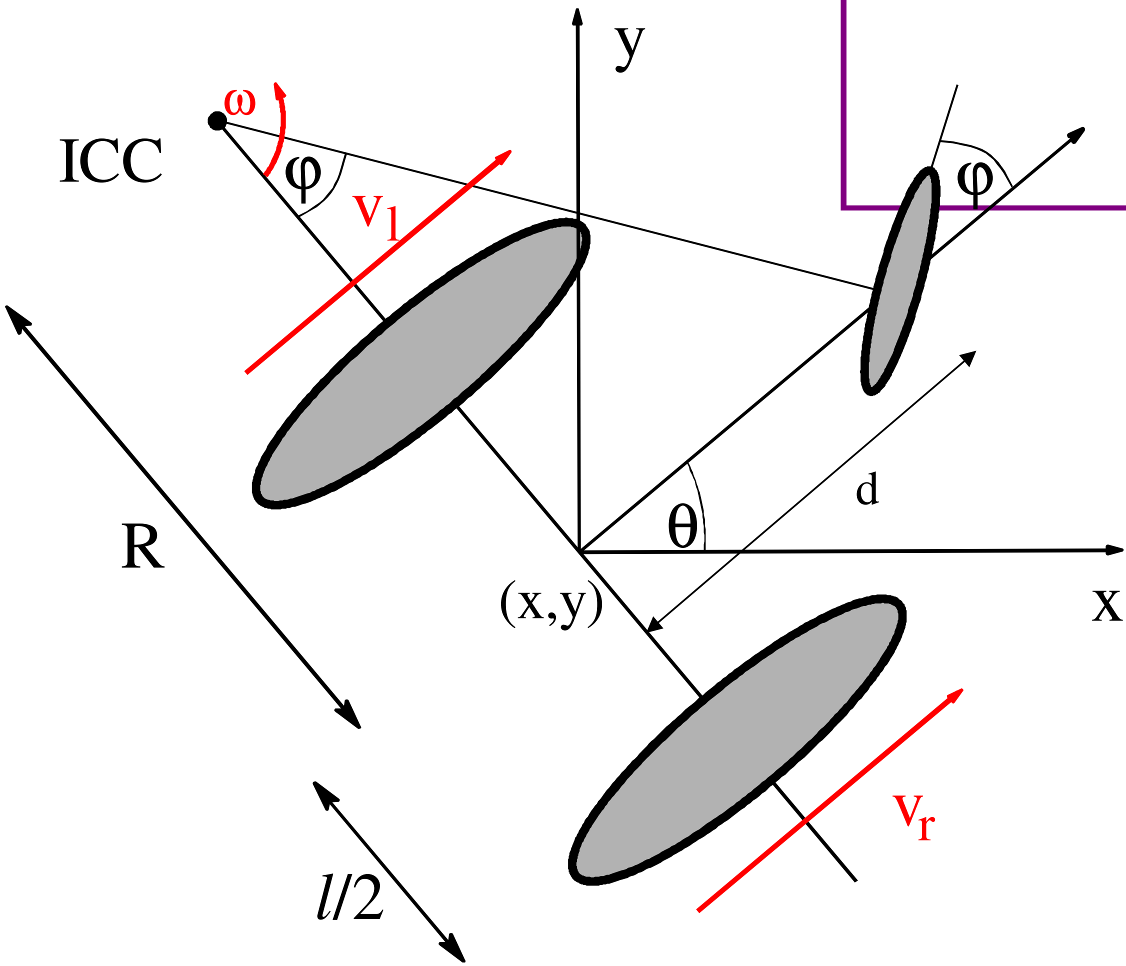
Examples for differential drive robot



Ackermann Drive

$$ICC = [x - R\sin\theta,\ y + R\cos\theta]$$

$$R = \frac{d}{\tan\varphi}$$



$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

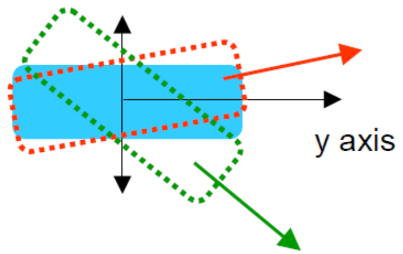
$$R = \frac{l}{2}\frac{(v_l + v_r)}{(v_r - v_l)}$$

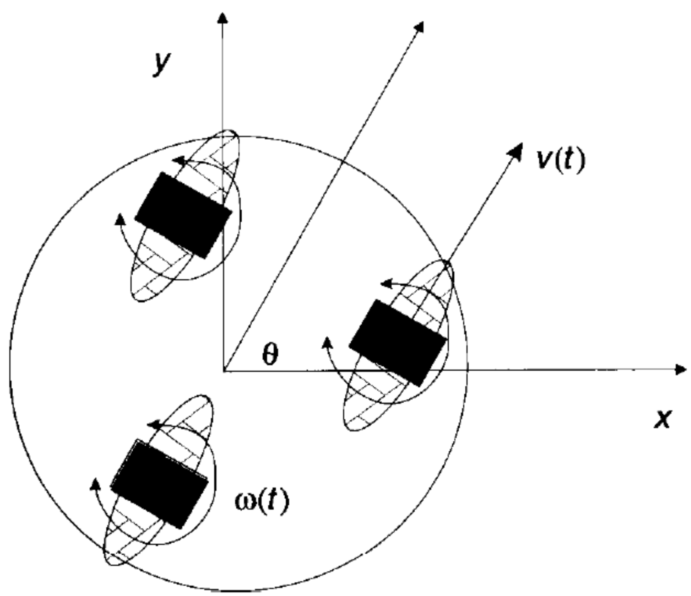
$$\omega = \frac{v_r - v_l}{l}$$

Synchonous Drive

• Steered wheel

The orientation of the rotation axis can be controlled





In a synchronous drive robot, each wheel is capable of being driven and steered.

- Typical configurations

– Three steered wheels arranged as vertices of an equilateral triangle often surmounted by a cylindrical platform

– All the wheels turn and drive in unison

- This leads to a holonomic behaviour

Synchonous Drive

• All the wheels turn in unison

• All of the three wheels point in the same direction and turn at the same rate
– This is typically achieved through the use of a complex collection of belts that physically link the wheels together

• The vehicle controls the direction in which the wheels point and the rate at which they roll

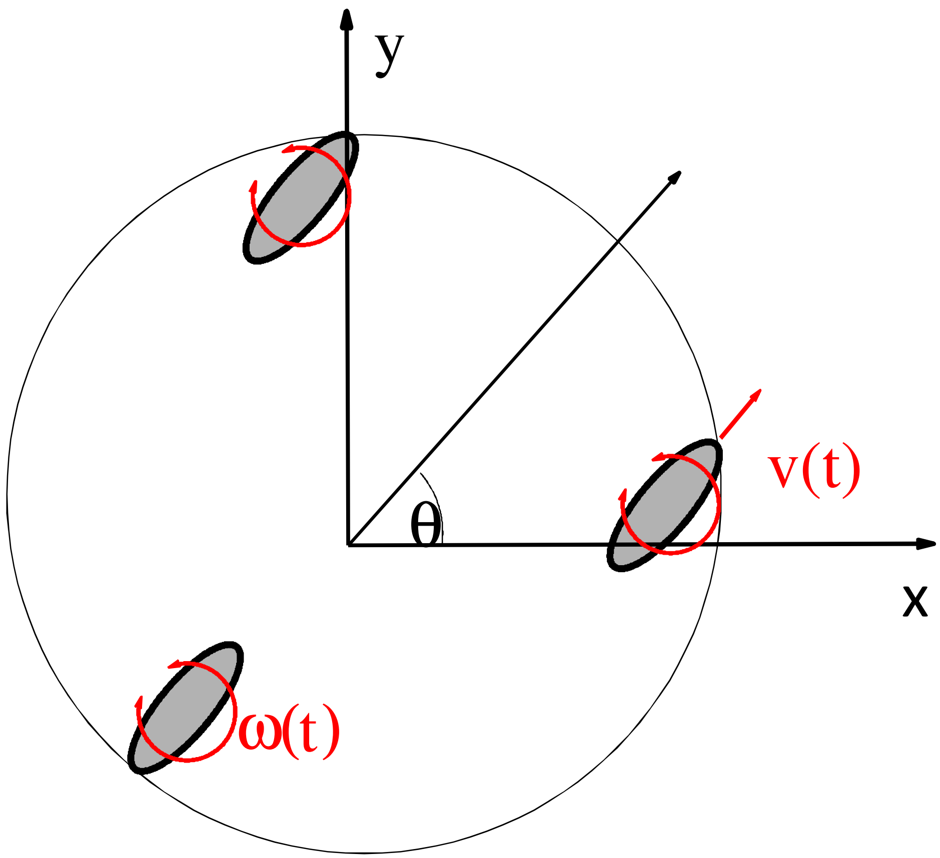
• Because all the wheels remain parallel the synchro-drive always rotate about the centre of the robot

• The synchro drive robot has the ability to control the orientation of their pose directly.

Synchonous Drive

Control variables (independent)

– *v(t), w(t)*



$$x(t) = \int_0^t v(t')\ \cos[\theta(t')]dt'$$

$$y(t) = \int_0^t v(t')\ \sin[\theta(t')]dt'$$

$$\theta(t) = \int_0^t \omega(t')\ dt'$$