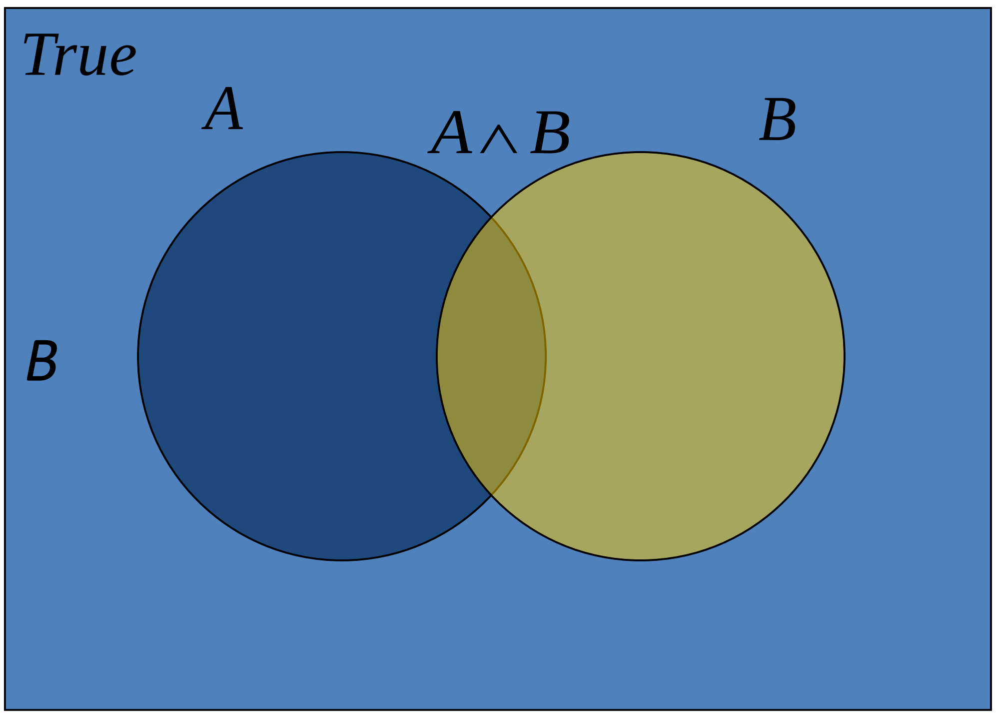
Robotics Group Project - 5CCS2RGP

Lecture 4: Fundamentals for Probabilistic Robotics

Fundamentals in Probability

$$\Pr(A \vee B) = \Pr(A) + \Pr(B) - \Pr(A \wedge B)$$



$$\Pr(A) + \Pr(\text{Ø}A) = 1$$

$$\Pr(\text{Ø}A) = 1 - \Pr(A)$$

Joint and Conditional Probability

- $P(X{=}x \text{ and } Y{=}y) = P(x,y)$

- If X and Y are independent then

$$P(x,y) = P(x)\, P(y)$$

- $P(x \mid y)$ is the probability of x given y

$$P(x \mid y) = P(x,y) / P(y)$$

$$P(x,y) = P(x \mid y)\, P(y)$$

- If X and Y are independent then

$$P(x \mid y) = P(x)$$

Law of Total Probability, Marginals

Discrete case

$$\sum_x P(x) = 1$$

$$P(x) = \sum_y P(x, y)$$

$$P(x) = \sum_y P(x \mid y) P(y)$$

Continuous case

$$\int p(x)\, dx = 1$$

$$p(x) = \int p(x, y)\, dy$$

$$p(x) = \int p(x \mid y)\, p(y)\, dy$$

Law of Total Probability, Marginals

Discrete case

$$\sum_{x} P(x) = 1$$

$$P(x) = \sum_{y} P(x, y)$$

$$P(x) = \sum_{y} P(x \mid y) P(y)$$

X1= move forward
X2=move backward
X3= stop

X1= move forward

When Y= green light
Or
When Y=yellow light
Or
When Y= red light

Bayes Formula

$$P(x, y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$\boxed{P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$$

Normalization

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)} = h\,P(y|x)P(x)$$

$$\sum_x P(x|y) = 1$$

Normalization

$$P(x|y) = \frac{P(y \mid x)P(x)}{P(y)} = \eta P(y \mid x)P(x)$$

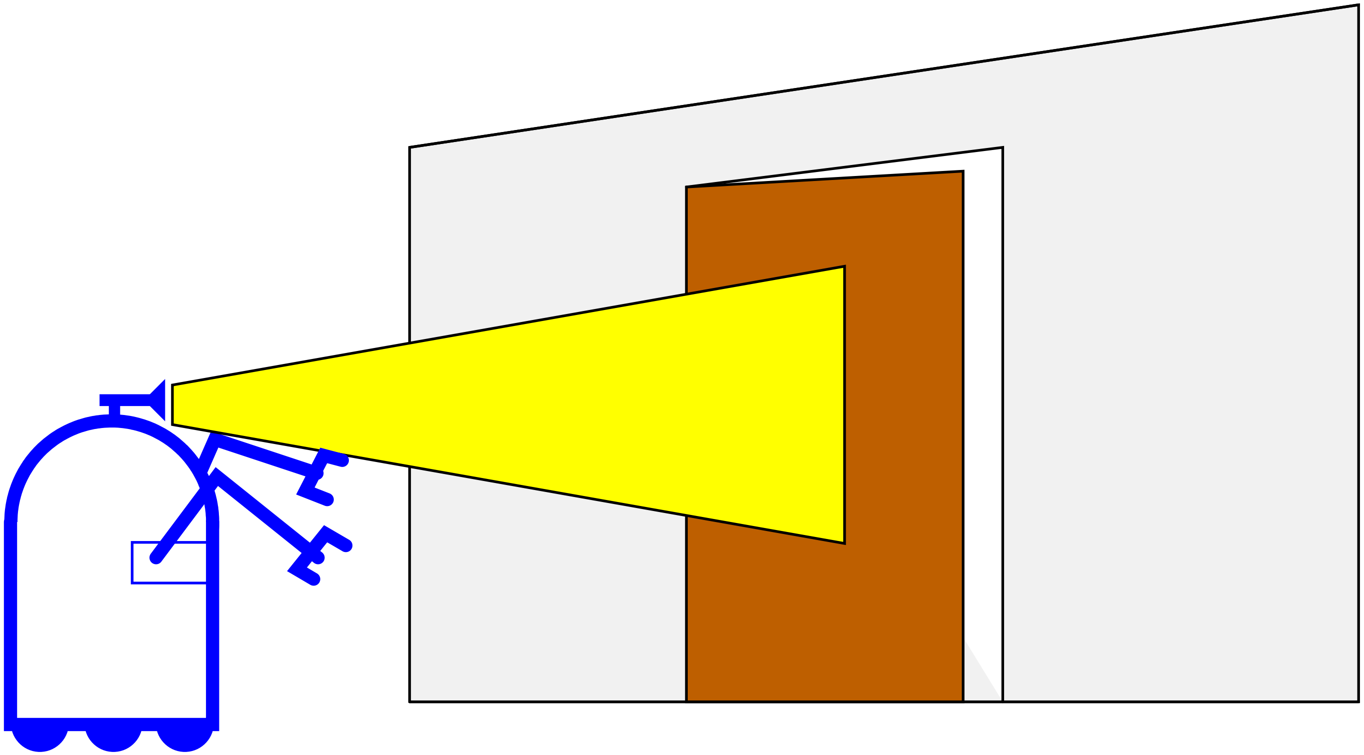
$$\sum_x P(x \mid y) = 1$$

$$\eta \sum_x P(y \mid x)P(x) = 1$$

$$\eta = P(y)^{-1} = \frac{1}{\sum_x P(y \mid x)P(x)}$$

Simple Example of State Estimation

- Suppose a robot obtains measurement z
- What is $P(open|z)$?



Causal vs. Diagnostic Reasoning

- *P(open|z)* is diagnostic.
- *P(z|open)* is causal.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge:

count frequencies!

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

Example

- $P(z|open) = 0.6$ $\qquad$ $P(z|\neg open) = 0.3$
- $P(open) = P(\neg open) = 0.5$

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z \mid open)\,p(open) + P(z \mid \neg open)\,p(\neg open)}$$

$$P(open \mid z) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

Z raises the probability that the door is open.

Combining Evidence

- Suppose our robot obtains another observation z_2.

- How can we integrate this new information?

- More generally, how can we estimate $P(x| z_1...z_n)$?

Actions

- Often the world is **dynamic** since
  - **actions carried out by the robot**,
  - **actions carried out by other agents**,
  - or just the **time** passing by

  change the world.

- How can we **incorporate** such **actions**?

Typical Actions

- The robot **turns its wheels** to move
- The robot **uses its manipulator** to grasp an object
- Plants grow over **time**…

- Actions are **never carried out with absolute certainty**.
- In contrast to measurements, **actions generally increase the uncertainty**.

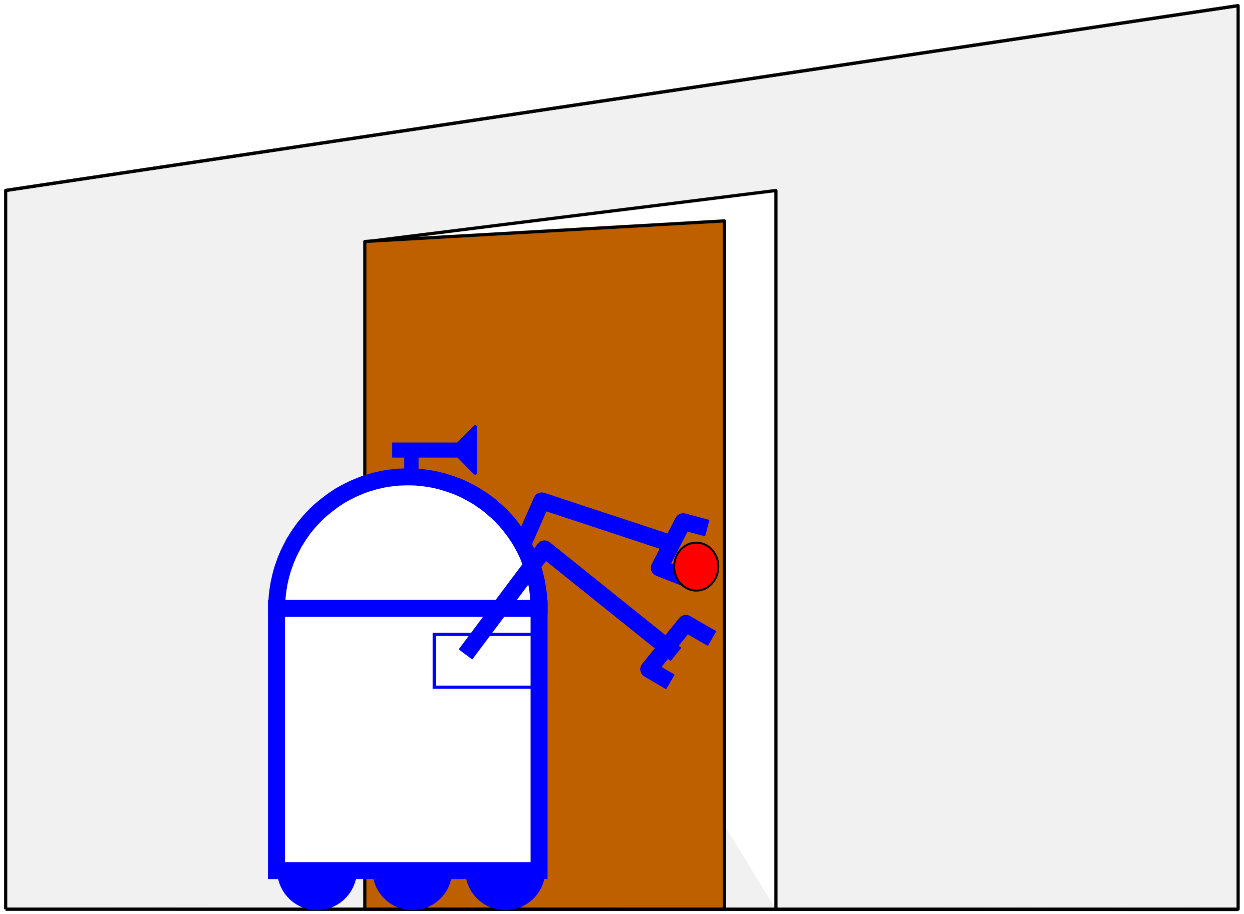
Modeling Actions

- To incorporate the outcome of an action *u* into the current "belief", we use the conditional pdf(probability density function)

$$P(x|u, x')$$

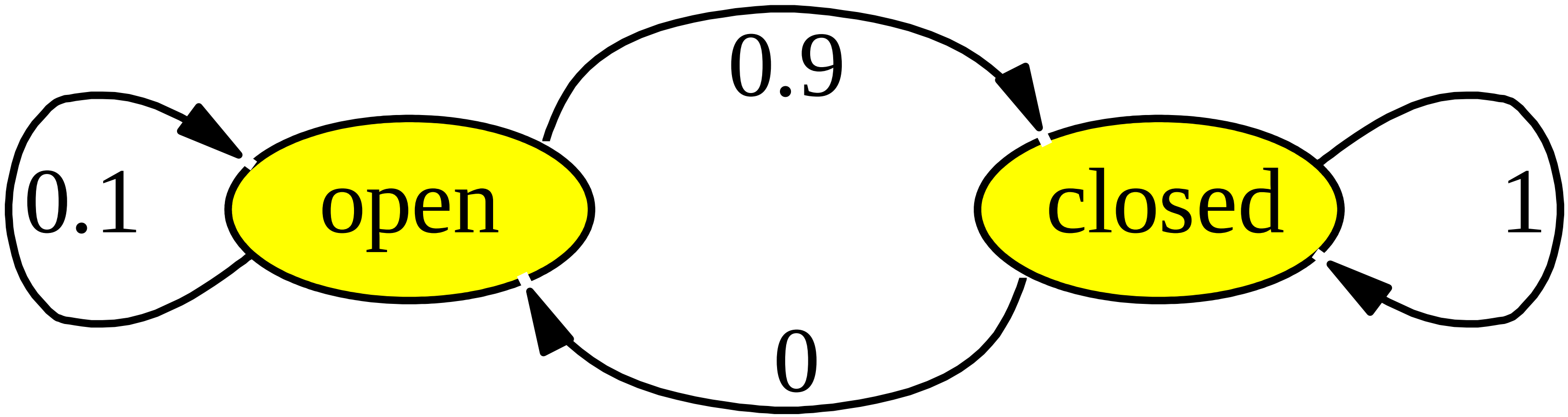
- This term specifies the pdf that **executing *u* changes the state from *x'* to *x*.**

Example: Closing the door



State Transitions

P(x|u,x') for *u* = "close door":



If the door is open, the action "close door" succeeds in 90% of all cases.

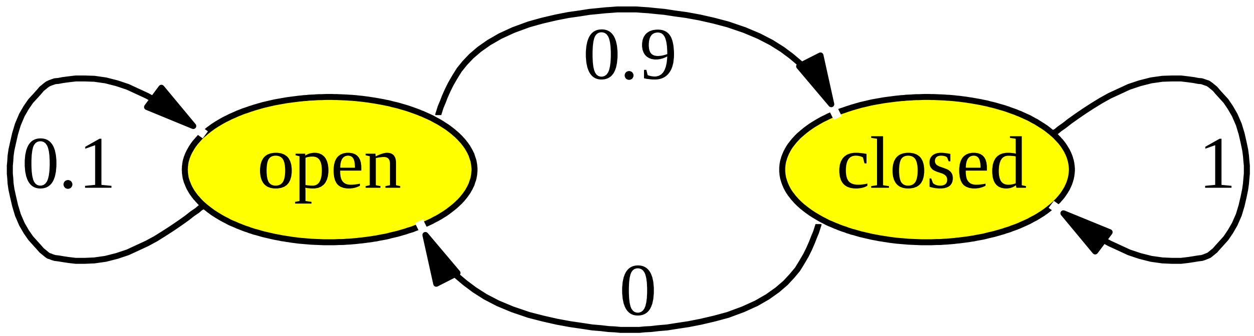
Integrating the Outcome of Actions

Continuous case:

$$P(x \mid u) = \int P(x \mid u, x')P(x')dx'$$

Discrete case:

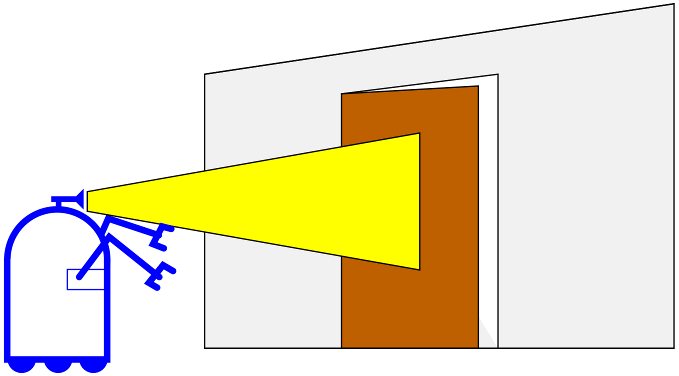
$$P(x \mid u) = \sum_{x'} P(x \mid u, x')P(x')$$

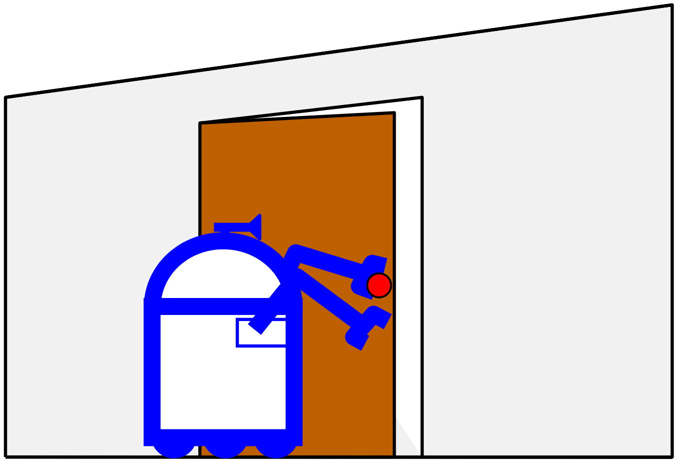


Initial condition: Open =0.5, close = 0.5

P(x|u,x') for *u* = "close door":

P(Open)= P(open/u)P(open)+P(open/u)P(close)
= 0.1*0.5+0*0.5
= 0.05
P(Close) = P(close/u)P(open)+P(close/u)P(close)
= 0.9*0.5+0.5
=0.95





Initial condition: Open =0.5, close = 0.5

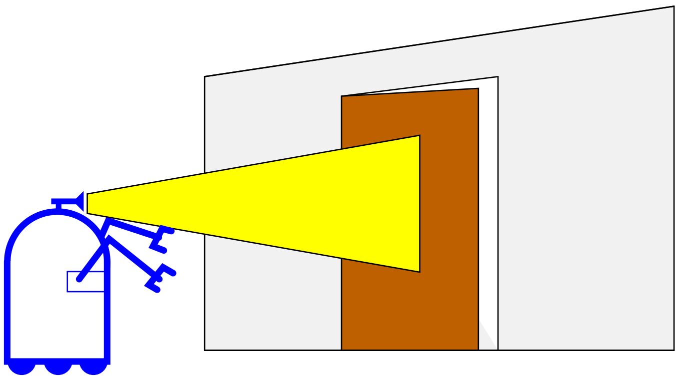
After observation: Open =0.67, close = 0.33

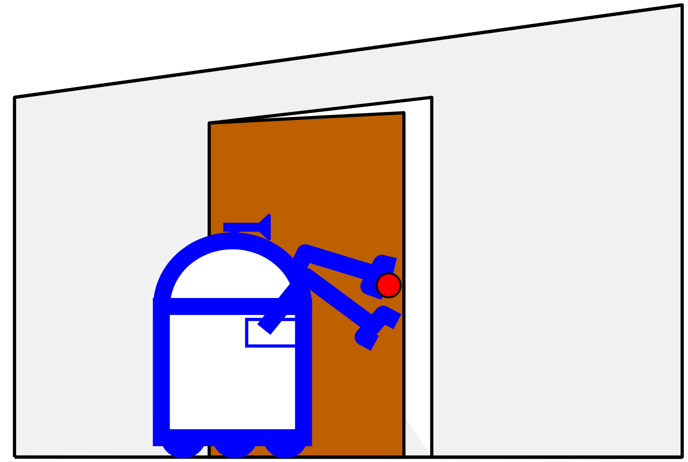
Initial condition: Open =0.5, close = 0.5

After one action: Open =0.05, close = 0.95

Question

- How to iterate the process to cope with continued motions of a robot





Initial P : Open =0.5, close = 0.5

updated P: Open =0.67, close = 0.33

After observation P: Open =0.67, close = 0.33

After one action P: Open =? , close = ?

updated probability

Question

- How to extend the binary state to multiple states (in our case, discretized grids)?