Consumer Search and Firm Strategy with Multi-Attribute Products*

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Abstract

I study a model of directed search in which a consumer inspects products whose valuations are correlated through shared attributes. The consumer discovers her valuation for the attributes of the inspected products and adapts her search strategy based on what she has learned. The consumer anticipates the optimal paths that arise after different realizations; this generates a search rule that accounts for learning systematically. In this search environment, a multiproduct seller commits to a menu of horizontally differentiated products. The seller can exploit the fact that the emerging search paths reveal the consumer's preferences: by setting different prices for *ex ante* identical products, the seller can encourage specific paths to arise and exploit the information that the consumer learned through search. In some cases, the seller optimally limits the set of available products.

Keywords: consumer search, directed search, learning, multiproduct monopoly, pricing, product portfolio

JEL Codes: D42, D83, L12, L15

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1. Introduction

Multiproduct firms are important players in many economic environments. The wide array of strategic choices at their disposal, however, makes them a difficult subject to study. Much has been written about the risk multiproduct firms run to have their products cannibalize each others' demand.[1] Less attention has been devolved to the synergy arising when the products offered by such a firm are correlated and to the effect this has on how consumers interact with the product menu. To study this dimension of the firm's strategic considerations, I develop a framework that allows products to be correlated through shared attributes. In this environment, I study the optimal pricing and menu composition that is chosen by a multiproduct monopolist, and how these choices affect consumer learning when there are search frictions.

The consumer search literature has highlighted the role of search frictions as determinants of market outcomes.[2] The effect of these frictions for intra-firm search, however, has been so far understudied.[3] I contribute to the literature by incorporating correlation across the products offered by a single firm to study how this affects consumers' optimal search and how firms would condition their strategic decisions on it. In particular, I consider consumers that value products based on their attributes (Lancaster, 1966). Initially, consumers observe all products and respective attributes, but they do not know how much they value the different attributes. For example, laptops may differ in their processing speed and graphical capabilities, which depend on the processor and the graphic card that are installed.

Consumers decide which products to search for and inspect, and then, based on their findings, adapt their strategy accordingly for their next search. The reasoning is as follows: if two products share an attribute, consumers value them identically with respect to that attribute. Through the search process, consumers learn their preferences for attributes and, depending on what they learn about specific attributes, can redirect their subsequent search because they know which products share the same attributes and which ones do not. The result of any given inspection makes consumers update her expectations for the remaining products based on which attributes they share. This, in turn, instructs the next inspection.

In many circumstances, these learning dynamics represent well consumer search behavior: if a consumer learns that she dislikes a certain attribute in a product, she would rationally try to avoid other products that share that attribute. For example, Hodgson and Lewis (2020) shows evidence of "spatial learning" in search: consumers inspect more differentiated products early and get closer to the eventually purchased option as search

[1]This consideration is prominently pointed out, for example, in Nocke and Schutz (2018).

[2]Consumers generally do not make consumption choices with perfect information; evidence of this can be found in the empirical industrial organization literature (Sovinsky Goeree, 2008) and in the marketing literature (Mayzlin and Shin, 2011).

[3]Prominent exceptions are Petrikaitė (2018) and Nocke and Rey (2023).

progresses. I show that this multi-attribute structure generates a version of Weitzman (1979)'s optimal search in an environment with correlated products. Further, I show that, in this environment, "backtracking" to a previously inspected and abandoned attribute can be optimal.

A multiproduct monopoly firm commits to a menu and posts products' prices anticipating the consumer optimal search process. Prices are posted and contribute to determining the order in which consumer search for their preferred option.[4] Because the outcome of each inspection instructs the next, each inspection reveals the consumer's learned preferences. The firm can price products differently to encourage consumers to self-sort based on the preferences they learn about through the search process, a mechanism reminiscent of that highlighted in Mayzlin and Shin (2011). Unlike in Mayzlin and Shin (2011), however, different prices can emerge in my framework even if products are *ex ante* identical from the consumer's perspective.

Differential prices might induce the consumer to deviate from the seller's preferred order of search. I show that in some cases, when the product menu is relatively small, the seller has an incentive to restrict the supply by removing specific products from the menu and, with them, alternative search paths available to the consumer.[5] The menu restriction induces the firm's preferred order of search to arise, and it is an optimal strategy when the likelihood of a positive realization is high and search is cheap. Whenever this is the case, the seller strictly prefers a uniform pricing strategy over setting different prices for different products. Therefore, both uniform and differential prices can arise in equilibrium.

The results highlight the ability of a multiproduct firm to steer consumers through strategic menu selection. By anticipating how a consumer would react after observing a product, the seller can encourage search towards better suited products, and profit off the consumer's incentive to find good matches. The seller wants the consumer to keep searching whenever possible: what is learned through inspection of a product makes the consumer fine-tune her selection.[6] The seller can increase profits by setting higher prices along paths consistent with positive realizations without discouraging the consumer to search on paths consistent with negative ones.

I propose strategic menu selection as a new mechanism through which consumers can be steered in their consumption choices. To the best of my knowledge, this paper is the first to study the implications of menu selection for consumer steering. In particular, the dynamics presented in this paper highlight the possibility to steer consumers through

[4]Price advertisement also resolves hold-up problems arising with monopoly pricing in the presence of search frictions as shown in Anderson and Renault (2006); a more in-depth analysis on the matter can be found in Robert and Stahl (1993) and Konishi and Sandfort (2002).

[5]This strategic choice is fundamentally different from that highlighted by Johnson and Myatt (2002): The authors consider menu pruning in response to entry, while no competitor is ever present in my framework.

[6]The learning component, then, leads to an outcome opposite to that shown in Petrikaitė (2018); the multiproduct monopoly firm studied by the author has an incentive to obfuscate options to increase the probability of selling more expensive alternatives.

recommendation systems in environments when a single agent has control over the product menu. Additionally, the paper unearths a novel possible instance of price discrimination based on consumers' search history.

The rest of the paper is structured as follows: after reviewing the related literature, I present the framework (Section 2) and characterize the optimal search process with multiple attributes and the learning process they imply in a simplified version of the model (Section 3). Afterwards, I solve the problem of a monopoly seller that selects which products to make available and their prices (Section 4). I present a general version of the search model and provide the equilibrium pricing for the infinite products case in Section 5. After exploring extensions and limitations in Section 6, I conclude in Section 7.

Related literature This paper relates to several strands of literature. First, it contributes to the ordered consumer search literature pioneered by Weitzman (1979). Weitzman characterizes the optimal process for a consumer costly searching among n independent boxes. Each box is characterized by a reservation value, a score representing the value that would make the consumer indifferent between opening the box and keeping a certain reward equal to the score. The optimal search order has the consumer opening boxes from the highest to the lowest score. The consumer optimally stops when no unopened box has a score higher than the highest past realization.

The role of search order on market outcomes has been studied extensively in oligopoly settings: Choi et al. (2018) and Haan et al. (2018) study the effect of posted prices on search order. Because sellers want to undercut each other to gain prominence in the search order, pinning down an equilibrium requires consumers to be heterogeneous enough, specifically in the form of different mean expected qualities. Anderson et al. (2020) obtains similar results by introducing heterogeneity through the search cost distribution. The features instructing the order of search in these models are, however, never shared between products. Because the multiproduct monopoly seller I focus on does not have an incentive to undercut himself, moreover, heterogeneity in consumers' characteristics is not necessary in my setting.

Other authors have incorporated correlation in products in the presence of search frictions: Shen (2015) and Armstrong and Zhou (2011) embed the search process in a Hotelling framework so that, in both settings, the available products are perfectly negatively correlated. Ke and Lin (2022) and Bao et al. (2022) study optimal search in a simple framework in which a discrete number of products share one of their two attributes. Ke and Lin (2022) provides conditions under which correlation in search leads to complementarity of the products available. Bao et al. (2022), instead, studies Bayesian updating when the consumer cannot distinguish the role of each attribute in the *ex post* utility each product grants. Conditional search order is also at the core of a recent contribution by Doval (2018): The paper extends Weitzman's search process by allowing the consumer to consider all uninspected products as viable outside options, which changes the relative value of the available options and, therefore, the optimal consumer search process.

Weitzman (1979)'s result relies on the assumption that boxes are independently distributed. I relax this assumption and propose a tractable, history-dependent scoring system that incorporates the value of searching beyond the target of inspection. The score is determined accounting for the paths that would be optimally taken by the consumer after the realization they refer to and, therefore, reflect the full value of inspecting new attributes and the respective continuation value. Through this scoring system, I show that a dynamic, adaptive version of Weitzman (1979)'s optimal search policy can be characterized in this environment. Therefore, the paper relates to the growing literature of learning in search (Garcia and Shelegia, 2018, Greminger, 2022, Preuss, 2023).

The paper further contributes to the wide literature on multiproduct firms. Earlier contributions addressed several possible strategies available to this kind of seller. Some, like Mussa and Rosen (1978), focused on price discrimination with vertically differentiated products. Others, like Eaton and Lipsey (1979), discuss market pre-emption through introduction of horizontally differentiated options. Other notable example relate to R&D expenditure (Lin, 2004, Lambertini and Mantovani, 2009) and bundling of products (McAfee et al., 1989).

This paper contributes to the literature on the interaction between menu selection and pricing (Brander and Eaton, 1984, Johnson and Myatt, 2002, Nocke and Schutz, 2018). Novel to the literature is the inclusion of correlation across the products offered by the firm. Correlated products allow consumers to learn their preferences as they inspect options and, therefore, the presence of correlated products affects the value of inspecting each product in isolation.

Finally, the paper contributes to the literature of pricing in search. The seminal Wolinsky (1986) model, and most of the literature that followed, focuses on competitive settings.[7] Instead, I study within-firm directed search in a monopoly setting as in recent contributions by Petrikaitė (2018) and Nocke and Rey (2023). The latter studies the incentives of a multiproduct seller to "garble" product information to induce consumers to search longer. Because search costs are assumed to be fixed, the firm has no incentive to price discriminate. Petrikaitė (2018), instead, shows that a multiproduct seller can steer consumers towards expensive products by obfuscating cheaper options. In my framework, steering can arise in the form of differential prices being optimally set by the seller without strategic obfuscation of the products made available. The paper, then, is related to the growing steering literature as well (e.g. Ichihashi, 2020, who also considers a monopoly setting).[8]

[7] This is true also for most papers studying multiproduct firms; see for example Zhou (2014), Rhodes (2015), Rhodes et al. (2021)

[8] Other notable contributions, although less closely related, can be found in De Corniere and Taylor (2019), Teh and Wright (2022), and Heidhues et al. (2023).

2. Simplified Framework

The products. I consider an industry with products differentiated with respect to two attributes.[9] A product (i, j) is identified by attributes $A_i \in A$ and $B_j \in B$. I consider first a simplified framework in which A and B come in two variants each and follow a simple binomial distribution. In Section 5, I propose a more general framework to build on the intuition of the simplified one. Each attribute A_i can be found combined with all attributes B_j, $i, j \in \{1, 2\}$, and *vice versa*. One can visualize the products as displayed in a grid, with the rows representing the A attributes, the columns representing the B attributes, and the cells representing products defined by a specific combination of A and B attributes as depicted in Figure 1. Notice that products are only differentiated horizontally through their attribute compositions and are otherwise identical in quality.

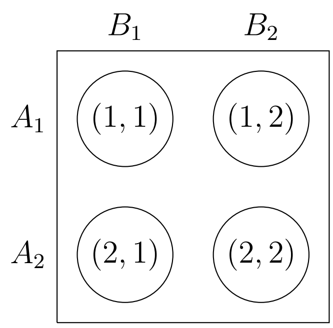

Figure 1: *Products in the same row (resp. column) share attribute A_i (resp. B_j).*

The consumer. A representative, risk-neutral consumer (she) has unit demand, is aware of the available products and their attribute composition, and can inspect the products in any order she likes. The consumer has no prior knowledge of her preferences over the available attributes; she learns the realization of each attribute separately by inspecting a product characterized by it. In line with existing models,[10] I assume that *ex post* utility generated by a generic product (i, j) takes the form:

$$u(A_i, B_j) = A_i + B_j = u_{i,j}.$$

I assume that attributes follow a Binomial distribution: each attribute $y \in A \cup B$ is either a match, generating *ex post* utility one with probability $\alpha \in (0, 1)$, or it is not, generating utility zero instead. The assumption that attributes enter $u_{i,j}$ additively crucially implies that there are no complementarities between attributes: once an attribute is discovered, its realized value affects all products that are defined by it in the same way. The expected utility of an unsampled product (i, j) is then:

$$E[u_{i,j}] = \alpha + \alpha = 2\alpha.$$

Expected utility of a product (i, j) sharing an attribute with a previously sampled product,

[9] The framework is adapted from Smolin (2020).

[10] For example: Choi et al. (2018) and Greminger (2022).

say A_i, but not the other, is instead:

$$E[u_{i,j}] = A_i + \alpha.$$

In this environment, I study the optimal sequential search process with free recall: a consumer can always go back to a previously inspected product at no additional cost. The cost of inspecting a product is indexed by the constant $s \in (0, 2\alpha)$. The consumer learns the value of each attribute separately after inspecting a product defined by it. Finally, the consumer's outside option is normalized to $u_0 = 0$.

The seller. A multiproduct monopoly seller (he) selects which of the possible products to make available to the representative consumer (that is, he selects $\tilde{N} \subseteq N$), and their respective prices. He is also aware of the match probability α and search costs s. The seller can influence the search pattern over available products through prices. Prices are set before the search process starts, cannot be changed, and are observed costlessly by the consumer before she starts searching. All production costs are equal to zero.

Timing and equilibrium concept. The timing of the interaction can be summarized as follows:

1. The seller selects $\tilde{N} \subseteq N$ products to make available and price vector $\mathbf{p}(\tilde{N})$.
2. The consumer observes $\tilde{N}$, $\mathbf{p}(\tilde{N})$, chooses between searching and her outside option, and, if she searches, what to inspect.
3. After each inspection, the consumer chooses between stopping and keeping searching (and what to inspect next) until she either purchases an inspected product or leaves without making a purchase.

I consider Subgame Perfect Equilibria: because the seller commits to menu and prices before the search process starts, and because prices are posted, there is no need to model beliefs explicitly in this environment.

3. A Simple Model of Multi-Attribute Search

Because $A_i \in \{A_1, A_2\}$, $B_j \in \{B_1, B_2\}$, the product space N consists of four products:

$$N = \{(1,1), (1,2), (2,1), (2,2)\}.$$

To illustrate the search dynamic in isolation, I start with the assumption that prices are exogenously set at zero; this assumption will be relaxed in the next section. The consumer can inspect any product in $\tilde{N}$; I start from the case in which $\tilde{N} \equiv N$. At any given point of the search sequence, the set of available products can be partitioned in the set of inspected products, I, and uninspected products, $\tilde{N} \setminus I$.

Updating expected utilities. Suppose that the consumer already inspected one of the products. Because all products are *ex ante* identical, inspecting $(1,1)$ first is without loss of generality.[11] Whenever the product to inspect can be chosen randomly without loss of generality, I assume that products are inspected in increasing order of their indices. After the first inspection, the consumer has learned realizations A_1 and B_1. Which of the remaining products should be inspected next, if any?

In this simplified framework, it is straightforward to show that the consumer would want to search keeping an attribute she has learned to have positive valuation for (if search costs are low enough), and ignoring one for which she has valuation zero. Formally, given realization $u_{1,1} = A_1 + B_1$, the consumer updates her expectations for the remaining product according to:

$$E(u_{1,2}|I = \{(1,1)\}) = A_1 + \alpha, \qquad E(u_{2,1}|I = \{(1,1)\}) = \alpha + B_1,$$
$$E(u_{2,2}|I = \{(1,1)\}) = 2\alpha.$$

The consumer would next choose to inspect the product with the highest updated expected value as long as:

$$\max_{(i,j)\in N\setminus I} E(u_{i,j}|I) - s > \max_{(i,j)\in I} u_{i,j},$$

which immediately leads to the optimal follow-up search for each possible realization of $(1,1)$:

- if $A_1 = B_1 = 0$, $(2,2)$ is searched next; no other search can take place because $A_2 \geq A_1$ and $B_2 \geq B_1$.
- if $A_1 = B_1 = 1$, the consumer stops at $(1,1)$ because $A_1 \geq A_2$ and $B_1 \geq B_2$.
- if $A_1 > B_1$, $(1,2)$ is searched next (if $\alpha > s$); no other search can take place because $A_1 \geq A_2$ and B_2 is shared between $(1,2)$ and $(2,2)$.
- if $A_1 < B_1$, $(2,1)$ is searched next (if $\alpha > s$); no other search can take place because $B_1 \geq B_2$ and A_2 is shared between $(2,1)$ and $(2,2)$.

[11] All products share each attribute that characterizes it with another product, and for all products there is one other product that shares no attributes with it. Therefore, all product are *ex ante* identical as long as prices are uniform

Expected utility of searching. Different realizations lead to different search paths being taken every time a new product is inspected. These conditional search paths emerge predictably, and all realizations generate unambiguously an optimal path forward. In turn, this implies that a rational consumer would account for the likelihood of these different paths emerging, and the expected utility they are associated with, when deciding whether to start searching or not. From the above, therefore, we obtain the expected utility of searching given the available products and the optimal search paths that can emerge:

$$E(u_{i,j}|I \equiv \emptyset) = 2\alpha^2 + 2\alpha(1-\alpha)\max\{1, 2\alpha + (1-\alpha) - s\} + (1-\alpha)^2(2\alpha - s) - s.$$

The first term refers to (i, j) being the best possible match ($u_{i,j} = 2$, with probability α^2). The second term refers to the eventuality of the consumer liking only one of the two attributes, and incorporates the possible second search that outcome would entail, which only takes place if $s \leq \alpha$. The third refers to the case in which $u_{i,j} = 0$ so that the product sharing no attributes with it would be inspected next. Figure 2 exemplifies the optimal search pattern.

4. Seller's Optimal Strategy

The seller's problem is twofold: he must set up prices to maximize profit, and he must select $\tilde{N}$ to generate trade opportunities. The two decisions are related. The consumer search path depends on the price she observes, and which prices would deter her from searching depend on the available products. In particular, the consumer is willing to search a product priced above its myopic expected value $2\alpha - s$ as long as the expected utility of searching from that point onward is non-negative. As shown above, this can be achieved when products that share attributes with each other are made available. A seller can, in principle, price products above their myopic expected value as long as he made available enough products to justify it.

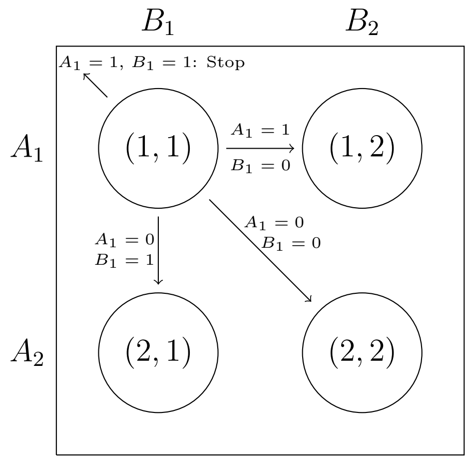

Figure 2: *Optimal search with binomial distribution and all products available, starting from* $(1, 1)$.

The two decisions - menu selection and pricing - interact in non-obvious ways. Uniform prices, for example, cannot induce an order of search different from the one characterized above. If these uniform prices are too high, however, some search paths could end prematurely: even if products are identical *ex ante*, the first one searched - $(1, 1)$ in the example above - carries more new information than every subsequent search that could arise. It follows that the highest price that would make two products not sharing attributes worth searching is different. If prices are not uniform, however, the consumer could adapt her optimal order of search in response: between a more expensive product for which she has positive information and a cheaper one for which she has no information, that she would inspect the former first is not obvious.

To study these different interactions, I solve the menu and pricing game of the seller considering uniform and differential prices separately. I show that the seller can always manipulate prices to induce a specific ordering of the consumer search. Moreover, I show that the seller has an incentive to strategically restrict the menu of available products to induce his preferred order of search to arise when search is cheap.

4.1. Uniform Prices

Under uniform prices, the seller's trade-off is clear-cut. He wants to raise prices to capitalize on any positive outcome of the consumer search, and he wants to lower prices to incentivize inspections after negative outcomes. The seller is indifferent regarding which product is ultimately purchased, as long as one is. For this reason, I start by assuming that all products are available: $\tilde{N} \equiv N$. I then show the seller's incentive to restrict the menu and the effect this choice has on consumer search.

Consider a generic uniform price level p^u. The seller wants to set the highest level p^u conditional on certain constraints implied by the consumer search process not being violated. Given the optimal search pattern identified in the section above, the expected utility of performing the first inspection is:

$$\begin{aligned}&E(u_{1,1}|I \equiv \emptyset) = \alpha^2 \max\{2 - p^u, 0\} - s \\ &+ 2\alpha(1-\alpha)\max\{1 - p^u, \alpha \max\{2 - p^u, 0\} + (1-\alpha)\max\{1 - p^u, 0\} - s, 0\} \\ &+ (1-\alpha)^2 \max\{\alpha^2 \max\{2 - p^u, 0\} + 2\alpha(1-\alpha)\max\{1 - p^u, 0\} - s, 0\}.\end{aligned} \tag{1}$$

That is: the value of inspecting $(1,1)$ is equal to the expected value generated by the search paths that are induced by the possible different realizations. These in turn depend on the relative value of s and α, over which the seller has no control over, and p^u.

At $p^u = 0$, the search problem of the consumer is identical to the one explored in the example above. As prices grow, however, some search paths become inaccessible. The first search path to be prevented by high prices is the one that arises conditional on a bad first match. Indeed, given observation $u_{1,1} = 0$, $(2,2)$ is searched as long as

$$E(u_{2,2}|u_{1,1} = 0) = \alpha^2 \max\{2 - p^u, 0\} + 2\alpha(1-\alpha)\max\{1 - p^u, 0\} - s \geq 0. \tag{2}$$

It is straightforward to show that there exists values p^u such that this condition is not satisfied but $E(u_{1,1}|I \equiv \emptyset)$ is positive: even if the consumer would not search after a bad realization of $(1,1)$, the presence of products sharing attributes with it makes it more likely to find something worth purchasing. As long as p^u is such that $E(u_{1,1}|I \equiv \emptyset)$ is non negative, the consumer can rationally start inspecting products. With this inspection, the consumer can discover that she likes both attributes, after which she always stop searching because she can find no better match. Alternatively, if the consumer likes only one attribute, she is interested in inspecting the other available product that shares it. Suppose $A_1 = 1$, $B_1 = 0$, and $p^u \leq 1$. The consumer would want to perform this additional search if and only if:

$$u_{1,1} = 1 - p^u \leq 1 + \alpha - s - p^u = E(u_{1,2}|I = \{(1,1)\}),$$

which is always satisfied if $s \leq \alpha$, that is, if inspecting a single attribute is worth the necessary search cost. If $s > \alpha$, that is, if s is higher than the expected gain of inspecting one attribute in isolation, the consumer would only ever inspect a product she knows nothing about. In this case, the presence of correlated products is immaterial: because no product can be reached after inspecting a different product with which it shares an attribute, the expected gain of inspecting a product is only ever its expected value. Therefore,

$$p^M = \frac{2\alpha - s}{\alpha(2-\alpha)},$$

(where the superscript M stands for "myopic") is the optimal price when $s > \alpha$.

Suppose now that $s \leq \alpha$. The seller can select one of two pricing profiles: on one hand, he can elect to price products in a way that encourages a follow-up search after a first bad realization. These prices must make a product just myopically worth searching, or, they must solve equation (2) with equality:

$$\mathbf{p^E} = \begin{cases} p_L^E = p^M & \text{if } \alpha^2 \leq s \leq \alpha, \\ p_H^E = \frac{2\alpha^2 - s}{\alpha^2} & \text{if } 0 < s < \alpha^2, \end{cases}$$

where E stands for "encourage", L stands for "low", and H stands for "high".

Alternatively, the seller can select higher prices that discourage search after a bad first realization. These prices must be strictly higher than the encouraging ones and lead to a lower probability of trade, but a higher return conditional on the consumer finding something to purchase. These prices are such that $E[u_{1,1}]|_{I \equiv \emptyset} = 0$, because for any higher price the consumer would not start searching:

$$\mathbf{p^D} = \begin{cases} p_L^D = \frac{2\alpha(1+(1-\alpha)(\alpha-s))-s}{\alpha(2-\alpha)} & \text{if } \frac{3\alpha^2-2\alpha^3}{1+2\alpha-2\alpha^2} \leq s \leq \alpha, \\ p_H^D = \frac{2\alpha(\alpha(3-2s)-(1-\alpha)s)-s}{\alpha^2(3-2\alpha)} & \text{if } 0 < s < \frac{3\alpha^2-2\alpha^3}{1+2\alpha-2\alpha^2}, \end{cases}$$

where D stands for "discourage".

Uniform price selection. Lower prices can always be selected whenever higher ones do not prevent the consumer from searching. The seller is, however, not interested in his products being inspected, but in his products being purchased. Trade is maximized for $p \leq 1$: any higher price requires the consumer to like both attributes in a product to purchase it. Notice that it holds:

$$p_L^E > 1 \iff 0 < s < \alpha^2.$$

Therefore, the price that maximizes search and trade can be identified as the minimum between p_L^E and 1. To simplify the notation, I define:

$$p_T = \min(p_L^E, 1),$$

where T stands for "trade", as p_T is the price that maximizes the probability of trade. Overall, when selecting p^{u*} among the candidate equilibrium prices displayed above, the seller chooses between maximizing search efforts, maximizing per-sale revenue, and maximizing probability of trade. Higher prices discourage search and reduce probability of trade for a given search pattern; lower prices encourage search but lead to lower revenue conditional on trade taking place.

By plugging in the various (feasible) prices for the various combinations of α and s and following the search path different prices induce according to Equation (1), one can obtain the expected profit of the seller. These profits can then be directly compared and lead to a unique equilibrium price for all possible combinations of α and s. In particular, when $s > \alpha$, the only candidate price and relative expected profit is:

$$p^M < 1 \quad \to \quad \pi^M = p_M \left(1 - (1-\alpha)^4\right).$$

Instead, when $s \leq \alpha$, the candidate prices obtained above lead to expected profits:

$$p_T \leq 1 \quad \to \quad \pi_L^E = p_T \left(1 - (1-\alpha)^4\right),$$

which maximizes probability of trade and is always valid,

$$p_L^D < 1 \quad \to \quad \pi_I^D = p_L^D \left(1 - (1-\alpha)^2\right),$$

which prevents any further inspection after a bad first realization if $0 < s < \frac{3\alpha^2 - 2\alpha^3}{1+2\alpha-2\alpha^2}$, but generates trade if any one inspected attribute is appreciated,

$$p_H^E > 1 \quad \to \quad \pi_H^E = p_H^E \left[\alpha^2(1 + 2(1-\alpha) + (1-\alpha)^2)\right],$$

which always allows for a second inspection if $0 < s < \alpha^2$, but requires the consumer to find a product to like in both attributes to lead to a purchase, and

$$p_H^D > 1 \quad \rightarrow \quad \pi_H^D = p_H^D \left[\alpha^2(1 + 2(1-\alpha))\right],$$

which does not allow for another search after a bad first realization. In all cases, expected profit is calculated as price times the probability of trade generated because production costs are assumed to be equal to zero. The candidate prices reflect the relative importance of encouraging search and extracting rent conditional on search taking place. In particular, the seller trades off higher probability of trade by encouraging search and revenue conditional on trade taking place by discouraging it.

Intuitively, higher prices are preferable, for the seller, for low search costs and high probability of a match α. For such parameters the consumer is easily encouraged to start searching. If $s > \alpha$, there is only one candidate price, p^M, the lowest of the candidate prices. For $s \leq \alpha$, instead, which of the four candidate prices is selected depends on the relative value of s: the lower s is, the higher prices can be set without impeding search.

Implications for the optimal menu selection. Because prices can encourage or discourage search, they also determine the optimal product menu selection. When all products are available, any product can be rationally selected to be the first to inspect by the consumer. For high enough prices, however, not all products can be inspected after fixing a starting point. From the above discussion it emerges that if the seller optimally selects $p^{u*} = \mathbf{p^D}$, conditional on the consumer starting from $(1,1)$, inspection of $(2,2)$ could not rationally take place. Indeed, $(2,2)$ would only be inspected after a bad first realization, but $p^{u*} = \mathbf{p^D}$ prevents this search altogether. When the seller selects a price that prevents search after a bad first realization, introducing three or four products is equivalent from the seller's perspective. Because this equivalence is a byproduct of the unrealistic assumption of zero fixed costs associated with each product, it is sensible to assume that, in this case, only three products would be introduced.

Notice that this does not affect the expected utility of search if inspection starts from the right product. If $(2,2)$ were to be removed, search starting from $(1,1)$ would be unaffected. Starting from any other product, however, would generate negative expected utility of search. Suppose for example that $(2,2)$ was removed and that the consumer started from $(1,2)$ or $(2,1)$. Then, not only she would not rationally inspect the unrelated product, but she would not be able to inspect $(2,2)$ after learning something positive about it. This cannot be optimal.

By removing a product, the seller effectively "locks" the consumer into a specific search path. The values α, s and p^u determine which search paths can be taken; given these search paths, products are introduced. For example: if it $s > \alpha$, inspection of a single attribute is never rational. Then, the only feasible search paths affect products that share no attributes. It follows that, in this case, only products that share no attribute would be introduced. The discussion motivates the following result:

Proposition 1. *Consider a multiproduct seller selecting optimal menu $\tilde{N} \subseteq N$ and uniform pricing p^u of multi-attribute products. In equilibrium:*

- *If $s > \alpha$: $p^{u*} = p^M$, $|\tilde{N}| = 2$, and the consumer can start searching from any available product.*
- *If $s < \alpha$ and $p^{u*} = \mathbf{p^E}$: $\tilde{N} \equiv N$, and the consumer can start searching from any available product.*
- *If $s < \alpha$ and $p^{u*} = \mathbf{p^D}$: $|\tilde{N}| = 3$, and the consumer is steered toward a specific search path.*

Proof. All calculations and precise cut-offs for α and s can be found in Appendix A. ■

Discussion. The seller values higher probabilities of trade taking place: because prices are uniform, the seller is not concerned with which product is purchased as long as one is. Selecting prices that do not hinder the probability of trade is often optimal. Raising prices is only worth it if the loss of a potential trade is compensated when trade does take place. In particular, α must be high enough that the chances of not liking the first product inspected are low, and s must be low enough that search is not discouraged. Whenever this is the case, the seller can raise price and not introduce all possible variants; as a consequence, there is a loss in trade efficiency. When the supply is restricted, moreover, the seller effectively induces a specific order of search. strategic menu selection can give rise to endogenous prominence based on the relative position of the products.

At uniform prices the consumer retains some positive expected value from search when the seller has an incentive to maximize trade by keeping prices low. Whenever this is the case, moreover, the consumer is free to start from any of the available products. As I will show in the next section, however, the seller generally has a profitable deviation if he is allowed to set different prices for these products and soften the trade-off between encouraging search after bad realizations and profiting whenever fine-tuning after a good, but not great, match is possible.

4.2. Differential Prices

When prices are assumed to be uniform, the choice of the seller is between keeping prices low to maximize search, and raising them to capitalize on good realizations. Ideally, the seller wants both: low prices to make the consumer keep searching after bad realizations, and high prices to profit off the consumer learning what she likes. This can be achieved if the seller can price products differently.

The trade-off of the seller under uniform prices refers to different search paths. Low prices encourage further search whenever the consumer finds nothing to like with her first inspection. High prices generate higher profits when the consumer partially likes at least the first option inspected. By pricing along these paths differently, the seller can achieve both higher probability of trade compared to the high uniform price case, and higher expected profit compared to the low uniform price case.

To see why, consider again the uniform price p_T that generates the maximum probability of trade but low rent extraction. When this price is optimally selected, it allows the consumer to keep searching after a bad first realization, and trade is likely to take place. In particular, what is needed is that the first product inspected, say $(1, 1)$, and the product that would be searched next conditional on $A_1 = B_1 = 0$, $(2, 2)$, to be priced at p_T. On this path, if the other products were priced higher than p_T, nothing would change because $(1, 2)$ and $(2, 1)$ would not be considered even at uniform prices, as long as the consumer can rationally start searching.

If the consumer, instead, learns that she likes an attribute inspected in the first search, she would like to search next along that attribute. This is clearly true if prices are uniform. Suppose, however, that $(1, 2)$ and $(2, 1)$ were priced slightly higher than $(1, 1)$. If the consumer has learned that she likes A_1 (resp. B_1), and if the price difference is not too high, she would still want to search the more expensive product. Going backwards: the consumer would start her search from the cheaper option given that products are *ex ante* identical. As long as the price differential is not too high, the consumer has no incentives to stop searching early, nor to deviate towards a different search path. By pricing $(1, 1)$ and $(2, 2)$ at $p = p_T$, and the remaining products at a higher price the seller can then achieve both higher prices and higher probability of trade. In doing so, the seller erodes at the consumer expected utility without preventing search. When considering the equilibrium strategy of the seller, the following result emerges:

Proposition 2. *Consider a multiproduct seller selecting optimal menu $\tilde{N} \subseteq N$ and pricing $\mathbf{p}(\tilde{N})$ of multi-attribute products. There exist values $\underline{\alpha} \in (0, 1)$ and $\underline{s} \in (0, \alpha)$ such that, in equilibrium:*

- *For $\alpha \in (0, \underline{\alpha}]$:*
  - *all products are introduced at different prices for $s \in (0, \alpha]$, and*
  - *two uncorrelated products are introduced and priced at $p = p^M$ for $s \in (\alpha, 2\alpha)$.*
- *For $\alpha \in (\underline{\alpha}, 1)$:*
  - *three products are introduced and priced at $p \in p^D$ for $s \in (0, \underline{s}]$,*
  - *all products are introduced at different prices for $s \in (\underline{s}, \alpha]$, and*
  - *two uncorrelated products are introduced and priced at $p = p^M$ for $s \in (\alpha, 2\alpha)$.*

Proof. All calculations and precise cut-off values for $\underline{\alpha}$ and $\underline{s}$ can be found in Appendix A. ■

Determining the optimal pricing vector with differential prices is challenging in this environment. In particular, the difference in prices can induce the consumer to adapt their search strategy to avoid the more expensive product and retain some expected utility. We are interested in finding out the optimal price spread from the seller's point of view, in which cases this spread does not affect the optimal search order, and, when it does, what is the seller optimal "reply". Henceforth, I assume that $(1, 1)$ and $(2, 2)$ have lower prices and therefore act as possible starting points; furthermore, I keep the assumption of products over which the consumer is indifferent to be searched in increasing order of their indices.

First, consider the optimal price spread. The search rules determine two separate constraints. Prices must be such that search can start. Moreover, prices must be consistent with the search process as it unfolds. The price increase being profitable relies on the consumer learning about which attribute she likes: a higher price can arise only on a path dictated by the consumer finding an attribute to keep. Suppose the consumer inspects $(1, 1)$ and observes $A_1 = 1, B_1 = 0$. Suppose moreover that the optimal base price selected by the seller is $p_T \leq 1$. Conditional on inspecting one attribute being worth the cost of inspection ($s < \alpha$), the consumer would want to search $(1, 2)$ if:

$$u_{1,1} = 1 - p_{1,1} \leq 1 + \alpha - s - p_{1,2} = E(u_{1,2}|I = \{(1, 1)\}),$$

which implies $p_{1,2} \leq p_{1,1} + \alpha - s$, where $p_{1,1}$ and $p_{1,2}$ are the observed prices for $(1, 1)$ and $(1, 2)$, respectively. The higher price $p_{1,2}$ effectively captures the expected gain of searching that product after learning positive information about it by inspecting a different product. Because the seller is interested in the highest price that does not dissuade the search, the following candidate prices profile arises:

$$p_{1,1} = p_{2,2} = p^* = p_T \quad p_{1,2} = p_{1,2} = p^{**} = p_T + \alpha - s = p_T + \delta_L,$$

if $\alpha^2 < s < \alpha$, and:

$$p_{1,1} = p_{2,2} = p^* = p_H^E > 1 \quad p_{1,2} = p_{1,2} = p^{**} = 2 - \frac{s}{\alpha},$$

if $0 < s < \alpha^2$. The latter can be found following the same steps as the former, accounting for the fact that at these prices only a product that the consumer likes in both its attributes can be purchased.

Given search as characterized above, these pricing structure lead to the same probability of trade as their uniform counterparts. Compared to them, however, they lead to higher expected profit because the more expensive products are purchased with positive probability. Notice that this deviation preserves the internal consistency of the search process because the consumer would always inspect the cheapest product first if she has no information on any of the available products.

Consumer adaptation and firm response. Differential prices can distort the optimal search order of the consumer after the first realization. In particular, the consumer could find it optimal to ignore the more expensive product even if she learns that she likes something about it. In this case, the consumer would search $(2,2)$ hoping to find a good realization instead, and would only inspect the more expensive product if she knows she likes both of its attributes and nothing else. Consider again the candidate prices profile $p_{1,1} = p_{2,2} = p_T$, $p_{1,2} = p_{1,2} = p_T + \delta_L$. After realization $A_1 = 1$, $B_1 = 0$:

$$u_{1,1} = 1 - p_{1,1}, \quad E(u_{1,2}|I = \{(1,1)\}) = 1 + \alpha - s - p_{1,2},$$

$$E(u_{2,2}|I = \{(1,1)\}) = \alpha^2(2 - p_{2,2}) + (1-\alpha)(1 - p_{2,2}) + \alpha(1-\alpha)(2 - s - p_{1,2}) - s$$

When prices are uniform, a consumer would always want to inspect $(1,2)$ after learning $A_1 = 1$, $B_1 = 0$. This is not necessarily the case. It is possible that the consumer, observing the different prices, decides to change the order in which to inspect the remaining products. In particular, she could elect to inspect $(2,2)$ first and learn her realizations for all attributes. Then, the consumer could discover that $u_{2,2} = 2$, which she would not be able to by inspecting $(1,2)$. If she were to learn that $A_2 = 0$ and $B_2 = 1$, instead, then and only then would she inspect $(1,2)$ and purchase it.

For α high enough and s low enough, inspecting $(2,2)$ before $(1,2)$ is a rational deviation: search in this case is cheap, and the likelihood of liking both attributes A_2 and B_2 is relatively high. This deviation is at the detriment of the seller: the more expensive products now are reached with lower probability. The seller can optimally reply in three ways:

- the seller can let the consumer search $(2,2)$ first, and further increase $p_{1,2}$ and $p_{2,1}$ to $(p_{1,1} + 1 - s)$, or
- the seller can reduce prices $p_{1,2}$ and $p_{2,1}$ to encourage his preferred order of search to arise, or
- the seller can remove $(2,2)$ to induce his preferred order of search and keep the same prices for all other products.

The first reply further highlights the ability of the seller to condition prices on search behavior. If the consumer has an incentive to search $(2,2)$ after $(1,1)$ conditional on $A_1 + B_1 = 1$, the seller knows that the other two products would only be reached if they are the only product generating utility equal to 2. The probability of this happening, however, is lower than in the optimal price profile. Alternatively, the seller can make $(1,2)$ and $(2,1)$ cheaper. Because the consumer is interested in first searching $(2,2)$ because the alternative is too expensive, this deviation re-establishes the most profitable search order. Because the prices need to be lower, however, these paths are now less profitable than without the deviation. Finally, removing $(2,2)$ forces the consumer to take the path that the seller wants her to. This, however, reduces the probability of trade. These deviation are only necessary as long as $(\alpha, s) \in (0,1) \times (0, \alpha^2)$: when $s > \alpha^2$, search costs are too high for the consumer to be interested in searching $(2,2)$ when the seller would want her to inspect $(1,2)$ or $(2,1)$.

Each of the above strategies generates different expected profits for the seller. Given $p^* = p_T$, $p^{**} = p_T + \delta_L$:

- if the seller allows the consumer to deviate and raises p^{**} to $\overline{p} = p^* + 1 - s$,

$$\overline{\pi} = (1 - (1-a)^4)p^* + 2\alpha^2(1-\alpha)^2(\overline{p} - p^*);$$

- if the seller reduces p^{**} to $\underline{p}$ to induce seller preferred order,

$$\underline{\pi} = (1 - (1-a)^4)p^* + 2\alpha^2(1-\alpha)(\underline{p} - p^*);$$

- if the seller removes $(2,2)$ to prevent the deviation deviation,

$$\widehat{\pi} = (1 - (1-a)^2)p^* + 2\alpha^2(1-\alpha)(p^{**} - p^*).$$

All three options are optimal for some combinations of α and s. The same exercise can be applied to the alternative pricing profile $p_{1,1} = p_{2,2} = p_H^E > 1$, $p_{1,2} = p_{1,2} = 2 - \frac{s}{\alpha}$: in this case, deviation by the consumer is always feasible, and so the seller must react accordingly as well. In particular, for this alternative profile, removing $(2,2)$ always dominates the other two strategies.

Comparison. The feasible expected profits under differential prices must be compared to the highest expected profit under uniform prices obtained in the previous subsection. Two results emerge. First, whenever the seller has an incentive to select uniform prices that encourage search, he has an incentive to differentiate prices. This is intuitive: the lowest prices when products are priced differently are the same as the trade-maximizing uniform price. Because prices are set up to generate strictly higher profits while maintaining the same probability of trade, it is clearly an improvement to set differential prices.

Second, whenever it is optimal to remove a product to prevent deviation by the consumer, the high uniform prices generate higher profits. This, too, is straightforward: when the seller's best option is to give up on an inspection in case of a bad first match, uniform prices generate higher expected profits because, when prices are different, the consumer always starts from the cheaper one.

The leftmost graph in Figure 3 summarizes the equilibrium menu and pricing selection for all feasible combinations of α and s. The two decisions are intertwined. The seller has an incentive to make all products available only if they can all be reached, and purchased, with positive probability.

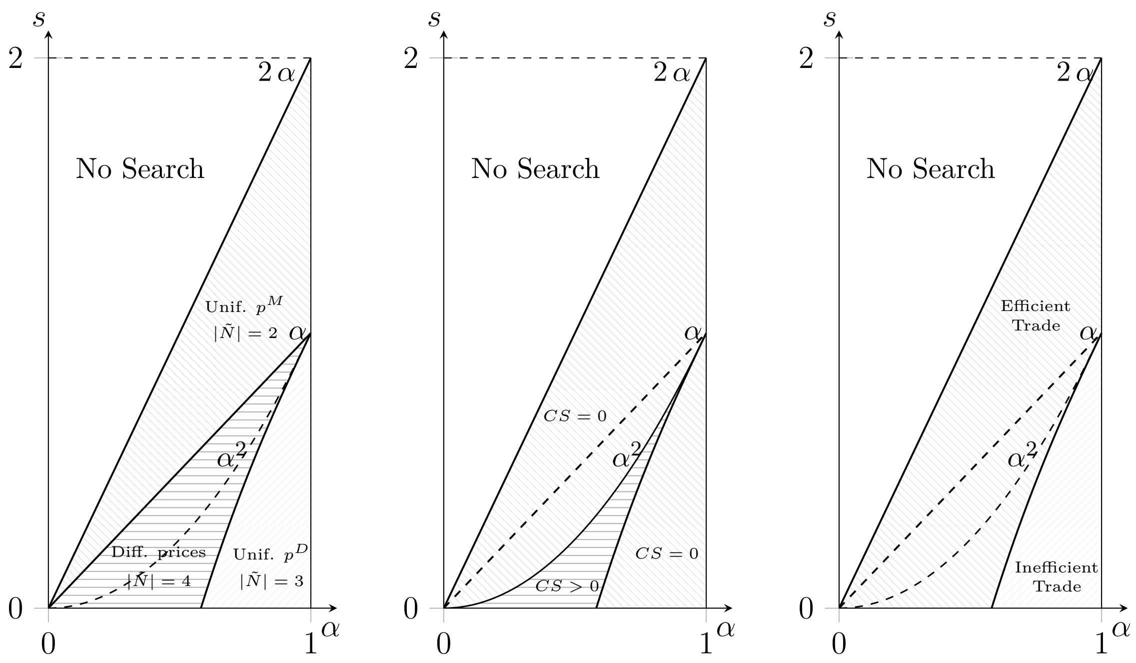

Figure 3: *Equilibrium monopoly menu selection and pricing (leftmost), expected consumer surplus (center), and trade efficiency (rightmost) for all feasible combinations of* $\alpha \in (0, 1)$ *and search costs* $s \in (0, 2\alpha)$

When search costs are very high, only a bad first realization induces the consumer to keep searching: introducing more than two products allows the consumer to randomize her starting point but at no benefit to the seller. On the other hand, when search costs are very low, the seller prefers to set prices that prevent some search paths to arise if probability of a match is relatively high. Lower search costs do not necessarily translate to more product variety, nor to efficient trade: when uniform high prices are selected, probability of trade is not as high as it could feasibly be because the menu is strategically restricted as well.

Finally, whenever all products are introduced, they are never priced uniformly. This pricing structure allows the monopolist to more efficiently extract rent while maintaining the highest probability of trade. Only when the consumer can deviate and force a reaction in the monopolist optimal pricing (that is, for $s < \alpha^2$), the consumer preserves some positive expected utility as long as the menu is not optimally restricted by the monopolist. Otherwise, the monopolist is able to capture it all through strategic pricing and menu selection.

4.3. Discussion of the Results

The results of this section highlight the incentives of a multiproduct seller to strategically determine the menu of available products to extract rent efficiently. To do so, he leads consumers towards specific search paths consistent with different outcomes of past inspections. With differential prices the seller is able to profit off the learning component of search in this environment.

This finding is at odds with the standard prediction of search models with multiproduct firms. In environments in which inspection of a product does not inform consumers of their taste for alternatives, strategic obfuscation of alternatives is the general outcome. Petrikaitė (2018), for example, shows that a multiproduct seller, like the one studied here, has an incentive to increase search cost of inspecting one product to induce consumers to inspect the easier-to-find, and more expensive, alternative first. Strikingly, the prediction goes in the opposite direction in the framework presented here. The learning component induces the seller to display some products more prominently, at a lower price, to let consumers learn about their tastes. Encouraging search, rather than discouraging it, allows the seller to sell more expensive products.

A possible application of this framework relates to the practice of businesses to offer free samples of new products to attract interest. In particular, by making some products prominent and easy to assess, a firm can encourage potential buyers to learn about their taste for novelties and alternatives that they might not have considered otherwise. In doing so, the firm can use the positive experience associated with the sampling to increase the willingness to pay of consumers unaware of their preferences for said products. Together with the strategic ordering shown above, this points at the importance of menu selection and positioning of options in environments with search frictions.

The model carries implications for digital markets, particularly in relation to recommendation systems and price discrimination based on consumers' search history. Recommendation systems have been objects of great interest and scrutiny in the past few years because of their crucial role in the digital economy. A good recommendation system reduces frictions and, therefore, increases efficiency of trade. It is clear, however, that such systems can be objects of manipulation. The results of the model imply that the learning component relevant when searching products sharing attributes creates incentives to bias recommendations. The seller modelled here does not want to make prominent the best match possible. Rather, he wants the consumer to start from a subpar match and then self-select towards a more expensive product after learning her preferences because she might be discouraged from inspecting an expensive product without any information about it.

Consumers self-selecting based on taste also creates the incentive to condition pricing on their search history. Algorithmic pricing, the practice of pricing items automatically to adapt to the state of the market, are more and more commonly used in the digital world.[12] The model highlights the role that a product's position in the attribute space plays in their pricing. Consumers are willing to search more expensive products only if they have already learned something positive about them by inspecting a different option.

[12] Airline companies and, more recently, e-commerce retailers are prime examples of this practice being widely in use.

Equivalently, one can imagine a reactive pricing system that adapts as search unfolds. If two products sharing attributes are inspected in sequence, the ordering signals that the consumer has learned something positive about those attributes. The price of the second, then, can be safely raised by an algorithm trained to recognize these patterns. On the other hand, if two products not sharing any feature are inspected in sequence, both should be priced low to maintain the consumer engaged with the search.

5. Optimal Search with Multi-Attribute Products

The simplified framework analyzed above hints at the mechanics underlying optimal search in this environment. A consumer inspects different products after different realizations, and the value of inspecting two products depends on the order in which they are inspected even if they are *ex ante* identical. Further, the value of inspecting a product depends on the other products that share attributes with it.

I now generalize this intuition. I rethink the problem in a way that allows to apply standard search logic and therefore reduce the search problem to a set of rules reminiscent of Pandora's optimal search policy. Weitzman (1979)'s result is not immediately applicable to this environment due to correlation: because products share attributes, it is not possible to assign to each one a score that only depends on the product itself. I show how this can be achieved by building "nests" of products to be scored as a single "box" and letting their score update following certain realizations to account for changes in the optimal search that would follow.[13]

The nests relevant for the search process can be considered effectively independent from each other at the moment of search. By leveraging this structure, static scores can be constructed following the same steps highlighted in seminal work by McCall (1970) and Kohn and Shavell (1974). These static scores can then be combined to account for possible changes in the structure of the search process in response to certain realizations, which generates the appropriate reservation values instructing the search process.

Adapting the framework I assume now A and B to come in infinite variants so that the number of products available for purchase is infinite as well. Further, I assume each attribute to be an i.i.d random variable distributed according to a cumulative distribution function F: given a generic attribute $y \in A \cup B$, $F(y)$ is assumed to have support $[0, \hat{y}]$ for some positive $\hat{y}$, and to be twice-differentiable everywhere on it. The cost of inspecting a product is still indexed by the constant s, and the consumer's outside option is still normalized to $u_0 = 0$. The timing of the interaction is unchanged.

[13]This approach was inspired by the work contained in Anderson et al. (2021); I thank Daniel Savelle for his many helpful comments.

5.1. A General Search Process

Consider first a simpler case, illustrated in Figure 4. The two products available share one attribute (A_1) and are independent along the other (B_j, $j \in \{1, 2\}$).

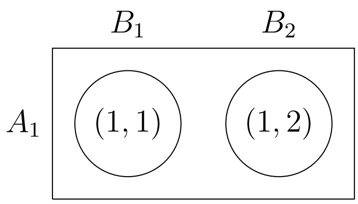

Figure 4: *Two products available*

Suppose that the consumer already inspected $(1, 1)$: she has learned her valuation for A_1, shared by both products, and B_1. She still does not know her valuation for B_2. At this stage, it is clear that choosing between stopping at $(1, 1)$ and costly inspecting $(1, 2)$ is governed by the standard myopic search process illustrated in Weitzman (1979).[14] In particular, $u_{1,1}$ is known, and $(1, 2)$'s value is only unknown in B_2. Therefore, we can express the value of inspecting $(1, 2)$ using Weitzman (1979)'s reservation value. In particular, the certain equivalent that makes a consumer indifferent between that value and costly discovering realization B_2 is the value z that solves:

$$s = \int_z^{\hat{y}} (B_2 - z)\, dF(B_2).$$

Then, the reservation value of inspecting $(1, 2)$ when A_1 is known is simply:[15]

$$r_{1,2} = A_1 + z.$$

Following Weitzman (1979), the consumer would inspect $(1, 2)$ if and only if $B_1 < z$, or $u_{1,1} < r_{1,2}$. Figure 5 illustrates. We cannot go backwards and apply the same myopic logic to the choice of inspecting $(1, 1)$: because the reservation value of each individual product depends on the other, we cannot apply Pandora's search algorithm.

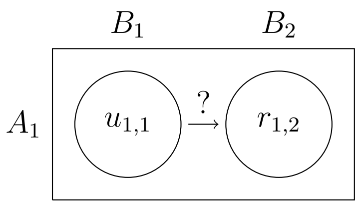

Figure 5: *Myopic search after inspecting* $(1, 1)$

Suppose however that the products were in a bigger box, and that the consumer had to decide whether to open one box containing $(1, 1)$ and a nested box containing $(1, 2)$, or nothing at all. The action of opening this "compound" box, that I refer to as $X_{1,1}$, can be scored.

If the consumer opens the box she discovers $u_{1,1}$, the implied reservation value $r_{1,2}$, and searches accordingly. Because we know how search takes place inside this box, $X_{1,1}$ can be scored in a way that reflects not just the value of inspecting $(1, 1)$ but also the value of the information learned through the possibility of correcting towards $(1, 2)$. When applied to each product sepa-

[14]This intuition can be found, for example, in Ke and Lin (2022).
[15]Notice that this is the same utility structure studied in Choi et al. (2018).

rately, this intuition generates an environment in which products sharing attributes can be appropriately scored to reflect the information they carry.

The consumer could also want to inspect $(1,2)$ first. We can imagine another compound box, $X_{1,2}$, containing $(1,2)$ and a nested box containing $(1,1)$. The two are *ex ante* identical before either is opened and, once one is opened, the other becomes the smaller nested box contained in the one inspected first. Maintaining the assumption that the consumer inspects unknown attributes in increasing order of their index when indifferent is still without loss of generality.

The value of a compound box. First, define $X_{i,j}$ a box containing products (i,j), immediately available upon opening the box, and all products $(i, j' \neq j)$, $(i' \neq i, j)$, inside smaller boxes that must be opened by paying an additional search cost s. In words, $X_{i,j}$ contains all products defined by A_i, B_j, or both. Notice that all products are contained in multiple boxes. Through this feature, correlation can be handily accounted for.

Consider any compound box $X_{i,j}$ in isolation. To define optimal search inside of it (that is, assuming no other compound box is available), we keep the assumption that, whenever indifferent, the consumer inspects closed boxes in ascending order of their index. Suppose the consumer is about to open $X_{i,i}$ and pay the relative search cost s. Let A^H, B^H be the highest past realization of the previously inspected $A_{j<i}$, $B_{j<i}$. If $\max\{A^H, B^H\}$ is low enough, the choice of searching $X_{i,i}$ is unaffected by all realizations that took place before. On the other hand, if either or both A^H and B^H are high enough, the choice is predictably affected by said realization.

Consider box $X_{i,i}$. The consumer is aware that inside she will find product (i,i) and will have the option to stop or inspect products $(i, j \neq i)$, $(j \neq i, i)$. How would she do so? All attributes $A_{j<i}$, $B_{j<i}$ have already been inspected and are known. Suppose the consumer already opened the box. The consumer can choose between

- stopping at (i,i), generating utility $u_{i,i} = A_i + B_i$, or
- searching again keeping A_i and
  - inspect a product defined by $B_{j<i}$, whose realization is already known, after paying cost s: $u_{i,j<i} = A_i + B_j - s$,
  - search a product defined by $B_{j>i}$, whose realization is unknown after paying cost s: $E[u_{i,j>i}] = A_i + E[B_j] - s$,
- searching again keeping B_i and:
  - search a product defined by $A_{j<i}$, whose realization is already known, after paying cost s: $u_{i,j<i} = A_j + B_i - s$,
  - search a product defined by $A_{j>i}$, whose realization is unknown after paying cost s: $E[u_{j>i,i}] = E[A_j] + B_i - s$.

After opening $X_{i,i}$, these choices can be ranked according to the classic result of Weitzman (1979). In particular, after $X_{i,i}$ has been opened, the remaining options are independent of each other because all attributes are assumed to be i.i.d. Therefore, we can assign a score to all of the options above by finding the certain equivalent of each. Stopping and inspecting a product whose realization is fully known trivially has certain equivalent matching the known ex post utility: $r_{i,i} = u_{i,i}$, $r_{i,j<i} = u_{i,j<i} - s$, $r_{j<i,i} = u_{j<i,i} - s$. Keeping the classic nomenclature, I refer to this as "reservation values" of these options.

Notice that these closed boxes, that I henceforth refer to as "nested" as they are accessible in this form only inside a compound box, are only unknown in one attribute after $X_{i,i}$ has been opened. This is the same object whose reservation value is provided in Choi et al. (2018).[16] In particular, because all attributes $y \sim F(y)$ with support $[0, \hat{y}]$, the certain equivalent of spending a search cost s to discover the realization of any unknown attribute y is z that solves:

$$s = \int_z^{\hat{y}} (y - z)\, dF(y) \tag{3}$$

and, therefore: $r_{i,j>i} = A_i + z$, $r_{j>i,i} = z + B_j$.

Notice that the choice between moving forward towards $(i, j > i)$ or $(j > i, i)$ and going backward to any known $(i, i' < i)$, $(i' < i, i)$ is resolved again simply by applying Weitzman (1979)'s optimal search policy: if there is at least one product $(i, j < i)$ (or $(j < i, i)$) such that $u_{i,j<i} - s > A_i + z$ (or $u_{j<i,i} - s > z + B_i$), no nested box with score $r_{i,j>i} = A_i + z$ (or $r_{j>i,i} = B_i + z$) would be opened, and the product generating the highest $u_{i,j<i} - s$ (or $u_{j<i,i} - s$) would be inspected and selected. This happens if $A^H > z + s$ (or $B^H > z + s$). Otherwise, all products $(i, j < i)$ (or $(j < i, i)$) would be ignored. Because A^H, B^H are the highest past realizations, they are known before $X_{i,i}$ is opened. Therefore, the consumer opens $X_{i,i}$ knowing already whether she would go forwards (that is, open nested boxes $(i, j > i)$ or $(j > i, i)$) or backwards (that is, inspecting a product $(i, j < i)$ or $(j < i, i)$) if she decides to search again).

This observation implies that unopened compound boxes that are constructed around products not sharing attributes are *de facto* independent for all values of A^H, B^H at the moment of making the choice of opening a new compound box. The possible configurations in which compound boxes can be found is illustrated in Figure 6. Depending on A^H and B^H, the effective path inside each unrelated box does not cross. We can then compute the expected value of searching box $X_{i,i}$ in isolation by tracing the optimal search therein for different values of A^H, B^H. This, in turn, means that each configuration as in Figure 6 can be solved independently. The value of opening these compound boxes in each configuration, then, can be combined to obtain the actual, history dependent reservation value governing optimal search.

[16] In Appendix B I show that once a nested boxes is optimally opened, the consumer either stops or opens more nested boxes depending on the current realized payoff, making this branch of the optimal search policy myopic in nature.

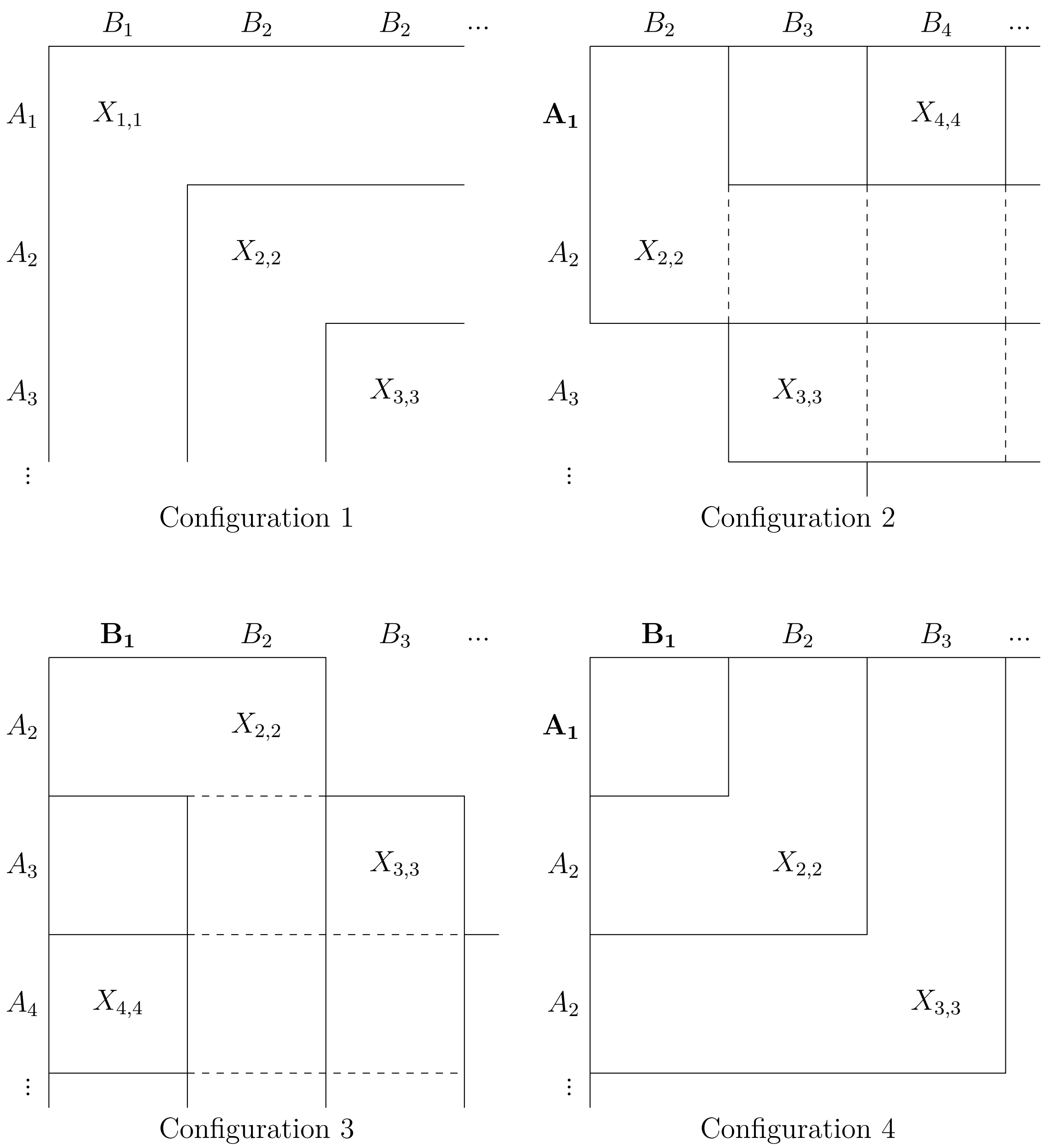

Figure 6: *Possible configurations of compound boxes. Configuration* 1 *represents boxes when* $\max\{A^H, B^H\} < z + s$*; Configurations* 2 *and* 3 *represent boxes when* $A_1 = A^H > z + s > B^H$ *and* $B_1 = B^H > z + s > A^H$ *respectively and, therefore, reroute search towards themselves; Configuration* 4 *represents boxes in which* $A_1 = A^H > z + s$ *and* $B_1 = B^H > z + s$.

5.2. Four Independent Configurations

Configuration 1. If $\max\{A^H, B^H\} < z + s$, the consumer will either stop at (i, j) or open nested boxes with unknown content. In this configuration the consumer searching inside a compound box always keeps the highest between A_i, B_j and either stops if the lowest is above z or opens nested boxes paying search cost s. In the latter case, the consumer will keep doing so until she finds something that beats z paying a search cost for each inspection.

To leverage the independence of compound boxes locked in a given configuration, it is necessary to obtain the distribution of values the consumer expect to find inside of it. Let w be the expected payoff of a consumer opening a compound box in this configuration and $H(w)$ be its CDF. Given optimal search inside the compound box, $w_{i,i}$ of opening box $X_{i,i}$ is:

$$w_{i,i} = \max\{A_i, B_i\} + \max\{z, \min\{A_i, B_i\}\}.$$

To compute $H(w)$, we must consider how different realizations for A_i and B_i interact. Fix a generic value B_i. If $B_i < z$, it is kept if and only if $B_i > A_i$. Otherwise, A_i is kept. In this case, $w = \max\{A_i, B_i\} + z$. If $B_i > z$, it is always kept over nested boxes $(i, j > i)$; A_i is also kept if its realization is above z, or: $w = B_i + \max\{A_i, z\}$.

Because $F_a(A) \equiv F_b(B) \equiv F(y)$ have support $[0, \hat{y}]$, $H(w)$ has support $[z, 2\hat{y}]$ and can be expressed as:

$$\begin{aligned} H(w) = &\int_0^z F_a \left(\int_0^B F_b\left(w - z\right)\, dF_a(A) + \int_B^{\hat{y}} F_b\left(w - z\right)\, dF_a(A) \right) dF_b(B) + \\ &+ \int_z^{\hat{y}} F_a \left(\int_0^z F_b\left(w - z\right)\, dF_a(A) + \int_z^{\hat{y}} F_b\left(w - A\right)\, dF_a(A) \right) dF_b(B). \end{aligned}$$

Suppose that the all compound boxes are "locked" in this configuration as in Figure 6 (top left corner).[17] The optimal search in this simplified case can be obtained through definition of a value function as shown in McCall (1970) and Kohn and Shavell (1974). In particular, we want to find $\underline{W}$ that solves the dynamic programming problem:

$$\underline{W} = -s + \max\{w, E[\underline{W}]\}, \tag{4}$$

where w follows the cumulative distribution function $H(w)$, and $\underline{W}$ is the maximum return the consumer would obtain after opening a compound box (and searching optimally therein if she stopped there). In this case, the optimal process sees the consumer stopping and keeping $w \geq E[\underline{W}]$ and searching if $w < E[\underline{W}]$. Because compound boxes locked in a configuration are effectively independent objects, this problem bears the same solution as Weitzman (1979). In particular, the relevant threshold value above which a box is kept is

[17] That is, imagine boxes to be unchangeable and such that the value of its content always follows w without possibility of being updated.

$\underline{W}$ that solves:

$$s = \int_{\underline{W}}^{2\hat{y}} (w - \underline{W})\, dH(w), \tag{5}$$

Configurations 2, 3, and 4. The same procedure allows to obtain static reservation value associated with boxes locked in different configurations. Consider first configuration 2: if $\max\{A^H, B^H\} > z + s > \min\{A^H, B^H\}$, the consumer will not open nested boxes along one attribute but would do so along the other. W.L.O.G., assume $A^H > z + s > B^H$ so that after opening $X_{i,i}$, the consumer would always go back to a product $(j < i, i)$ rather than opening nested boxes $(j > i, i)$ (but could still open nested boxes $(i, j > i)$). In particular:

- if $B_i > z$, the consumer chooses between keeping (i, i), $u_{i,i} = A_i + B_i$, and returning to $(j < i, i)$, $u_{j<i,i} = A^H + B_i - s$
- if $B_i < z$, instead, the consumer chooses between $(j < i, i)$ and inspecting nested boxes $(i, j > i)$, $r_{i,j>i} = A_i + z$.

Let $w_a(A^H)$ be the expected payoff of a consumer opening a compound box in this configuration and $H_a(w_a(A^H))$ be its CDF.[18] Assuming again that unopened compound boxes are locked in this configuration (illustrated in the top right corner of Figure 6), their static reservation value is $\underline{W}_a(A^H)$ that solves:

$$s = \int_{\underline{W}_a(A^H)}^{\hat{y}} (w^a - \underline{W}_a(A^H))\, dH_a(w^a). \tag{6}$$

The same exact exercise leads to $\underline{W}_b(B^H)$ (bottom left corner of Figure 6), relevant when $A^H < z + s < B^H$.

Consider now configuration 4 (bottom right corner of Figure 6): if $\min\{A^H, B^H\} > z+s$, the consumer will not open any nested box. In particular:

- if $B_i > B^H - s$ and $A_i > A^H - s$, the consumer stops,
- if $B_i > B^H - s$ and $A_i < A^H - s$, the consumer inspects and keeps (i', i), $u_{i',i} = A^H + B_i - s$,
- if $B_i < B^H - s$ and $A_i > A^H - s$, the consumer inspects and keeps (i, i'), $u_{i,i'} = A_i + B^H - s$,
- if $B_i < B^H - s$ and $A_i < A^H - s$, instead, the consumer chooses between (i', i) and (i, i'), depending on which has the highest utility.

[18] A closed form expression for this and all subsequent CDFs can be found in Appendix B.

Labeling $w_{a,b}(A^H, B^H)$ and $H_{a,b}(w_{a,b}(A^H, B^H))$ the expected payoff and CDF of boxes locked in this configuration, their reservation value is $\underline{W}_{a,b}(A^H, B^H)$ that solves:

$$s = \int_{\underline{W}_{a,b}(A^H,B^H)}^{\hat{y}} (w^a - \underline{W}_{a,b}(A^H, B^H))\, dH_{a,b}(w^{a,b}). \tag{7}$$

The final step requires to combine these thresholds to account for the fact that compound boxes are not locked in any given configuration but, rather, can move from one configuration to the next depending on the realizations A^H and B^H found along the search process.

5.3. Optimal Search Process

The values $\underline{W}$, $\underline{W}_a(A^H)$, $\underline{W}_b(B^H)$, and $\underline{W}_{a,b}(A^H, B^H)$ can be appropriately combined to obtain the reservation values of unopened compound boxes when they are not locked in any given configuration. Once again, which of this values is relevant depends on past realizations: if some $A_{j<i} > z + s$ and/or some $B_{j<i} > z + s$ is found, this affects the value of all future boxes because by construction all compound boxes contain at least one product defined by all attributes.

The relevant value of the unopened compound boxes can evolve only in one direction, from configuration 1 to 4, and never backwards. Indeed, once $A_{j<i} > z + s$ is found, it can never be forgotten: once the relevant reservation value of the current configuration of $X_{i,i}$ changes from $\underline{W}$ to $\underline{W}_a(A^H)$, it can never revert to $\underline{W}$ or change to $\underline{W}_b(B^H)$. From this point onward, it can only stay at $\underline{W}_a(A^H)$ or change to $\underline{W}_{a,b}(A^H, B^H)$. Moreover, once configuration 4 is reached, all unopened compound boxes will keep this configuration.

Suppose all closed boxes reached configuration 4. This implies that $\min\{A^H, B^H\} > z + s$. Suppose the consumer has observed these A^H and B^H and must choose whether to open the next box. If boxes were to be locked, with any future A and B realization not being able to affect the next, the value of all closed boxes would be $\underline{W}_{a,b}(A^H, B^H)$. However, this does not capture the search dynamics appropriately.

Suppose the next box were to be opened and that $A_i > A^H$ was be found. The next compound box would have a different reservation value, $\underline{W}_{a,b}(A_i, B^H)$. The expected value of future boxes given the current values A^H, B^H can be obtained recursively. Let $\underline{W}^*_{a,b}(A^H, B^H)$ be the expected equivalent of costly opening the next box on the search path. This can be rewritten in terms of expected new values $\tilde{A}^H$ and $\tilde{B}^H$:

$$\begin{aligned}
\underline{W}^*_{a,b}(A^H, B^H) =& \underline{W}_{a,b}(A^H, B^H) \int_0^{B^H} \int_0^{A^H} dF_a(A) dF_b(B) + \\
&+ \int_0^{B^H} \int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B^H)\, dF_a(A) dF_b(B) + \\
&+ \int_{B^H}^{\hat{y}} \int_0^{A^H} \underline{W}_{a,b}(A^H, B)\, dF_a(A) dF_b(B) + \\
&+ \int_{B^H}^{\hat{y}} \int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A) dF_b(B) \\
&= \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)
\end{aligned}$$

Consider the choice of the consumer. If she chooses to open the next compound box, $X_{i+1,i+1}$, she knows that she will stop only if $w_{i+1,i+1} > \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$. All future boxes will have this updated value. Then, the gain she expects from searching the next box can be written as:

$$\begin{aligned}
&- s + \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H) \int_{\max\{A^H, B^H\}}^{\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)} dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H) + \\
&+ \int_{\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)}^{2\hat{y}} w_{a,b}\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)
\end{aligned}$$

The certain equivalent of this first inspection, then, is simply $\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$ that solves:

$$s = \int_{\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)$$

Notice that $\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$ is strictly higher than $\underline{W}_{a,b}(A^H, B^H)$ because $\underline{W}_{a,b}(A^H, B^H)$ is increasing in A^H and B^H. This threshold captures not only the value of inspecting the next box, that by itself would have had reservation value $\underline{W}_{a,b}(A^H, B^H)$, but also that of the updating that opening the box might lead to.

We can repeat the same exact exercise with the other configurations. For configuration 2, we write:

$$\begin{aligned}
\underline{W}^*_a(A^H) =& \underline{W}_a(A^H) \int_0^{z+s} \int_0^{A^H} dF_a(A) dF_b(B) + \\
&+ \int_0^{z+s} \int_{A^H}^{\hat{y}} \underline{W}_a(A)\, dF_a(A) dF_b(B) + \\
&+ \int_{z+s}^{\hat{y}} \int_0^{A^H} \underline{W}_{a,b}(A^H, B)\, dF_a(A) dF_b(B) + \\
&+ \int_{z+s}^{\hat{y}} \int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A) dF_b(B) \\
=& F(z+s)\, \underline{W}_a(\tilde{A}^H) + (1 - F(z+s))\, \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)
\end{aligned}$$

The certain equivalent of this first inspection, then, is the linear combination $F(z+s)\underline{W}_a(\tilde{A}^H) + (1 - F(z+s))\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$ that solves:

$$
\begin{aligned}
s =& F(z+s) \int_{\underline{W}_a(\tilde{A}^H)}^{2\hat{y}} (w_a - \underline{W}_a(\tilde{A}^H)\, dH_a(w_a, \tilde{A}^H) + \\
&+ (1 - F(z+s)) \int_{\underline{W}_{a,b}(\tilde{A}^H,\tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)
\end{aligned}
$$

which is both higher than $\underline{W}_a(A^H)$ and the hypothetical $\underline{W}_a(\tilde{A}^H)$ one would compute ignoring the possibility that the next box could change in value. An equivalent formulation can be found for configuration 3.

Finally, for configuration 1, we can write:

$$
\begin{aligned}
\underline{W}^* =& \underline{W} \int_0^{z+s} \int_0^{z+s} dF_a(A) dF_b(B) + \\
&+ \int_0^{z+s} \int_{z+s}^{\hat{y}} \underline{W}_a(A)\, dF_a(A) dF_b(B) + \\
&+ \int_{z+s}^{\hat{y}} \int_0^{z+s} \underline{W}_b(B)\, dF_a(A) dF_b(B) + \\
&+ \int_{z+s}^{\hat{y}} \int_{z+s}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A) dF_b(B) \\
=& F(z+s)^2 \underline{W} + F(z+s)(1 - F(z+s))(\underline{W}_a(\tilde{A}^H) + \underline{W}_b(\tilde{B}^H)) + \\
&+ (1 - F(z+s))^2 \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^h)
\end{aligned}
$$

so that the reservation value of the next box inspected when all boxes are in the first configuration is the linear combination of values that that solves:

$$
\begin{aligned}
s =& F(z+s)^2 \int_{\underline{W}}^{2\hat{y}} (w - \underline{W})\, dH + \\
&+ F(z+s)(1 - F(z+s)) \int_{\underline{W}_a(\tilde{A}^H)}^{2\hat{y}} (w_a - \underline{W}_a(\tilde{A}^H))\, dH_a(w_a, \tilde{A}^H) + \\
&+ F(z+s)(1 - F(z+s)) \int_{\underline{W}_b(\tilde{B}^H)}^{2\hat{y}} (w_b - \underline{W}_b(\tilde{B}^H))\, dH_b(w_b, \tilde{B}^H) + \\
&+ (1 - F(z+s))^2 \int_{\underline{W}_{a,b}(\tilde{A}^H,\tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)
\end{aligned}
$$

By taking into account all possible configurations, and all the ways in which this configurations can evolve into one another, we can then write the reservation value of all unopened boxes as:

$$\mathcal{W}(A^H, B^H) = \begin{cases} \underline{W}^* & \text{if} \quad \max\{A^H, B^H\} < z + s, \\ \underline{W}_a^*(A^H) & \text{if} \quad A^H > z + s > B^H, \\ \underline{W}_B^*(B^H) & \text{if} \quad B^H > z + s > A^H, \\ \underline{W}_{a,b}^*(A^H, B^H) & \text{if} \quad \min\{A^H, B^H\} > z + s. \end{cases}$$

The values above reflect the value of inspecting any given compound box given all information learned so far and anticipating how the game could change given future realizations: $\mathcal{W}(A^H, B^H)$ incorporates the value of searching along all possible paths, defined by the number of attributes found above $z + s$. Once a path is taken, that path can never be left. Each path is built through the optimal search process when boxes are in the appropriate state through $\underline{W}_\cdot$, which is defined based on the optimal search process inside the compound box as per its current configuration, captured by $w_\cdot(i, j)$ for each $X_{i,j}$. Because each branch is optimized, the whole process is too.

While the optimal search order cannot be determined *ex ante* because of the learning component, whenever the consumer must choose what to do there is no ambiguity regarding the value of her possible options:

Proposition 3. *Let* $A^H = \max\{y \in A \cap I\}$, $B^H = \max\{y \in B \cap I\}$ *be the highest discovered realization for* A *and* B. *Optimal search is characterized as follow:*

- ***Compound box selection:*** *compound boxes are opened until the expected payoff according to the optimal search policy inside of it,* $w_{i,j}$, *is higher than the reservation value of all unopened compound boxes,* $\mathcal{W}(A^H, B^H)$.
- ***Search inside the selected compound box:*** *Given selection of compound box* $X_{i,j}$, (i, j) *is kept if* $u_{i,i} > \max\{r_{i,k\neq j}, r_{k\neq i,j}\}$; *otherwise, the next box opened is the one with the highest* $r_{i,k}$ *or* $r_{k,i}$.
- ***Stopping rule:*** *Boxes (compound or nested) are opened until all unopened (compound or nested) boxes have updated reservation value below the highest realized payoff.*

Proof. All calculations and closed form equations for $H(w_\cdot)$ can be found in Appendix B. ■

The multi-attribute structure proposed here allows one to score search options appropriately by leveraging the fact that at any given point compound boxes can be thought of as effectively independent object along the search path. This, in turn, generates an environment in which the standard intuition behind optimal search can be adapted. This process can be thought of as a consumer sampling unrelated products until at least an attribute worth keeping is found. When this happens, the consumer ignores all remaining compound boxes and searches inside the one that let her find that first attribute to keep in order to find an appropriate other one to pair with it.

Notice that the structure of the compound boxes reflect the internal consistency of the search process: opening a compound box always carries more information than a nested box inside a previously opened one. Therefore, if a compound box is selected, the attribute that is kept when searching inside of it must have had a realization high enough to compensate for the lower informational value of not inspecting two new attributes.

5.4. Optimal Pricing with Infinite Products

In order to solve for the optimal pricing scheme in this more complex environment, we leverage once again the structure of compound boxes. This structure presented above can be readily adapted to incorporate prices. In particular, the value associated with each product must be reduced by the posted price; this new value can be used to score compound boxes appropriately, accounting for the price of all products on the relevant search paths. In other words, prices affect the values $w_{\cdot}$ of opening any compound box; the effect cascades to the reservation values $\mathcal{W}$, which allows to solve for optimal pricing.

Consider the compound box $X_{1,1}$ built around product $(1,1)$ priced at $p_{1,1}$; the box contains all products $(1,j)$, priced at $p_{1,j}$, and all products $(i,1)$, priced at $p_{i,1}$. Suppose the consumer opened $X_{1,1}$ and decided to search in it keeping attribute A_1. Then, she would inspect next the product $(1,j)$ that satisfies:

$$\max_j(A_1 + z - p_{1,j}) \geq A_1 + B_1 - p_{1,1},$$

Three things are worth noticing: first, if $p_{1,j}$ is not uniform, the consumer would always select to inspect products $(1,j)$ in increasing order of price. Second, for $X_{1,1}$ to be inspected before all other $(1,j)$ products, it must have been the cheapest of them. Third, if $p_{1,1} \neq p_{1,j}$, $(1,j)$ would be inspected next if and only if:

$$B_1 \leq z - (p_{1,j} - p_{1,1}) < z.$$

The same structure governs inspection of products $(i,1)$.

In principle, all products $(1,j)$ could be priced differently. Suppose that prices where increasing in j and always strictly below z. Then, if the consumer decided to inspect $(1,2)$ after discovering A_1, B_1, he would expect to either keep it if it beats the reservation value of $(1,3)$, or keep searching, and so on for all subsequent inspections. The total value associated with this path given vector of prices $\mathbf{p_{1,k}}$ of all products $(1,k>1)$ is then:

$$y(\mathbf{p_{1,j}}) = \sum_{k=1}^{\infty} F(z-(p_{1,k+1}-p_{1,k}))^k \int_{z-(p_{1,k+1}-p_{1,k})}^{\hat{y}} (y - p_{1,k+1})\, dF(y).$$

To see the effect of prices, it is useful to compute the expected value of a compound box when the products therein have prices posted. Consider the generic compound box $X_{i,j}$ in the first configuration for simplicity and for illustrative purposes. Let $\Delta_{i,k} \equiv p_{i,k+1} - p_{i,k}$ and $\Delta_{k,j} \equiv p_{k+1,j} - p_{k,j}$; further, let:

$$\bar{y}_{i,k} = E[y|y > z - \Delta_{i,k}], \quad \bar{y}_{k,j} = E[y|y > z - \Delta_{k,j}].$$

Then:

$$\begin{aligned} E[w_{i,j}(\mathbf{p_{i,j}})] =& [1 - F(z - \Delta_{i,i+1})][1 - F(z - \Delta_{j+1,j})](\bar{y}_{i,i+1} - \bar{y}_{j+1,j} - p_{i,j}) \\ &+ [1 - F(z - \Delta_{i,i+1})]F(z - \Delta_{j+1,j})(\bar{y}_{i,i+1} + y(\mathbf{p_{i,k}})) \\ &+ F(z - \Delta_{i,i+1})[1 - F(z - \Delta_{j+1,j})](y(\mathbf{p_{k,j}}) + \bar{y}_{j+1,j}) \\ &+ F(z - \Delta_{i,i+1})F(z - \Delta_{j+1,j})\left(y(\mathbf{p_{i,k}}) + y(\mathbf{p_{k,j}})\}\right), \end{aligned}$$

While a high price that does not make a product never worth inspecting makes it more profitable to sell, it also pushes the product attached to it further away from the optimal starting point of the consumer. Suppose all products were priced a some uniform level p^u and one was slightly more expensive. Then, not only the more expensive product would have lower value in any search path in which it could be found, but all compound boxes that contain it would also have a lower $E[w(\mathbf{p})]$, which translates to a lower reservation value. None of the boxes associated with this product, then, would ever be inspected as there are infinite better alternative for the consumer.

Another difficulty relates to the updating process described in the pages above. Attributes can still have realizations that reroute search towards themselves, and in a way that is much more cumbersome to keep track of when prices are accounted for. Moreover, because the relationship between the different possible scores $\mathcal{W}$. depends on the specific realization or realizations that triggered the update, the updating could lead to all unopened boxes to become less valuable than they originally were, which could lead the consumer to end his search prematurely.

Both concerns can be addressed, and the following result emerges:

Proposition 4. *Consider a multiproduct seller pricing infinite products defined by two infinite sets of i.i.d. attributes. There exist a unique, uniform equilibrium pricing vector such that* $p_{i,j} = p^* = \underline{W}^*$, $\forall (i,j)$.

Proof. All calculations can be found in Appendix C. ■

Proposition 4 states that the only possible equilibrium features uniform pricing. In the simplified framework of Section 4, different prices could be optimal because the second product searched was generally the last one: if $(2,2)$ was inspected after $(1,1)$, the consumer would either purchase it or stop searching because no other products were available to inspect. To make it worth searching, it had to be priced accordingly. In the infinite attributes case, instead, all compound boxes are of infinite size at the beginning of the search process. It follows that "discounting" some products to encourage search after a bad realization is unnecessary.[19]

To further highlight the role of menu size, notice that $p^* = \underline{W}^*$ is at once the "encouraging" and "discouraging" price in the words of the simplified framework. It encourages search because it does not prevent search after a bad realization because compound boxes do not shrink in this framework. At the same, it matches the value of opening the compound box exactly, just like the discouraging price did in the simplified framework. This suggests that the relationship between optimal pricing and menu selection depends not only on the menu composition, but also on its size.

The fact that compound boxes do not shrink does not necessarily imply that products cannot be priced differently. In principle, given the reservation value of a compound box, different products could be priced differently to capitalize on the information learned through inspection just like it was the case for the simplified framework. In Appendix C, I show that this cannot be optimal. The intuition is as follows: suppose that compound box $X_{1,1}$'s products were priced according to $p_{1,1} = p$ for some $p > 0$ and $p_{1,j} = p_{i,1} = p + \delta$ for some $\delta > 0$.[20] Plugging in these prices in the score of the compound box, one finds:

$$\begin{aligned} E[w_{1,1}(\mathbf{p_{1,1}})] =& [1 - F(z-\delta)]^2(2\bar{y}_\delta - p) \\ &+ 2F(z-\delta)[1-F(z-\delta)](\bar{y}_\delta + z - (p+\delta)) \\ &+ F(z-\delta)^2\left(\underline{y}_\delta + z - (p+\delta)\right), \end{aligned}$$

where $\underline{y}_\delta$ is the expected value of the highest of two realizations below $z - \delta$.

Studying $E[w_{1,1}(\mathbf{p_{1,1}})]$ reveals that any positive δ would be detrimental to the expected profit of the seller. On one hand, the probability that the consumer finds a realization that induces her to keep searching after inspecting $(1,1)$ shrinks as δ increases because $F(z-\delta)$ is decreasing in δ. On the other hand, the participation constrain implied by the fact that the consumer must decide to open the first box becomes tighter as δ increases.

To see why, notice that the expected value of opening a compound box net of prices is equivalent to that of opening the same box when search costs are higher, and in particular $s' > s$ such that $z' = z - \delta$. It follows that $\delta > 0$ makes starting the search process less valuable, which tightens the consumer participation constraint and, therefore, how high prices that do not discourage search can be.

That $p^* = \underline{W}^*$, the initial reservation value of any compound boxes, follows from the updating dynamic detailed in the previous subsection. In particular, it follows from the fact that all updates increase the value of subsequent boxes rather than shrink it. In particular, the lowest value of a compound box after any updating can be shown to

[19] A more in depth discussion about the finite number of attributes case can be found in the Extensions.

[20] In the Appendix, I show that if an equilibrium with differential prices exists, it must have prices following this structure.

be $\underline{W}^*_{a,b}(A^H, B^H) \geq \underline{W}^*_{a,b}(z+s, z+s) = \underline{W}^*$. Therefore, the highest prices that the monopolist can set is the highest price that does not prevent search from taking place and, in particular, $p^* = \underline{W}^*$.

6. Extensions

6.1. More than Two Attributes

In the baseline model, two attribute products are scored by building fictitious boxes including the product itself and closed boxes with all other products sharing attributes with it. The same logic can be applied to three attribute products. With three attributes, two kinds of closed boxes must be included in the compound box with the product it represents. On one hand, all products sharing exactly two attributes with the central product can be represented as small nested boxes equivalent to the ones contained in the two attributes case.

On the other, products that share only one attribute with the central one are unknown in two dimensions, and must be placed in two-dimensional boxes equivalent to the compound boxes of the baseline model. These "intermediate" boxes themselves contain infinite small nested boxes as well. One can imagine multiple grids representing two attribute products side by side to resemble a cube, with the intermediate boxes representing search along one of the sides, and the small boxes representing search along one of the edges as in Figure 7.

We can conceptualize the same process to find the optimal search path for a consumer searching in this environment. First, it is necessary to rethink the structure of a generic nested box $X_{i,i,i}$. If $X_{i,i,i}$ is built around product (i,i,i), it contains all products that share at least one attribute with it. Therefore, it can be represented as the three edges of a cube and the sides delimited by them. Each side can be thought of as a two-dimensional grid in which "intermediate" compound boxes akin to the ones defined in the main model can be found. The choice of opening these boxes is governed by the same $\mathcal{W}$. functions defined above. The choice of opening a different three-dimensional box, instead, requires computing the reservation value of the possible "locked" configurations this box can come out of. All configurations will always be made of three edges and three sides. Whether the edges stretch forward, towards undiscovered attributes, or backward, to known past realizations, depends once again on whether single attributes are found above or below $z+s$, or combinations of two attributes above or below $\mathcal{W}$.

6.2. Purchase Without Inspection

It is assumed throughout the paper that consumers must expend a search cost to inspect any product. Because products in this environment share attributes some uninspected

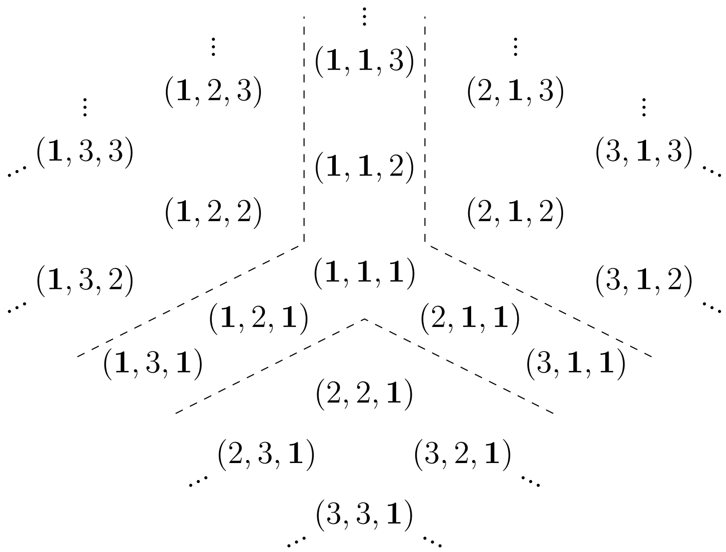

Figure 7: *Graphical representation of a three attribute compound box centered around* $(1, 1, 1)$*. Products that share two attributes with* $(1, 1, 1)$ *can be displayed along the edges of a cube (north for products sharing* A_1*,* B_1*, south-west for products sharing* A_1*,* C_1*, south east for products sharing* B_1*,* C_1*); Products that share one attribute with* $(1, 1, 1)$ *can be displayed along the sides of the cube (north-west for products sharing only* A_1*, north-east for products sharing only* B_1*, south for products sharing only* C_1*).*

products could be fully revealed without being inspected. If search is understood as the physical action of finding a product, this distinction is immaterial. If, however, one were to interpret search as the time and effort necessary to ascertain the quality of the match of a product, it would be sensible to suggest that products uninspected but nonetheless known in their realization should not need search costs to be expended. In this extension I explore the implications of this alternative interpretation.

If taking a product whose attribute have been fully independently discovered is free, the only optimal search process would be one that involves searching new attributes in pairs until the highest realization for each attribute is such that they, together surpass the value of all uninspected products. This can be accomplished by modifying the way reservation values update after each observation. The lowest realization that reroutes search towards itself inside all unopened compound boxes is (without loss of generality) $A_1 > z$ rather than $A_1 > z + s$. With this change, the choice of keeping an attribute is always dominated because all products sharing an attribute with an inspected product would be contained, at zero additional cost, in all unopened compound boxes, and affects them all through the same updating detailed above.

The pricing game in Section 4 would be affected by this change. Recall that the price the multiproduct seller can impose is restricted by the search cost and by the opportunity of searching additional products. If the consumer would always search on the diagonal before selecting a combination of known attributes to keep, the seller would have an incentive to increase the price of all products off the diagonal to capitalize on the consumers' ability to correct his choice for free. The change in interpretation does not affect the result qualitatively, but the mechanical change to the search process suggests that prices would be more dispersed in equilibrium under this alternative interpretation of search costs.

6.3. Limitations and Directions Forward

Finite number of products In the baseline general model I consider an infinite number of variants for each attribute: once the consumer starts searching in one direction, she can continue to do so without ever changing until she finds something to keep, which happens with probability one. Restricting the environment to finite sets of variants beyond the simple example reported in Section 3 introduces new challenges. The logic underneath the structure of the compound boxes and the search process itself is, however, unchanged.

Consider a box like the one in Figure 8. This box can be scored following the same logic used for the infinitely large boxes: if upon opening the box $\min\{A_1, B_1\} > z$, the consumer would stop. If $\max\{A_1, B_1\} > z > \min\{A_1, B_1\}$ or $z > \max\{A_1, B_1\}$, the highest would be kept.

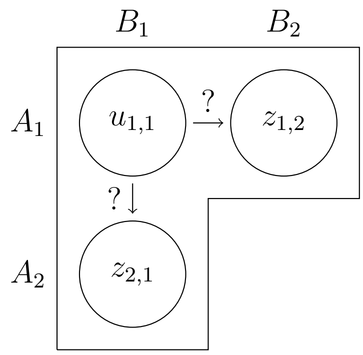

Figure 8: *Three products available*

Differently from before, it is possible now for the consumer to search keeping one attribute fixed and, after exhausting all products sharing that attribute, switching to products characterized by the other. Consider again Figure 8, and suppose A_1 and B_1 both had very low realizations such that $z > A_1 > B_1$. The consumer will inspect $(1, 2)$ next. In the infinite attributes case, the consumer would never run out of $(1, j)$ products to inspect.

The consumer would optimally inspect $(2, 1)$ next if it holds:

$$B_1 + z > A_1 + \max\{B_1, B_2\}.$$

If $B_1 > B_2$, this is trivially true because $z > A_1 > B_1$. Otherwise, if it holds:

$$B_2 < z - (A_1 - B_1),$$

then the consumer would optimally inspect $(2, 1)$ next and keep the highest of all three realizations.

Finding one or more attributes above $z + s$ affects all subsequent boxes in the same way as they did before: such an attribute beats all remaining unopened boxes in the same dimension, and reroutes search towards itself in every unopened compound box. This, in turn, allows for the same updating detailed in the baseline model to take place.

The final difference with the infinitely large boxes of the baseline model also makes the problem likely intractable and follows from the fact that when compound boxes are opened and discarded, the following boxes “shrink” by one variant per attribute. Suppose $X_{1,1}$ contained products characterized by n variants of A and $m-1$ variants of B. Further, suppose $\max\{A_1, B_1\} < z+s$. Then, $X_{2,2}$ would effectively contain products characterized by $n-1$ variants of A and $m-1$ variants of B. Assuming consumers search in increasing order of the index, then, the size of each subsequent compound box $X_{i,i}$ would have $n+1-i$ variants of A and $m+1-i$ variants of B.

The implication of this last remark is that while thinking about boxes as locked in some configuration achieves the same conceptual independence between objects, now every subsequent choice is “discounted” by the value associated with one more variant for each attribute. Effectively, this means that the choice of searching now and searching again later can never be the same. While in principle this could be accounted for, as the structure resembles of that of Weitzman (1979), combining the resulting locked reservation values to generate adaptive ones to take the place of $\mathcal{W}$ quickly leads to a computationally intractable problem.

Different distributions. In principle, removing the assumption of attributes following the same distribution can be accommodated. One can imagine a variant of the model above in which all A attributes were i.i.d and all B attributes were too, but the two sets followed a different distribution. This does not affect the analysis significantly. Far more challenging is accounting for different distributions across different variants of the same attribute in the general framework. The reason stems from the way compound boxes are constructed: with different distributions come different reservation values z for the same search cost s, which means that the expected value of searching along one dimension is not straightforward to compute.

A possible solution might be to use the EPT as characterized by Armstrong (2017) and Choi et al. (2018) to pin down said value, and the value of all other dimensions. A general solution of this more complex problem, however, becomes quickly intractable, and is therefore left for future research.

Competition. It is natural to ask what would be the features of an hypothetical equilibrium with competing firms. The results above rely strongly on the seller’s ability to coordinate product menu and pricing. Restricting the supply by means of strategic de-listing would clearly not be possible when products are introduced and priced by separate agents. Competition should lead to more variety as a consequence. Additionally, the seller studied here is interested in eliciting specific search patterns, but he is indifferent regarding which product acts as starting point. Conditional on a certain variant being the first one visited, however, the remaining available products do not generate the same expected profit.

While this is irrelevant for a monopolist, competing sellers would likely try to gain prominence through undercutting strategies. This incentive to undercut, however, makes pinning down an equilibrium with competing firms exceedingly complex and well beyond the scope of this paper. Nonetheless, if such an equilibrium exists it should feature lower, uniform prices when consumers have the same prior considered here.

7. Conclusion

In this paper, I study the implications of product correlation through shared attributes for directed search and the associated incentives of a seller to introduce different products and prices to capitalize on consumer learning. The framework highlights a novel interaction between pricing and optimal order of inspection in directed search: consumers have an incentive to find better matches in their search process as they learn what they like. This dictates their strategy predictably. On one hand, this allows to rethink the problem in a way that generates threshold values of searching different available options in a way reminiscent of Weitzman (1979)'s optimal search policy. On the other hand, it highlights that a multiproduct seller is able to profit off the learning process by setting differential prices to let consumers self-select based on their preferences.

The framework's predicted search patterns align well with recent evidence of spatial learning in search: Hodgson and Lewis (2020) reports evidence of search for digital cameras to be characterized by a learning process consistent with the one in this framework. Consumers are shown to inspect a broader set of attributes early only to close in on their preferred alternatives in later stages, getting closer and closer to the product they ultimately choose to purchase. This pattern cannot be easily reconciled with standard search models, but is well in line with the prediction of this framework. Further, the model presented here can more easily rationalize the pervasive tendency of consumers to retrace their steps while searching for products.

The implications of this model for recommendation systems and algorithmic pricing schemes have been addressed in an earlier section. It is worth stressing out, however, that these implications go beyond the specific market structure studied here. Coordination of menu and pricing allows a multiproduct seller to induce specific search paths to arise. Equivalently, one can imagine e-commerce platforms to do the same through manipulation of the options presented to captured consumers and the information therein. This is especially true in a world in which data on consumers' decisions, consumption and search patterns is abundant, and algorithmic pricing and recommendation systems are ever more effective at predicting human behavior. In line with recent work on consumption steering and self-preferencing, then, the model's results suggest the need for meticulous regulatory oversight over the algorithms determining what consumers shopping online are shown, and when.

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Appendix

A. Simplified Framework: Monopoly Pricing

Uniform prices As in the main text, I start by assuming $\tilde{N} \equiv N$ and obtain equilibrium prices for different combinations of α, s. Then, I show the optimal restriction of $\tilde{N}$ conditional on the optimal prices.

The seller is interested in finding prices that maximize probability of trade times price. Given expected utility of search as per Equation 1:

$$\begin{aligned}
&E[u_{1,1}]|_{I\equiv\emptyset} = \alpha^2 \max(2-p^u,0) - s \\
&+ 2\alpha(1-\alpha)\max(1-p^u,(1-\alpha)\max(1-p^u,0)+\alpha\max(2-p^u,0)-s,0) \\
&+ (1-\alpha)^2\max(\alpha^2\max(2-p^u,0)+2\alpha(1-\alpha)\max(1-p^u,0)-s,0)
\end{aligned}$$

the highest prices that make consumers start search can be computed as prices that make the expression reach a value of zero:

$$\mathbf{p^D} = \begin{cases} p^M = \frac{2\alpha-s}{\alpha(2-\alpha)} & \text{if } \alpha \leq s < 2\alpha \\ p^D_L = \frac{2\alpha(1+(1-\alpha)(\alpha-s))-s}{\alpha(2-\alpha)} & \text{if } \frac{3\alpha^2-2\alpha^3}{1+2\alpha-2\alpha^2} \leq s < \alpha \\ p^D_H = \frac{2\alpha(\alpha(3-2\alpha)-(1-\alpha)s)-s}{\alpha^2(3-2\alpha)} & \text{if } 0 < s < \frac{3\alpha^2-2\alpha^3}{1+2\alpha-2\alpha^2} \end{cases}$$

The highest prices that allows for inspection after a bad first realization, instead, are:

$$\mathbf{p^E} = \begin{cases} p^E_L = \frac{2\alpha-s}{\alpha(2-\alpha)} & \text{if } \alpha^2 \leq s < 2\alpha \\ p^E_H = \frac{2\alpha^2-s}{\alpha^2} & \text{if } 0 < s < \alpha^2 \end{cases}$$

In each segment identified among the two sets of prices above, lower prices are always feasible, as they generate positive expected utility of search. Lower prices can induce more extensive search and higher probability of trade. Therefore, we look for profitable price reductions for each segment in consideration.

If $\alpha \leq s < 2\alpha$, only $p^M = p^E_L$ is feasible among the candidates above. Furthermore, it can be shown that:

$$\alpha \leq s < 2\alpha \;\rightarrow\; p^M < 1$$

By plugging in p^M in equation 1, one sees that at this prices the consumer stops and purchase if $u(a,b) \neq 0$, and is willing to search again if $u_{1,1} = 0$. It is clear that no deviation from p^M can be profitable: if prices are any higher, expected utility of search would be negative and search would not start; if prices were any lower, no additional probability of trade would be generated. Therefore, in this segment, $p^{u*} = p^M$.

If $\frac{3\alpha^2-2\alpha^3}{1+2\alpha-2\alpha^2} \leq s < \alpha$, both p^D_L and p^E_L are feasible. Moreover, it holds $p_M = p^E_L < p^D_L$

for the whole segment. Therefore, it is sufficient to compare expected profits under p_L^E and p_L^D. Notice that p_L^D is such that searching again after a bad first realization is not possible. In this segment:

$$\alpha^2(2 - p_L^D) + 2\alpha(1-\alpha)(1 - p_L^D) - s < 0$$

Therefore, the seller compares:

$$\pi_L^E = (1 - (1-\alpha)^4)p_L^E$$

$$\pi_I^D = \alpha^2(1 + 2(1-\alpha))p_L^D$$

Direct comparison indicates that p_L^D is selected for some combination of high α and relatively low s:

$$\pi_I^D > \pi_L^E \iff \frac{4\alpha^2 - 2\alpha}{3\alpha - 1} < s < \alpha$$

p_L^E is selected otherwise.

If $0 < s < \frac{3\alpha^2 - 2\alpha^3}{1+2\alpha-2\alpha^2}$, several distinctions must be made. First, $p_L^E < 1 \iff \alpha^2 < s < \alpha$. Therefore, for $s < \alpha^2$, $p_T = 1$ becomes a feasible deviation as it is the price that maximizes probability of trade. Further, p_H^D is now a feasible price to select: it only leads to a purchase if an inspected product is liked in both attributes, and allow for a second search after finding one liked attribute but not after a bad first realization. p_H^E also requires two attributes to be liked by the consumer, but always allow for a follow up search. p_H^E, which is always true in this segment, only allows for a follow-up search if $0 < s < \alpha^2$. This final segment must be split in two sub-segments.

If $\alpha^2 < s < \frac{3\alpha^2 - 2\alpha^3}{1+2\alpha-2\alpha^2}$, $p_L^E < 1$ is always the best choice:

$$\pi_L^E > \pi_H^D = p_H^D(\alpha^2(1 + 2(1-\alpha))$$

If $0 < s < \alpha^2$, $p_H^E > p_T$; the choice is between:

$$\pi_T = (1 - (1-\alpha)^4)p_T$$

$$\pi_H^E = (\alpha^2(1 + 2(1-\alpha) + (1-\alpha)^2)p_H^E$$

$$\pi_H^D = (\alpha^2(1 + 2(1-\alpha))p_H^D$$

Direct comparison indicates that all three pricing levels can be optimal: π_T is optimal for:

$$\min\left(\frac{3\alpha^3 - 6\alpha^2 + 2\alpha}{\alpha - 2}, \frac{-\alpha^4 + 8\alpha^3 - 12\alpha^2 + 4\alpha}{2\alpha^2 - 2\alpha - 1}\right) < s < \alpha^2$$

π_H^E is optimal for:

$$0 < s < \min\left(\frac{3\alpha^3 - 6\alpha^2 + 2\alpha}{\alpha - 2}, \frac{2\alpha^2}{3}\right)$$

and α high enough. Otherwise, π_H^D is optimal.

All feasible combinations of $\alpha \in (0,1)$ and $s \in (0, 2\alpha)$ are then accounted for when restricting the seller to a uniform pricing strategy.

Differential prices It must be shown that the price deviations shown in the main text lead to a higher expected profit. Consider $p^u = p_L^E$. As long as at this price level consumers have a strictly positive expected utility of search, the seller can introduce differential prices profitably. In particular, consider pricing such that:

$$p_{1,1} = p_L^E < 1 \quad p_{2,2} = p_L^E < 1 \quad p_{1,2} = p_L^E + \alpha - s \quad p_{2,1} = p_L^E + \alpha - s$$

Which is valid for $p_L^E < 1$ or, $\alpha^2 < s$. As shown in the main text, for $s > \alpha$ the consumer has no reason to search again after finding something she likes, and indeed would lead to a lower, rather than higher, price level for $p_{1,2}$ and $p_{2,1}$. In this segment ($\alpha^2 < s < \alpha$), such prices lead to strictly higher expected profits. Indeed, when the consumer starts from $(1,1)$ (equivalently, $(2,2)$), she only searches the more expensive product if she already knows that she likes it in some attribute. The consumer cannot start from any other product: if she starts from the more expensive product, her expected utility of search in this segment is negative.

Finally, the difference in prices do not induce changes in the optimal search path. To see why, consider the optimal deviation available to the consumer on the path in which she would want to inspect $(1,2)$: inspecting $(2,2)$ leads to utility equal to two with probability α^2, and allows to correct to $(1,2)$ if she learns that she likes B_2 but not A_2, which happens with probability $\alpha(1-\alpha)$. The expected utility along this alternate path is equal to:

$$(\alpha^2(2 - p_L^E) + (\alpha(1-\alpha) + (1-\alpha)^2)(1 - p_L^E) + \alpha(1-\alpha)(2 - s - (p_L^E + \alpha - s)) - s$$

which is lower than the expected utility of searching $(1,2)$ directly if $s > \alpha^2$. Therefore, no deviation is possible in this segment.

If $s < \alpha^2$, two changes must be accounted for. First, p_T is the preferred option, because $p_L^E > 1$ does not lead to trade taking place. In turns, this implies that because base prices are lower than the myopic expected value of inspecting a product, consumer surplus is above zero if $s < \alpha^2$ under differentiated prices. Further, the consumer would want to search the cheaper $(2,2)$ first, because search costs are low. The seller can react by:

- letting the consumer do so, increase the price of $(1,2)$ to $p_T + 1 - s$
- reducing the price $(1,2)$ to induce his preferred order of search
- removing $(2,2)$.

The first reaction re-establishes the equilibrium: the consumer now inspects the more expensive product only if he knows it is the only product that leads to utility equal to two. Because this is the case, its price can be increased, because the search process took away all uncertainty about it. This product is purchased with probability $\alpha^2(1-\alpha)^2$ and leads to expected profit:

$$\overline{\pi} = (1-(1-\alpha)^4)p_T + 2\alpha^2(1-\alpha)^2(1-s)$$

which is still a strictly higher expected profit than the respective uniform price strategy.

The second reaction also re-establishes the equilibrium: by setting a lower price for $(1,2)$, the seller makes sure that the consumer has no incentive to deviate. Because $s < \alpha^2$, the baseline price is $p = p_T$ and the level p that prevents the deviation solves:

$$\alpha(2-p) - s = \alpha^2 + (1-\alpha)\alpha(-p-s+2) - s \iff p = 1 + s\left(\frac{1-\alpha}{\alpha}\right)$$

which leads to expected profits:

$$\underline{\pi} = (1-(1-\alpha)^4)p_T + 2\alpha^2(1-\alpha)\left(s\left(\frac{1-\alpha}{\alpha}\right)\right)$$

Finally, removing $(2,2)$ prevents the deviation from taking place at all. Because no follow-up search in case of a bad first realization is possible without $(2,2)$, however, overall probability of trade decreases. Expected profits in this case are:

$$\widehat{\pi} = (1-(1-\alpha)^2)p_T + 2\alpha^2(1-\alpha)(\alpha - s)$$

By direct comparison, one finds that all three can be optimal for different values of α, s. In particular, $\widehat{\pi}$ is optimal for α high enough, that is, for:

$$0 < s < \min\left(\frac{3\alpha^2+\alpha-2}{2\alpha^2}, \frac{1}{2}\left(\alpha^2+3\alpha-2\right)\right)$$

$\underline{\pi}$ is optimal for:

$$\max\left(\frac{\alpha}{\alpha+1}, \frac{1}{2}\left(\alpha^2+3\alpha-2\right)\right) < s < \alpha^2$$

while $\overline{\pi}$ is optimal otherwise.

The same argument can be applied to the trade-off between p_H^E and p_H^D when $0 < s < \alpha^2$. In this segment, p_H^E is such that trade only happens if the consumer learns that she likes both attributes about a product, but the parameters encourage the consumer to search again after a bad first realization. Here, too, the seller can choose an intermediate strategy between uniform prices at p_H^E and uniform prices at p_H^D. Suppose the consumer inspected $(1,1)$ and learned $A_1 = 1$, $B_1 = 0$. Then, she would want to inspect $(1,2)$. She does so as long as:

$$\alpha(2-p_{1,2}) - s \geq 0 > 1 - p_H^E$$

which implies:

$$p_{1,2} = 2 - \frac{s}{\alpha}$$

It can be shown that the consumer always reacts to this price level by inspecting $(2,2)$ instead of $(1,2)$. Indeed, if $0 < s < \frac{\alpha^2}{1+\alpha}$, it holds:

$$\alpha^2 (2 - p_I) + (1-\alpha)\alpha \left(-\left(2 - \frac{s}{\alpha}\right) - s + 2 \right) - s > \alpha \left(2 - \left(2 - \frac{s}{\alpha}\right)\right) - s = 0$$

Once again, the seller can react by allowing the deviation and further increasing $p_{1,2}$ to $2 - s$, reducing $p_{1,2}$ to $\frac{2\alpha^3 - 2\alpha^2 - 2\alpha - 3\alpha^2 s + 5\alpha s - s}{(\alpha-2)\alpha}$ to make the consumer search according to his preferred order, or remove $(2,2)$.

Unlike in the previous case, the latter option is always optimal. When the sellers selects differentiated prices, then, for α high and s low the consumer has an incentive to adapt in a way that makes the seller restrict the menu of available products.

Comparison Comparison between the optimal uniform price strategy and the deviation shown above is straightforward. First, it is trivial that whenever $p^{u*} = p_T$, all deviations are strictly preferable: indeed, the strategy with differentiated prices preserves the total probability of trade but generates higher profits for some positive probability. To compare the above strategy with the other uniform prices the seller can optimally select, direct comparison of the profit is sufficient. The same applies to the case in which $p^{u*} = p_L^D$ and $0 < s < \alpha^2$.

Two results emerge: when selecting p_T as base product and the consumer does not adapt their search strategy, this is always optimal. Second, when there is adaptation by consumer and seller, those profits must be compared with the relevant uniform price in the segment, that is, p_H^D.

Direct comparison indicates that p_H^D dominates different prices whenever the optimal reply of the seller to the consumer adapting his search strategy is to restrict the supply. This follows from the fact that, with different prices, consumers always search the cheapest one first. Therefore, the only comparisons left are between π_H^D and the best between $\overline{\pi}$ and $\underline{\pi}$ when $p^* = p_T$. It holds:

$$\underline{\pi} > \pi_H^D \iff \frac{\alpha^4 - 8\alpha^3 + 12\alpha^2 - 4\alpha}{2\alpha^3 - 6\alpha^2 + 4\alpha + 1} < s < \alpha^2$$

$$\overline{\pi} > \pi_H^D \iff \frac{\alpha^4 + 4\alpha^3 - 10\alpha^2 + 4\alpha}{2\alpha^4 - 4\alpha^3 + 4\alpha^2 - 2\alpha - 1} < s < \alpha^2$$

Which delimit the lower right area in Figure 3 in the main text.

B. General Model: Search Dynamics

Proof of Proposition 3

Step 1: The first compound box Let $X_{1,1}$ be the compound box containing $(1,1)$ and infinitely many nested boxes containing $(1, j > 1)$, $(i > 1, 1)$. Suppose the consumer had already opened the box. Her current payoff is $k = \max\{u_0, A_1 + B_1\}$. To determine how she would act afterwards, consider the value function:

$$\begin{aligned} V(k) = \max\{k, -s + E[V(\max\{k, A_i + B_1\})], \\ - s + E[V(\max\{k, A_1 + B_j\})]\}. \end{aligned}$$

Suppose $V(k) = k$. Then:

$$\begin{aligned} k &> -s + E[V(\max\{k, A_i + B_1\})] = -s + E[\max\{V(A_i + B_1), k\}], \\ s &> E[\max\{V(A_i + B_1) - k, 0\}] = \int_k^{\hat{y}} (V(A_i + B_1) - k)\, dF(y). \end{aligned}$$

$$\begin{aligned} k &> -s + E[V(\max\{k, A_1 + B_j\})] = -s + E[\max\{V(A_1 + B_j), k\}], \\ s &> E[\max\{V(A_1 + B_j) - k, 0\}] = \int_k^{\hat{y}} (V(A_1 + B_j) - k)\, dF(y). \end{aligned}$$

Therefore, there exist values r_A, r_B such that if $k > \max\{r_A, r_B\}$, $V(k) = k$. Suppose that $-s + E[V(\max\{k, A_i + B_1\})] > \max\{k, -s + E[V(\max\{k, A_1 + B_j\})]\}$. Then:

$$\begin{aligned} V(k) &= -s + E[\max\{V(A_1 + B_j), V(k)\}], \\ s &= E[\max V(A_1 + B_j, V(k))] \quad \rightarrow \quad V(k) = r_A \end{aligned}$$

Suppose now that $-s+E[V(\max\{k, A_1+B_j\})] > \max\{k, -s+E[V(\max\{k, A_i+B_1\})]\}$. Then:

$$\begin{aligned} V(k) &= -s + E[\max\{V(A_i + B_1), V(k)\}], \\ s &= E[\max V(A_i + B_1, V(k))] \quad \rightarrow \quad V(k) = r_B \end{aligned}$$

To compute r_A and r_B, the optimal policy conditional on $V(k) = r_A$ and $V(k) = r_B$ respectively must be defined. Assuming that the consumer inspects products in increasing order of their indices when indifferent, I make the following:

Claim 1. *If $V(\max\{u_0, A_1 + B_1\}) = r_A$, $V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) = \max\{A_1 + B_2, r_A\}$; if $V(\max\{u_0, A_1+B_1\}) = r_B$, $V(\max\{u_0, A_1+B_1, A_2+B_1\}) = \max\{A_2+B_1, r_B\}$.*

By contradiction, suppose that $V(\max\{u_0, A_1 + B_1\}) = r_A$. Then:

$$\begin{aligned}V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) = \max\{&u_0, A_1 + B_1, A_1 + B_2,\\ &- s + E[V(\max\{k, A_i + B_1\})], -s + E[V(\max\{k, A_1 + B_j\})]\}.\end{aligned}$$

This is immediate: because $V(\max\{u_0, A_1 + B_1\}) = r_A$, it must hold:

$$-s + E[V(\max\{k, A_1 + B_j\})] > \max\{u_0, A_1 + B_1, -s + E[V(\max\{k, A_i + B_1\})]\}.$$

To see why, suppose $\max\{u_0, A_1 + B_1\} = A_1 + B_1$. Then, by the same argument as above, for $V(\max\{u_0, A_1 + B_1\}) = r_A$ it must be that $k < r_A$. If $\max\{u_0, A_1 + B_1\} > A_1 + B_2$, the same condition applies. Otherwise, if $\max\{u_0, A_1 + B_1\} < A_2 + B_2$, then for $V(\max\{u_0, A_1 + B_1, A_2 + B, 2\}) = r_A$ to be true, $A_2 + B_2$ must also be below r_A. Because $V(\max\{u_0, A_1 + B_1\}) = r_A$, it must be that $A_1 + B_1 < r_A$. It follows that $V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) = \max\{A_1 + B_2, r_A\}$.

In words: if it is optimal to inspect $A_1 + B_2$ after opening the compound box, it must also be optimal to inspect $A_1 + B_3$ if the consumer does not want to stop searching. Therefore, the optimal policy conditional on $V(\max\{u_0, A_1 + B_1\}) = r_A$ is a myopic policy in which the current highest realization is compared to the value of inspecting the next product. The consumer is indifferent between stopping at $(1, j)$ and inspecting $(1, j + 1)$, $j \geq 1$, if:

$$\begin{aligned}A_1 + B_j =& - s + A_1 + B_j \int_0^{B_j} dF(y) + \int_{B_j}^{\hat{y}} B_{j+1}\, dF(y),\\ s =& \int_{B_j}^{\hat{y}} (B_{j+1} - B_j)\, dF(y).\end{aligned}$$

Let z be the value of B_j that satisfies the condition above. It follows that $r_A = A_1 + z$. In the same fashion, from $V(\max\{u_0, A_1 + B_1\}) = r_A$ one obtains that $r_B = z + B_1$. It follows that the value function representing the choice of opening the compound box $X_{1,1}$ and searching optimally in it is:

$$V(u_0) = \max\Big\{u_0, \max\Big\{u_0, \max\{A_1, B_1\} + \max\{z, \min\{A_1, B_1\}\}\Big\}\Big\},$$

because the consumer would always select $A_1 + z$ over $B_1 + z$ if and only if $A_1 > B_1$, and will stop at $(1, 1)$ if $\min\{A_1, B_1\} > z$ She would also take her outside option, u_0, if she opens the box and none of these options had value above it. The consumer is indifferent between opening the compound box and not opening if:

$$\begin{aligned}u_0 =& - s + u_0 \int_0^{u_0} dF(y) + \int_{u_0}^{2\hat{y}} w\, dH(w),\\ s =& \int_{u_0}^{2\hat{y}} (w - u_0)\, dH(w),\end{aligned}$$

where $w = \max\{A_1, B_1\} + \max\{z, \min\{A_1, B_1\}\} \in (z, 2\hat{y})$, and its CDF satisfies:

$$H(w) = \int_0^z F_a \left(\int_0^B F_b\,(w-z)\; dF_a(A) + \int_B^{\hat{y}} F_b\,(w-z)\; dF_a(A) \right) dF_b(B) + \\ + \int_z^{\hat{y}} F_a \left(\int_0^z F_b\,(w-z)\; dF_a(A) + \int_z^{\hat{y}} F_b\,(w-A)\; dF_a(A) \right) dF_b(B).$$

Keeping the standard nomenclature, I refer to the value u_0 that satisfies the above equation as the reservation value of the the compound box.

Result 1. *Let $\underline{W}$ be the reservation value of $X_{1,1}$ and z the reservation value of any $y \in A \cup B$. The optimal policy with only one compound box $X_{1,1}$ is:*

- *Open $X_{1,1}$ if $u_0 < \underline{W}$, otherwise keep u_0,*
- *if $u_0 > \max\{A_1, B_1\} + \max\{z, \min\{A_1, B_1\}\}$, stop and keep the outside option, otherwise:*
  - *if $\max\{z, \min\{A_1, B_1\}\} = \min\{A_1, B_1\}$, stop and keep $A_1 + B_1$,*
  - *if $\max\{z, \min\{A_1, B_1\}\} = z$, inspect $(1, j)$ until $B_j \geq z$ is found if $A_1 > B_1$, and inspect $(i, 1)$ until $A_i \geq z$ is found if $A_1 < B_1$.*

Step 2: Uncorrelated compound boxes Let $\tilde{X}_{i,i}$, $i \geq 1$, be the compound box containing (i, i) and infinitely many compound boxes containing $(i, j > i)$, $(j > i, i)$. We want to show that, in this environment, the optimal search policy follows a myopic optimal policy such that if $u_0 < \underline{W}$, the consumer starts searching and stops after finding a product with *ex post* utility higher than the reservation value of all closed boxes.

Suppose the consumer opened $X_{1,1}$. Let $k = \max\{u_0, A_1 + B_1\}$; consider the value function:

$$V(k) = \max\{k, -s + E[V(\max\{k, A_i + B_1\})], \\ - s + E[V(\max\{k, A_1 + B_j\})], -s + E[V(\max\{k, A_i + B_j\})]\}.$$

Compared the value function of the last paragraph, we must now also compare the first three options with the last one. Suppose once again that $V(k) = k$. Then, it holds:

$$k > -s + E[V(\max\{k, A_i + B_j\})] = -s + E[\max\{V(A_i + B_1), k\}], \\ s > E[\max\{V(A_i + B_1) - k, 0\}].$$

which once again implies that there exist a value $R_{2,2}$ such that if $k > \max\{r_A, r_B, R_{2,2}\}$, $V(k) = k$.

We must verify that r_A andr_B are still the value associated with a myopic policy. This is once again immediate: if $r_A > \max\{k, r_B, R_{2,2}\}$ (resp., $r_B > \max\{k, r_A, R_{2,2}\}$), then

$V(\max\{u_0, A_1+B+1, A_1+B_2\}) = \max\{A_1+B_1, r_A\}$ (resp., $V(\max\{u_0, A_1+B+1, A_1+B_2\}) = \max\{A_1+B_1, r_B\}$). In words: if keeping A_1 or B_1 and inspecting B_2 or A_2 has value higher than searching $(2,2)$, keeping the same attribute and inspecting A_3 or B_3 must also have a higher value. This implies that once an attribute is optimally kept, it is never abandoned.

To fully characterize the search process, we must compute the optimal policy after opening $X_{2,2}$. To do so, we prove the following:

Claim 2. *If $V(\max\{u_0, A_1+B_1\}) = R_{2,2}$:*

$$V(\max\{u_0, A_1+B_1, A_2+B_2\}) = \max\{A_2+B_2, -s+E[V(\max\{A_2+B_2, A_i+B_2\})], \\ -s+E[V(\max\{A_2+B_2, A_2+B_j\})], -s+E[V(\max\{A_2+B_2, A_i+B_j\})]\},$$

where once again A_i, B_j are unsampled attributes.

Since currently all compound boxes are uncorrelated, opening $X_{2,2}$ does not generate any new information about the content of $X_{1,1}$. This, will not be the case when we prove the statement in the final step of the proof. For now: since we know that optimally keeping an attribute leads to an myopic optimal policy, the result follows from the same observation than before. In particular:

- If $V(\max\{u_0, A_1+B_1, A_2+B_2\}) = -s+E[V(\max\{A_2+B_2, A_2+B_j\})] = A_2+z$, or $V(\max\{u_0, A_1+B_1, A_2+B_2\}) = -s+E[V(\max\{A_2+B_2, A_i+B_2\})] = B_2+z$ the consumer will open nested boxes myopically forever,
- If $V(\max\{u_0, A_1+B_1, A_2+B_2\}) = A_2+B_2$, the consumer would stop,
- If $V(\max\{u_0, A_1+B_1, A_2+B_2\}) = -s+E[V(\max\{A_2+B_2, A_i+B_j\})]$, the consumer would open the next compound box.

It must hold that $V(\max\{u_0, A_1+B_1, A_2+B_2\}) \neq \max\{u_0, A_1+B_1, A_1+z, B_1+z\}$ because $V(\max\{u_0, A_1+B_1\}) = R_{2,2}$. It follows that the optimal policy after opening $X_{2,2}$ is to myopically select between $\max\{A_2, B_2\} + \max\{z, \min\{A_2, B_2\}\}$ and $R_{3,3}$. Since this was the same policy the consumer followed at $(1,1)$, the consumer is once again following a myopic policy. Therefore, $R_{2,2} = \underline{W}$, which proves the claim.

Result 2. *Let $\underline{W}_{i,i} = \underline{W}$ be the reservation value of uncorrelated compound boxes $\tilde{X}_{i,i}$ and z the reservation value of any $y \in A \cup B$. The optimal policy with infinitely many $\tilde{X}_{i,i}$ is:*

- *Open $\tilde{X}_{1,1}$ if $u_0 < \underline{W}$, otherwise keep u_0,*
- *if $\max\{A_1, B_1\} + \max\{z, \min\{A_1, B_1\}\} > \underline{W}$:*
    - *if $\max\{z, \min\{A_1, B_1\}\} = \min\{A_1, B_1\}$, stop and keep $A_1 + B_1$,*

  - *if* $\max\{z, \min\{A_1, B_1\}\} = z$, *inspect* $(1, j)$ *until* $B_j > z$ *is found if* $A_1 > B_1$ *and inspect* $(i, 1)$ *until* $A_i > z$ *is found if* $A_1 < B_1$,

- *if* $\max\{A_1, B_1\} + \max\{z, \min\{A_1, B_1\}\} < \underline{W}$, *open* $\tilde{X}_{2,2}$ *and go back to the second point.*

Step 3: General model We now remove the assumption of compound boxes being uncorrelated. This implies that the consumer can move freely on the grid of products inspecting one or two new attributes as she sees fit. We want to show that the optimal search process still follows a process that can fully characterized with threshold rules.

Suppose for now that the consumer never optimally goes back to a previously discarded attribute. That is, suppose that $k = \max\{u_0, A_1 + B_1, A_1 + B_2\}$. Then:

$$\begin{aligned} V(k) = \max\{&k, -s + E[V(\max\{k, A_1 + B_j\})], \\ &- s + E[V(\max\{k, A_i + B_2\})], -s + E[V(\max\{k, A_i + B_j\})]\}. \end{aligned}$$

Notice that this value function does not allow the consumer to inspect combinations of discovered attributes. This will be addressed shortly. For now, we want to show that:

Claim 3. *If the consumer cannot backtrack to a combination of discovered attributes,* $V(\max\{u_0, A_1 + B_1\}) = r_A$ *implies:*

$$V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) = \max\{A_1 + B_2, -s + E[V(\max\{A_1 + B_2, A_1 + B_j\})],$$

where B_j *are unsampled* B *attributes.*

In words: we want to show that the if the consumer cannot backtrack to a combination of undiscovered attributes, keeping an attribute and still searching is still myopic.

From the last paragraph, we know that if $V(\max\{u_0, A_1 + B_1\}) = r_A$, it must hold:

$$\begin{aligned} V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) \neq \max\{&u_0, A_1 + B_1, \\ &- s + E[V(\max\{k, A_i + B_1\})], -s + E[V(\max\{k, A_i + B_j\})]. \end{aligned}$$

We must now prove that $r_A > R_{2,2}$ implies that $V(\max\{u_0, A_1 + B_1, A_1 + B_2\}) \neq -s + E[V(\max\{k, A_i + B_2\})]$, that is, inspecting $(2, 2)$ after $(1, 2)$ cannot be optimal. Suppose by contradiction that the optimal policy was such that, after inspecting $(1, 2)$, the consumer would optimally inspect $(2, 2)$ with probability $q \in (0, 1)$ and then follow the optimal policy from there. Then, it must hold:

$$
\begin{aligned}
r_A =& qR_{2,2} + (1-q)(r_A - R_{2,2}) > R_{2,2},\\
r_A q =& 2qR_{2,2} - R_{2,2},\\
r_A =& 2R_{2,2} - \frac{R_{2,2}}{q} > R_{2,2},\\
q >& 1
\end{aligned}
$$

Which is clearly a contradiction: if $(1,2)$ is optimally picked over $(2,2)$, it can never be optimal to inspect $(2,2)$ afterwards. This proves the claim.

When compound boxes share products, each combination of known attributes, inspected or not, becomes effectively an outside option. Suppose that the consumer opened $X_{1,1}$, $X_{2,2}$, and $X_{3,3}$. Then, the highest available payoff for the consumer is:

$$
\begin{aligned}
k = \max\{&u_0, A_1+B_1, A_2+B_2, A_3+B_3,\\
&A_1+B_2-s, A_1+B_3-s, A_2+B_3-s,\\
&A_2+B_1-s, A_3+B_1-s, A_3+B_2-s\}.
\end{aligned}
$$

Only a subset of these can ever be relevant. Consider A_2+B_3-s and A_1+B_3-s. If the consumer decides to backtrack to one of these two available payoffs after opening $X_{3,3}$ and observing the realization B_3, it is clear that she woulds select the former if $A_2 > A_1$ and the latter otherwise. Importantly, this information is known to the consumer before opening the compound box $X_{3,3}$.

From the last paragraph, we know that if the consumer cannot backtrack and chooses to search keeping an attribute fixed, the optimal policy that follows is myopic. Therefore, the choice of backtracking is pinned down by the highest past realization. Suppose the consumer opens $X_{3,3}$. If she chooses to keep A_3, she chooses between:

- A_3+B_3, readily available,
- A_3+z, opening nested boxes,
- A_3+B_1-s or A_3+B_2-s, backtracking.

Without loss of generality, suppose $B_2 > B_1$. Suppose further that $B_3 < z$. Then, the consumer chooses $\max\{A_3+z, A_3+B_1-s\}$. Notice that:

$$
\max\{A_3+z, A_3+B_1-s\} = A_3+B_1-s \quad \iff \quad B_1 \geq z+s.
$$

Because the realization B_1 is known before opening the box, the consumer is already aware of whether she would backtrack or go forward. Moreover, if $V(k) = A_3+z$, it is clear that the consumer would never choose to backtrack afterwards. This confirms that the optimal policy conditional on inspecting a single attribute is myopic.

We can finally prove the main statement. In words, we now use the predictability of the search process when the highest past realization of A attributes, A^H, and B attributes, B^H, are above or below $z+s$ to define these thresholds.

Formally, suppose the consumer needs to decide whether to open $X_{i,i}$. let $k = \max\{u_0, \max_{j<i}\{u_{j,j}\}, A^H + B^H - s\}$ be the current highest sure payoff for the consumer. Define:

$$
\begin{aligned}
V(k) = \max\{k, -s + E[V(\max\{k, A_i + B_k\})], -s + E[V(\max\{k, A_k + B_i\}), \\
- s + E[V(\max\{k, A_k + B_k\})]], \quad \forall k > i.
\end{aligned}
$$

We already established that there exist a value $\hat{k}$ such that if $k > \hat{k}$, $V(k) = k$. Further, we proved above that if $V(k) = -s + E[V(\max\{k, A_k + B_i\})$ or $V(k) = -s + E[V(\max\{k, A_i + B_k\})$, the optimal policy has the consumer only opening nested boxes keeping the same attribute A_i or B_i fixed. Finally, we established that if A^H and/or B^H are above $z+s$, either one or both $-s+E[V(\max\{k, A_k+B_i\})$ and $-s+E[V(\max\{k, A_i+B_k\})$ will be dominated by backtracking. With these considerations we can define the value $\mathcal{W}$ such that the consumer opens compound box $X_{i,i}$ if and only if $\mathcal{W} > \max\{k, A_{i-1} + B_i, A_i + B_{i-1}\}$.

If $\max\{A^H, B^H\} < z + s$, the myopic (but incorrect) value of inspecting the next compound box is the same is in the last paragraph: $\underline{W}$. Suppose $X_{i,i}$ is opened next and $A_i > z + s$. The next box will have a different reservation value. The correct reservation value must account for the possibility of discovering attributes that change the search from that point onward. To do so, we must first compute the value of inspecting a compound box when $\max\{A^H, B^H\} > z + s$. We first do so assuming, as in the last paragraph, that boxes are uncorrelated but in different configurations depending on the number of attributes above $z + s$ that were found. Then, we combine them appropriately to produce the correct reservation values.

Suppose an attribute A^H was found above $z + s$. The consumer would go back to it rather than opening nested boxes unknown in their A component. Let $w_a(A^H)$ be the expected payoff of opening a compound box locked in this configuration. Let this box be $X_{i,i}$. Suppose $B_i < z$ is found. Then, the consumer must choose between opening nested boxes with score $r_{i,j>i} = A_i + z > A_i + B_i = u_{i,i}$ and backtracking to a box with score $r_{i,j<i} = A^H - s + B_i$. Therefore, the consumer inspects nested boxes if $A_i > A^H - s - (z - B_i)$, and backtrack otherwise. If $B_i > z$, the consumer stops at (i, i) if $A_i > A^H - s$ and backtrack otherwise. The CDF then can be obtained by fixing the value B_i and then integrating for it as it was done for configuration 1:

$$\begin{aligned}H_a(w_a) = &\int_0^z F_a\left(\int_0^{A^H-s-(z-B)} F_b\left(w_a-(A^H-s)\right)dF_a(A)+\right.\\&\left.+\int_{A^H-s-(z-B)}^{\hat{y}} F_b\left(w_a-z\right)dF_a(A)\right)dF_b(B)+\\&+\int_z^{\hat{y}} F_a\left(\int_0^{A^H-s} F_b\left(w_a-(A^H-s)\right)dF_a(A)+\right.\\&\left.+\int_{A^H-s}^{\hat{y}} F_b\left(w_a-A\right)dF_a(A)\right)dF_b(B).\end{aligned}$$

The equivalent formulation for $H_b(w_b)$, relevant if $A^H > z+s > B^H$ is:

$$\begin{aligned}H_b(w_b) = &\int_0^z F_b\left(\int_0^{B^H-s-(z-A)} F_a\left(w_b-(B^H-s)\right)dF_b(B)+\right.\\&\left.+\int_{B^H-s-(z-A)}^{\hat{y}} F_a\left(w_b-z\right)dF_b(B)\right)dF_a(A)+\\&+\int_z^{\hat{y}} F_b\left(\int_0^{B^H-s} F_a\left(w_b-(B^H-s)\right)dF_b(B)+\right.\\&\left.+\int_{B^H-s}^{\hat{y}} F_a\left(w_b-B\right)dF_b(B)\right)dF_a(A).\end{aligned}$$

Suppose now $\min\{A^H,B^H\} > z$: the consumer will not inspect any nested box of which he does not already know the value of. Now, the relevant thresholds determining whether something is kept or not are A^H-s and B_H-s: (i,i) is only kept if both $A_i > A^H-s$ and $B_i > B^H-s$. Otherwise, the highest between A^H-s+B_i and A_i+B^H-s is kept. Therefore:

$$\begin{aligned}H_{a,b}(w_{a,b}) = &\int_0^{B^H-s} F_a\left(\int_0^{A^H-B^H+B} F_b\left(w_{a,b}-(A^H-s)\right)dF_a(A)+\right.\\&\left.+\int_{A^H-B^H+B}^{\hat{y}} F_b\left(w_{a,b}-(B^H-s)\right)dF_a(A)\right)dF_b(B)+\\&+\int_{B^H-s}^{\hat{y}} F_a\left(\int_0^{A^H-s} F_b\left(w_{a,b}-(A^H-s)\right)dF_a(A)+\right.\\&\left.+\int_{A^H-s}^{\hat{y}} F_b\left(w_{a,b}-A\right)dF_a(A)\right)dF_b(B).\end{aligned}$$

When boxes are assumed to be locked in any of these configurations, the value function governing search is exactly the same as the one in the last paragraph since all boxes are independent. Then, we construct myopic reservation values:

$$s = \int_{\underline{W}_\kappa}^{2\hat{y}} (w_\kappa - \underline{W}_\kappa)dH_\kappa(w_\kappa).,$$

with $\kappa \in \{\{a\},\{b\},\{a,b\}\}$.

Let $\underline{W}^*_{a,b}(A^H, B^H)$ be the expected equivalent of costly opening the next box on the search path. This can be rewritten in terms of expected new values $\tilde{A}^H$ and $\tilde{B}^H$:

$$\begin{aligned}\underline{W}^*_{a,b}(A^H, B^H) = \underline{W}_{a,b}(A^H, B^H)\int_0^{B^H}\int_0^{A^H} dF_a(A)dF_b(B)+ \\ +\int_0^{B^H}\int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B^H)\, dF_a(A)dF_b(B)+ \\ +\int_{B^H}^{\hat{y}}\int_0^{A^H} \underline{W}_{a,b}(A^H, B)\, dF_a(A)dF_b(B)+ \\ +\int_{B^H}^{\hat{y}}\int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A)dF_b(B) = \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)\end{aligned}$$

Consider the choice of the consumer. If she chooses to open the next compound box, $X_{i+1,i+1}$, she knows that she will stop only if $w_{i+1,i+1} > \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$. All future boxes will have this updated value. Then, the gain she expects from searching the next box can be written as:

$$\begin{aligned}&-s + \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)\int_{\max\{A^H,B^H\}}^{\underline{W}_{a,b}(\tilde{A}^H,\tilde{B}^H)} dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)+ \\ &+\int_{\underline{W}_{a,b}(\tilde{A}^H,\tilde{B}^H)}^{2\hat{y}} w_{a,b}\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)\end{aligned}$$

The certain equivalent of this first inspection, then, is simply $\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$ that solves:

$$s = \int_{\underline{W}_{a,b}(\tilde{A}^H,\tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)$$

For configuration 2, we write:

$$\begin{aligned}\underline{W}^*_a(A^H) =&\underline{W}_a(A^H)\int_0^{z+s}\int_0^{A^H} dF_a(A)dF_b(B)+ \\ &+\int_0^{z+s}\int_{A^H}^{\hat{y}} \underline{W}_a(A)\, dF_a(A)dF_b(B)+ \\ &+\int_{z+s}^{\hat{y}}\int_0^{A^H} \underline{W}_{a,b}(A^H, B)\, dF_a(A)dF_b(B)+ \\ &+\int_{z+s}^{\hat{y}}\int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A)dF_b(B) \\ =&F(z+s)\underline{W}_a(\tilde{A}^H) + (1 - F(z+s))\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)\end{aligned}$$

The certain equivalent of this first inspection, then, is the linear combination $F(z +$

$s)\underline{W}_a(\tilde{A}^H) + (1 - F(z+s))\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)$ that solves:

$$
\begin{aligned}
s =& F(z+s)\int_{\underline{W}_a(\tilde{A}^H)}^{2\hat{y}} (w_a - \underline{W}_a(\tilde{A}^H)\, dH_a(w_a, \tilde{A}^H) + \\
& + (1 - F(z+s)) \int_{\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)
\end{aligned}
$$

An equivalent formulation can be found for configuration 3. Finally, for configuration 1, we write:

$$
\begin{aligned}
\underline{W}^* =& \underline{W}\int_0^{z+s}\int_0^{z+s} dF_a(A)dF_b(B) + \\
& + \int_0^{z+s}\int_{z+s}^{\hat{y}} \underline{W}_a(A)\, dF_a(A)dF_b(B) + \\
& + \int_{z+s}^{\hat{y}}\int_0^{z+s} \underline{W}_b(B)\, dF_a(A)dF_b(B) + \\
& + \int_{z+s}^{\hat{y}}\int_{z+s}^{\hat{y}} \underline{W}_{a,b}(A,B)\, dF_a(A)dF_b(B) \\
=& F(z+s)^2\underline{W} + F(z+s)(1 - F(z+s))(\underline{W}_a(\tilde{A}^H) + \underline{W}_b(\tilde{B}^H)) + \\
& + (1 - F(z+s))^2\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^h)
\end{aligned}
$$

so that the reservation value of the next box inspected when all boxes are in the first configuration is the linear combination of values that that solves:

$$
\begin{aligned}
s =& F(z+s)^2\int_{\underline{W}}^{2\hat{y}} (w - \underline{W})\, dH + \\
& + F(z+s)(1 - F(z+s))\int_{\underline{W}_a(\tilde{A}^H)}^{2\hat{y}} (w_a - \underline{W}_a(\tilde{A}^H))\, dH_a(w_a, \tilde{A}^H) + \\
& + F(z+s)(1 - F(z+s))\int_{\underline{W}_b(\tilde{B}^H)}^{2\hat{y}} (w_b - \underline{W}_b(\tilde{B}^H))\, dH_b(w_b, \tilde{B}^H) + \\
& + (1 - F(z+s))^2\int_{\underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H)}^{2\hat{y}} (w_{a,b} - \underline{W}_{a,b}(\tilde{A}^H, \tilde{B}^H))\, dH_{a,b}(w_{a,b}, \tilde{A}^H, \tilde{B}^H)
\end{aligned}
$$

Therefore:

$$
\mathcal{W}(A^H, B^H) = \begin{cases}
\underline{W}^* & \text{if} \quad \max\{A^H, B^H\} < z+s, \\
\underline{W}_a^*(A^H) & \text{if} \quad A^H > z+s > B^H, \\
\underline{W}_B^*(B^H) & \text{if} \quad B^H > z+s > A^H, \\
\underline{W}_{a,b}^*(A^H, B^H) & \text{if} \quad \min\{A^H, B^H\} > z+s.
\end{cases}
$$

C. General Model: Monopoly Pricing

The proof of Proposition 4 comes in two steps. First, I show that if a non-uniform equilibrium price vector exists, it must be such that lower uniform prices are set for exactly one product characterizing all attributes, and higher uniform prices are set for all other products. Next, I show that all such price vectors are dominated by the uniform price as per Proposition 4.

Optimal differentiated price vector Suppose the seller wanted to set differential prices for his infinite products. First, it is obvious that at least one product must be prices differently than all others. For notational clarity, I define $p_1 < p_2 < p_3$ as a set of three price levels. I show that any optimal differential price vector must be such that a set of products sharing no attributes with each other must be priced at p_1 and all other products must be priced at either p_2 or p_3, but there cannot be any vector with more than two price levels.

First suppose that more than one product sharing an attribute A_i has price set at p_1. The geometry of the product space implies that there must be one attribute B_j for which the same applies. For example, if $(1, 1)$ and $(1, 2)$ were priced at p_1, $(1, 2)$ and $(2, 2)$ would also need to be. Then, the consumer would optimally start her search process from $(1, 2)$ because compound box $X_{1,2}$ contains the most cheap products. If the consumer then wanted to open a new compound box, she would optimally select $X_{3,3}$ and proceed along the diagonal.

If p_1 is such that the consumer would want to open $X_{1,2}$ but not $X_{3,3}$ without updating, the seller would have the incentive to set a lower p_1 to all products on the diagonal and increase the price of $(1, 2)$; on the other hand, if the consumer is willing to open $X_{3,3}$ without updating, then $p_{1,2} = p_1$ implies that with positive probability the consumer will choose to keep either A_1 or B_2 and purchase $(1, 1)$ or $(2, 2)$ at a lower price that he would have been willing to. Therefore, the seller would have the incentive to increase $p_{1,2}$ to re-establish the canonical order of search. This intuition extends to any number $n > 1$ of products for each attribute, and to all attributes. Therefore, at most one product per attribute can be optimally set to be cheaper than the others.

Suppose now that a strict subset of attributes has all associated products priced at either p_1 or p_2, while all other attributes follow the pricing detailed above. If the selected price is p_1, all products with such attributes are cheaper than all others, and are therefore more valuable to the consumer. If the consumer is willing to exhaust these products and still inspect the attributes with differentiated prices, with positive probability the seller sells at a lower price than the consumer was willing to pay. If the selected price is p_2, all such attributes would be pushed to the end of the search order and never reached because the consumer has infinite better alternatives available.[21]

[21] This implies that the pricing scheme with infinite products has infinite payoff-equivalent equilibria in

Finally, suppose that exactly one product per attribute is priced at p_1 and all others are priced at either p_2 or p_3. Suppose first that a finite subset of attributes has products priced at either p_1 or p_2 and all other attributes have products priced at either p_1 or p_3. A consumer that optimally decides to start searching will search first the compound box or boxes in which the most cheap products can be found. If he is willing to keep searching the boxes until only the ones with the highest number of expensive products and stop without updating, having the latter group cannot be optimal, and all products should belong to the former group. If the consumer is still interested in searching, instead, all products should belong to the latter group.

Suppose now that all attributes are such that one product is priced at p_1, a finite subset of products is priced at p_2 and all others are priced at p_3. If the consumer optimally elected to keep an attribute after inspecting a product priced at p_1, she would select to inspect the ones priced at p_2 first. If after exhausting them she would stop, all other products should also have been priced at p_2. Otherwise, all products should have been priced at p_3. The result immediately extends to any number of price levels larger than two. The result follows.

Optimality of uniform prices Next, I show that for any vector of differential prices structured as above, there exist an uniform price vector that preserves probability of trade and returns strictly higher expected profit. As discussed in the main text (and detailed in the next part of the proof), probability of trade conditional on the consumer starting to search depends on the probability of finding realizations such that the resulting updating of unopened compound boxes makes the consumer stop searching and not purchase anything. The highest uniform price is such that:

$$\mathcal{W}(\mathbf{p^{unif}}) = \mathcal{W} - p^{unif} = 0, \quad \forall (i,j).$$

Where $\mathcal{W}$ is the initial value of inspecting a closed nested box net of prices. From the discussion above and from the proof of proposition 3, all updating to $\mathcal{W}$ is upward and, therefore, probability of trade in case of the highest uniform price is 1. Therefore, it must be shown that no pricing scheme with differential prices can generate a higher expected profit than $\mathcal{W}$.

To do so, it is sufficient to show that when differential prices are set, the value of inspecting any closed compound box is lower than with uniform prices. Suppose this is the case: the threshold above which a compound box is kept when locked in configuration 1 would then be lower. This cascades into a reduction of the overall value of searching and, therefore, reduces expected profits of the firm.

which the seller sets a high price for all products defined by a finite subset of attributes which are never reached.

Notice that, as per the main text, without prices it holds:

$$w_{i,i} = \max\{A_i, B_i\} + \max\{z, \min\{A_i, B_i\}\},$$

when all products are priced uniformly, purchasing any of the products inside the compound box is equivalent. When they are not, instead, the price spread affects when consumers would keep searching instead of stopping at (i, i). Suppose $p_{i,i} = p$ and $p_{i,j\neq i} = p_{j\neq i,i} = p+\delta$ for some $\delta > 0$. Then, a consumer would elect to inspect nested boxes if $z - \delta > \min\{A_i, B_i\}$, because now the prices associated with the nested boxes is higher. notice that this implies that $w_{i,i}$ when differential prices are set is equivalent to $w_{i,i}$ with uniform prices when search costs are higher, or:

$$\max\{A_i, B_i\} + \max\{z - \delta, \min\{A_i, B_i\}\} = \max\{A_i, B_i\} + \max\{z', \min\{A_i, B_i\}\},$$

where z' solves:

$$s' = \int_{z'}^{\hat{y}} (y - z')\, dF(y)$$

and because $z' = z - \delta$ and z is decreasing in s, the result follows.

Therefore, differential prices reduces the value of search when there are infinitely many attributes, which in turn means that any pricing vector with differential prices limits the value of search compared to an equivalent one with uniform prices. Therefore, any differential pricing vector that makes the consumer indifferent between searching or not (which makes it equivalent to $p^* = \mathcal{W}$) must generate lower expected profits than its equivalent counterpart.

Optimal uniform prices vector Finally, it must be shown that $p^* = \mathcal{W}$ is indeed optimal. To do so, it sufficient to show that $\mathcal{W}_{a,b}(z+s, z+s)$ is the lowest updated value a compound box can ever have and that $\mathcal{W}_{a,b}(z+s, z+s) \geq \mathcal{W}$.

For the former, recall that it holds:

$$\begin{aligned}\mathcal{W}_{a,b}(A^H, B^H) = \underline{W}_{a,b}(A^H, B^H) \int_0^{B^H} \int_0^{A^H} dF_a(A) dF_b(B) + \\ + \int_0^{B^H} \int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B^H)\, dF_a(A) dF_b(B) + \int_{B^H}^{\hat{y}} \int_0^{A^H} \underline{W}_{a,b}(A^H, B)\, dF_a(A) dF_b(B) + \\ + \int_{B^H}^{\hat{y}} \int_{A^H}^{\hat{y}} \underline{W}_{a,b}(A, B)\, dF_a(A) dF_b(B)\end{aligned}$$

$$\begin{aligned}\mathcal{W}_a(A^H) = \underline{W}_a(A^H) \int_0^{z+s} \int_0^{A^H} dF_a(A) dF_b(B) + \int_0^{z+s} \int_{A^H}^{\hat{y}} \underline{W}_a(A)\, dF_a(A) dF_b(B) + \\ + \int_{z+s}^{\hat{y}} \int_0^{A^H} \mathcal{W}_{a,b}(A^H, B)\, dF_a(A) dF_b(B) + \int_{z+s}^{\hat{y}} \int_{A^H}^{\hat{y}} \mathcal{W}_{a,b}(A, B)\, dF_a(A) dF_b(B)\end{aligned}$$

(which has an equivalent counterpart for $\mathcal{W}_b(B^H)$), and

$$\mathcal{W} = \underline{W}\int_0^{z+s}\int_0^{z+s} dF_a(A)dF_b(B) + \int_0^{z+s}\int_{z+s}^{\hat{y}} \mathcal{W}_a(A)\, dF_a(A)dF_b(B) + \\ + \int_{z+s}^{\hat{y}}\int_0^{z+s} \mathcal{W}_b(B)\, dF_a(A)dF_b(B) + \int_{z+s}^{\hat{y}}\int_{z+s}^{\hat{y}} \mathcal{W}_{a,b}(A,B)\, dF_a(A)dF_b(B),$$

and notice that if $A^H = z+s$ (equivalently, $B^H = z+s$), it holds:

$$H_{a,b} = H_a (= H_b) = H,$$

$$\underline{W}_{a,b}(z+s, z+s) = \underline{W}_a(z+s)(= \underline{W}_b(z+s)) = \underline{W}.$$

and that all are weakly increasing in A^H and/or B^H, with $\underline{W}$ being constant in both and the other being strictly increasing in either or both.

Therefore:

$$\mathcal{W}_{a,b}(z+s, z+s) = \underline{W}_{a,b}(z+s, z+s)\int_0^{z+s}\int_0^{z+s} dF_a(A)dF_b(B) + \\ + \int_0^{z+s}\int_{z+s}^{\hat{y}} \underline{W}_a(A)\, dF_a(A)dF_b(B) + \\ + \int_{z+s}^{\hat{y}}\int_0^{z+s} \underline{W}_b(B)\, dF_a(A)dF_b(B) + \int_{z+s}^{\hat{y}}\int_{z+s}^{\hat{y}} \underline{W}_{a,b}(A,B)\, dF_a(A)dF_b(B),$$

$$\mathcal{W} = \underline{W}\int_0^{z+s}\int_0^{z+s} dF_a(A)dF_b(B) + \\ + \int_0^{z+s}\int_{z+s}^{\hat{y}} \underline{W}_a(A)\, dF_a(A)dF_b(B) + \\ + \int_{z+s}^{\hat{y}}\int_0^{z+s} \underline{W}_b(B)\, dF_a(A)dF_b(B) + \int_{z+s}^{\hat{y}}\int_{z+s}^{\hat{y}} \underline{W}_{a,b}(A,B)\, dF_a(A)dF_b(B).$$

Therefore, $\underline{W}_{a,b}(z+s, z+s) = \underline{W}$ proves the result.

First, we must compute CDFs for all four configurations in isolation. For configuration 1, both are provided in the main text:

$$w_{i,i} = \max\{A_i, B_i\} + \max\{z, \min\{A_i, B_i\}\}.$$

$$H(w) = \int_0^z F_a\left(\int_0^B F_b\left(w-z\right)\, dF_a(A) + \int_B^{\hat{y}} F_b\left(w-z\right)\, dF_a(A)\right) dF_b(B) + \\ + \int_z^{\hat{y}} F_a\left(\int_0^z F_b\left(w-z\right)\, dF_a(A) + \int_z^{\hat{y}} F_b\left(w-A\right)\, dF_a(A)\right) dF_b(B).$$

Consider now configuration 2: in this configuration, an attribute A^H was found above $z+s$. Therefore, the consumer would go back to it rather than opening nested boxes unknown in their A component. Let $w_a(A^H)$ be the expected payoff of opening a compound box locked in this configuration. W.L.O.G., let this box be $X_{i,i}$. Suppose $B_i < z$ is found. Then, the consumer must choose between opening nested boxes with score $r_{i,j>i} = A_i + z > A_i + B_i = u_{i,i}$ and backtracking to a box with score $r_{i,j<i} = A^H - s + B_i$. Therefore, the consumer inspects nested boxes if $A_i > A^H - s - (z - B_i)$, and backtrack otherwise. If $B_i > z$, the consumer stops at (i,i) if $A_i > A^H - s$ and backtrack otherwise. The CDF then can be obtained by fixing the value B_i and then integrating for it as it was done for configuration 1:

$$\begin{aligned} H_a(w_a) = \int_0^z F_a \Bigg(\int_0^{A^H-s-(z-B)} & F_b\left(w_a - (A^H - s)\right) dF_a(A) + \\ & + \int_{A^H-s-(z-B)}^{\hat{y}} F_b\left(w_a - z\right) dF_a(A) \Bigg) dF_b(B) + \\ + \int_z^{\hat{y}} F_a \Bigg(\int_0^{A^H-s} F_b\left(w_a - (A^H - s)\right) dF_a(A) & + \int_{A^H-s}^{\hat{y}} F_b\left(w_a - A\right) dF_a(A) \Bigg) dF_b(B). \end{aligned}$$

The equivalent formulation for $H_b(w_b)$ is:

$$\begin{aligned} H_b(w_b) = \int_0^z F_b \Bigg(\int_0^{B^H-s-(z-A)} & F_a\left(w_b - (B^H - s)\right) dF_b(B) + \\ & + \int_{B^H-s-(z-A)}^{\hat{y}} F_a\left(w_b - z\right) dF_b(B) \Bigg) dF_a(A) + \\ + \int_z^{\hat{y}} F_b \Bigg(\int_0^{B^H-s} F_a\left(w_b - (B^H - s)\right) dF_b(B) & + \int_{B^H-s}^{\hat{y}} F_a\left(w_b - B\right) dF_b(B) \Bigg) dF_a(A). \end{aligned}$$

Consider now configuration 4: in this configuration, both an attribute A^H and an attribute B^H were found above $z+s$. The consumer will not inspect any nested box of which he does not already know the value of. Now, the relevant thresholds determining whether something is kept or not are $A^H - s$ and $B_H - s$: (i,i) is only kept if both $A_i > A^H - s$ and $B_i > B^H - s$. Otherwise, the highest between $A^H - s + B_i$ and $A_i + B^H - s$ is kept. Therefore:

$$\begin{aligned} H_{a,b}(w_{a,b}) = \int_0^{B^H-s} F_a \Bigg(\int_0^{A^H-B^H+B} & F_b\left(w_{a,b} - (A^H - s)\right) dF_a(A) + \\ & + \int_{A^H-B^H+B}^{\hat{y}} F_b\left(w_{a,b} - (B^H - s)\right) dF_a(A) \Bigg) dF_b(B) + \\ + \int_{B^H-s}^{\hat{y}} F_a \Bigg(\int_0^{A^H-s} F_b\left(w_{a,b} - (A^H - s)\right) dF_a(A) & + \int_{A^H-s}^{\hat{y}} F_b\left(w_{a,b} - A\right) dF_a(A) \Bigg) dF_b(B). \end{aligned}$$

Given any of these CDFs, we can define and solve the value function governing search following McCall (1970) and Kohn and Shavell (1974). In particular, we want to find $\underline{W}_{\cdot}$ that solves:

$$\underline{W}_{\cdot} = -s + \max\{w_{\cdot}, E[\underline{W}_{\cdot}]\}.$$

It is immediate to see that locking boxes in one of the above configuration, the optimal strategy was the consumer stop after finding $w_{\cdot} > E[\underline{W}_{\cdot}]$ and keep searching otherwise. Therefore, we just need to compute $E[\underline{W}_{\cdot}]$. To do so, we proceed as follows. First, because locking the boxes makes them independent, the probability of each compound box opened being the one at which the consumer stops is the same. Formally, let $E[\underline{W}_{\cdot}|N]$ be the expected value of the payoff of the consumer after he accepts the N^th offer he receives, it holds:

$$\begin{aligned}
E[\underline{W}_{\cdot}|N] &= E[w_{\cdot}|N] - sN; \\
E[\underline{W}_{\cdot}] &= E[E[w_{\cdot}|N]] - sE[N] \\
&= E[w_{\cdot}|w_{\cdot} > \underline{W}_{\cdot}] - \frac{s}{Pr[w_{\cdot} > \underline{W}_{\cdot}]}.
\end{aligned}$$

Therefore, $\underline{W}_{\cdot}$ satisfies:

$$s = \int_{\underline{W}_{\cdot}}^{2\hat{y}} (w_{\cdot} - \underline{W}_{\cdot}) dH_{\cdot}(w_{\cdot}).$$

For all four configurations. These static reservation values can then be combines as per the main text to obtain the actual reservation values $\mathcal{W}$.

Proof of Proposition 3 We are now in a position to prove Proposition 3. To do so, it must be shown that for all configurations it is optimal to keep searching after finding $w_{\cdot} < \mathcal{W}_{\cdot}$ and stop otherwise. I consider configuration 4 first because it is the last possible configuration of all unopened compound boxes, then argue that the result implies the same outcome for the configurations that can turn into it. To ease notation, I drop the suffix.

Notice first that, for any A^H, B^H, it holds $\mathcal{W}(A^H, B^H) > \underline{W}(A^H, B^H)$: indeed, $\mathcal{W}(A^H, B^H)$ includes any update generated by realizations higher than A^H or B^H. Then, opening a compound box leads to one of three outcomes. First, if $w < \underline{W}(A^H, B^H)$, the consumer would keep searching. This follows from the definition of $\underline{W}(A^H, B^H)$ in the locked configuration setting. If $w > \mathcal{W}(A^H, B^H)$, the consumer would clearly stop at that compound box. It must be shown that the consumer should keep searching if $\underline{W}(A^H, B^H) < w < \mathcal{W}(A^H, B^H)$ is found.

Suppose the consumer is about to open compound box $X_{i,i}$ and found $\underline{W}(A^H, B^H) < w < \mathcal{W}(A^H, B^H)$. If this is true, it must be that either $A_i > A^H$, $B_i > B^H$, or both. Therefore, the next box will be defined by "locked" reservation value:

$$\underline{W}(\max\{A_i, A^H\}, \max\{B_i, B^H\}) > \underline{W}(A^H, B^H),$$

and dynamic reservation value

$$\mathcal{W}(\max\{A_i, A^H\}, \max\{B_i, B^H\}) > \mathcal{W}(A^H, B^H).$$

If the consumer were to stop searching, he would do so solving a dynamic programming problem with an unchanging value function, namely one that assumes all future boxes to have value $\underline{W}(\max\{A_i, A^H\}, \max\{B_i, B^H\})$. Instead, the continuation value of the search process should account for the possibility of opening the next box to update the value of all unopened boxes to $\underline{W}(\max\{A_i, A_{i+1}, A^H\}, \max\{B_i, B_{i+1}, B^H\})$. But this is exactly the definition of $\mathcal{W}(\max\{A_i, A^H\}, \max\{B_i, B^H\})$. Therefore, the consumer should only stop after finding $w_{i,i} > \mathcal{W}(\max\{A_i, A^H\}, \max\{B_i, B^H\})$.

The same logic can be applied backwards: configurations 2 and 3 both have a dynamic reservation value higher than their "locked" reservation value. Moreover, the dynamic reservation value includes the probability that opening $X_{i,i}$ changes the configuration of unopened boxes from 2 or 3 to 4 and the value of searching on boxes with that configuration from that point onward. Therefore, the dynamic reservation value correctly captures the continuation value of the search process: $\mathcal{W}_a$ and $\mathcal{W}_b$ are the appropriate reservation values governing search. The same applies to configuration 1 as well.

Once a compound box is selected, optimal search inside of it (no matter the configuration therein) follows standard myopic search logic: a nested box is inspected if its reservation value $r_{i,j}$ or $r_{j,i}$ is higher than the highest realized payoff $u_{i,i}$, $u_{i,j}$, or $u_{j,i}$. Because it holds:

$$r_{i,j} = \begin{cases} A_i + B_j - s & \text{if} \quad j < i \\ A_i + z & \text{if} \quad j > i \end{cases}$$

$$r_{j,i} = \begin{cases} A_j + B_i - s & \text{if} \quad j < i \\ B_i + z & \text{if} \quad j > i \end{cases}$$

if the highest $A_{j<i} > z + s$ (resp. $B_{j<i} > z + s$), the consumer would backtrack to a product known to beat all closed nested boxes both known and unknown in their realization. Otherwise, unknown nested boxes would be opened until a second attribute is found to beat the reservation value of all subsequent ones.

To finally prove Proposition 3, it must be shown that when a compound box is optimally kept, it is never abandoned. Suppose $X_{i,i}$ is kept and, in particular, A_i is kept. If B_i is optimally kept as well, the search process stops. Suppose instead $B_i < z$ so that the consumer inspects nested boxes $(i, j > i)$. Suppose $B_{i+1} > z$ is found. It must be shown that the consumer does not want to open any other box. If some $\hat{A}_{j<i} > z + s$ was found, this follows from the fact that if $\hat{A}_{j<i} + z$ was better than the next closed compound box, it would have been kept. The consumer optimally keeping A_i implies that $A_i + z > \underline{W}_a > \hat{A}j < z + z$. Therefore, it must only be shown that $A_i + B_{i+1} > z + B_{i+1}$, or that $A_i > z$ is necessary for A_i to be kept.

Suppose by contradiction that was not the case. Then, since $A_i \leq z$ was kept but $B_i < z$ was not, it must hold that $2z \geq \mathcal{W}$. Notice however that it holds:

$$2s = \int_{\tilde{z}}^{2\hat{y}} (2y - \tilde{z}) dF(y) \quad \iff \quad \tilde{z} = 2z.$$

Since $2z$ is the certain equivalent of spending two times the search cost to obtain two times the reward associated with one attribute, $2z < \underline{W} < \mathcal{W}$ because $\underline{W}$ is the certain equivalent of discovering the realization of two i.i.d. attributes for one search cost. This contradicts $2z \geq \mathcal{W}$, which proves the result.