Layer-skipping connections facilitate training of layered networks using equilibrium propagation.

Jimmy Gammell   Sae Woo Nam   Adam N. McCaughan

July 28, 2020

Motivation: appeal of equilibrium propagation

- Equilibrium propagation:[1] a biologically-motivated learning framework

[1] Benjamin Scellier and Yoshua Bengio. *Equilibrium Propagation: Bridging the Gap Between Energy-Based Models and Backpropagation*. 2016. arXiv: 1602.05179 [cs.LG].

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  3. Seems unlikely in biological systems

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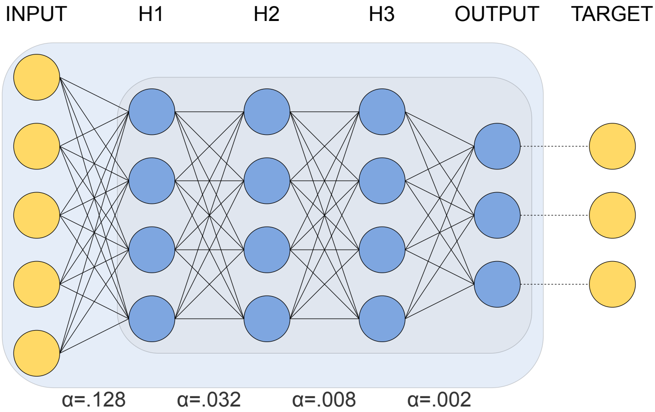
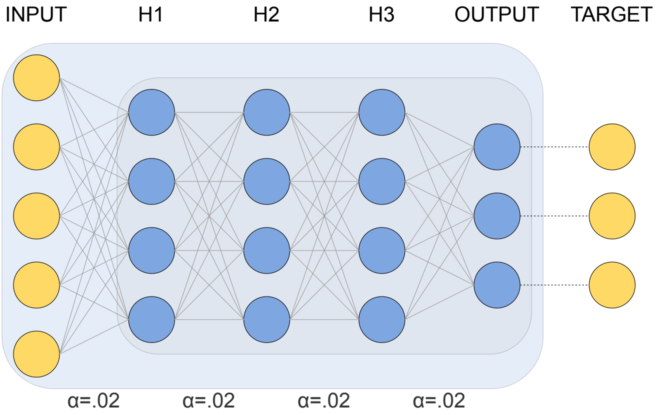
- Improves network training speed
- Increases uniformity with which each layer trains
- Performance similar to that of per-layer rates

Original topology



- From original paper
- Per-layer learning rates

Our topological modifications



- Starting point: original topology
- One learning rate for all layers

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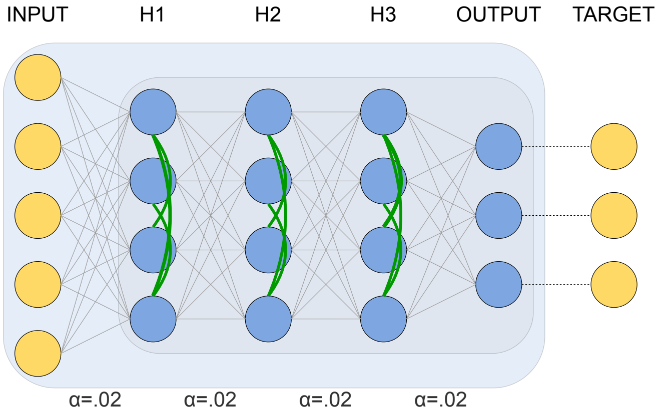

- Hidden layers fully-connected

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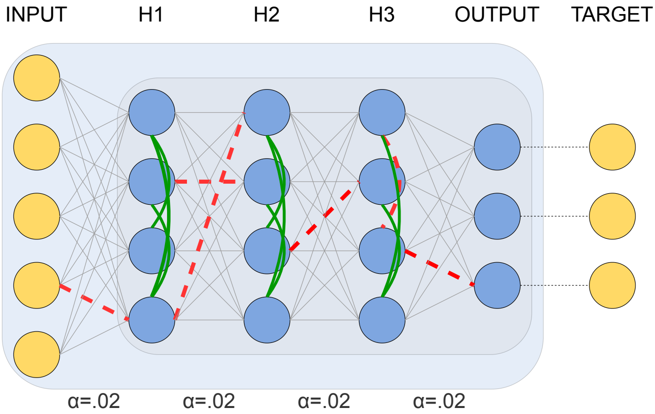

- Consider each connection
- Remove with probability p

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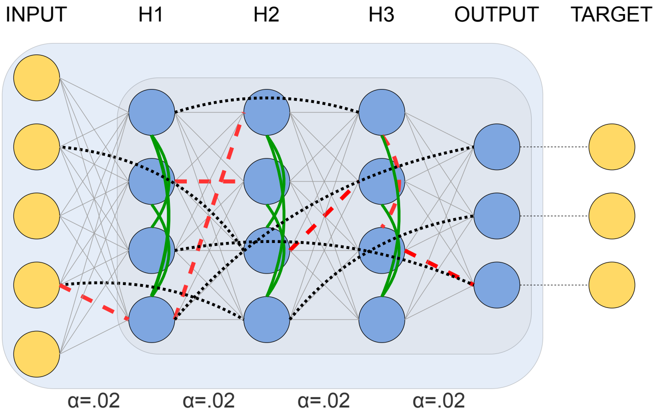
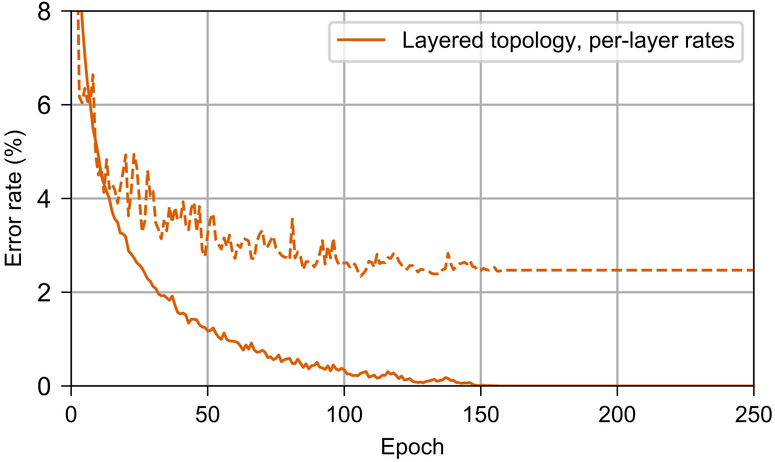


- For each removed connection, randomly connect different pair
- No connections in input or output layers

Results: performance of network with layer-skipping connections



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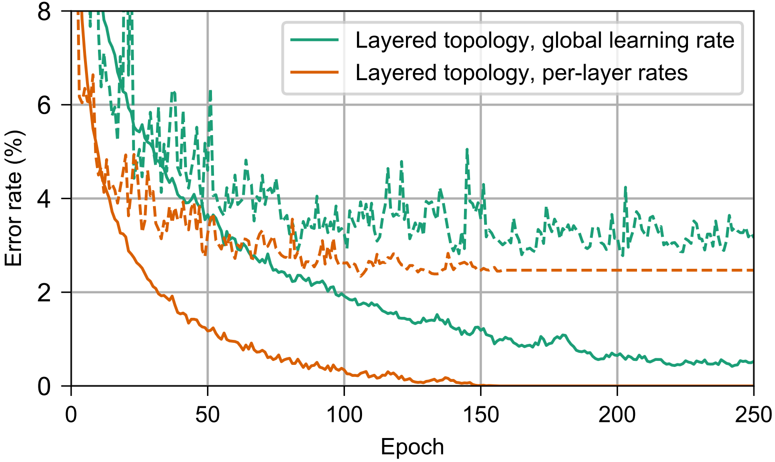

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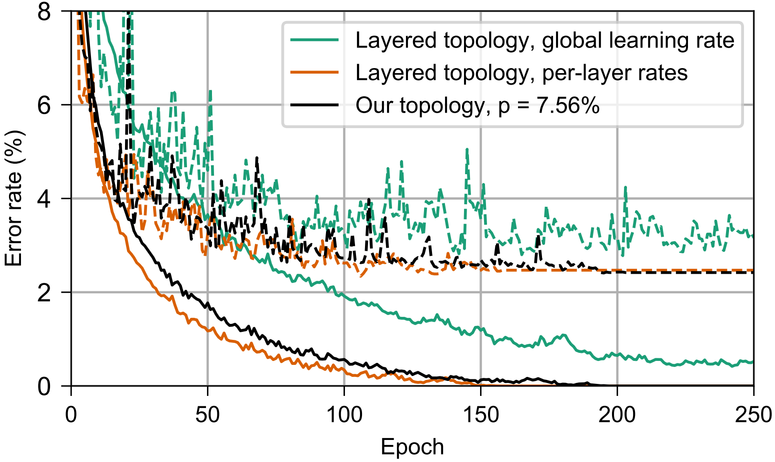

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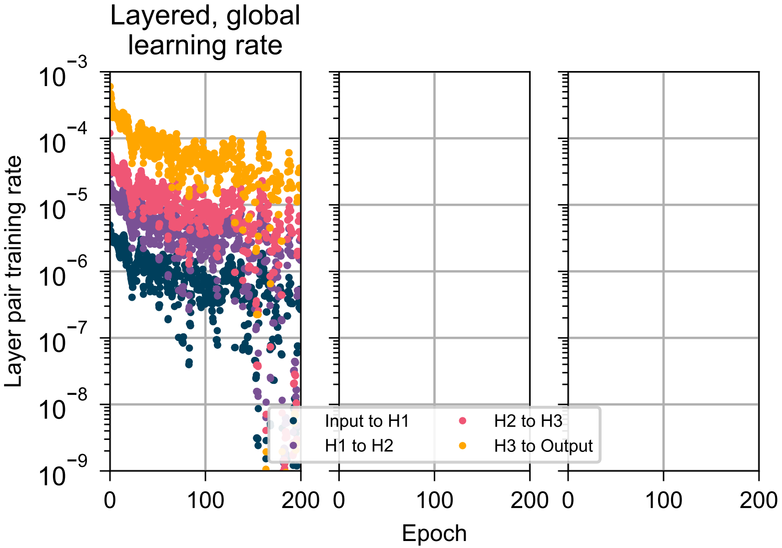

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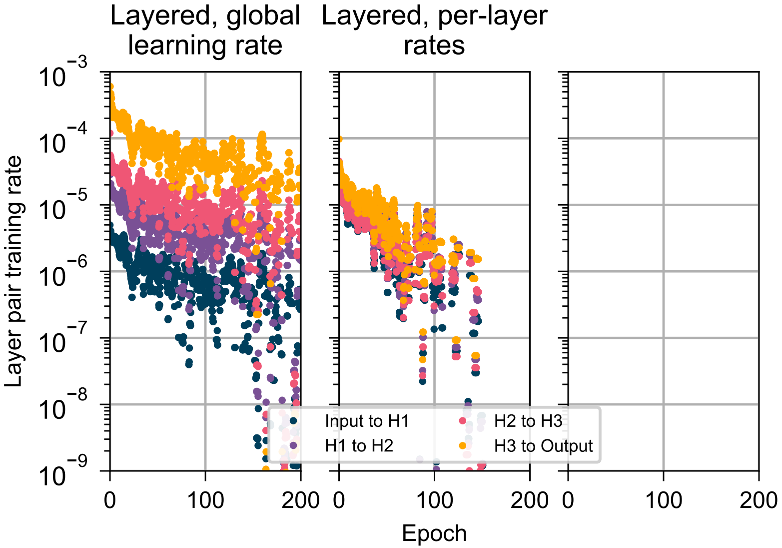

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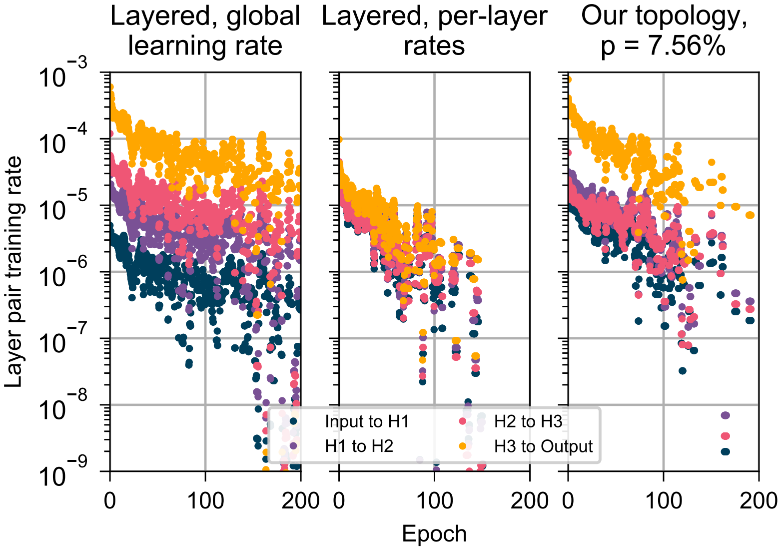

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- Good solution where simplicity, biological plausibility are important

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- Try training a network with added layer-skipping connections, then removing them afterwards