Layer-skipping connections facilitate training of layered networks using equilibrium propagation.

Jimmy Gammell   Sae Woo Nam   Adam N. McCaughan

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  - Neurons and connections perform few distinct tasks

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  - Like in Hopfield network

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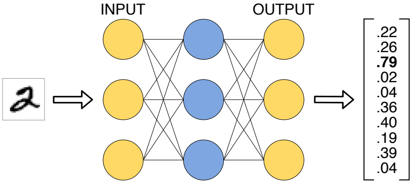

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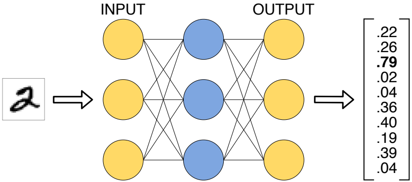

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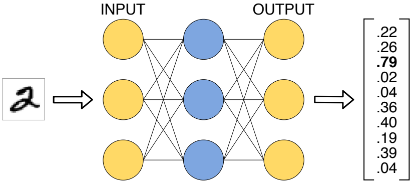

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  - Prediction: output activations at equilibrium

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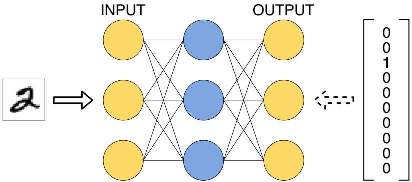

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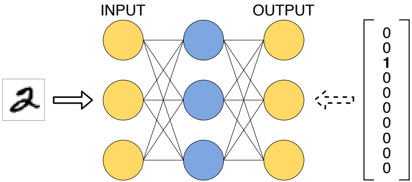

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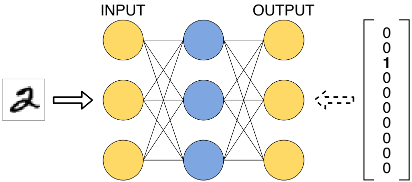

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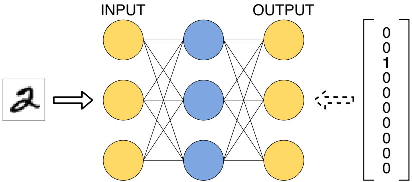

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- Differences between equilibrium states can be used to compute gradient

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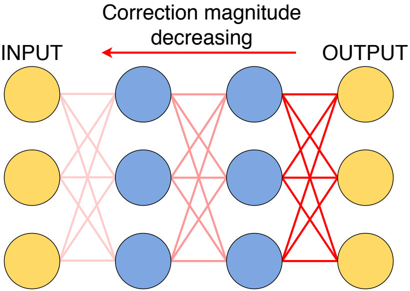
  - Same connections usable in both phases of training

Background: vanishing gradient problem



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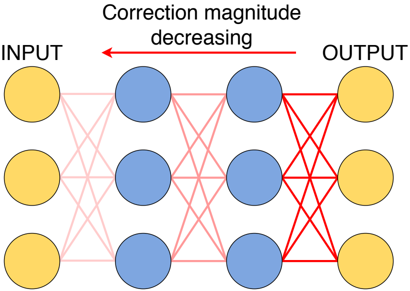

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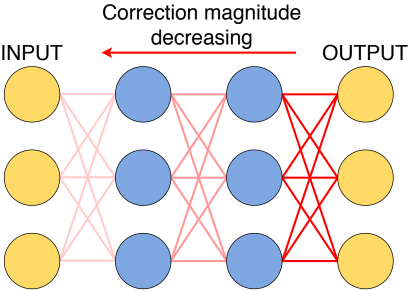

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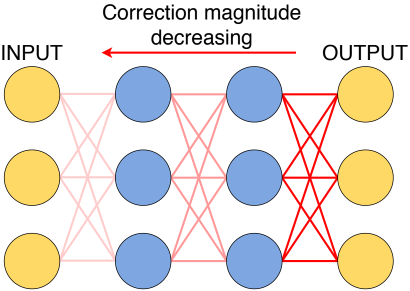

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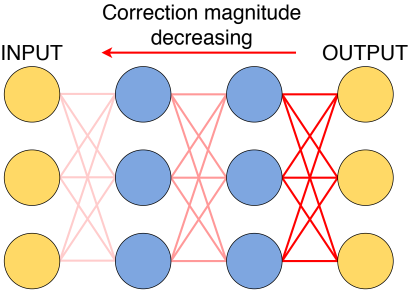

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- Need to solve - deep networks better than shallow networks
- Solved for digital computers, but no simple solution for analog implementations

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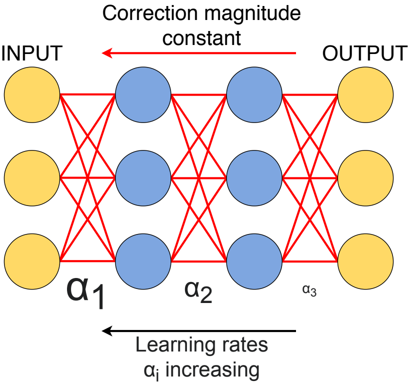

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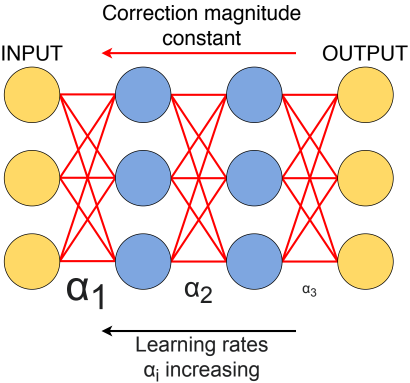

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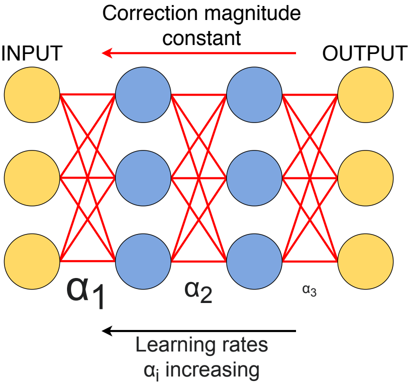

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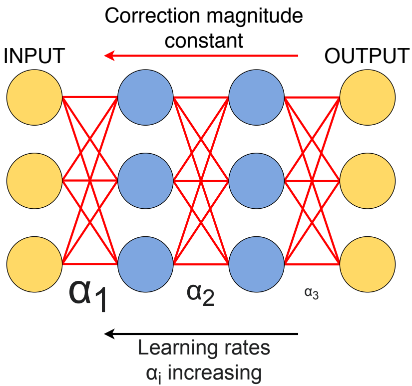

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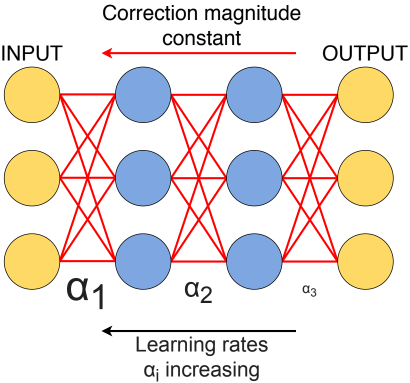

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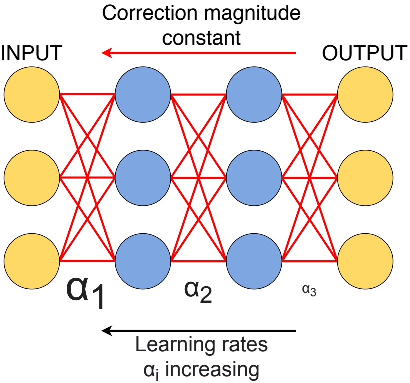

- Original paper: independent learning rates for each layer
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  3. Seems unlikely in biological systems

Our solution: layer-skipping connections

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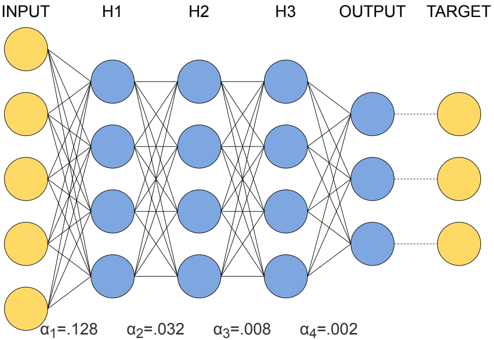
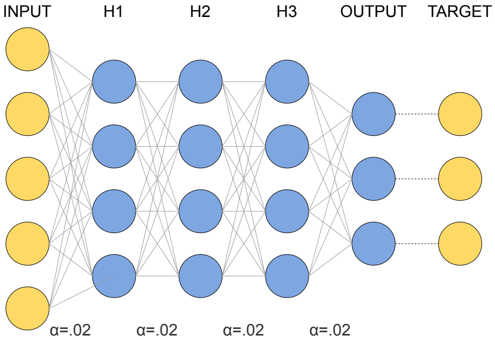
- Topology inspired by small-world networks

Original layered topology



- From original paper
- Per-layer learning rates

Our topological modifications



- Starting point: original topology
- One learning rate for all layers

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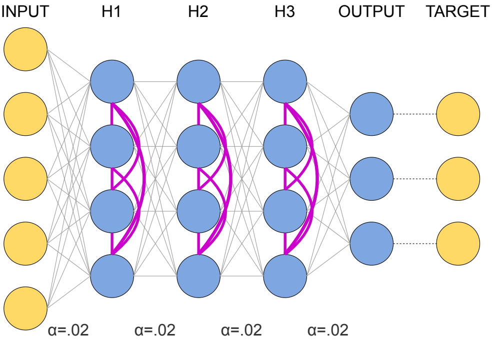

- Hidden layers fully connected

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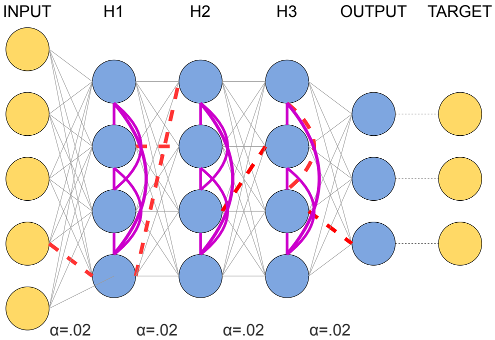

- Consider each connection
- Remove with probability p

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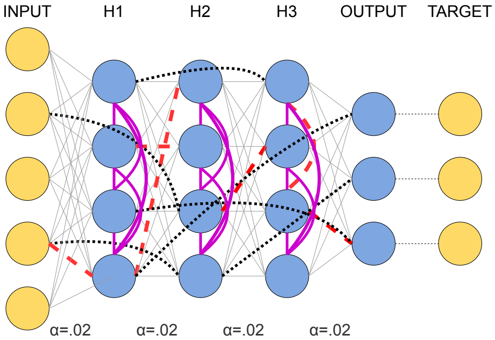

- For each removed connection, randomly connect a different pair
- No connections within input or output layers

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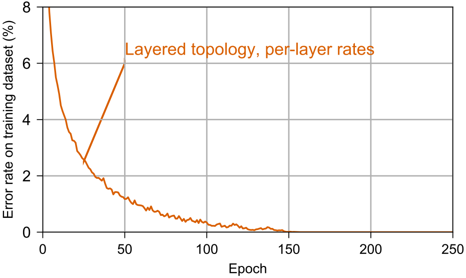
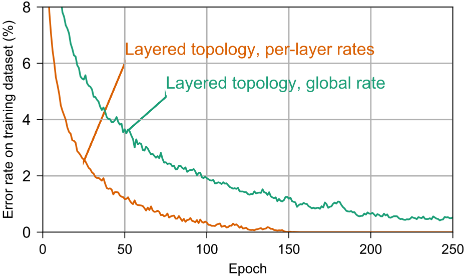
- Mitigates vanishing gradient problem

Results: training error of layered network with per-layer learning rates



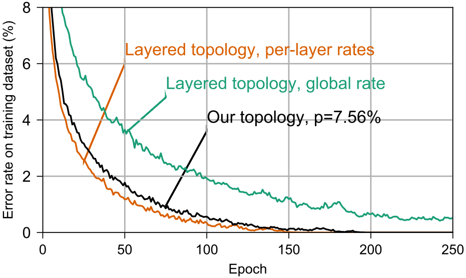
- Network with per-layer rates
- Evaluated on MNIST

Results: training error of layered network with single global learning rate



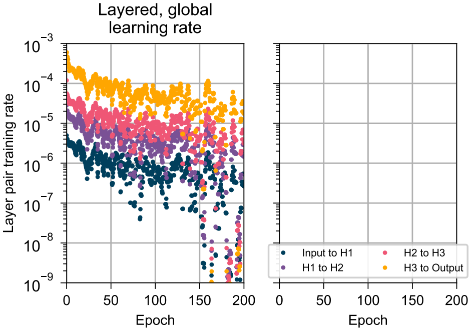
- Network with one global learning rate
- Training slows down

Results: training error of network with our topology



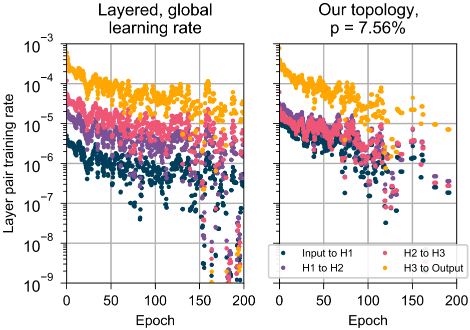
- Network with our topology (still one global learning rate)
- Trains significantly faster than layered network
- Performance similar to original network

Results: vanishing gradient in layered network with single global learning rate



- Vanishing gradient problem when one learning rate is used

Results: vanishing gradient in layered network with our topology



- Our topology mitigates vanishing gradient problem
- Shallowest weights train faster due to lack of layer-skipping connections to target

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- Good solution where simplicity, biological plausibility important

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- Effect of p on network performance
- Effectiveness on deeper networks
- Try training a network with added layer-skipping connections, then removing them afterwards

Acknowledgments

- Adam N. McCaughan
- Sae Woo Nam
- Sonia Buckley
- Alex Tait
- Zach Grey

Thanks!

- Questions? Email me at `jimmy.i.gammell@gmail.com`