Layer-skipping connections facilitate training of layered networks using equilibrium propagation.

Jimmy Gammell   Sae Woo Nam   Adam N. McCaughan

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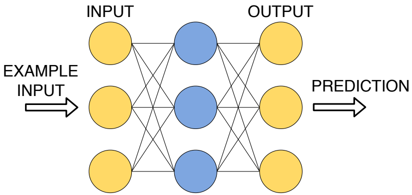
- Want learning framework requiring simple hardware
  - Neurons and connections perform few distinct tasks

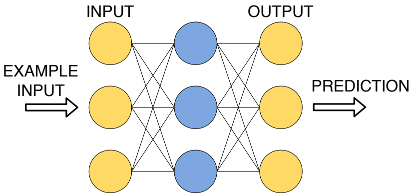
Background: equilibrium propagation



- Equilibrium propagation: a biologically motivated learning framework

Scellier and Bengio, Equilibrium Propagation: Bridging the Gap Between Energy-Based Models and Backpropagation

Background: equilibrium propagation



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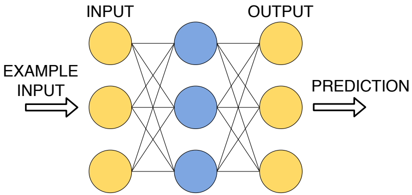

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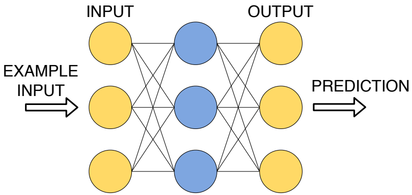

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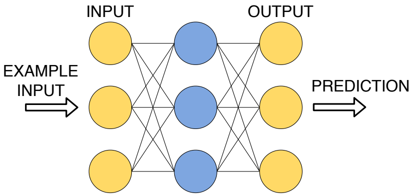

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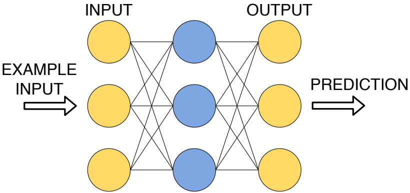

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  - Prediction: output activations at equilibrium

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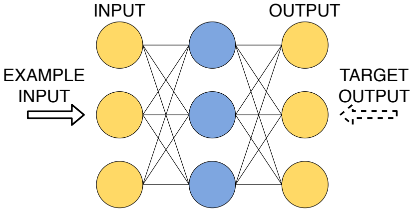

- Second phase of training: weakly-clamped phase

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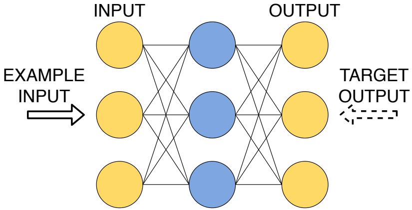

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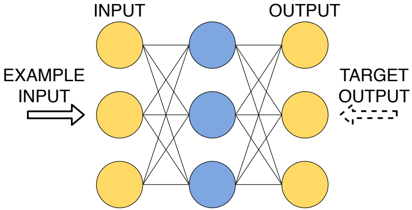

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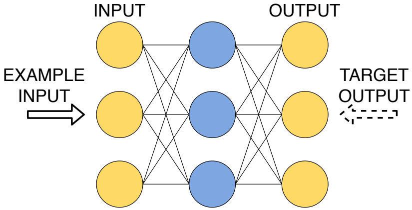

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- Differences between equilibrium states can be used to compute gradient

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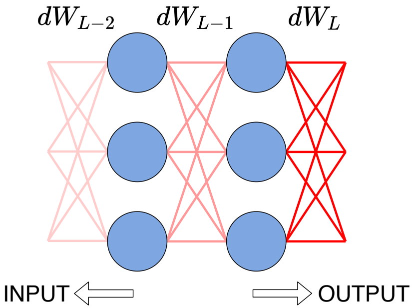
  - Biologically-plausible (relative to backpropagation)
  - Implementable in neuromorphic analog hardware

Background: vanishing gradient problem



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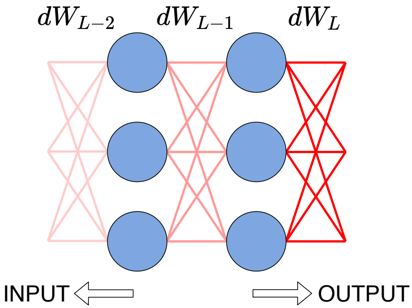

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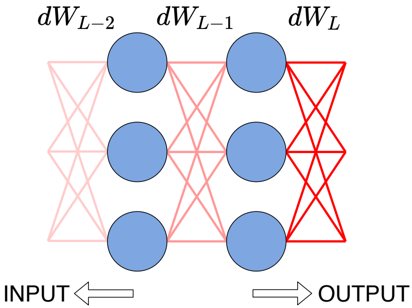

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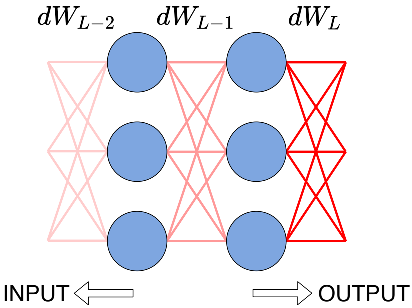

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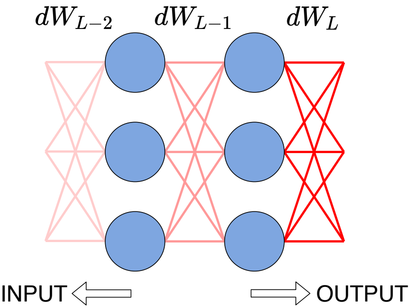

- Problem: vanishing gradients in layered networks
  - Slow training
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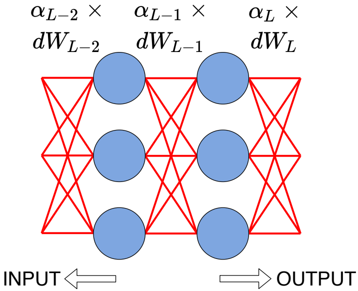
- Need to solve - deep networks better than shallow networks
- Not yet solved in simple, biologically-plausible manner

Background: vanishing gradient problem



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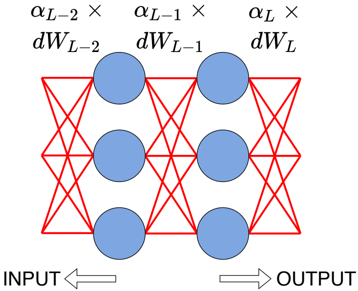

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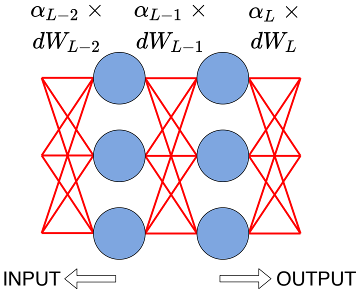

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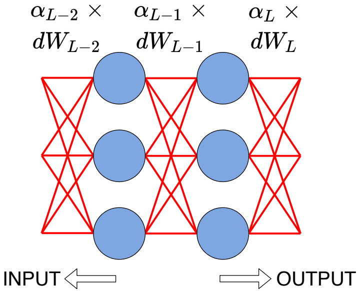

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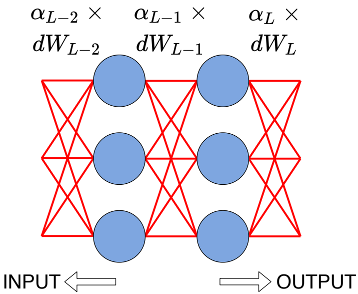

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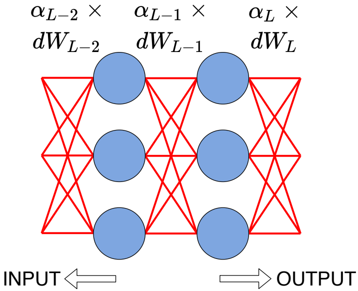

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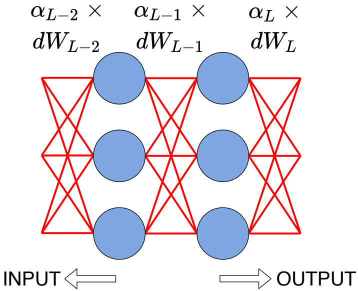

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- Problem can be mitigated by instead using topological modification based on layer-skipping connections

Our solution: layer-skipping connections

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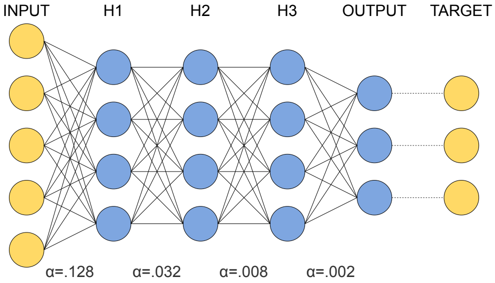
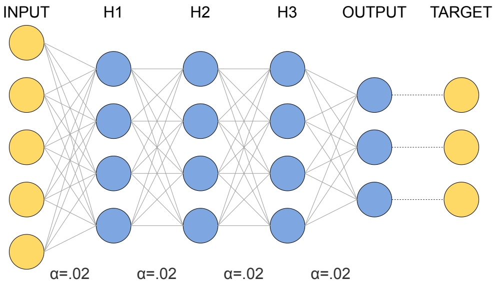
- Topology inspired by small-world networks

Original layered topology



- From original paper
- Per-layer learning rates

Our topological modifications



- Starting point: original topology
- One learning rate for all layers

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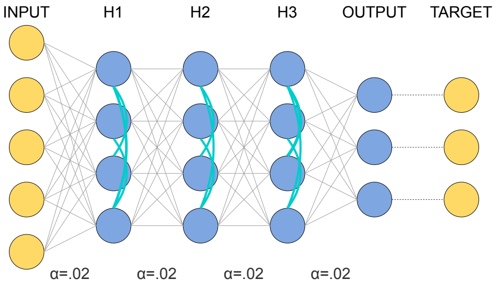

- Hidden layers fully connected

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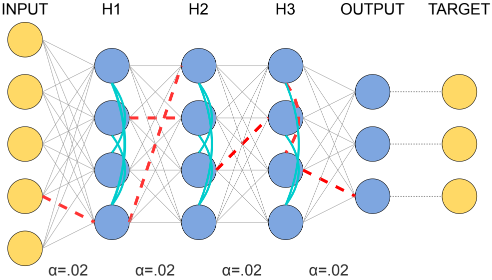

- Consider each connection
- Remove with probability p

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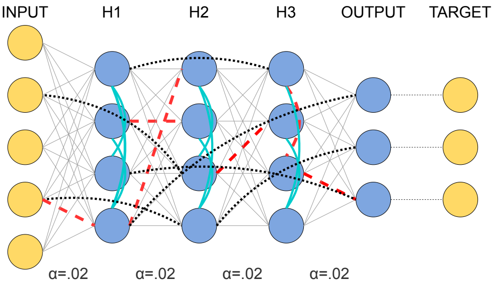
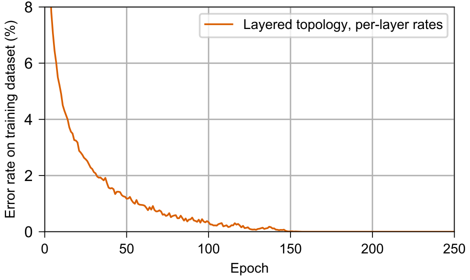


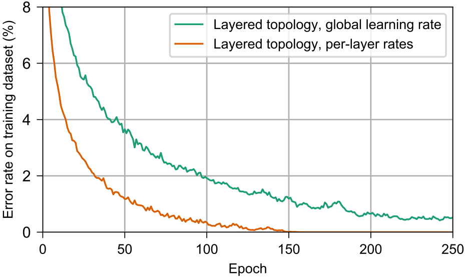
- For each removed connection, randomly connect a different pair
- No connections within input or output layers

Results: training error of layered network with per-layer learning rates



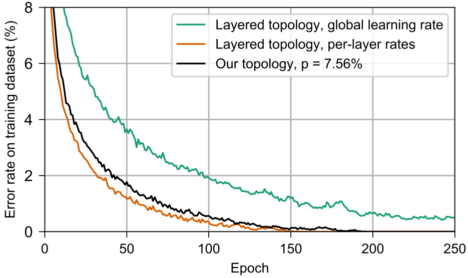
- Baseline performance: network from original paper
- Per-layer learning rates

Results: training error of layered network with single global learning rate



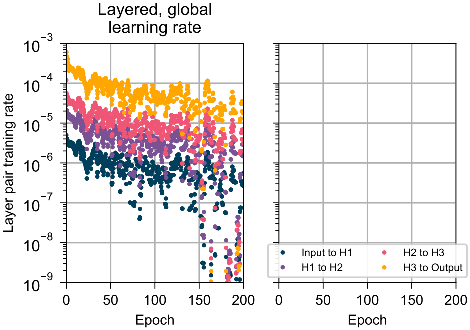
- Network with one global learning rate
- Training slows down

Results: training error of network with our topology



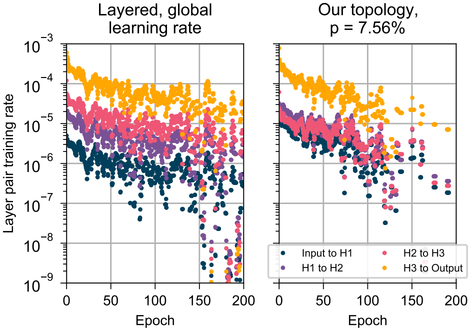
- Network with our topology (still one global learning rate)
- Trains significantly faster than layered network
- Performance similar to original network

Results: vanishing gradient in layered network with single global learning rate



- Vanishing gradient problem when one learning rate is used

Results: vanishing gradient in layered network with our topology



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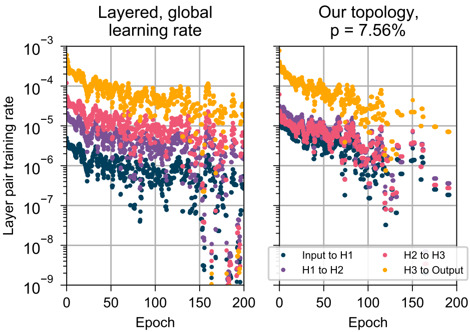
Results: vanishing gradient in layered network with our topology



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- Shallowest weights train faster due to lack of layer-skipping connections to target

Conclusions

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  1. Only two new hyperparameters; constant with depth
  2. Small-world networks have been observed in biological brains
  3. Easy to implement in networks with configurable connectivity
- Good solution where simplicity, biological plausibility important

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- Effect of p on test error
- Effectiveness on deeper networks
- Try training a network with added layer-skipping connections, then removing them afterwards

Acknowledgments

- Sonia Buckley
- Zach Grey
- Adam N. McCaughan
- Sae Woo Nam
- Alex Tait