Chapter 21

Fluids

Fluids yield their form yet never break apart,
They flow where force persuades, a patient art,
From silent depths to currents swift and wide,
Law shapes the paths through which their motions glide

Fluid mechanics studies the behavior of substances that can flow, namely liquids and gases. Unlike solids, fluids deform continuously under the action of applied forces. This chapter develops the basic concepts of fluid statics and fluid dynamics with emphasis on ideal fluid flow.

21.1 Properties of Fluids

A fluid is a substance that offers no permanent resistance to shear stress.

21.1.1 Density and Pressure

The mass density ρ of a fluid is defined as mass per unit volume

$$\rho = \frac{m}{V}$$

Pressure is defined as the normal force per unit area exerted by the fluid

$$p = \frac{F}{A}$$

Pressure at a point in a fluid at rest acts equally in all directions.

21.1.2 Variation of Pressure with Depth

Consider a fluid of uniform density ρ under gravity. The pressure at depth h below the surface is given by

$$p = p_0 + \rho g h$$

where p_0 is the pressure at the surface.

21.2 Fluid Statics

Fluid statics deals with fluids at rest.

21.2.1 Pascal's Law

Pascal's law states that an external pressure applied to a confined fluid is transmitted undiminished throughout the fluid.

If pressures p_1 and p_2 act on areas A_1 and A_2 respectively

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This principle is used in hydraulic systems.

21.2.2 Buoyancy and Archimedes' Principle

A body immersed in a fluid experiences an upward buoyant force equal to the weight of the displaced fluid

$$F_b = \rho V g$$

where V is the volume of fluid displaced.

21.3 Fluid Flow

Fluid dynamics concerns fluids in motion.

21.3.1 Types of Flow

Fluid flow may be classified as

- Steady or unsteady
- Laminar or turbulent
- Compressible or incompressible

In this chapter, attention is restricted to steady, incompressible flow.

21.3.2 Streamlines

A streamline is a curve whose tangent at any point is in the direction of the fluid velocity.

The velocity field of the fluid is described by

$$\mathbf{v} = \mathbf{v}(x, y, z, t)$$

In steady flow, the velocity field is time-independent.

21.4 Equation of Continuity

The equation of continuity expresses conservation of mass in fluid flow.

For steady incompressible flow through a tube of varying cross-sectional area

$$Av = \text{constant}$$

For two sections of the flow

$$A_1 v_1 = A_2 v_2$$

21.5 Bernoulli's Equation

Bernoulli's equation relates pressure, velocity, and height along a streamline for an ideal fluid.

21.5.1 Derivation

For steady incompressible flow without viscosity, energy conservation yields

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

This equation applies along a single streamline.

21.5.2 Applications of Bernoulli's Equation

Between two points in a flowing fluid

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Bernoulli's principle explains phenomena such as lift, venturi flow, and atomizers.

21.6 Viscosity and Real Fluids

Real fluids experience internal resistance to flow known as viscosity.

21.6.1 Coefficient of Viscosity

For laminar flow between parallel layers, the shear stress τ is proportional to the velocity gradient

$$\tau = \eta \frac{dv}{dy}$$

where η is the coefficient of viscosity.

21.6.2 Poiseuille's Law

For laminar flow of a viscous fluid through a cylindrical tube of radius r and length l, the volume flow rate is

$$Q = \frac{\pi r^4}{8\eta l}\Delta p$$

where Δp is the pressure difference between the ends.

21.7 Reynolds Number

The nature of fluid flow is characterized by the Reynolds number

$$\mathrm{Re} = \frac{\rho v D}{\eta}$$

where D is a characteristic length scale.

Low Reynolds numbers correspond to laminar flow, while high values indicate turbulent flow.

21.8 Closing Remarks

Fluid statics explains pressure and buoyancy in stationary fluids. Fluid dynamics describes the motion of fluids through conservation laws and energy principles. These ideas form the foundation for advanced studies in aerodynamics, hydrodynamics, and applied fluid mechanics.