

CHAPTER 25

QUANTUM MECHANICS

*States evolve where chance and law entwine,
Waves of thought replace the classical line,
Measured truths collapse from what may be,
In silence ruled by probability*

Quantum mechanics provides a fundamental description of physical systems at atomic and subatomic scales. Unlike classical physics, it is inherently probabilistic and requires a wave-based formulation of matter and energy.

25.1 FAILURE OF CLASSICAL PHYSICS

Classical mechanics fails to explain phenomena such as blackbody radiation, photoelectric effect, and atomic stability. These discrepancies motivated the development of quantum theory.

25.2 WAVE-PARTICLE DUALITY

Quantum mechanics is founded on the dual nature of matter and radiation.

25.2.1 DE BROGLIE HYPOTHESIS

Louis de Broglie proposed that particles possess wave-like properties.

The wavelength associated with a particle of momentum p is

$$\lambda = \frac{h}{p}$$

where h is Planck's constant.

25.3 WAVE FUNCTION AND PROBABILITY INTERPRETATION

25.3.1 WAVE FUNCTION

The state of a quantum system is described by a complex-valued wave function $\psi(x, t)$.

The wave function itself has no direct physical meaning.

25.3.2 BORN INTERPRETATION

The probability density of finding a particle at position x at time t is given by

$$P(x, t) = |\psi(x, t)|^2$$

The wave function must satisfy the normalization condition

$$\int |\psi(x, t)|^2 dx = 1$$

25.4 OPERATORS AND OBSERVABLES

Physical observables are represented by linear operators acting on the wave function.

25.4.1 POSITION AND MOMENTUM OPERATORS

The position operator is

$$\hat{x} = x$$

The momentum operator in one dimension is

$$\hat{p} = -i\hbar \frac{d}{dx}$$

where $\hbar = \frac{h}{2\pi}$.

25.4.2 EXPECTATION VALUES

The expectation value of an observable represented by operator \hat{A} is

$$\langle A \rangle = \int \psi^* \hat{A} \psi dx$$

25.5 THE SCHRÖDINGER EQUATION

25.5.1 TIME-DEPENDENT SCHRÖDINGER EQUATION

The evolution of the wave function is governed by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x, t) \psi$$

where $V(x, t)$ is the potential energy.

25.5.2 TIME-INDEPENDENT SCHRÖDINGER EQUATION

For a time-independent potential, separation of variables leads to

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x) \psi = E \psi$$

where E is the energy eigenvalue.

25.6 QUANTIZATION AND ENERGY EIGENVALUES

Solutions of the Schrödinger equation exist only for discrete values of energy.

25.6.1 PARTICLE IN A ONE-DIMENSIONAL BOX

For a particle confined to a box of length L , the allowed energy levels are

$$E_n = \frac{n^2 h^2}{8mL^2}$$

where n is a positive integer.

25.7 UNCERTAINTY PRINCIPLE

The Heisenberg uncertainty principle imposes a fundamental limit on simultaneous measurement of position and momentum

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

This limitation is intrinsic to nature and not due to experimental imperfections.

25.8 COMMUTATION RELATIONS

The non-commutativity of operators underlies quantum uncertainty.

For position and momentum

$$[\hat{x}, \hat{p}] = i\hbar$$

25.9 MEASUREMENT AND WAVE FUNCTION COLLAPSE

Upon measurement, the wave function collapses into an eigenstate of the measured observable.

The probability of obtaining a particular eigenvalue is determined by the projection of the wave function onto the corresponding eigenstate.

25.10 CORRESPONDENCE PRINCIPLE

In the limit of large quantum numbers, quantum predictions approach classical results, ensuring consistency with classical physics.

25.11 CLOSING REMARKS

Quantum mechanics replaces deterministic trajectories with probabilistic descriptions governed by wave functions and operators. Its principles form the foundation of modern physics, chemistry, and emerging quantum technologies.