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# ASSIGNMENT 2

## Problem 1

- Camera located at $(0,1,0)$, looking at the point $(1,0,0)$
- View up vector $(1,1,0)$

1) Vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ of camera coordinate frame

$$e = (0,1,0)$$
$$\vec{g} = (1,0,0) - (0,1,0) = (1,-1,0)$$
$$\vec{t} = (1,1,0)$$

We have camera space coordinate frame:

- origin $e = (0,1,0)$
- $\vec{w} = \frac{-\vec{g}}{\|\vec{g}\|} = \frac{(-1,1,0)}{\sqrt{2}} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$
- $\vec{v} = \frac{\vec{t}}{\|\vec{t}\|} = \frac{(1,1,0)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$
- $\vec{u} = \vec{v} \times \vec{w} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} \times \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = (0\ 0\ 1)$

2) 4x4 homogenous transformation matrix converts coordinates in camera coordinate frame to coordinates in the world frame?

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} & e \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) 4x4 homogeneous transformation matrix that converts coordinates in the world frame to coordinates in the camera coordinate frame

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} & e \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \vec{u} & -\vec{u}\vec{e} \\ \vec{v} & -\vec{v}\vec{e} \\ \vec{w} & -\vec{w}\vec{e} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Tilt the camera up by rotating it by angle $\theta$ in counterclockwise direction about the axis passing along $u$ axis in the camera frame.

$$M' = R_{\vec{u}}(-\theta).M.$$

$$= R_x(-\theta).M \quad (\vec{u} \text{ is the } x \text{ axis in camera frame})$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} M$$