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Small Worlds Among Interlocking Directors: Network Structure and Distance in Bipartite Graphs

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Abstract

We describe a methodology to examine bipartite relational data structures as exemplified in networks of corporate interlocking. These structures can be represented as bipartite graphs of directors and companies, but direct comparison of empirical datasets is often problematic because graphs have different numbers of nodes and different densities. We compare empirical bipartite graphs to simulated random graph distributions conditional on constraints implicit in the observed datasets. We examine bipartite graphs directly, rather than simply converting them to two 1-mode graphs, allowing investigation of bipartite statistics important to connection redundancy and bipartite connectivity. We introduce a new bipartite clustering coefficient that measures tendencies for localized bipartite cycles. This coefficient can be interpreted as an indicator of inter-company and inter-director closeness; but high levels of bipartite clustering have a cost for long range connectivity. We also investigate degree distributions, path lengths, and counts of localized subgraphs. Using this new approach, we compare global structural properties of US and Australian interlocking company directors. By comparing observed statistics against those from the simulations, we assess how the observed graphs are structured, and make comparisons between them relative to the simulated graph distributions. We conclude that the two networks share many similarities and some differences. Notably, both structures tend to be influenced by the clustering of directors on boards, more than by the accumulation of board seats by individual directors; that shared multiple board memberships (multiple interlocks) are an important feature of both infrastructures, detracting from global connectivity (but more so in the Australian case); and that company structural power may be relatively more diffuse in the US structure than in Australia.

Keywords: interlocking directors, bipartite graphs, small world, global network structure

Small world investigations are the most common of recent studies investigating the global features of network structures. Small world research (Kochen, 1989; Watts, 1999a, 1999b; Watts and Strogatz, 1998) permits an understanding of how network connections can traverse large social distances, such as along paths of acquaintances across a large population. Milgram's (1967) innovative study suggested that the US population might be connected at "six degrees of separation", with the phrase now common currency (e.g., Guare, 1990). Studies of connectivity among webpages across the rapidly burgeoning World Wide Web also demonstrate "small worldness", with the average number of links between any two pages surprisingly small (Albert et al., 1999).

In this paper we examine small world and other global properties of affiliation networks. Affiliation networks involve a bipartite graph with two types of nodes. We simulate from a uniform conditional random bipartite graph distribution to investigate global network features such as the short path lengths and high clustering characteristic of small world networks. We then compare the observed affiliation networks of corporate boards and directors in the US and Australia in 1996 to these conditional random distributions.

Our direct examination of the bipartite graph is unusual. Typically, small-world studies of social networks have examined *networks* involving only one type of social entity (e.g. individuals or webpages). These social networks can be represented as a 1-*mode* graph, where the term 1-*mode* refers to the fact that there is only one type of social entity or *actor*. In the graph, actors are represented by *nodes* with *edges* representing the presence of a *social tie* between actors. A *path* in a network, then, is a connected sequence of ties, and small world properties include short average path lengths.

Networks of interlocking directors are another, more complex type of network, *affiliation networks*. Affiliation networks involve at least two distinct types of social entity: commonly, individuals and the groups of which they might be members. Interlocking corporate boards are typical affiliation networks with clearly defined memberships and a sprinkling of joint position holders (interlocking directors) who link boards. An affiliation network can be represented as a 2-*mode* or *bipartite graph*, with two types of nodes, and edges only possible between nodes of different types. In this article, the two types of nodes refer to individuals (directors) and groups (company boards), with an edge indicating that this director sits on this board.

Bipartite structures have had limited attention in small world research, with perhaps the best known example being the “Kevin Bacon game”, where ties between famous movie actors are determined by co-appearances in the same movie (Watts, 1999a). Other work includes Newman’s (2001a, 2001b) detailed studies of the global properties of a scientific collaboration affiliation network. Important results are also summarized in Albert and Barabási (2002) and in Newman et al. (2001). The social network literature more generally has a long tradition of examining affiliation networks, both substantively and methodologically (e.g. Breiger, 1974; Davis et al., 1941; Galaskiewicz, 1985; Iacobucci and Wasserman, 1990; Skvoretz and Faust, 1999; Snijders and Stokman, 1987; Wasserman and Iacobucci, 1991), including studies of interlocking among boards of directors of large companies (e.g. Carroll and Alexander, 1999; Davis and Mizruchi, 1999; Mizruchi, 1996; Mintz and Schwartz, 1985; Scott, 1997; Stokman et al., 1985; Windolf, 2002). A small amount of work has been done on tripartite networks with three social entities (e.g. Fararo and Doriean, 1984).

In analyzing an affiliation network, one type of node may attract most interest, while the other may be somewhat neglected. From any affiliation network, two 1-mode graphs can be derived. For instance, the actor-to-movie affiliation network is typically condensed to the actor-to-actor 1-mode graph. A 1-mode focus makes good sense when theoretical attention is on one type of node. Most studies of interlocking directorates concentrate on the intercorporate network, with interest often in the hierarchy of corporate power structures (e.g., Mintz and Schwartz, 1985). In the example of scientific collaboration networks, the principal concern may lie in the collaborations rather than in co-authored papers (the collaboration network examined by Newman, 2001a, 2001b deals with both collaborations and papers). Borgatti and Everett (1997) noted that many bipartite data sets are collected with immediate intention of conversion to a 1-mode network, in which case there is no need to develop techniques specifically to address the bipartite data. Standard network approaches will suffice.

But without a theoretical imperative to concentrate on one of the two social entities, 1-mode analyses may not be appropriate. Analyses of 1-mode graphs necessarily force a focus (either totally or sequentially) on one or other type of node. Breiger (1974) argued that the *duality* of social entities in affiliation structures implied no reason to privilege one 1-mode network over the other (see also Breiger and Pattison, 1986). Nor is there any compulsion to decompose Breiger's dual structure into its 1-mode derivatives. As Borgatti and Everett (1997) further observed, certain bipartite data sets are collected with the explicit intention that both modes of the data remain relevant; and in that case we need techniques that can work with the 2-mode data directly. In line with this recommendation, as our interest in this article is to compare the global structures of the interlocking directorship networks of the US and Australia, we directly examine the bipartite graphs themselves rather than work solely with the derived 1-mode data.

Borgatti and Everett (1997) present several valuable methods to investigate affiliation networks, including vizualisation techniques and correspondence analysis. They describe how to adapt standard network centrality measures so as to investigate which nodes are important in the bipartite graph. They discuss a bipartite version of graph centralization—a measure of how much the graph is centralized around one node—and present extensions of network techniques to reveal cohesive subgroups among the nodes. Our focus is somewhat different. We are not interested in the identities of individual directors or companies, or how they might be categorized into meaningful subgroups. Rather, we wish to examine aspects of global structure of the graph in relation to certain local patterns of director/company linkages. Because small world considerations have sharpened understanding of what might count as useful global properties of networks, we use that focus for our investigation, although our interest is not restricted to small world considerations. Our analytic strategy—based as it is on comparisons of observed data against simulated graph distributions—is also different from that of Borgatti and Everett. Yet, as described below, there are some interesting intersections between our approach and the 2-mode analyses of Borgatti and Everett.

The argument in the rest of this paper is developed as follows. We begin by reviewing aspects of global network properties, in particular small world properties, for 1-mode networks. We then discuss features of interlocking directorate networks and propose several theoretical ideas specific to affiliation networks. These ideas include a specification of *social neighbourhoods* and *social settings* for networks of interlocking directorates and the generic definition of *network infrastructure* in affiliation networks. We describe our methodology, based on comparison of observed data against a simulated distribution of random graphs conditional on the known constraints of the observed data sets. We present results for two network infrastructures, derived from US and Australian corporate data for 1996. We conclude by discussing the common variations from the simulated graph distributions, and some differences, between the two observed data sets.

1. Global Structures, Local Processes, and 1-Mode Graphs

As Milgram (1967) noted, apparent network closeness in terms of short path lengths is not that remarkable in itself. For instance, if each individual were to have as few as ten acquaintances—and provided those acquaintances were distinct—six degrees of separation would imply connections from one person to a million others. (Even so, an individual's ability to *find* a path to a particular other among the million is surprising—Watts et al., 2002.) But of course the choices of acquaintances are never distinct, for in the human social world, individuals tend to *cluster* into clique-like structures. On a planetary scale, the average propensity for pairs of individuals to form a tie is very low, but network partners of any one individual are quite likely to do so. As Watts (1999a) pointed out, it is not immediately apparent that the three small-world properties—a low propensity to form ties relative to the entire network (i.e., low network *density*), a high tendency for clustering, and average short path, or *geodesic*, length[1]—can coexist in the one network. Through simulation studies, however, Watts (1999a) showed that a very structured network with high clustering but long mean geodesics might be transmuted into a short mean geodesic network with the addition of a relatively small number of random links. Watts demonstrated that this could be achieved without the formation of a "hub", a node with particularly high degree, which is one fairly obvious way to shorten path-lengths.

As a result of Watt's work, there has been an upsurge of interest in small-world and other global network properties, particularly in large-scale graphs such as the internet. Albert and Barabási (2002) and Strogatz (2001) provide excellent reviews. In our own recent work (Robins et al., in press), we noted that much of this renewed attention to small world issues has focussed on global properties of networks without much consideration of local social processes that might generate such networks.

The emergence of global system properties from purely localized interactions is a burgeoning field of scientific discussion that embraces network formation in its ambit (Buchanan, 2002). Human social networks may be largely, if not wholly, emergent from localized interactions (Boyd and Jonas, 2001), within *social neighborhoods* (Pattison and Robins, 2002). *Social neighborhoods* are sites of social interaction for small subsets of network actors, implicating the potential network ties among those actors. By postulating particular types of social neighborhood, a researcher in effect hypothesizes that the graph of the network is built up from an agglomeration of small localized subgraphs, corresponding to the neighborhood types. For instance, a tendency to clustering can emerge from a tendency to form triangles within neighborhoods of three actors.

Additionally, social neighbourhood processes may be structured and constrained by their *social setting structure* (Pattison and Robins, 2002; see also Feld, 1981; Mische and White, 1998; Mische and Robins, 2002; White, 1995). The *social setting* is a spatiotemporal or sociocultural space wherein people come together, making interaction possible. In other words, tie formation may emerge from neighbourhood-type processes, but these processes themselves may be constrained to occur in localized settings that provide a venue for the process. For instance, clustering may principally be observed within groups (*settings*) of strong friends, with little clustering between such groups (Granovetter, 1973). The substantive form of the *setting structure* may be quite loose and non-constraining, as in a local residential neighbourhood, or it may be highly circumscribed as in a work organisation.

Different forms of *social neighborhoods* and *setting structures* allow us to postulate different sets of localized conditions and behaviours that, when generalized across a network produce distinct global outcomes and network structures. Robins et al. (in press) showed that simulations of locally-specified Markov random graph models (Frank and Strauss, 1986) could result in small world networks, and indeed in other global "worlds", depending on the size of parameter values specified for relevant social neighborhood processes. Based on the results of these simulations, they suggested that small world properties could arise in 1-mode networks when (i) individuals sought multiple partners, (ii) but the costs to individuals of many ties progressively increased, establishing a tendency against having very many partners, (iii) partners tended to agree about other possible partners (i.e. clustering), (iv) but this clustering tendency was not too strong (else the network becomes too clique-like). The point is not that these simple local processes are necessarily the basis of small world social networks (although such processes seem plausible), but that it is possible to generate small world networks, and other types of networks, based on localized specifications of possible human behaviors.[2]

Robins et al. (in press) used Markov random graphs to build models because they are locally specified. They are one class of what the network analytic literature terms *p* models* or *exponential random graph models* (Wasserman and Pattison, 1996; Wasserman and Robins, in press). This class of models assumes that the graph is an agglomeration of localized network subgraphs or *configurations*, with parameters specifying the tendency for the configurations to be observed. The configurations involve only a handful of possible edges, emphasizing their localized nature. Each can be construed as emergent from a neighbourhood process, as individuals engage in behaviors that result in localized relational structures in the form of the configuration. For instance, a clustering effect emerges if individuals tend to introduce their friends to each other, resulting in the formation of triangles of acquaintances. Markov random graph configurations include single edges, triangles and star-like structures where two or more edges are centred on the one node.[3] Such configurations are useful in examining clustering, because a natural clustering measure for a graph is $3 \times T/S_2$, where T is the number of triangles in the graph and S_2 the number of two-stars[4] (Newman, 2001a; Newman et al., 2001). In investigating bipartite graphs, we continue to use counterparts of 1-mode Markov graph configurations.

Under Watts's (1999a) definition, a small world combines the features of *short* average geodesics and *high* clustering. A small-world investigation then needs to decide on criteria that can define *short* and *high*. Graphs with edges observed independently of one another but with a fixed probability p across the graph (Erdös and Renyi, 1959) typically have short mean geodesics and low clustering, a feature of graphs with high levels of "randomness". A distribution of graphs with constant probability p is referred to as a Bernoulli graph distribution (Frank, 1981; Frank and Nowicki, 1993). These distributions then can provide criteria for *short average paths* and *high clustering*, with the definitions of *short* and *high* taken as relative to the Bernoulli distribution. Consistent with Watts (1999a), Robins et al. (in press) used these properties to examine graphs of given density, by simulating a Bernoulli graph distribution with mean number of edges corresponding to that density. From this distribution, they derived distributions of median geodesics and clustering coefficients. If the given graph had a median geodesic that was not extreme in the derived distribution, and a clustering coefficient that was extreme,[5] they argued that the observed graph had small-world features: short average geodesics and high clustering.

Our approach below in part follows that of Robins et al. (in press). The major difference is that in this article we have observed graphs about which we wish to make inferences, so we are not investigating a simulated distribution of Markov graphs. As well, there are constraints in the observed data, so rather than compare the observed graphs against Bernoulli graph distributions, we simulate conditional uniform random graph distributions that reflect these constraints, as described below. Where we follow Robins et al. is in using counts of relevant local configurations, geodesic distributions, degree distributions, and clustering coefficients as appropriate features of the observed graph, for comparison against a suitable graph distribution that is otherwise unstructured apart from the constraints in the data. Before we describe this approach in detail, we review issues relating to bipartite graphs and interlocking directorships.

2. Bipartite Graphs: Interlocking Directorships

To examine interlocking directorships, researchers collect the names of board members of a population of companies. They then compare names to identify where the same person is serving on multiple boards. This data can be represented as a bipartite graph with the companies as one set of nodes, the persons as a second set of nodes, and each affiliation (person-to-company) as an edge. Direct interpretation of this bipartite graph is difficult. Most commentators resort to discussion of the derived 1-mode graphs, the network of intercorporate connections or the network of interpersonal connections.

We propose a different reading of the bipartite graph. In the interpersonal graph actors are gathered into discrete, but overlapping groups. These groups are the corporate boards. The interpersonal graph thus has the structure of the intercorporate graph incorporated within it and retains information about the intercorporate graph. This understanding of the bipartite graph can then be related to the generic concepts of *social neighbourhood* and *social setting* proposed by Pattison and Robins (2002). Each board could be construed as a social neighbourhood. It is however, a very unusual one. We know exactly who is and is not a member. Its boundaries are clear, not fuzzy. Furthermore, all members know one another, so it always has a local density of 1.00, the maximum possible.

The powers and limitations of corporate boards illustrate the concept of *social setting*. Boards make the full range of decisions about the business operations of the company. They are also able to set out matters relating to their own constitution, membership and management. The activity of corporate boards can vary widely in different systems. For example, a basic aspect of the affiliation network of interlocking directors is the number of board seats available. If companies appoint large boards there are more people in the system and less constraints on the appointment of outside, interlocking directors. The setting structure of the US and Australia differ markedly here: the US companies in our dataset have an average board size of 13.3 seats, compared to an average of 8 for Australian companies. Boards also decide whether to appoint insiders to the company who have no outside board positions or outsiders with positions in other companies. They may also assess the worth of an appointment in terms of the number of outside positions the appointee holds. They may want an outside director with sufficient positions to bring experience and expertise to a board, yet they may avoid someone with too many positions, unable to give the time or interest to the board. Thus the *social neighbourhoods* of boards are visible and distinct, but we also see the external parameters of corporate governance and custom, which constitute the *social setting*, constraining activity within these social neighbourhoods.

Researchers of interlocking directorates are mostly motivated by specific hypotheses about a particular dataset. They seek to find structures in these networks that reflect and validate their understanding of the functions and motives of interlocking and its history within the national context they examine. US studies, for example, have made much of the centrality of commercial banks in the system of interlocks in that country and thus find their decreased prominence in recent times highly significant (Mintz and Schwartz, 1985; Davis and Mizruchi, 1999).

Wider theoretical debates in this area considered the possible opposition of two motives of interlocking. Inter-organizational analysis, best represented by resource dependency theory (Pennings, 1980), saw interlocking as a reflection of underlying structures of inter-corporate resource dependencies of which control relations were the extreme instance. These theorists looked at intercorporate ties as directional when the executive of one corporation sat on the board of another as a non-executive director. Class theorists presented an opposing view. They saw interlocking as a mechanism to facilitate communication and coordination among directors. Thus the network of interlocks provided an infrastructure of class organisation and laid the potential for consensus formation across the boardrooms of networked corporations (Useem, 1984). In this view, the particular pattern of inter-organizational links is not critical, just the volume of interlocking.

In the 1980s these theories were mediated by two very influential "replacement" studies (Palmer, 1983; Ornstein, 1982, 1984). These studies considered interlocks that had been accidentally broken by the unexpected retirement or death of a director and the number of times that the replacement director reconstituted the inter-corporate interlocks of their predecessor. The results of both studies were almost the same. Both found that approximately half of inter-corporate ties were reconstituted and half not. They found, however, that directionality had little influence on the probability of reconstitution whereas multiple interlocks (more than one director from one board serving on a second board) did. Thus, insofar as underlying corporate relations can be seen as affecting the affiliation network of interlocking directorates it will do so by increasing the level of multiple interlocking.

In this paper we analyse the networks of interlocking directorates only as undirected ties. This is in line with the findings of these replacement studies. However, because we consider them as affiliation networks we have the ability to see how the multiplicity of inter-corporate linkages appears also in the multiplicity of interpersonal links (the same persons meeting on two or more boards). Multiplicities are not often studied in interlocking director studies but it is possible to investigate the structure of a particular system in terms of the groupings of companies networked at increasing levels of multiplicity (Carroll and Alexander, 1999).

The impetus of debate that culminates in these replacement studies was a concern to show the extent to which underlying organizational links, particularly relations of ownership and control, were the basis of interlocks. While the lesson of these studies is that these relations account for only a small proportion of interlocks, a separate process at the interpersonal level may act to create multiple links between boards. This is the process of personal referral. If a board is looking to make an appointment, one of its current members can endorse a candidate who they meet sitting on a second board. If the appointing board appoints that candidate it creates two interlocks between those two boards, although there may be no organizational link between the two companies. Multiple linkages between boards may thus stem from interpersonal, as well as inter-organizational processes.

In this paper, rather than investigating the affiliation network of all directors of all boards, we examine the *network infrastructure* (Alexander, 2003). We define the network infrastructure as a subgraph of the full network. It is the affiliation network made by persons who hold two or more positions, that is, person nodes with bipartite degree two or greater.[6] These persons are the "interlockers" of the company-to-company graph. They are, simultaneously, "networkers" in the person-to-person graph. The network infrastructure is an affiliation network in its own right. We can derive 1-mode networks for the affiliation network as needed. A tie exists between directors when they are members of the same board, and a tie is present between any two boards with a common director.

The concept of a bipartite *network infrastructure* makes immediate sense in the context of the company-to-company 1-mode graph. Directors holding just one board seat make no interlocks. It is only multiple positions holders who need to be considered when charting a company-to-company interlock network. At the same time, any company in the network infrastructure necessarily has at least one interlocker on its board.

The case for not considering single position holders in the context of the person-to-person graph is more complex. In terms of path length, the person-to-person graph has a structure resembling a connected (contracted) caveman graph (Watts, 1999a). It can be represented as an "inner circle" of the cross-board connections made by the joint memberships of the interlockers. Attached to each of the affiliations of the interlockers is a group of single position holders and, occasionally, an affiliation of another interlocker (Alexander, 2003). For all the directors holding just one board seat all paths to persons outside their home board must pass through an interlocker on that board. The average distance among interlockers (i.e. the distance in the bipartite network infrastructure) is thus the dominant element in determining the path lengths in the person-to-person network.

3. Global Properties of Bipartite Graphs

We now present a formal description of a bipartite graph and describe some of the statistics that we examine later. We investigate geodesics, although the distinctive nature of the bipartite graph implies that there are three types of geodesics, not just one as in the case of a 1-mode graph. There are also several clustering coefficients. We examine counts in the graph of various localized configurations, as well as degree distributions.

3.1. Data Structure

A bipartite graph has two types of nodes, which we will label A and P, for associations—in our case, companies—and persons (directors), respectively, where $A = \{a, b, c, \ldots\}$ and $P = \{p, q, r, \ldots\}$. Edges (or *ties*) can only be observed between A and P nodes, not between nodes of the same type. Let $X_{ap} = 1$ signify the presence of an edge, and $X_{ap} = 0$ its absence. Let n_A and n_P be the number of A and P nodes, respectively. Then the bipartite graph can be represented as a binary $n_A \times n_P$ matrix $\mathbf{X}$.

From the bipartite graph, two 1-mode graphs can be derived with nodes of only one type. For example, an edge between companies a and b is observed in the derived 1-mode graph when there is a person p such that $X_{ap} = X_{bp} = 1$. Notice that this definition removes information about the number of persons that might connect the two companies, so the binary 1-mode graph does not represent the multiplicity (or "tie strength") of intercorporate ties. We could investigate such valued 1-mode graphs by setting an edge between companies a and b to $\Sigma_p X_{ap} X_{bp}$, although this results in complexities in defining clustering coefficients.[7] So we limit the derived 1-mode graphs to binary edges, but we introduce (below) a statistic to measure the extent of multiple connections.

3.2. Geodesics

Because there are two types of nodes, there are three geodesic types: geodesics between companies, geodesics between persons, and geodesics from persons to companies. Within node-type geodesics in the bipartite graph are necessarily of even path length, twice the geodesic length in the relevant 1-mode graph. Between node-type geodesics are peculiar to the bipartite graph and are always an odd number. We examine the geodesic distribution of the bipartite graph—which include the between node-type geodesics—and also geodesic distributions of the two 1-mode graphs. This leaves open whether a bipartite graph might be small-world for one type of node but not for the other.

3.3. Degree Distributions

The bipartite graph has two degree distributions, one for A nodes and one for P nodes.

3.4. Graph Configurations

We count the bipartite graph configurations depicted in figure 1. We also count the configurations from the two derived 1-mode graphs in figure 2.

Edges (L): As described below, our simulation strategy fixes the number of edges to be equal to the observed graph.

Stars (S_{A2}, S_{P2}, S_{A3}, S_{P3}): A two-star in the bipartite graph corresponds to an edge in the relevant 1-mode graph, whereas a bipartite three-star corresponds to a 1-mode triangle.[8]

Three-paths (L_3): The number of three-paths in the bipartite graph is information lost if only 1-mode graphs are examined. Yet three-paths are important to connectivity, potentially

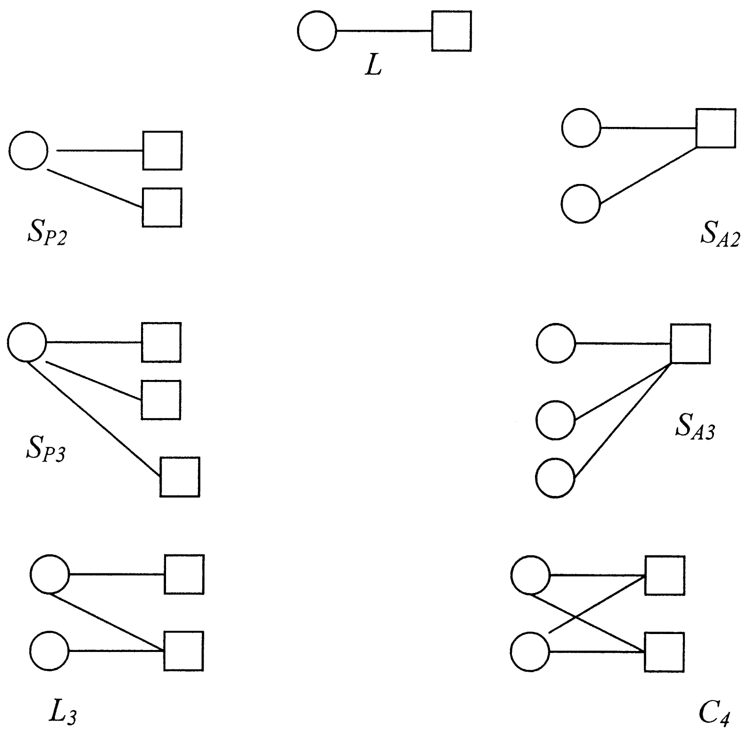

Figure 1. Configurations examined in the bipartite graph. NB: Circles represent P nodes (persons); squares A nodes (associations, in this case, companies).

providing short geodesics between directors and companies of which they are *not* members. Long paths across the network of course must comprise several of these short three-paths, so we argue that the three-paths are precursors of global connectivity. Whether they are efficient in ensuring network connectivity (with "efficiency" understood as providing connectivity with few edges) depends on how much redundancy occurs in these paths. Redundancy can take the form of various cycles.

Multiple links (C_4)[9]: This is the simplest and most local form of a cycle (and hence of redundancy.) If two directors share membership of the same two boards, then both the boards and directors have multiple links. These are instances of the multiple interlocks noted in the replacement studies of Palmer (1983) and Ornstein (1982; 1984). These multiplicities map as valued cells in both 1-mode graphs before they are dichotomized to binary graphs. However, as noted above, the majority of researchers reduce these 1-mode graphs to binary graphs, so losing this information. A count of the C_4 configuration measures multiple links directly in the bipartite graph.

In their discussion of bipartite cohesive subgroups, Borgatti and Everett (1997) introduced an analogue of a network clique, a *biclique*, as a complete bipartite subgraph, with the labelling of a (p, q) biclique to indicate a biclique with p nodes of one type and q nodes of the other type. Although they focus on (3,3) bicliques in their examples, our C_4 configuration is equivalent to their (2,2) biclique. It is the simplest biclique configuration with equal numbers of nodes of each type (apart from the trivial single edge configuration). In that sense, it can be viewed as an analogue of the triangle in a 1-mode network, which is the simplest 1-mode clique, apart from the trivial single edge.

Bipartite clustering coefficient. Note that the L_3 configuration is *lower order* to the C_4 configuration in that every C_4 configuration contains four L_3 configurations.[10] We could interpret the ratio $4 \times C_4/L_3$ as a bipartite clustering coefficient, analogous to the 1-mode clustering coefficient discussed above (three times the ratio of triangles to two-stars—the ratio of observed triangles to the number of lower order configurations, i.e., two-stars).

The level of C_4 formations indicates the extent to which directors re-meet one another on two or more boards, with dual and multiple interlocks being created between company boards. A high clustering coefficient suggests more frequent personal connections among pairs of directors. It may flow from some formal cooperation between companies or it may arise from individual directors introducing contacts from other boards to a new board as positions become vacant. High bipartite clustering indicates localized closeness and redundancy, just as is the case with triangles in 1-mode networks. In 1-mode networks of relatively low density with a fixed number of edges, many triangles can only form at the expense of possibly disconnecting the network. In other words, localized redundancy in the form of high clustering is at the cost of effective longer connectivity (which is why the small world is at all interesting.) For a bipartite graph, if the bipartite clustering coefficient is high, then many L_3 patterns are redundant. They do not provide new paths of connectivity across the bipartite graph. So for two bipartite graphs of similar size, the graph with the higher bipartite clustering ratio will show lower levels of connectivity.[11] Substantively, there are many possible implications depending on context. If, for instance, we consider the bipartite network as a possible vehicle for the spread of information or best practice among companies, high bipartite clustering may suggest that localized efforts at control or closeness are at the global cost of slowing or limiting that spread across the entire system.

In addition to bipartite clustering, we also calculate clustering coefficients for the derived 1-mode networks. As a result, we also count the numbers of two-stars and of triangles in the 1-mode graphs (figure 2).

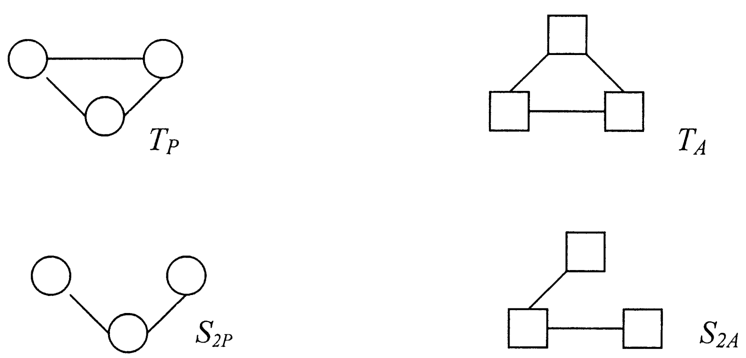

Figure 2. Configurations examined from the derived 1-mode graphs.

4. The Observed Data Sets and Methods of Comparison

The data sets for this study comprise the affiliations of directors in the top 200 non-financial and top 50 financial corporations in the US and Australia in 1996. The data was collected by the Social Networks Research Group at the Netherlands Institute for Advanced Study (NIAS) in 2000–2001.[12] Information was collected from business directories, annual reports and, occasionally, from direct enquiries to the companies. Directors' names were double checked to verify that the same names, indeed, identified the same person. The large component in each data set was extracted, resulting in 229 boards in the US and 198 in Australia. It is the network infrastructure of this large component that forms the observed datasets for this study.

The selection of the top 250 corporations as the basis of these data sets follows the common procedure used in corporate power structure research (Stokman et al., 1985). This procedure does not take account of the different sizes of the national economies. The top 250 companies in Australia cover a much greater proportion of the corporate sector than do the top 250 in the US. The US data are only the tip of a large iceberg while the Australian data are, in fact, the greater part of the iceberg itself.

This paper uses a "most different" strategy of cross-national comparison (Castles, 1991). We are interested to show that there is a common underlying structure of affiliation networks among the directors of the large companies of western capitalist countries. Comparisons against the simulations provide the basic mechanism of inference to this common structure. We thus expect to find that the observed networks in these countries are both close to the corresponding simulated networks or, if they differ from them, they both differ in similar ways. This finding would support a claim that the conditions governing network formation among corporate elites tend to be similar in capitalist democracies.

We examine the two data sets of corporate interlocks in the US and in Australia in 1996, collecting each of the statistics mentioned above for both. For each observed graph, we simulate a distribution of bipartite graphs randomly, conditional on a number of properties, to enable comparison. These properties include the same number of companies and persons as the observed data, as well as the constraints of the network infrastructure, that is, that the degree of each director is at least two, and of each company at least one. We also fix the number of edges in each graph of the simulation to be equal to the number of edges in the observed network infrastructure.

We use a bootstrap approach by starting from the observed graph, which automatically has the required constraints. We perform a large number of initial iterations, removing a randomly selected edge and adding a new edge between a previously untied director and company randomly selected, provided that the constraints are observed. With sufficient initial simulations, the result is a graph that is random conditional on the infrastructure constraints and with the same number of edges as the observed graph. We take this graph as the starting point for the second simulation run from which we collect data.[13]

With our new starting graph, the second simulation run uses the same iteration strategy as before. To be precise: at step t of the procedure we have bipartite graph $\mathbf{X}_t$. We select at random an a, b, p and q such that $X_{ap} = 1$ and $X_{bq} = 0$, and we define a new proposal graph $\mathbf{X}_t'$ as equal to $\mathbf{X}_t$ except that $X_{ap} = 0$ and $X_{bq} = 1$. $\mathbf{X}_t'$ and $\mathbf{X}_t$ obviously have the same number of edges, so we then check whether the other constraints are obeyed in $\mathbf{X}_t'$; namely that the degree of a remains greater than 0 and that the degree of p remains greater than 1. If these constraints are satisfied, the next graph in the simulation $\mathbf{X}_{t+1}$ is set as equal to the proposal graph $\mathbf{X}_t'$; but if the constraints are not satisfied, $\mathbf{X}_{t+1}$ is set as $\mathbf{X}_t$ (see Snijders and van Duijn, 2002, for a related algorithm for switching edges under complicated constraints). This process simulates a uniform distribution of random graphs conditional on the constraints, which serves as a basis of comparison to determine whether the observed graph has properties consistent with this distribution (see Pattison et al., 2000, for a more formal description of this general approach of comparing a particular graph against various graph distributions.)

From our second simulation run, we collect a sample of graphs and for each we examine the various graph statistics.[14] We thus have distributions of several statistics against which to compare the statistics from the observed graph. In particular, we follow the approach of Robins et al. (in press) in assuming that an observed geodesic statistic is "short" if it is not extreme compared to the geodesic distribution generated from the simulation; and that an observed clustering coefficient is "high" if it is extreme compared to the simulated distribution.

One important point about this simulation approach is that it avoids problems relating to the size of the networks. Here we are trying to compare two networks of different size, with different numbers of nodes and edges. In these circumstances, direct comparisons of graph statistics and degree and geodesic distributions may be meaningless. But by making comparisons first against appropriately sized reference distributions of graphs, and seeing in what ways the observed data sets differ from such distributions, we are able to draw conclusions comparing the two networks. This seems to us a principled way of making comparisons when networks differ substantially in size.

5. Results

5.1. The US Network Infrastructure, 1996

In 1996, the infrastructure of the bipartite graph in the US had 229 companies and 489 directors (interlockers/networks) with 1249 edges. Thus each board had an average of 5.45 interlockers/networkers while each interlocker/networker held an average of 2.55 board seats. (For further details, see Alexander, 2002). In Table 1, we present statistics from this observed bipartite graph with means and standard deviations from the simulated distributions. The Table presents the standardized z-score for each observed statistic based on the mean and standard deviation from the simulated distribution.

From Table 1, it is at once apparent that the observed US network infrastructure differs very substantially from the conditional random graph distribution. With a small number of exceptions, the z-scores are extreme,[15] especially for multiple links and bipartite clustering. The observed infrastructure is clearly not the result only of the constraints programmed into the simulation. Additional social processes are operating.

Table 1. Graph statistics for the United States network infrastructure. (Median geodesics and counts of configurations in the observed network; means and standard deviations for simulated graph distribution; standardized *z*-score for observed value; median of simulated median geodesics.)

Statistic	Observed	Simulated distribution	*Z*-score
S_{A2} (bipartite company two-stars)	3892	3387 (58)	+8.71
S_{A3} (bipartite company three-stars)	8696	6110 (326)	+7.93
S_{P2} (bipartite director two-stars)	1144	1132 (11)	+1.01
S_{P3} (bipartite director three-stars)	535	502 (31)	+1.06
L_3 (bipartite three-paths)	14786	12272 (248)	+10.14
C_4 (multiple links)	856	98 (19)	+39.89
C (bipartite clustering)	0.232	0.032 (0.006)	+32.75
Median geodesic (bipartite graph)	6	6	
1-mode Statistics			
S_{2A} (1-mode company two-stars)	11877	11974 (354)	−0.27
T_A (1-mode company triangles)	764	648 (38)	+3.05
C_A (1-mode company clustering)	0.193	0.162 (0.006)	+5.23
Median Geodesic (1-m. company)	3	3	
S_{2P} (1-mode director two-stars)	66425	50648 (2099)	+7.52
T_P (1-mode director triangles)	8743	6262 (334)	+7.43
C_P (1-mode director clustering)	0.395	0.371 (0.006)	+3.83
Median geodesic (1-m. director)	3	3	

In this observed network it is the corporations, not the persons, where the effects of social processes are most evident. Corporations have many more two- and three-stars than expected, but the numbers of director two- and three-stars are consistent with the random graph distribution. Corporate stars in the bipartite graph are created by 2, 3 or more directors sitting on the same boards. Newman et al. (2001: 14) found the same deviation from random models in their analysis of interlocks among the Fortune 1000 US corporations. They suggested that "bigshots run with other bigshots", arguing that this explained the deviation from randomness that they found.

We have a slightly different take on this result. The degree distributions of companies and directors presented in figure 3 are revealing. For directors the degree distribution is entirely consistent with the random distribution. The majority of interlockers link only two companies, and there are not many directors with degrees higher than four. There is little evidence here of directors with unusually high degrees acting as notable hubs within the graph. This result suggests that it is not directors with a large number of board positions (the "big linkers") that shape the distinctiveness of the structure. Some of these may indeed be "bigshots" or "big linkers" with 4, 5 or more board seats, but their number is entirely consistent with randomness given the constraints. In fact, we may infer that the director

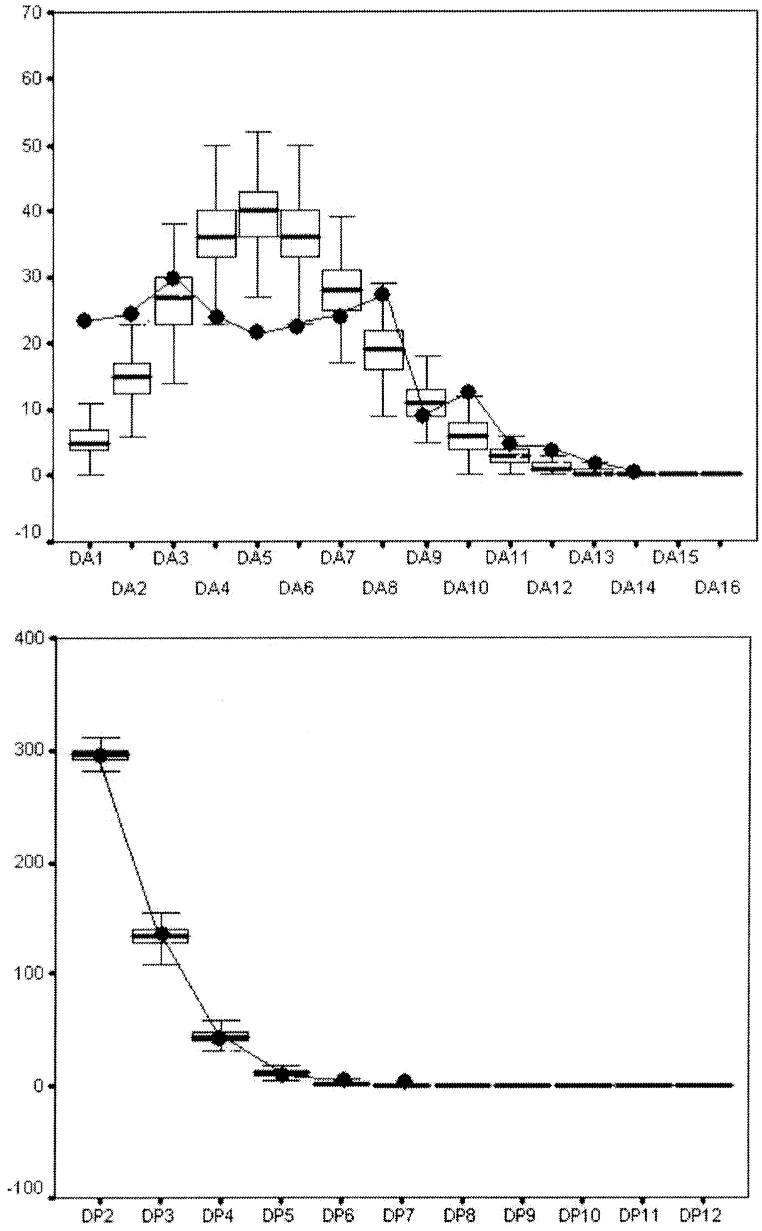

Figure 3. Degree distribution for companies (upper panel) and directors (lower panel) in the US network infrastructure, compared to conditional random graph distribution. (*Note*: Boxplots indicate ranges of nodes with a given degree across the distribution; dots represent degree distribution for the observed graph with lines connecting dots in observed degree distribution; DA*k* refers to degree *k* for companies, and DP*k* for directors. Outliers for boxplots not presented.)

degree distribution is explicable from properties implicit in the constraints (Pattison et al., 2000).

The picture is different for the corporate degree distribution, which shows the number of interlockers/networkers on boards. In the tail of the corporate degree distribution (e.g., companies with greater than seven interlockers), the number of companies with a given degree tends to be at the upper end of the range of the random distribution (except for boards with nine interlockers). There are more than expected companies acting as are moderate or high degree hubs. Interestingly, the numbers of companies with between 1 and 8 interlockers are approximately equal, an unexpected result that differs markedly from the random graph distribution, where degrees peak at five and then begin to decline. Just as there are more moderate to high degree corporations, there are more low degree corporations than expected, but fewer corporations with 4 to 6 interlockers.

High-degree corporate boards increase connectivity and short paths in the bipartite graph, as well as in the 1-mode director network. At the global level, however, the important contrast is that the network structure is determined primarily by the clustering of directors on corporate boards, not the accumulation of board seats by "big linkers". The graph structure is shaped more by decisions by some (not all) company boards to appoint moderate to high numbers of interlockers as directors, rather than by certain interlockers seeking to be on unusually large numbers of boards.[16] The constraints and opportunities of a bipartite structure thus produce *divergent* outcomes for the company-to-company and director-to director networks.

A second aspect of the observed affiliation network apparent from the statistics for the bipartite graphs is the impact of multiple connections, when the same interlockers sit on the same boards. There are many more multiple connections (C_4) than expected from the simulated graph distribution. Even though the number of three-paths (L_3) is also extreme, when we discount all three-paths created by the 856 multiple connections, there are fewer remaining than expected[17] (some two standard deviations lower than the mean from the simulated distribution). The bipartite clustering coefficient is very extreme compared to the simulated distribution, as so many of the three-paths are taken up in multiple connections. Substantively, there are vastly many more multiple connections between boards than would occur by chance. As suggested earlier, this finding is consistent with the bias toward multiple connections found in replacement studies and with a bias created by pre-existing personal contacts between boards.

The effect of additional multiplicities on the number of three-paths is important for network connectivity. As discussed earlier, a three-path that is not part of a multiple connection is an efficient means of providing connectivity in the graph. Because there are fewer of these paths, the high level of multiple connections is detracting from the potential connectivity of the bipartite network. This result is not substantial enough to affect median path length, which as Table 1 shows remains identical between the observed graph and the simulated distribution. But we have examined the bipartite geodesic distribution in greater detail (not reported in Table 1). For the observed graph, the 75th percentile of the geodesic distribution has a path length of 7, longer than that of 99% of graphs in the simulated distribution. So the decreased connectivity shows up not in the average but in the longer geodesics of the graph, which tend to be longer in the observed data than in the graphs of the random distribution.

In terms of network path length, two nodes that are "far apart" tend to be further apart in the observed data than in the random distribution.

Table 1 also presents summary information for the two 1-mode graphs. It provides confirmation of the divergent outcomes of the bipartite structure on the company and director 1-mode graphs. We see that in the 1-mode company graph the number of two stars (S_{2A}) is consistent with the random graph distribution, whereas the number of triangles (T_A) is extreme in that distribution, resulting in an extreme measure of clustering. At the same time, the observed median geodesic is consistent with the random graph distribution. We can say that this is a graph with small world properties, with median geodesics similar to a random graph but extreme clustering. In the 1-mode director graph, however, the divergence of two-stars and triangles is even greater, but the overall clustering is if anything somewhat less extreme than in the company 1-mode graph. Again, the median geodesic is consistent with the random graph distribution, so again we infer that the graph has small world features.

5.2. The Australian Network Infrastructure, 1996

In 1996, the bipartite graph generated by Australian company interlockers had 198 companies and 255 directors, with 675 edges. (For further details, see Alexander, 2003.) Compared to the US case, this is a slightly lower number of companies but substantially fewer directors, with consequent impact on the network infrastructure. In Table 2, we present the observed statistics from this bipartite graph together with means and standard deviations from the simulated distributions, as well as z-scores.

We can see that the patterns of the observed data relative to the conditional random distributions largely mirror those for the US network, although the strengths of effects may differ. Consistent with the observed US data, the bipartite company two- and three-stars are extremely high compared to the random graph distribution. But unlike the US situations where director two- and three-stars were consistent with the random graph distribution, the Australian director stars tend to be extreme. We see from figure 4 that the numbers of boards with high degree is greater than expected. Although it is difficult to see on the scale of the chart in figure 4, in the distribution of degree for directors it is, specifically, the number of directors with degrees 6 and 7 that are rather higher than most of the graphs in the random distribution (the trade-off is that there tends to be more degree 3 directors in random graphs). This "spike" in observed high degree nodes generates somewhat higher than expected numbers of director stars.

As with the US data, Australian three-paths (L_3) and multiple links (C_4) are extreme, but with three-paths less so. As a result, the impact on the three-paths is even more extreme than in the US. When we exclude the number of three-paths associated with the 584 C_4 formations, the resulting count is 2695, close to 12 standard deviations below the mean number of three-paths in the random distribution (allowing for the average 41 mean paths in the random distribution makes little difference). As a result, the Australian bipartite clustering coefficient is more extreme than for the US with a Z-score over double that of the US. So, multiple connections have an even more pronounced (negative) effect on connectivity in Australia than in the US. It is not surprising, therefore, that the observed

Table 2. Graph statistics for the Australian network infrastructure. (Median geodesics and configuration counts in observed network; means and standard deviations for simulated graph distribution; standardized z-scores; median of simulated median geodesics.)

Statistic	Observed	Simulated distribution	Z-score
S_{A2} (bipartite company two-stars)	1250	1100 (27)	+5.56
S_{A3} (bipartite company three-stars)	1735	1181 (99)	+5.60
S_{P2} (bipartite director two-stars)	677	656 (10)	+2.10
S_{P3} (bipartite director three-stars)	393	330 (29)	+2.17
L_3 (bipartite three-paths)	5031	4270 (133)	+5.72
C_4 (multiple links)	584	41 (14)	+38.78
C (bipartite clustering)	0.464	0.039 (0.01)	+33.87
Median geodesic (bipartite graph)	7	6	
1-mode Statistics			
S_{2A} (1-mode company two-stars)	4399	4920 (240)	−2.17
T_A (1-mode company triangles)	428	369 (32)	+1.84
C_A (1-mode company clustering)	0.292	0.225 (0.01)	+6.29
Median geodesic (1-m. company)	4	3	
S_{2P} (1-mode director two-stars)	12020	10226 (642)	+2.79
T_P (1-mode director triangles)	1687	1221 (102)	+4.57
C_P (1-mode director clustering)	0.421	0.358 (0.01)	+5.81
Median geodesic (1-m. director)	3	3	

median geodesic is longer than that expected from the random distribution. This contrasts to the US where we observed a lengthening of the geodesic only at the third quartile, not at the median.[18]

Inspection of the degree distributions in figure 4 reinforces these points. For companies, the degree distribution is broadly consistent with the random graphs, except in the tail where there are a small number of Australian companies with a large number of interlockers on their boards, and a somewhat smaller number of companies than expected with degrees between 3 and 5. So we have a distinct “long tail” effect where network connectivity relies disproportionately on a small number of highly connected hubs. Reliance of connectivity on a few hubs was not so marked in the US. In the US network, connectivity effects emerge not just from a small number of high degree companies but also from a substantial number of moderate degree companies.

Interestingly, observed director appointments as in the bottom panel of figure 4 present a similar pattern to the US counterpart in figure 3, although in the Australian case the tail of the observed degree distribution is somewhat higher than expected (i.e. in number of directors with 6 appointments, which exceeds all but 12% of random graphs, and in the number of directors with 7 appointments, which exceeds all but 8% of random graphs[19]). As noted above, these higher numbers in the tail explain the more extreme numbers of director two- and three-stars. So in the Australian case, connectivity in the bipartite graph

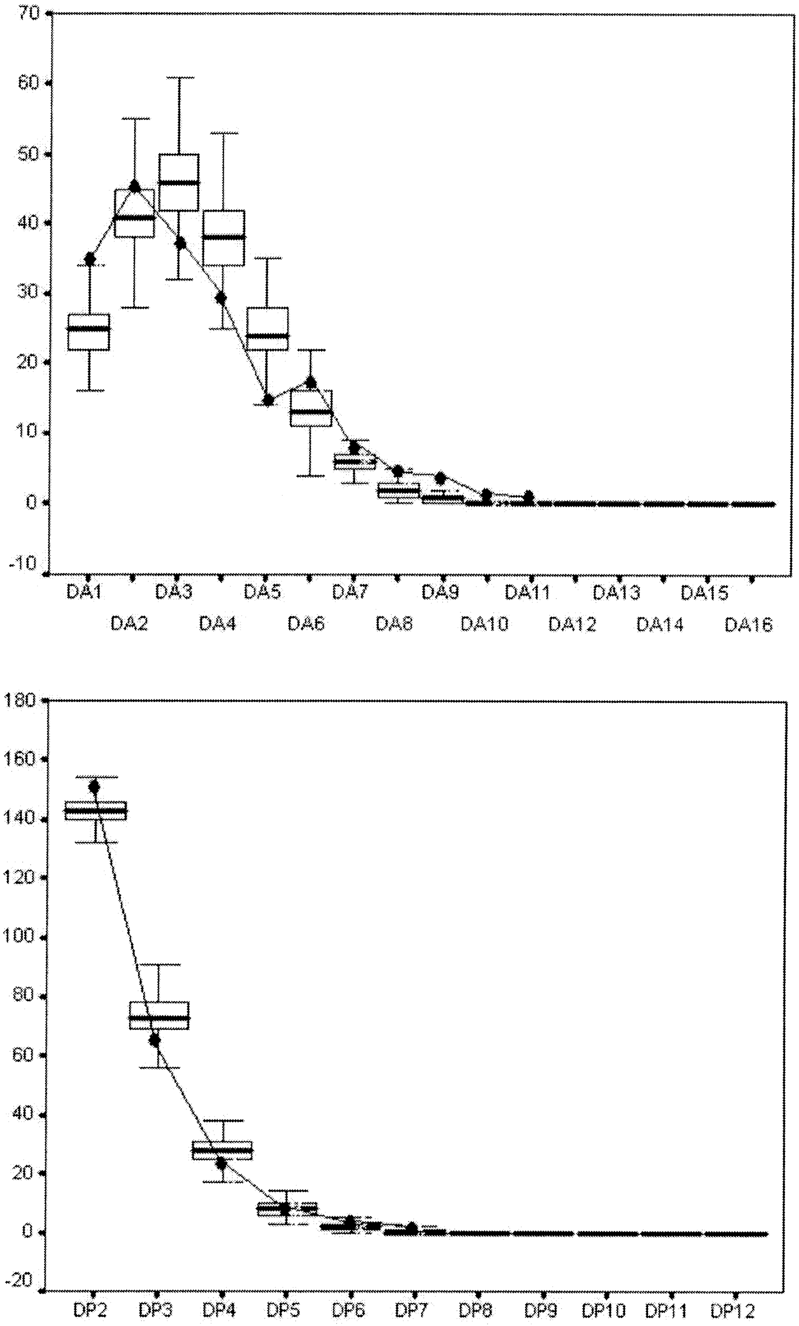

Figure 4. Degree distribution for companies (upper panel) and directors (lower panel) in the Australian network infrastructure, compared to conditional random graph distribution.

is enhanced by a small number of high degree directors as well as a small number of high degree companies. But these connectivity enhancements do not compensate for the decrease in connectivity occasioned by the number of multiplicities.

For the Australian data, the decrease in connectivity is observed in the 1-mode company statistics, where there are substantially fewer than expected two-stars. Moreover, many of these two-stars are involved in the somewhat larger than expected number of triangles, leading to a very high 1-mode clustering coefficient. Moreover there is a long median geodesic of 4 (all random graphs had a median geodesic of 3). So we do not have small world global features in the 1-mode intercorporate network of Australian companies, as path length tends to be longer than the random graph simulations. What is happening here is that Australian companies are establishing many localized and redundant connections among themselves but at the expense of longer range connectivity. The implication may be that it would take longer for certain types of information or practice to spread among Australian corporate boards. On the other hand, the other 1-mode graph, the network of directors, does, as in the US, present with small world features. It exhibits with high clustering, while the median path length is consistent with the random graph simulations.

6. Conclusions

We have presented a method that allows meaningful comparisons between affiliation networks of different sizes and density. We have applied this method to two observed affiliation networks of interlocking directorates with different numbers of companies and directors, and different densities of ties. Direct comparisons of these affiliations networks are usually problematic, but our strategy allows us to compare each observed dataset to a random graph distribution conditional on properties of the infrastructure of the observed data. Since we are dealing with the infrastructures every company has at least one interlocker on its board, and each interlocker is a member of least two boards. If the features of an infrastructure are consistent with distributions generated from these constraints, we argue that the structure of the observed network is emergent from random processes alone; no further processes are necessary to explain the data (Pattison et al., 2000). Where the observed network is different we argue this indicates the presence of social processes biasing the observed data away from the results that would be expected if only random processes were operating.

It is worthwhile commenting briefly on the choice of the network infrastructure as the data. As we have explained earlier, the infrastructure is at the "heart" of the entire intercorporate network, so an understanding of its structural properties is the crucial element in understanding that network. There is a vexed issue in network-related research about choice of criteria for inclusion of actors into a study, because in principle networks do not have "boundaries". The choice of the infrastructure can be seen as a feasible and informed measurement decision. Nevertheless, it is a decision that is determined by the data. Our simulation strategy enables us to draw useful conclusions about the infrastructures. If we were to take the further step of actually constructing a statistical model for an infrastructure, conclusions would need to be tempered by the knowledge that we were modeling a data-derived structure. Unless this fact was somehow incorporated into the model, formal statistical inference decisions would be undermined. Even so, such a model might be a useful exploratory tool.

In examining the two observed network infrastructures, the first and most important point to make is that they both exhibit considerable similarity. They both differ, in substantial and similar ways, from the random graph distributions. Accordingly, we conclude that many of their important properties do not merely emerge from the programmed constraints of the modeling process. Additional social processes are operating and these seem to be similar in the two observed datasets.

Both networks had higher than expected numbers of company stars. Company stars mean that both infrastructures contain a relatively large number of companies with a high number of interlockers on their boards, high degree hubs. Thus more boards than expected choose a high number of directors with external linkages. In these companies, *social selection* of new members seems to relate in part to the new director's pre-existing external connections. The global effect of this local decision making is, however, to enhance connectivity across the system.

But, at the same time, both bipartite networks had much higher than expected multiple connections (C_4). A very large number of pairs of interlockers find themselves sharing membership of more than one board. These muliple interlocks may arise from social (class-based) processes. New members with external connections may be introduced to corporations by existing board members who have experience with them on other boards. These multiple connections can also reflect intercorporate relations of trust and more formal alliances. Whatever the motives these lead to a clustering of interlockers on particular central boards, and the resulting multiplicity of shared board membership, measured through our C_4 count of multiple links, has implications at local and global levels. At the level of individual boards it may enhance strong relations, creating intercorporate links that might support strategic action; but in terms of longer range connections across the network, it diminishes connectivity in the affiliation networks, increasing path lengths and hence inhibiting small world effects.

This analysis of two countervailing effects of social selection procedures operating within boards—selecting as directors people with new external connections, and selecting directors who duplicate existing connections—reinforces our theoretical construal of boards as the *social neighbourhoods* wherein apparently local social processes play out without relation to the global structure of the network but still shape the emergent properties of the larger system. Decisions to appoint directors with outside links enhance global connectivity, potentially enabling swifter diffusion of information, practices, norms and opinions. On the other hand, selection of directors from the already existing pool of board contacts detracts from global connectivity, creating denser localized structures. The literature on interlocking directorates noted the existence of multiple interlocks between firms but the intercorporate perspective used by the replacement studies of the 1980s could only discount intercorporate relations as a possible explanation for its widespread occurrence. We have also suggested how interpersonal processes may contribute to this through a current director's endorsement of a new director, a practice that we can assume to be fairly widespread. Our comparisons show that the outcome of such a practice, multiple interlocks, is indeed evident in both countries we examine. Whether the higher level of multiple interlocking in Australia represents just the impact of this interpersonal networking, or may also contain a large element created by underlying patterns of intercorporate alliances and ownership is a matter for a further study.

We also observed some important differences between the two affiliation network infrastructures. We observe more than expected US companies with moderate number of interlockers, in contrast to the Australian data where, except for the extreme end, the degree distribution for companies is reasonably consistent simply with the infrastructure constraints. In the Australian data, enhanced connectivity rests with a small number of high degree company hubs, in contrast to the US structure with its relatively wider range of moderate to high degree companies. This suggests that proportionately there are fewer companies in the Australian data that are important to the connectivity of the system. As well, we observe more than expected Australian directors with high numbers of company memberships (even though the numbers in absolute terms are quite small), unlike the US data where the director membership distribution is entirely consistent simply with the expected levels given by the random graph distributions.

Higher corporate degrees have important connectivity effects through three-paths that provide directors with many connections to other companies via fellow board members. Each director in our study is a member of at least two boards, since we are dealing with affiliation network infrastructures, so boards with even a moderate number of interlockers create many paths of length three. As a result, the US network, with its relatively greater number of moderate and high degree boards, will have greater longer range connectivity than the Australian network, albeit not enough to prevail against the effects of multiple memberships. Thus, for the US, it was only longer range (third quartile) geodesics that have greater path length than expected from the infrastructure constraints, with median geodesics as short as those in the random graphs. By contrast in the less connected Australian network, it was the median geodesic that was longer than expected.

Our examination has used the conditional uniform random graph distribution as the basis of comparison. This is the most unstructured graph distribution that could be used consistent with the constraints implicit in the infrastructure. However, the observed data could also be compared against other graph distributions with known properties, along the lines proposed by Pattison et al. (2000), who suggested that a series of comparisons against a hierarchy of distributions might be revealing about the data. A reviewer suggested to us the incorporation of an additional constraint in our simulations by controlling for the (long-tailed) company degree distribution that seems to be an important primary feature of the data. We have done some preliminary work with this additional constraint. Initial results indicate some subtle differences, including that the number of three-paths in both datasets is less extreme, but that multiplicities and bipartite clustering remain very high. We infer that our general conclusions about the importance of multiple links in these data remain sound and that they are not the result of company decisions about the number of interlockers on their boards. But there are matters for further investigation, including with a wider range of datasets.

In summary, our methodology permits an investigation of global features of a bipartite graph relative to a random graph distribution conditional on certain properties. In this article, we have employed this methodology to examine features of two networks of interlocking directorships. Importantly, we examined features of the bipartite graph directly, rather than simply converting it into two 1-mode graphs. Our results have allowed us to infer many important structural similarities and differences between the two networks. We conclude that the structures tend to be influenced by the clustering of directors on boards, rather than the accumulation of many board seats by individual directors ("big linkers"). The number of shared multiple board memberships (multiple interlocks) is an important feature of both infrastructures well beyond the level consistent only with infrastructure constraints. These multiple memberships detract from global connectivity in both countries. Finally we suggest that company structural power may be relatively more diffuse and less concentrated in the US network than in Australia where a smallish number of Australian boards may be influential hubs within the network.

Acknowledgments

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Notes

1. A *geodesic* between two nodes i and j in a graph is the shortest path of edges connecting the two nodes. The geodesic length is the number of edges in a geodesic, and is taken to be infinite if there is no path from i to j. A graph with finite geodesics is said to be connected (i.e., there is a path between every pair of nodes.)
2. Moreover, in line with Watts' (1999a) original results, Robins et al. (in press) showed that large parameter values for the Markov random graph models resulted in highly structured long-geodesic global patterns, but that a scaling down of parameter values would see a rapid phase transition to more stochastic short-geodesic networks. There is some evidence that many human social networks remain stochastic but may be close to this phase transition point (Robins, 2003).
3. Markov random graphs are distinguished from other exponential random graph types in that all the edges in a configuration are adjacent to each other.
4. A k-star is a configuration of k edges centred on the one node. The factor of 3 is used in the clustering coefficient because any triangle comprises three 2-stars.
5. As "extreme" they used a criterion of the 95th percentile of the distributions derived from the Bernoulli graphs. Median geodesics were used, rather than means, in order to allow for the possibility of disconnected graphs. Moreover it is often useful to investigate the geodesic distribution in greater detail, by examining other order statistics (e.g. the 25th percentile, or first quartile; and 75th percentile, or third quartile, of geodesic length), as we do below. In this context, the use of the median is more natural than the mean.
6. The *degree* of a node is the number of edges adjacent to it. The *degree distribution* of a graph is the distribution of degrees across all nodes. A high degree node is sometimes called a *hub*.
7. In the extreme, one could define a separate clustering coefficient for each possible combination of valued edges in a triangle.
8. Of course there are other bipartite configurations that also produce 1-mode triangles, e.g. a 6-cycle, a path of length six through three directors and three companies. A bipartite three-star centred on a company indicates that three directors come together in the one social neighborhood, the company board. A six-cycle, on the other hand, indicates that three directors come together dyadically, in three separate company boards.
9. The L and C letters used in L_3 and C_4 refer to "Line" and "Circle", respectively.
10. To see that there are four L_3 configurations in one C_4 configuration, note that an L_3 pattern is a three path between a company and a director. For a given C_4 configuration, with two companies and two directors

implicated, there are four distinct possible pairings of a company and a director. Once the company and the director are chosen, the sequence of the three path is specified.

11. This is not to say that the more highly clustered graph will be disconnected, for such depends on the number of edges. But fewer pairs of nodes may be connected by k distinct paths for some k; i.e. the number of k-components will be greater.
12. The Australian data was collected by the second author as part of this group project. We would like to thank the other members of the Social Networks Research Group for making the US data available this analysis.
13. We chose the number of initial simulations so that each edge has a chance of being selected (and thereby changed) at least 10 times. For instance, for the US data, (1249 observed edges) we used an initial run of 15,000. We have experimented with different lengths of initial simulation runs. Initial runs of up to 1 million simulations did not produce markedly different results.
14. Typically, we examined every 1000th graph in a simulation run of 400,000, leading to samples of 400. By examining plots of the various statistics against the number of the iteration, we are confident that the process has "burned-in" to a stationary distribution. Examining the numbers of edges and the degree distributions of directors and companies across the sample confirms to us that the constraints are satisfied. We have also checked the simulation to see that all nodes participate in edge changes. Accordingly, we are satisfied that we are producing the desired conditional random bipartite graph distribution.
15. We take an absolute z-score value of 2 as extreme.
16. It is also the case that the boards with the most interlockers will also be the largest boards, since they have the greatest opportunity to appoint outside members. Furthermore these will be the largest companies since board size correlates with company size (Allen, 1974).
17. Recall that there are four three-paths implicit in any one multiple connection, so that subtracting the three-paths in the 856 multiple connections results in 11,362 three-paths not in C_4 patterns. This is well below the random graph mean of 12,272, even if one takes into account the 98 multiple connections expected in the random graph distribution.
18. The shorter geodesics (e.g. the 25th percentile) in the observed graph remain consistent with the random graph distribution (both with path length of 5).
19. For the observed US infrastructure, the number of directors with 6 appointments exceeds 74% of random graphs, and the number of directors with 7 appointments exceeds only 59% of random graphs.

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