



Universiteit Utrecht

*summerschool*  
**UTRECHT**

# Diffusion in Networks

Jiamin Ou

Summer School: Network Science

Day 5, Morning Session

2025

# Introducing ourselves

- Jiamin Ou: assistant professor, interested in dynamics of sustainable behavior, energy/emission models
- Vincent Buskens: prof. of sociology; interested in effects of social networks on behavior: cooperation, trust etc.



## Nicholas Christakis: The hidden influence of social networks



A network perspective to see  
“things” differently



Nicholas Christakis: The hidden influence of social networks

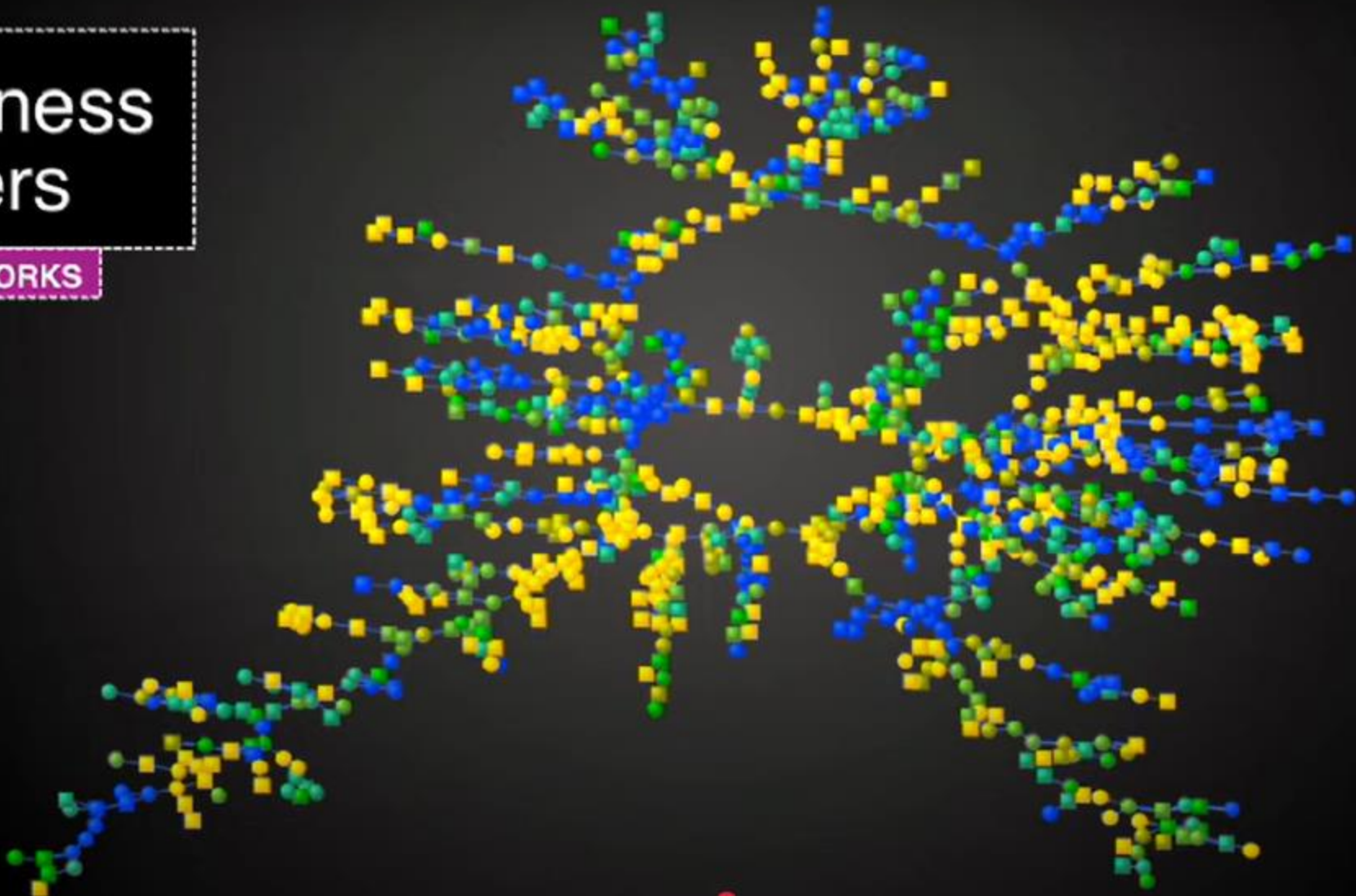
The Human Superorganism



Share

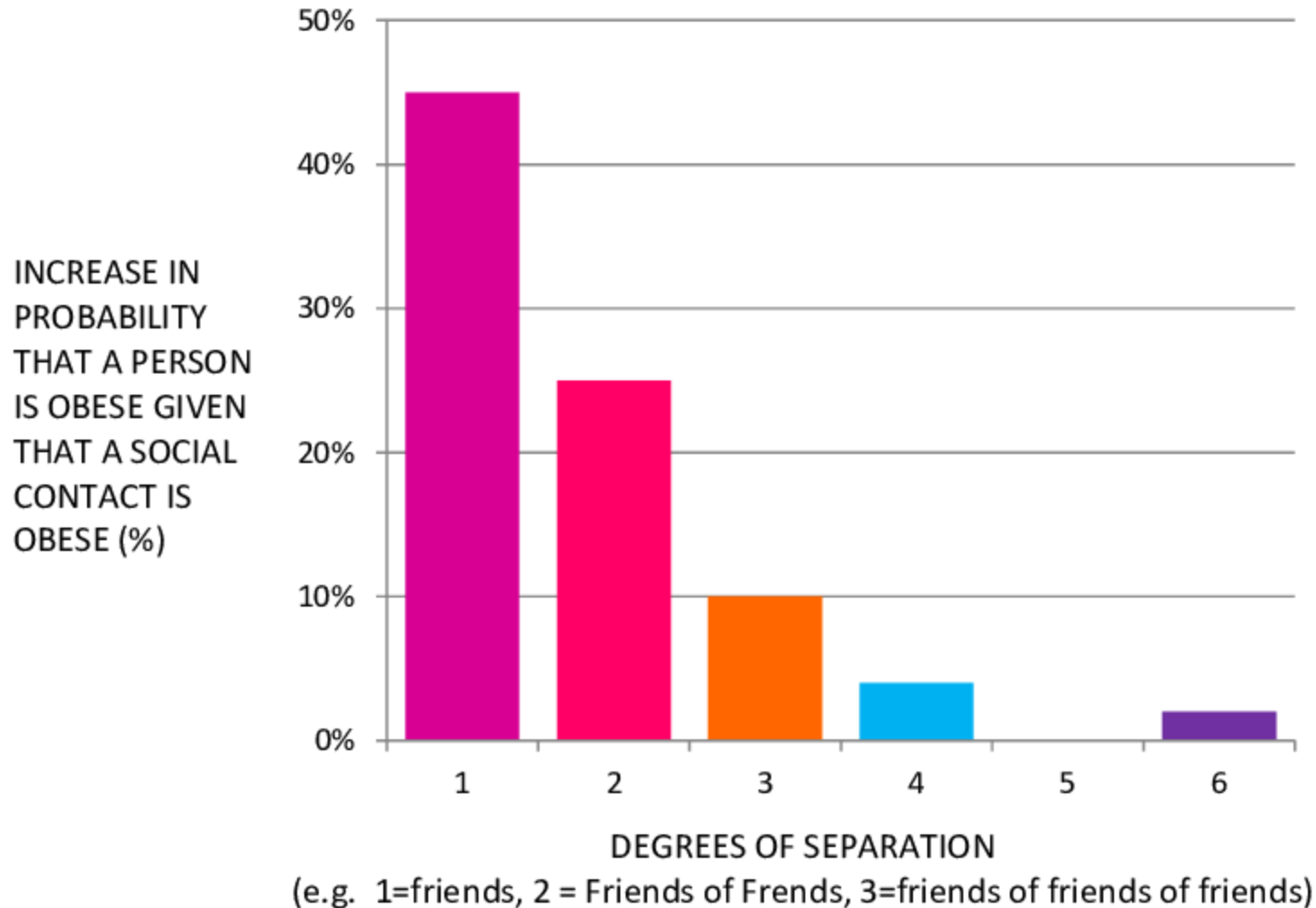
# Happiness Clusters

SOCIAL NETWORKS



MORE VIDEOS

# Degree of separation and body size



Results based on social network analysis of 12,067 people from 1971 to 2003 ("Framingham Heart Study")  
Adapted from "Nicholas Christakis: The hidden influence of social networks" <http://www.youtube.com/watch?v=2U-tOghblfE>, 04:02/18:44

What other phenomena could be better understood by examining them through a network perspective?

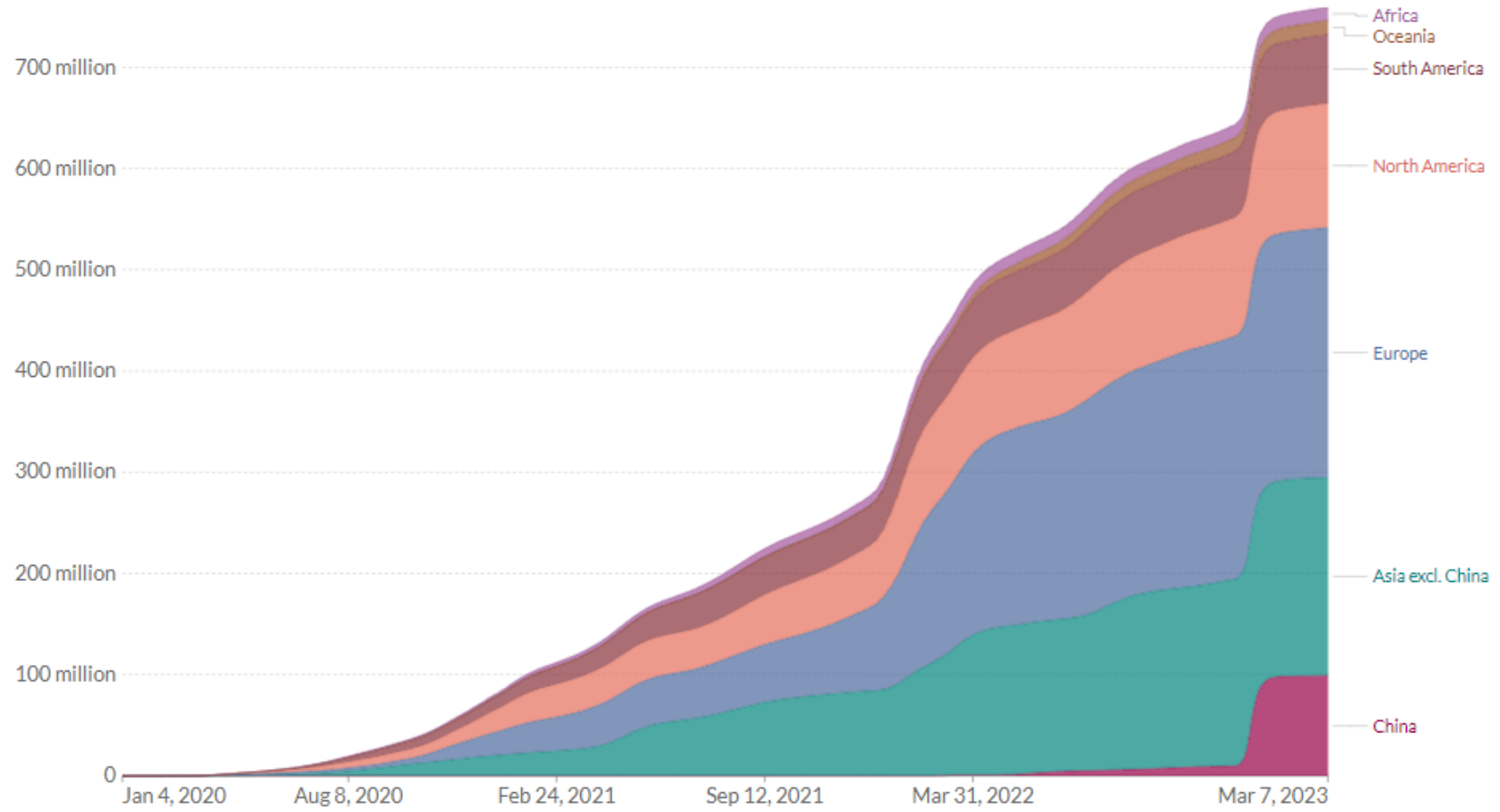
# The spread of COVID in human network

- **COVID 19:** From patient 0 in Dec 2019 to 700 million cases till 2023

## Cumulative confirmed COVID-19 cases by world region

7-day rolling average. Due to limited testing, the number of confirmed cases is lower than the true number of infections.

□ Relative



Source: WHO COVID-19 Dashboard

OurWorldInData.org/coronavirus • CC BY

► Jan 4, 2020 ◯ Mar 7, 2023



Many mechanism at work that cause *similarities between connected people*

- Contagion
- Selection
- Common context / third variables
- Social dynamics



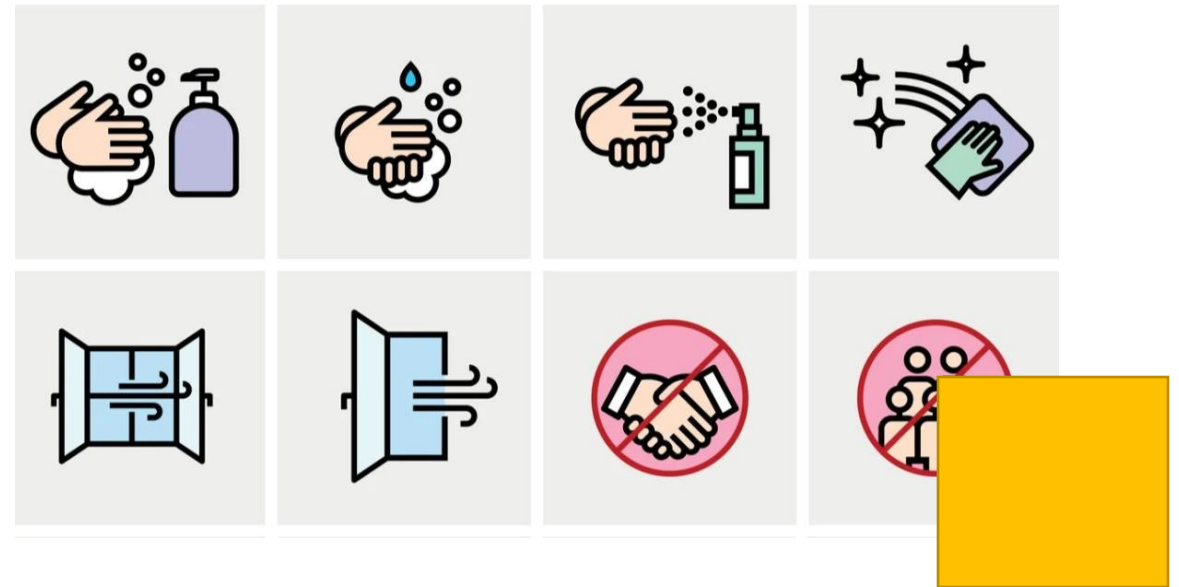
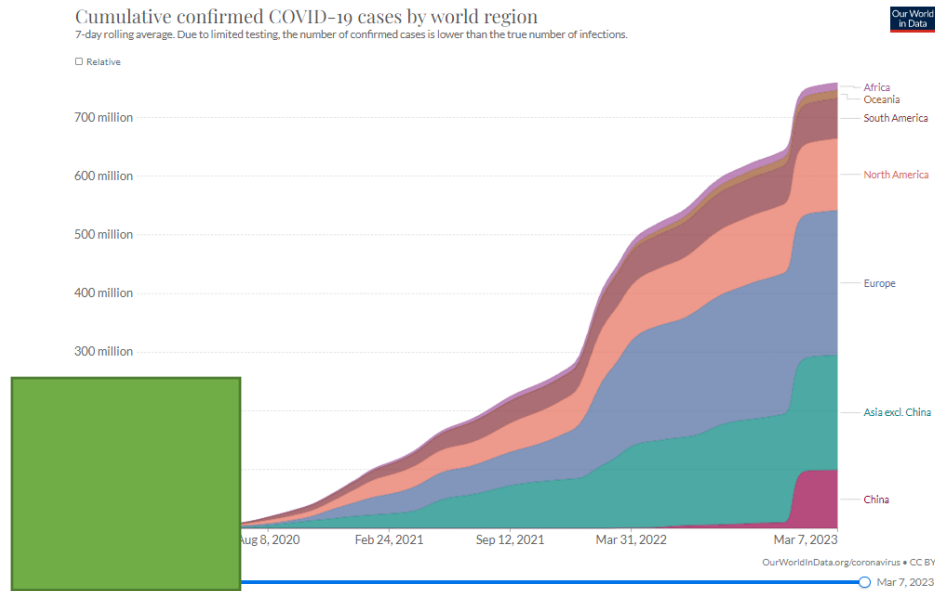
We will focus on the contagion mechanism today

# Which one is more difficult to spread in human network?

## Spread of COVID

vs

## Spread of hygiene, e.g., frequent handwashing



Let's try to find out:

Do viruses, information, behavioral change spread in the same way in social networks?

Can we depict the contagion process in numerical models?

# Today's programme

- **Simple contagion**  
Mechanism and the strength of 'weak ties'
- **Diffusion model for simple contagion**  
Independent cascade model
- **Complex contagion**  
Mechanism and the strength of 'strong ties'
- **Diffusion model for complex contagion**  
Threshold model

# Mechanism of simple contagion

- Single contact sufficient for transmission

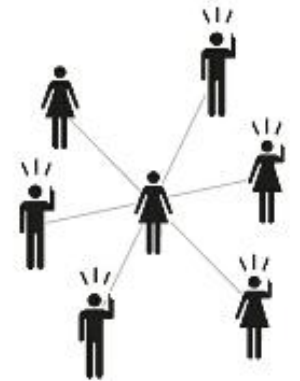
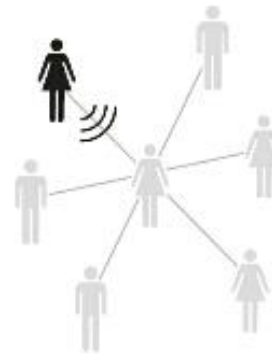


- Examples

Epidemic

Easily convincing rumors (one can costlessly repeat a story to many other)

Job information



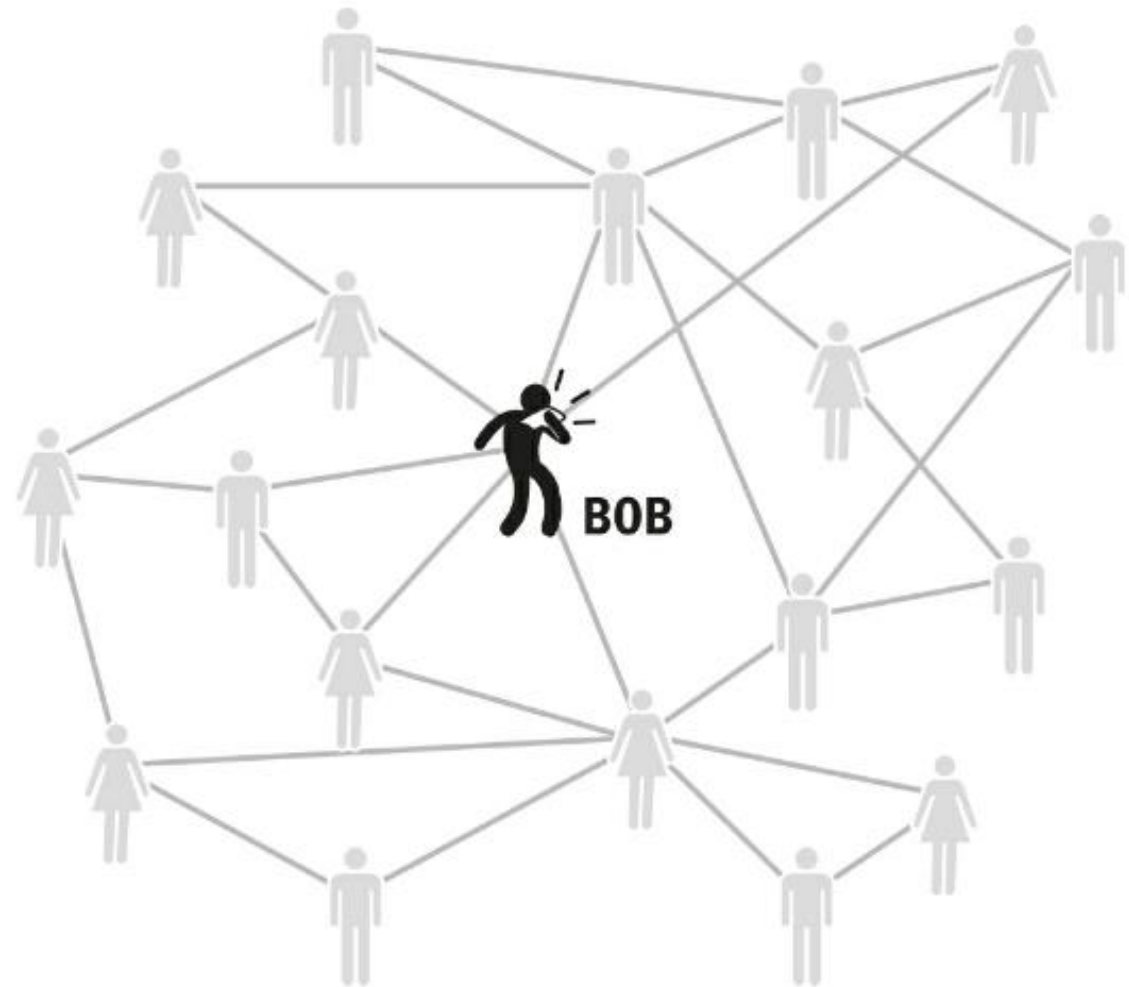
# Diffusion in social network under simple contagion

**Two states of people:**

**Inactivated**, susceptible to a contagion;

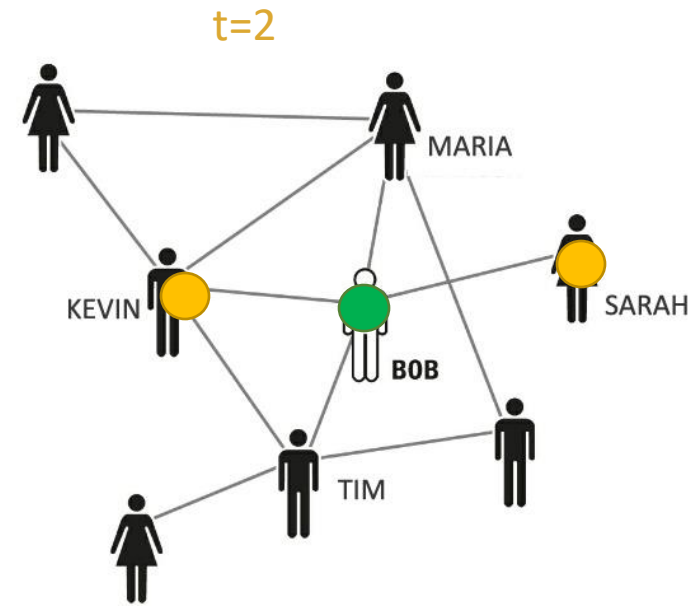
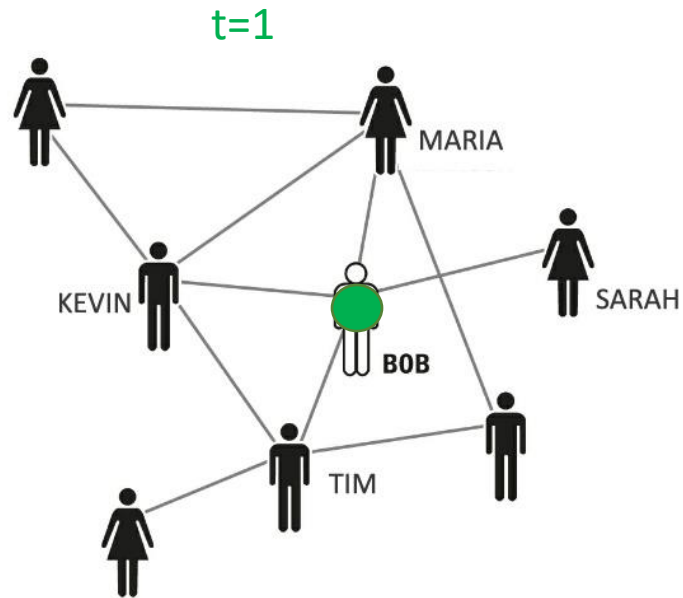
**Activated**, infected and can transmit the contagion to others

Bob is 'patient 0' and can pass the virus/information with probability  $\beta$



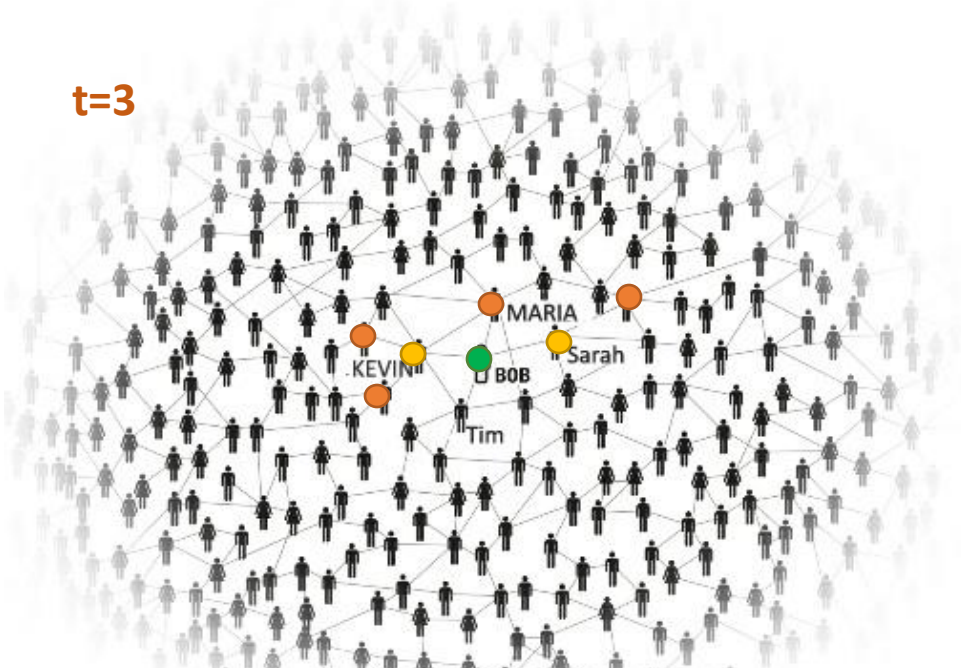
# Diffusion in social network under simple contagion

$\beta=50\%$

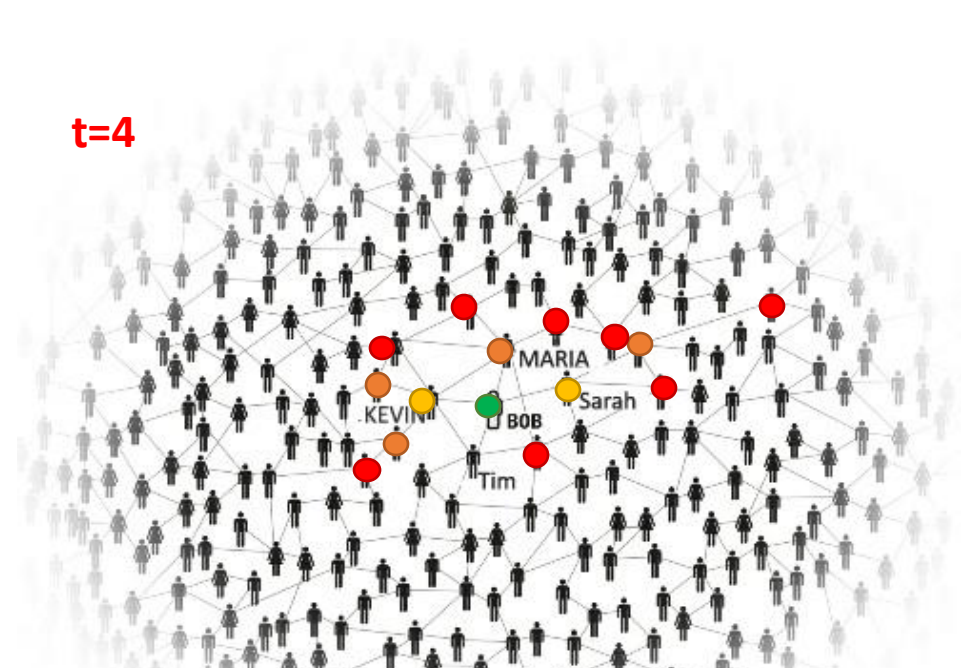




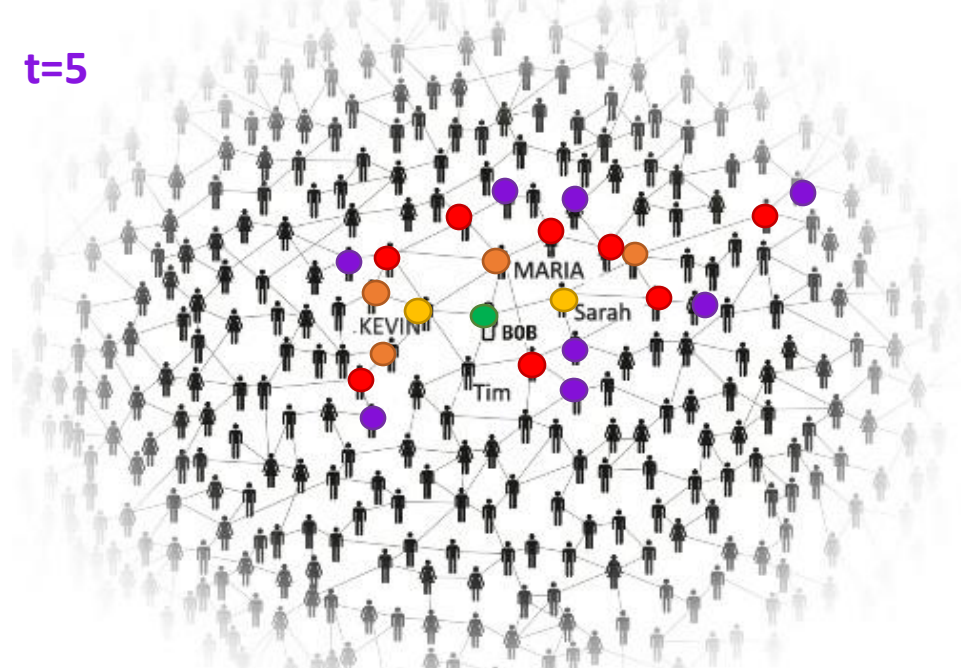
t=3



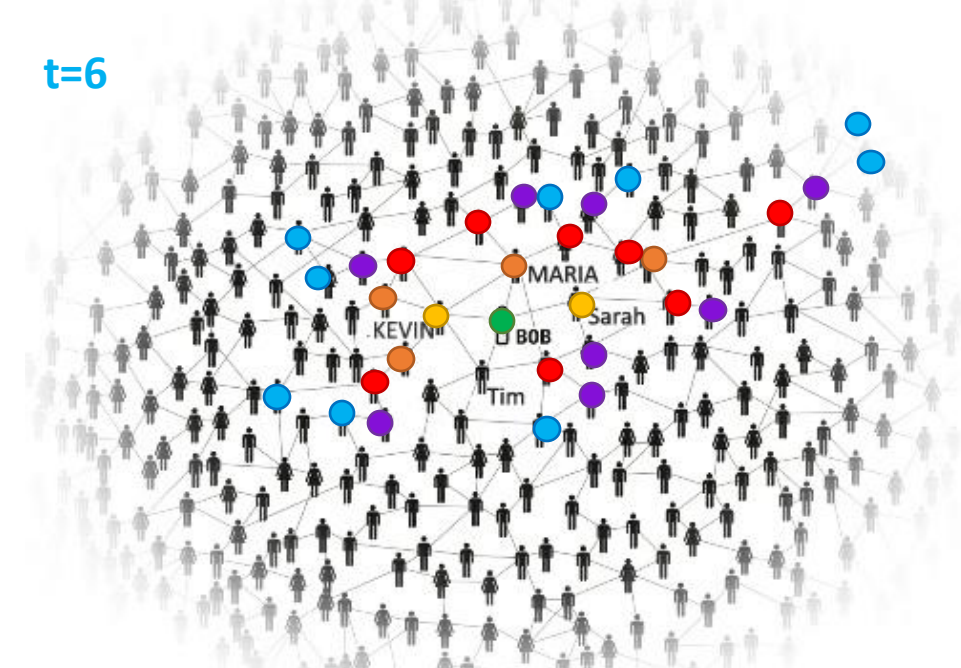
t=4



t=5



t=6





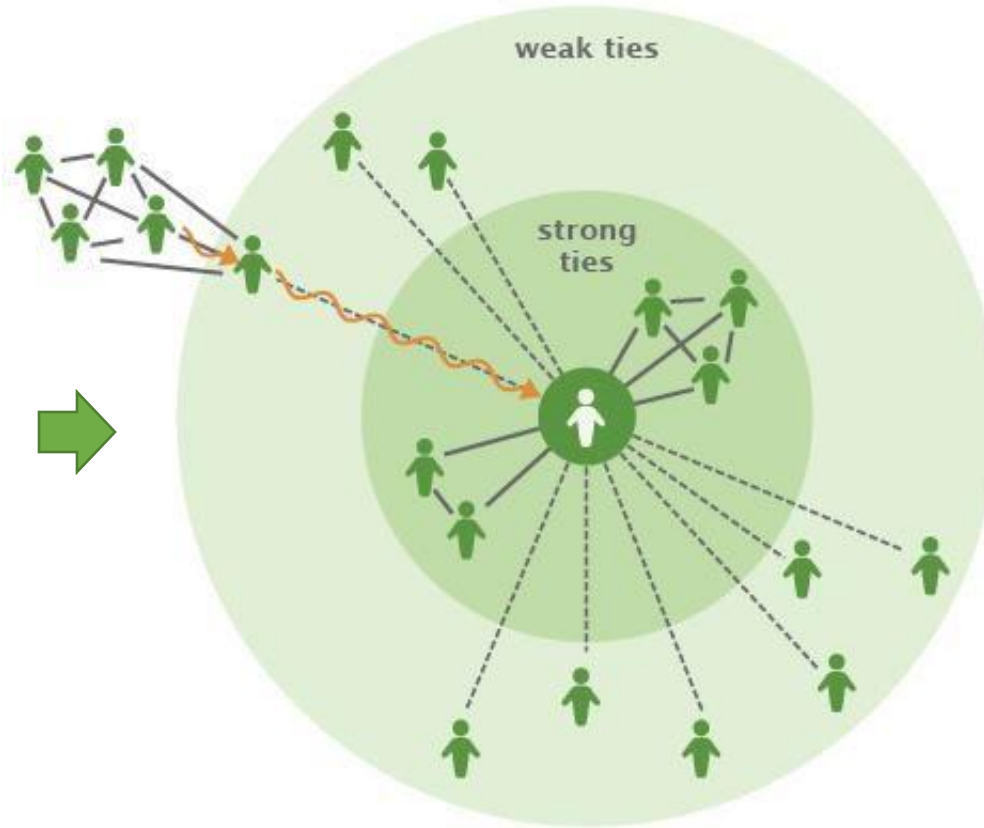
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Threshold model

# Weak and strong ties

Frequency of Contact

Emotional intensity and support



## Strong ties

Family, partner, close friends (Core discussion network)

## Weak ties

Distant friends, neighbors, colleagues

Interact less frequently, less invested in relationship

“The strength of weak ties”, Mark Granovetter, 1973

Most people got their current jobs through acquaintances rather than close friends



# Most people got their current jobs through acquaintances rather than close friends

- **Sample:**

A random sample of recent professional, technical and managerial job changers living in a Boston suburb, 1973

- **Procedure & Result:**

How often they saw the contact who communicated job offers to them:

Often (> twice a week), 16.7%

Occasionally ( more than once a year but less than twice a week), 55.6%

Rarely (once a year or less), 27.8%



***People that you don't meet often provide more efficient access to new information***

Mark. S. Granovetter, 1973. "The Strength of Weak Ties", American Journal of Sociology, 78 (6), 1360-1380.

# Is LinkedIn making you more successful?



- Mission of LinkedIn “connect the world’s professionals to make them more productive and successful.”
- Major social ties in LinkedIn: acquaintances or former colleagues
- Informational benefits from the usage of LinkedIn, Twitter, Facebook among a representative sample of Dutch online users
- Using LinkedIn significantly increased informational benefits. Using Twitter also resulted in a significant increase in informational benefits.
- Using Facebook resulted in significantly lower informational benefits.

LinkedIn, Facebook and Twitter:  
Weak or strong ties?

# Is LinkedIn making you more successful?



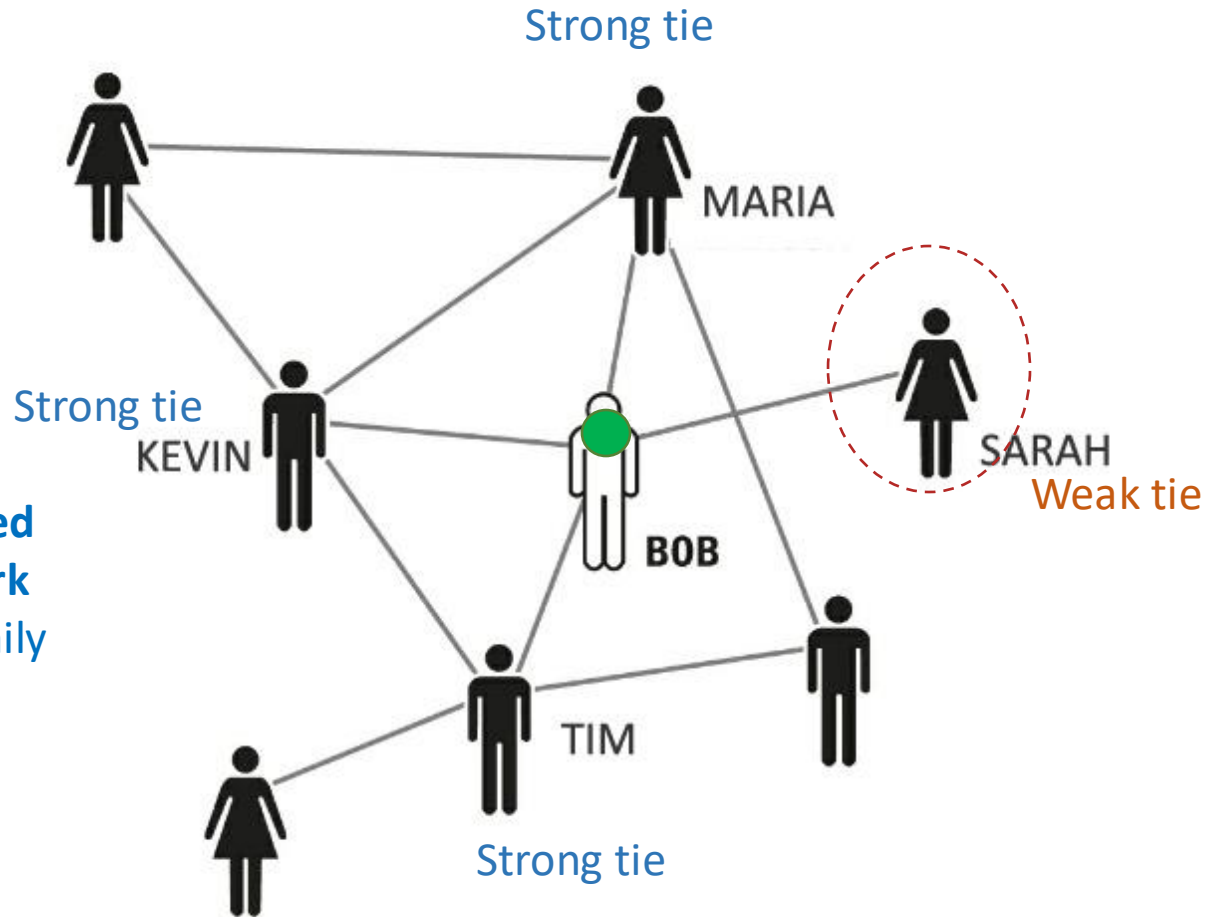
- Experiment with the “People You May Know”-algorithm
- Some receive more weak tie suggestions, others more strong tie suggestions.
- Weak (and medium) ties have the strongest effects on job transmission



Science, September 2022

# Why weak ties can accelerate information diffusion?

Among all the people connected to Bob, who is the weak tie?



A person's strong ties tend to be located within "closed triangles" in the network (your closed friends will know your family and some of your friends as well)

Now Bob needs to hire a new programmer and wants to use his “word of mouth” network to spread the news of job opening

1<sup>st</sup> round: Bob

2<sup>nd</sup> round: Bob → Kevin, Maria, Tim, Sarah

3<sup>rd</sup> round:

Kevin → Mila, Maria

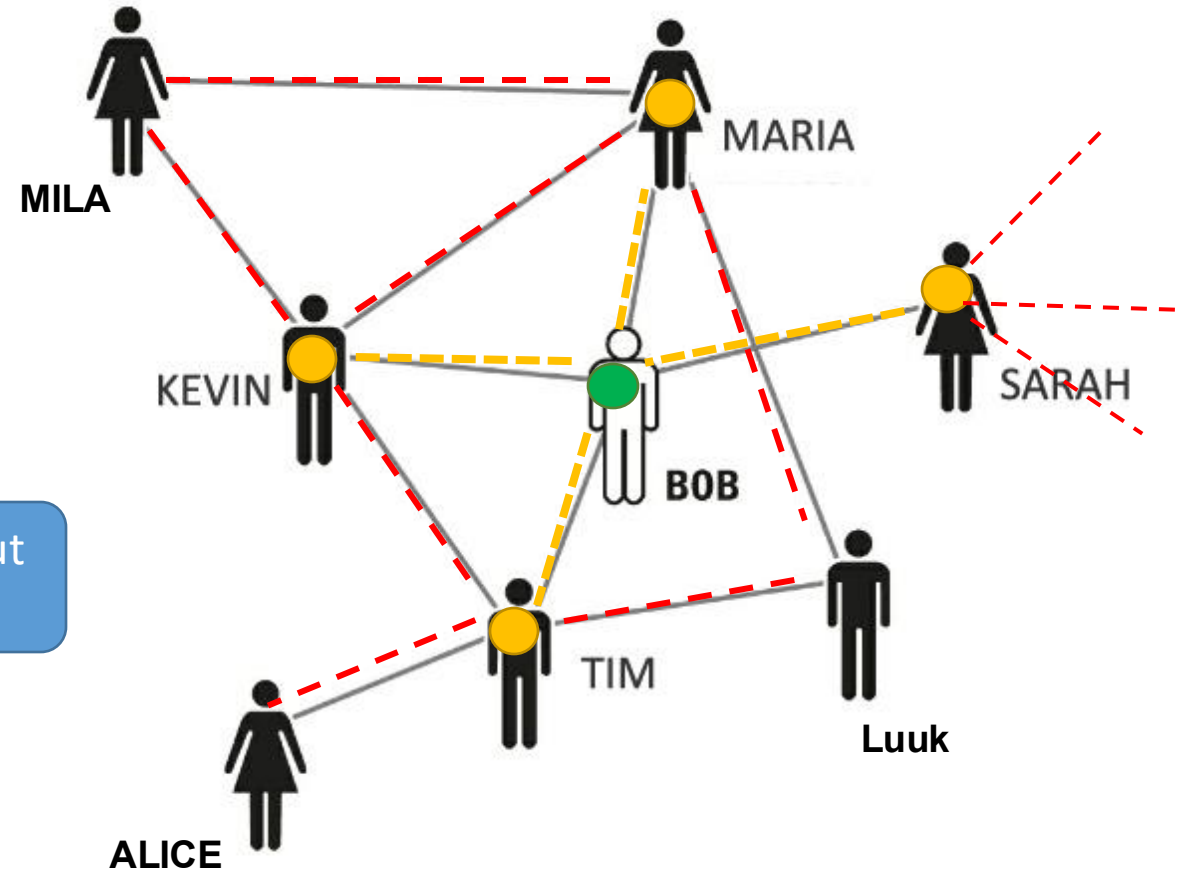
Maria → Kevin, Mila, Luuk

Tim → Kevin, Luuk, Alice

Sarah → to her other contacts

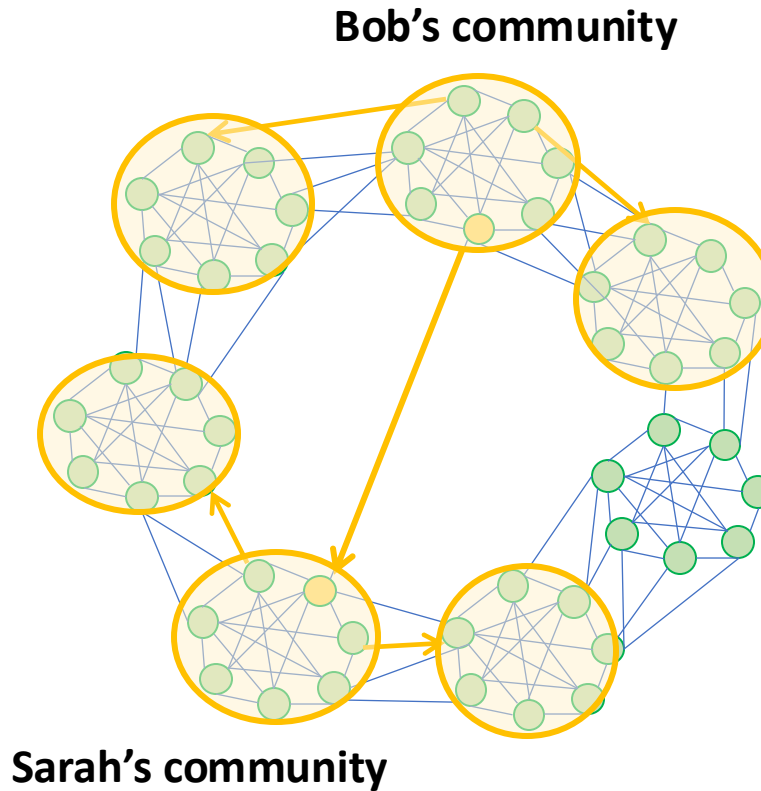
I've heard about it already!

A lot of *redundant information* within the close community of Bob, which is *not efficient*.





# Diffusion of simple contagion via weak tie



Under **simple contagion**, the single tie between Bob and Sarah **can** spread the rumor/virus to Sarah's community and fasten the diffusion process

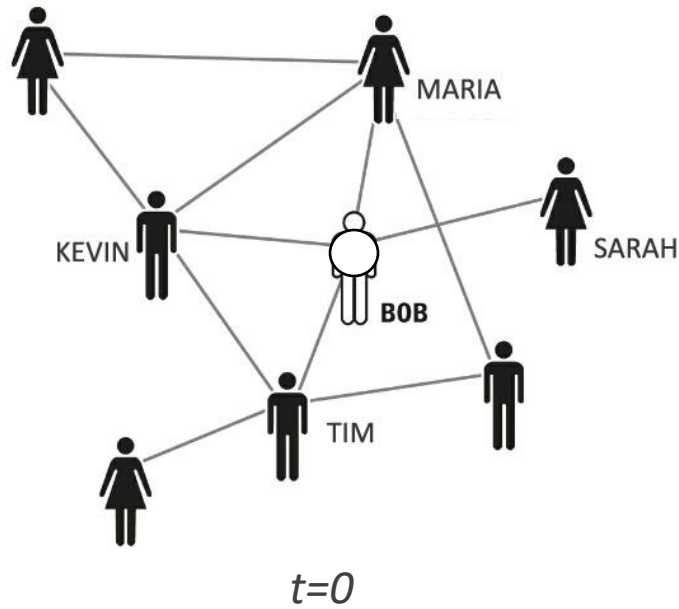
Now Bob tries out a new product and loves it.  
He shares with other friends.



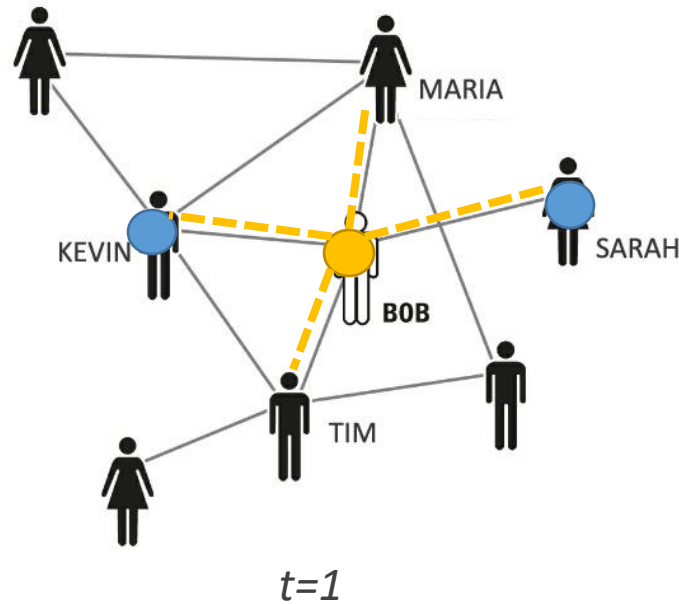
# Independent Cascade Model

- Nodes can have two states — active ( $S=1$ ) and inactive states ( $S=0$ ); once activated, can not be inactive again (e.g., Bob used the product)
- At time  $t=0$ ,  $k$  nodes are selected (i.e., activated). These nodes are called “*seed nodes*”
- When a node  $u$  is activated at time  $t$ , *it can activate neighbors  $v$  in subsequent time points* (Bob can talk about the same product with his friends again and again). The success depends on the probability  $p_{uv}$  assigned to the edge connecting  $u$  and  $v$ . ( $p_{uv}$  can be the same for every edge or different by edges)
- Stop when all the nodes are activated or the number of activated nodes is saturated.

$p_{uw} = 60\%$  for all edges

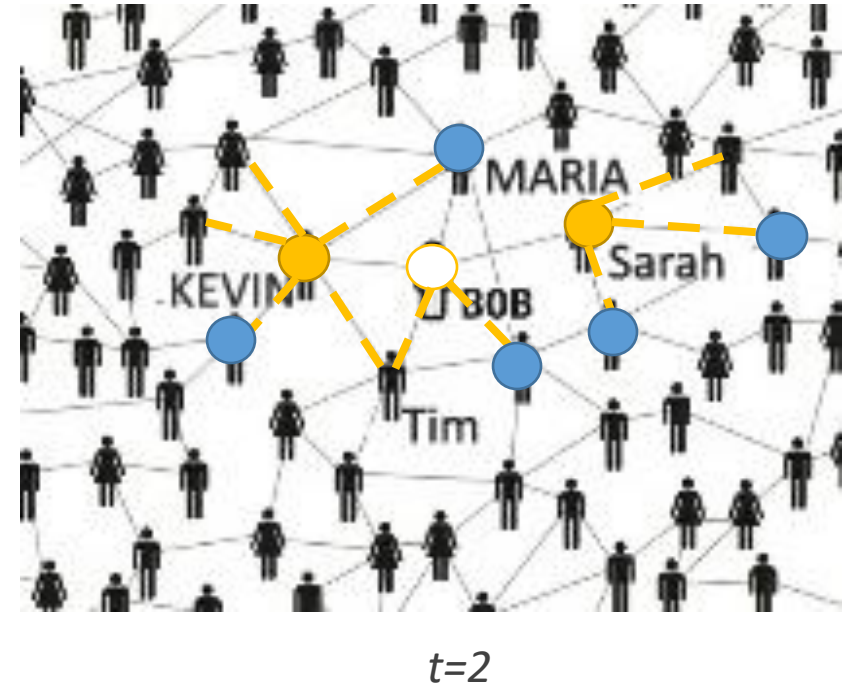


All nodes are inactive;  
Bob is activated.



Check out each neighbor of Bob:  
Generate a random number  $[0,1]$ , if smaller than 60%,  
activated; Otherwise, remains inactive.

Bob-Kevin: random number is 0.5, Kevin is activated;  
Bob-Tim: random number is 0.7, Tim is not activated;  
Bob-Sarah: random number is 0.1, Sarah is activated;  
Bob-Maria: random number is 0.9, Maria is not  
activated.



Bob and the activated neighbors Kevin  
and Sarah can now activate others again.

# Various other versions

- How long will somebody continue to convey the message: forever, one period, or something in between?
- Transition probabilities: homogeneous, heterogeneous?
- Epidemic models as the SIR model are also very similar:
  - **Susceptible (S)**: healthy people that can catch the virus from infected people, with probability  $\beta$ ;
  - **Infected (I)**: people who have been infected and are capable of infecting susceptible individuals;
  - **Recovered (R)**: people who have been infected and have recovered.

When will the strength of weak ties fail?  
(Are there any other mechanisms driving diffusion?)

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Independent cascade model and other variants
- **Complex contagion**  
Mechanism and the strength of 'strong ties'
- **Diffusion model for complex contagion**  
Threshold model