## Comparing formulae

Comparing path decomposition of  $T_{11}$  to analytical expressions ("harmonic decomposition") from Garcia-Suarez 2022 JMPS (can do up N=7, then it becomes too slow)

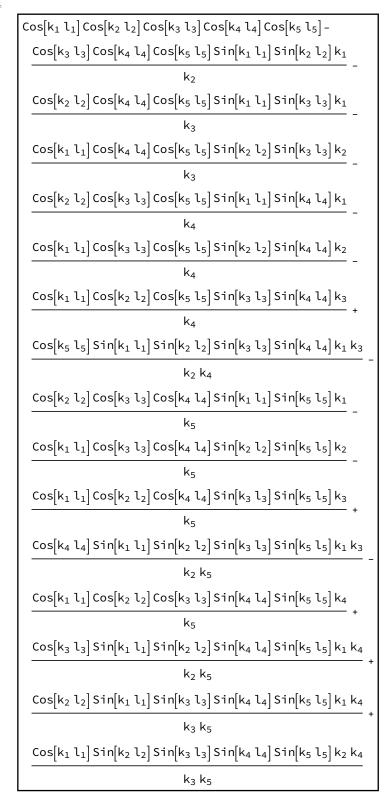
Number of layers

```
In[1]:= NN = 5;
```

## Harmonic decomposition

```
In[2]:= (*maximum number of tangents in any expression*)
maxNumberTerms = Floor[NN, 2];
(*number of factors in each group of expressions*)
numberTerms = 2 \pm \& /@ Range[\frac{Floor[NN, 2]}{2}];
(*number of addends belonging to each group*)
numberAddends = Binomial[NN, #] & /@ numberTerms;
indexVectors = If[NN > #[1]],
       Join[ConstantArray[1, #[1]], ConstantArray[0, NN - #[1]]],
       ConstantArray[1, #[1]]
      ] & /@ Transpose@{numberTerms, numberAddends};
indexSets = Flatten[Permutations[#] & /@ indexVectors, 1];
anTF = 1;
Do
   layers = Flatten@Position[indexSets[term], 1];
   Zs = k_{\sharp\sharp} \& /@ layers;
   sortedZs = Zs[\# ;; ; 2] \& /@ {1, 2};
   anTF = anTF + (-1)^{\text{Length[layers]/2}} * \left( \frac{\text{Times @@ sortedZs[1]}}{\text{Times @@ sortedZs[2]}} \right) \\ \text{Times @@ (Tan[k_{#} l_{#}] & /@ layers);}
   , {term, 1, Length@indexSets}];
anTF = (Times @@ (Cos[k_{\sharp} l_{\sharp}] \& /@ Range[NN])) * anTF;
oldMode = Expand[anTF];
Framed[oldMode]
```

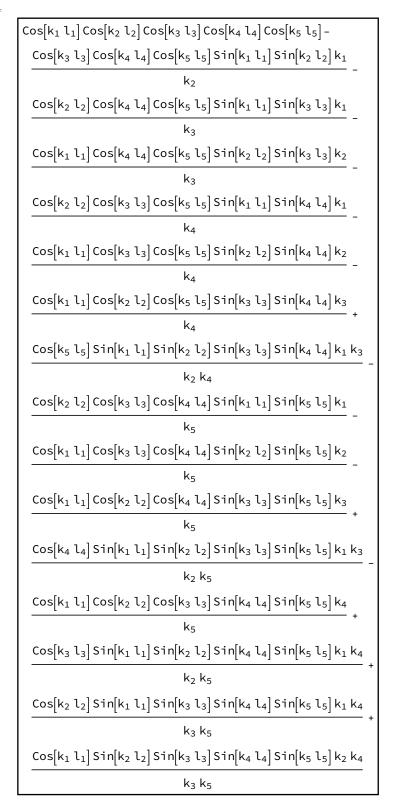
Out[11]=



## Path decomposition

```
In[12]:= ks = Table[Subscript[k, i], {i, 1, NN}];
 ls = Table[Subscript[l, i], {i, 1, NN}];
 (*1. Enumerate all directed paths: σ=±1 choice at each interface*)
  paths = Tuples[{-1, 1}, NN];
  paths = Select[paths, First[#] =!= -1 &];(*2^(NN-1) rows*)
 (*2. Amplitude coefficient A_j for each path-----*)
  amplitudes = Table
      \label{local_product_local_product_local} $$ \Pr \left[ 1/2 \left( 1 + paths[[p, i-1]] * paths[[p, i]] * ks[[i-1]] / ks[[i]] \right), (*interface factor*) $$ $$ \left( 1/2 \left( 1 + paths[[p, i-1]] * paths[[p, i]] * ks[[i-1]] / ks[[i]] \right). $$
       {i, 2, NN}], {p, Length[paths]}];
 cosineTerms = Cos[Dot[#, Table[kii lii, {ii, NN}]] & /@ paths];
  newMode = TrigExpand[Total[amplitudes * cosineTerms]];
  Framed[newMode]
```

Out[19]=



## Compare

In[20]:= Simplify[newMode - oldMode] Out[20]= 0