Utilizing analytical transfer functions to gauge the effect of velocity reversals

This notebook implements the calculations of the preprint entitled "Utilizing analytical transfer functions to gauge the effect of velocity reversals".

Preliminaries

Frequency range to take into consideration:

```
In[.]:= fmin = 0.01; fmax = 100;
```

```
In[.]:= (*Frequency interval*)
flist = Subdivide[fmin, fmax, 2000];
```

The parameters of the simulations

In[.]:= $\text{anTF} = \text{Cos}[r_1]\,\text{Cos}[r_2]\left(1 - \frac{\sqrt{\mu_1\,\rho_1}\ \text{Tan}[r_1]\ \text{Tan}[r_2]}{\sqrt{\mu_2\,\rho_2}} - \frac{i\left(-\sqrt{\mu_1\,\rho_1}\ \text{Tan}[r_1] - \sqrt{\mu_2\,\rho_2}\ \text{Tan}[r_2]\right)}{\sqrt{\mu_{\text{half}}\,\rho_{\text{half}}}}\right);$

Implementing the layer matrices

This list, when evaluated (see that as per definition the evaluation is delayed), generates the matrices for a given set of impedances ("alphas"), layer heights ("hs") and shear-wave velocities ("Vs").

The matrices relate the ground displacement to the displacement at the base of the last layer, the last one being the last one being the half-space.

In[.]:= $$\text{Llist} := \text{Table}\Big[\Big\{\Big\{\frac{1}{2}\left(1 + \text{alphas}[[jj]]\right) * \text{Exp}\Big[i * \frac{\omega * \text{hs}[[jj]]}{\text{Vs}[[jj]]}\Big],\ \frac{1}{2}\left(1 - \text{alphas}[[jj]]\right) * \text{Exp}\Big[-i * \frac{\omega * \text{hs}[[jj]]}{\text{Vs}[[jj]]}\Big]\Big\},$$
$$\Big\{\frac{1}{2}\left(1 - \text{alphas}[[jj]]\right) * \text{Exp}\Big[i * \frac{\omega * \text{hs}[[jj]]}{\text{Vs}[[jj]]}\Big],\ \frac{1}{2}\left(1 + \text{alphas}[[jj]]\right) * \text{Exp}\Big[-i * \frac{\omega * \text{hs}[[jj]]}{\text{Vs}[[jj]]}\Big]\Big\}\Big\},\ \{jj, 1, \text{Length}[\text{alphas}]\}\Big];$$

The parameters of the simulations

In[•]:=
```
Rho[V_] := 1800 + ((V - 200)/700) * 300;
dists = {{1, 4}, {2, 3}, {3, 2}, {4, 1}};
Vup = Range[400, 800, 100];
Vdown = Range[300, 800, 50];
```

In[•]:=
```
(*Bedrock Properties*)
Vhalf = 900;
rhohalf = Rho[Vhalf];
muhalf = rhohalf * Vhalf^2;
```

Computations

In[•]:=
```
NN = 2;
DE = 0.1;
dataTripletsNew = {};
AuxTableFiguresNew = {};
Do[(*kk: For each type thicknesses' combo*)
 Do[(*jj: For each VsL1 value*)
  Do[(*ii: For each VsL2 value*)
   (*Compute TF---------------------------------------*)
   (*Layer depth, average depth and thickness*)
   Depths = {dists[[kk]][[1]], dists[[kk]][[2]]} * 10.;
   hs = dists[[kk]] * 10;
   SiteName = "L1=" <> ToString[10 * dists[[kk]][[1]]] <>
     "m, L2=" <> ToString[10 * dists[[kk]][[2]]] <> "m, VsL1=" <>
     ToString[Vup[[jj]]] <> "m/s, VsL2=" <> ToString[Vdown[[ii]]] <> "m/s";
   Htotal = Total[hs]; (*always equal to 100 in this case*)
   (*The list is ordered from free surface to bottom*)
   (*Layer shear-wave velocities, densities, impedances and fundamental
    period estimate ------------------------------------------------*)
   Vs = {Vup[[jj]], Vdown[[ii]]};
   rhos = Rho[#] & /@ Vs;
   mus = Table[rhos[[uu]] * Vs[[uu]]^2, {uu, 1, Length[Vs]}];
   (*Frequency interval*)
   flist = Subdivide[fmin, fmax, 2000];
   alphas = Flatten@{(rhos[[1]] * Vs[[1]])/(rhos[[2]] * Vs[[2]]), (rhos[[2]] * Vs[[2]])/(rhohalf * Vhalf)};
   (*Kramer's*)
   L[ω_] = Llist[[2]].Llist[[1]];
   (*For[uu=2,uu≤Length[alphas],uu++,L[ω_]=Flatten[Llist[[uu]],1].L[ω]];*)
   TF[ω_] = 2/(L[ω][[1, 1]] + L[ω][[1, 2]]);
   Vbase = Vs[[-1]];
   TFKramer =
```

```
 Table[{flist[[uu]], Abs[TF[(2*π)/(Sqrt[1 + i*DE]) flist[[uu]]]]}, {uu, 1, Length@flist}];
(*plotList=AppendTo[plotList,TFKramer];*)
(*Compute Transfer Function (Aki and Richard's) --
 -----------------------------------*)
layerMatrix = hs[[#]] * {{0., 1/(mus[[#]] * (1 + i * DE))}, {-ω^2 * rhos[[#]], 0.}} & /@
  Range[Length@hs];
(*Layer matrices*)
exactExpFreq = {};
Do[
 (*Create the exponential matrices
  for each layer and put them in order to multiply them*)
 expMatList = func[layerMatrix[[#]] /. ω → 2 * π * flist[[ii]]] & /@
    Range[Length@hs] /. func → MatrixExp;
 (*Proceed with the multiplication*)
 exactExp = expMatList[[2]].expMatList[[1]];
 (*Do[exactExp=exactExp.expMatList[[jj]],
   {jj,2,Length@hs}];*)
 (*Add to the list of values*)
 exactExpFreq = AppendTo[exactExpFreq,
   {flist[[ii]], 2/(Abs[exactExp[[1, 1]] - i * (exactExp[[2,1]])/(2 π*flist[[ii]]*Sqrt[muhalf*rhohalf])])}]
 , {ii, 1, Length@flist}];
(*Compute and Evaluate Analytical ---------*)
(*Prepare to evaluate numerically*)
auxVar = (((anTF /. r_1 → (2 π * f * hs[[1]])/(Vs[[1]] Sqrt[1 + i * DE])) /. μ_1 ρ_1 → (rhos[[1]] * Vs[[1]])^2) /.
  μ_half ρ_half → (rhohalf * Vhalf)^2;
Do[
 auxVar2 = auxVar;
 auxVar = (auxVar2 /. r_i → (2 π * f * hs[[i]])/(Vs[[i]] Sqrt[1 + i * DE])) /. μ_i ρ_i → (rhos[[i]] * Vs[[i]])^2;
 , {i, 2, NN}];
auxVar = Expand[auxVar];
(*Make table evaluating numerically*)
myTF = Table[{flist[[k]], 2/(Abs[auxVar /. f → flist[[k]]])}, {k, 1, Length@flist}];
(*Save data*)
peak = Max[myTF[[All, 2]]];
dataTripletsNew = AppendTo[dataTripletsNew, {hs[[1]]/hs[[2]], (rhos[[2]] * Vs[[2]])/(rhohalf * Vhalf), peak}];
(*Plot Transfer
  function -----------------------------------------------------------------*)
TFPlot = ListLogLinearPlot[{
   TFKramer,
   exactExpFreq,
   myTF},
```

```
      PlotRange → {{fmin, fmax}, {0, 6}},
      Axes → False,
      PlotStyle → {{Automatic, Thickness[0.012]},
        {Red, Dotted, Thickness[0.015/3]}, {Orange, Dashed, Thickness[0.015/3]}},
      Frame → {{True, True}, {True, True}},
      Joined → True,
      FrameLabel → {{"u_top/S_i", None}, {"freq.[Hz]", Style[SiteName, FontSize → 8]}},
      PlotLegends → Placed[
        LineLegend[{"Kramer", "A&R", "Analytical"}, LegendLayout → "Column"], {0.2, Top}],
      RotateLabel -> False,
      ImageSize → Small,
      AspectRatio → 1];
    AuxTableFiguresNew = AppendTo[AuxTableFiguresNew, TFPlot];
    (*AuxTableFigures[[kk,jj]][[ii]]=TFPlot;*)
    , {ii, 1, 1 + 2 * jj}]
   , {jj, 1, Length@Vup}]
  , {kk, 1, Length@dists}]
```

AuxTableFiguresNrew contains all the images of transfer functions.

Verifying the lower bound:

```
In[•]:= Vslone = Table[ConstantArray[Vup[[ii]], 1 + 2 ii], {ii, 1, 5}];
impedanceList = N[(Rho[#] * #)/(Sqrt[rhohalf * muhalf])] & /@ Flatten[Vslone];
listEI = Flatten[Join[impedanceList, impedanceList, impedanceList, impedanceList]];

In[•]:= scalingPoints = Table[{listEI[[ii]], dataTripletsNew[[ii]][[3]]}, {ii, 1, 140}];

In[•]:= listColors = {Blue, Red, Green, Yellow, Gray};
colors = Table[ConstantArray[listColors[[ii]], 1 + 2 ii], {ii, 1, 5}];
colorList = Flatten[Join[colors, colors, colors, colors]];
listPoints2 = Table[{colorList[[ii]], Point[scalingPoints[[ii]]]}, {ii, 1, 140}];
```

Verifying the upper bound:

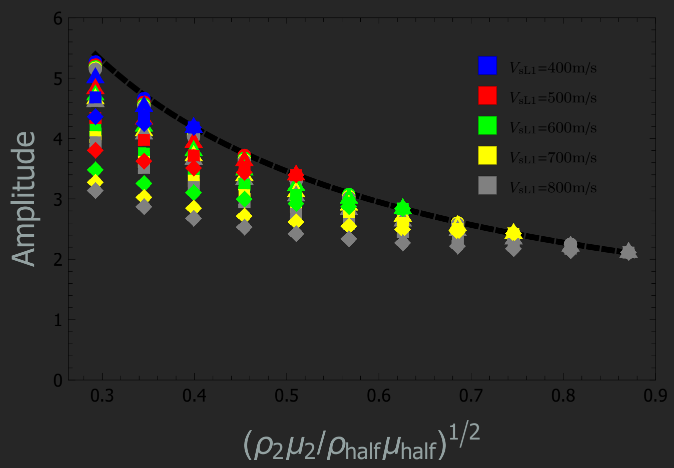
```
In[•]:= impedanceList2 = {};
Do[
 Do[
  AppendTo[impedanceList2, N[(Rho[Vdown[[jj]]] * Vdown[[jj]])/(rhohalf * Vhalf)]]
  , {jj, 1, 1 + 2 * ii}]
 , {ii, 1, Length@Vup}]

In[•]:= listZs = Flatten[Join[impedanceList2, impedanceList2, impedanceList2, impedanceList2]];

In[•]:= scalingPoints2 = Table[{listZs[[ii]], dataTripletsNew[[ii]][[3]]}, {ii, 1, 140}];

In[•]:= listPoints3 = Table[{colorList[[ii]], Point[scalingPoints2[[ii]]]}, {ii, 1, 140}];
```

```
In[•]:= listPoints3 = Table[{colorList[[ii]], Point[scalingPoints2[[ii]]]}, {ii, 1, 140}];
```

Images

```
In[•]:= (*Load the package code*)package = Import[
           "http://raw.github.com/AlexeyPopkov/PolygonPlotMarkers/master/PolygonPlotMarkers.m",
           "Text"];

        (*Install the package (existing file will be overwritten!)*)
        Export[FileNameJoin[{$UserBaseDirectory, "Applications", "PolygonPlotMarkers.m"}],
          package, "Text"];

In[•]:= Needs["PolygonPlotMarkers`"]

In[•]:= openForms =
          Graphics[{EdgeForm[], PolygonMarker[#, Offset[6]]}, AlignmentPoint → {0, 0}] & /@
           {"Circle", "Triangle", "Square", "Diamond"};
        markers = Table[ConstantArray[openForms[[ii]], 35], {ii, 1, 4}];
        markerList = Flatten[markers];

In[•]:= figUB = Show[
          Plot[2/(x + π/4 DE), {x, Min@scalingPoints2[[All, 1]], Max@scalingPoints2[[All, 1]]},
           PlotRange → {{0.9 * Min@scalingPoints2[[All, 1]], .9}, {0, 6}},
           PlotStyle → {Black, Dashed, Thickness[0.01]},
           Axes → False,
           Frame → True,
           FrameLabel → {{Style["Amplitude", FontSize → 16], ""},
             {Style["(ρ2μ2/ρhalfμhalf)^1/2", FontSize → 16], ""}},
           Epilog → Inset[SwatchLegend[{Blue, Red, Green, Yellow, Gray},
              Style["VsL1=" <> #, FontFamily → "LM Roman 8", FontSize → 8] & /@
               {"400m/s", "500m/s", "600m/s", "700m/s", "800m/s"}], Scaled[{0.8, 0.7}]]],
          ListPlot[{#} & /@ scalingPoints2, AspectRatio → 1, PlotStyle → colorList,
           PlotMarkers → markerList]]
```

Out[•]=




```
In[•]:= SystemOpen[DirectoryName[AbsoluteFileName["lower_bound_v2.pdf"]]]
```

In[∘]:=

```
figLB = Show[
  Plot[2/(x + π/4 DE), {x, Min@scalingPoints[[All, 1]], Max@scalingPoints[[All, 1]]},
   PlotRange → {{0.9 * Min@scalingPoints[[All, 1]], .9}, {0, 6}},
   PlotStyle → {Black, Dashed, Thickness[0.01]},
   Axes → False,
   Frame → True,
   FrameLabel → {{Style["Amplitude", FontSize → 16], ""},
     {Style["(ρ1μ1/ρhalfμhalf)^1/2", FontSize → 16], ""}},
   Epilog → Inset[SwatchLegend[{Blue, Red, Green, Yellow, Gray},
      Style["VsL1=" <> #, FontFamily → "LM Roman 8", FontSize → 8] & /@ {"400m/s", "500m/s",
        "600m/s", "700m/s", "800m/s"}, LegendLayout → "Row"], Scaled[{0.5, 0.15}]]],
  ListPlot[{#} & /@ scalingPoints, AspectRatio → 1, PlotStyle → colorList,
   PlotMarkers → markerList]]
```

Out[∘]=