Symmetric STDP

$$f(x) = e^{-\frac{|x|}{\tau_1}} \cos^2\left(\frac{x}{\tau_1}\right) - e^{-\frac{|x|}{\tau_2}} \sin^2\left(\frac{x}{\tau_2}\right) =$$

$$= \frac{\overbrace{e^{-\frac{|x|}{\tau_1}}}^{a(x)} + \overbrace{e^{-\frac{|x|}{\tau_1}} \cos\left(2\frac{x}{\tau_1}\right)}^{b(x)} - e^{\frac{|x|}{\tau_2}} + e^{\frac{|x|}{\tau_2}} \cos\left(2\frac{x}{\tau_2}\right)}{2}$$

$$a(x) = e^{-\frac{|x|}{\tau_1}} \Rightarrow a(0) = 1$$

$$a(x+\Delta t) = a(x) \cdot e^{-\frac{|\Delta t|}{\tau_1}}$$

$$b(x) = e^{-\frac{|x|}{\tau_1}} \cos\left(2\frac{x}{\tau_1}\right) \Rightarrow b(0) = 1$$

$$b(x+\Delta t) = b(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \cos\left(2\frac{\Delta t}{\tau_1}\right) - c(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \sin\left(2\frac{\Delta t}{\tau_1}\right)$$

$$c(x) = e^{-\frac{|x|}{\tau_1}} \sin\left(2\frac{x}{\tau_1}\right) \Rightarrow c(0) = 0$$

$$c(x+\Delta t) = c(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \cos\left(2\frac{\Delta t}{\tau_1}\right) + b(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \sin\left(2\frac{\Delta t}{\tau_1}\right)$$

$$f(x) = C \cdot e^{-\frac{|x|}{\tau_1}} \cos^2\left(\frac{x}{\tau_1} \cdot \frac{\pi}{2}\right) - L \cdot A \cdot e^{-\frac{2|x|}{\tau_2}} \cdot \sin^2\left(\frac{x}{\tau_2} \cdot \frac{\pi}{2}\right)$$

$\tau_1 \Rightarrow$ Peak distance

$C \Rightarrow$ central peak amplitude

$L \Rightarrow$ Lateral peak amplitude

$$\tau_2 = \frac{\tau_1}{\operatorname{atan}\left(\frac{\pi}{2}\right) \cdot \frac{2}{\pi}}$$

$$A = \frac{1}{e^{-\operatorname{atan}\left(\frac{\pi}{2}\right) \cdot 4/\pi} \cdot \sin^2\left(\operatorname{atan}\left(\frac{\pi}{2}\right)\right)}$$

$$\boxed{a(x) = e^{-\frac{|x|}{\tau_1}}} \Rightarrow \boxed{a(x+\Delta t) = e^{-\frac{|x+\Delta t|}{\tau_1}} = e^{-\frac{|x|}{\tau_1}} e^{-\frac{|\Delta t|}{\tau_1}} = a(x) \cdot e^{-\frac{|\Delta t|}{\tau_1}}}$$

$$\boxed{b(x) = e^{-\frac{|x|}{\tau_1}} \cos\left(2\frac{x}{\tau_1}\right)} \Rightarrow b(x+\Delta t) = e^{-\frac{|x+\Delta t|}{\tau_1}} \cdot \cos\left(2\,\frac{x+\Delta t}{\tau_1}\right) =$$

$$= e^{-\frac{|x|}{\tau_1}} \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \left(\cos\left(2\frac{x}{\tau_1}\right) \cdot \cos\left(2\frac{\Delta t}{\tau_1}\right) - \sin\left(2\frac{x}{\tau_1}\right) \cdot \sin\left(2\frac{\Delta t}{\tau_1}\right)\right) =$$

$$= \boxed{b(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \cos\left(2\frac{\Delta t}{\tau_1}\right) - c(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \sin\left(2\frac{\Delta t}{\tau_1}\right)}$$

$$\boxed{c(x) = e^{-\frac{|x|}{\tau_1}} \sin\left(2\frac{x}{\tau_1}\right)} \Rightarrow c(x+\Delta t) = e^{-\frac{|x+\Delta t|}{\tau_1}} \sin\left(2\,\frac{x+\Delta t}{\tau_1}\right) =$$

$$= e^{-\frac{|x|}{\tau_1}} \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \left(\sin\left(\frac{2x}{\tau_1}\right) \cos\left(2\frac{\Delta t}{\tau_1}\right) + \cos\left(\frac{2x}{\tau_1}\right) \cdot \sin\left(\frac{2\Delta t}{\tau_1}\right)\right) =$$

$$= \boxed{c(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \cos\left(2\frac{\Delta t}{\tau_1}\right) + b(x) \cdot e^{-\frac{\Delta t}{\tau_1}} \cdot \sin\left(2\frac{\Delta t}{\tau_1}\right)}$$