Cosmology with Astropy

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The Expansion of the Universe

- Hubble's law: $v = \mathrm{H}_0 D$
- Hubble's constant:

$$\mathrm{H}_0 = 100 \times h \mathrm{km/s/Mpc}, h \approx 70 \mathrm{km/s/Mpc}$$

- Scale factor: $r(t) = a(t)l$
- Hubble parameter:

$$\mathrm{H}(t) = \frac{\dot{a}}{a}$$

- Redshift:

$$1 + z = \frac{a_0}{a(t)} = \frac{\lambda_0}{\lambda(t)}$$

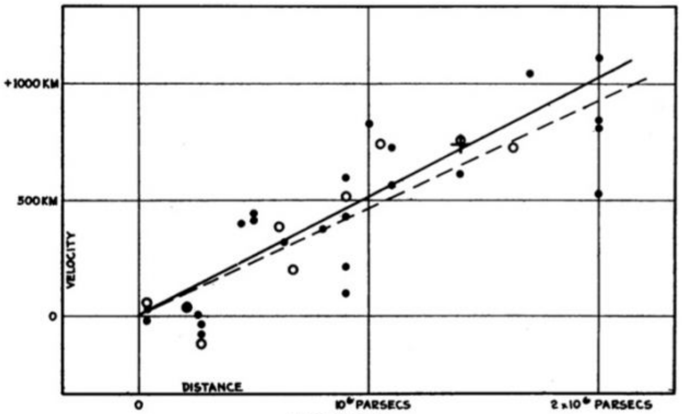
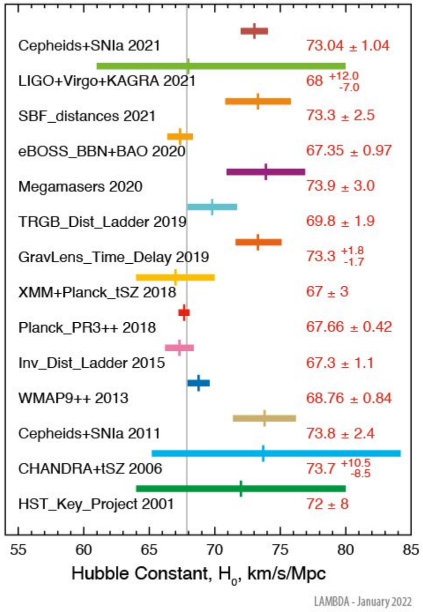


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.



The Friedmann Equations

- The Friedmann equation:

$$\mathrm{H}^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

- The Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho\frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- The Fluid equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

Density Parameters

- Critical density: $\rho_c(t) = \dfrac{3\mathrm{H}^2(t)}{8\pi G} = 1.88 \times 10^{-26} h^2 \mathrm{kg/m^3}$
- Density parameters:

$$\Omega_X(t) = \frac{\rho_X(t)}{\rho_c(t)}$$

$$\Omega_{M,0} \approx 0.310, \Omega_{\Lambda,0} \approx 0.689, \Omega_{\gamma,0} \approx 5.40 \times 10^{-5}, \Omega_{k,0} \approx 0.0$$

- Reworked Friedmann equation:

$$\Omega_M(z) + \Omega_\gamma(z) + \Omega_\Lambda(z) + \Omega_k(z) = 1$$

$$\mathrm{H}(z) = \mathrm{H}_0\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{\gamma,0}(1+z)^4 + \Omega_{\Lambda,0} + \Omega_{k,0}(1+z)^2}$$

The Equation of State

- Equation of State: $p = w\rho c^2$
- First linear order expansion:

$$w(a) = w_0 + w_a(1-a) = w_0 + w_a\frac{z}{1+z}$$

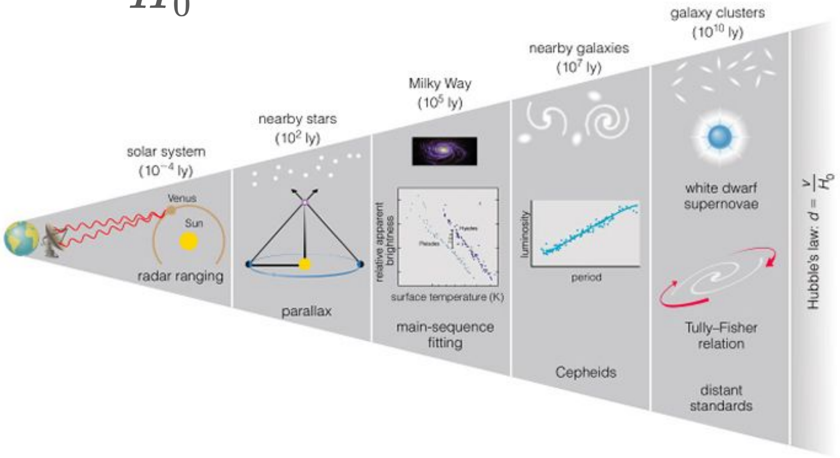
Value	Energy density scaling	Time scaling	Phenomena described
$w = 1$	$\rho \propto a^{-6}$	$a \propto t^{1/3}$	Free scalar field
$w = 1/3$	$\rho \propto a^{-4}$	$a \propto t^{1/2}$	Ultra-relativistic particles
$w = 0$	$\rho \propto a^{-3}$	$a \propto t^{2/3}$	Non-relativistic particles
$w = -1/3$	$\rho \propto a^{-2}$	$a \propto t$	Curvature
$w = -2/3$	$\rho \propto a^{-1}$	$a \propto t^2$	-
$w = -1$	$\rho \propto a^0$	$a \propto e^{Ht}$	Cosmological constant
$w < -1$	-	-	Phantom energy

Supernovae: standard candles

- End of star's lifetime: "explosion"
- Different types: spectroscopic classification
- SN Ia
  - Binary system: white dwarf + companion
  - WD accretes material until 1.4 solar masses (electron degeneracy pressure insufficient to stabilize WD)
  - Thermonuclear runaway
  - **Mass identical = energy release identical (≈ e+44 J) = absolute brightness identical**

Distance measures

- Luminosity distance: $d_L = (1+z)d_M(z) = \sqrt{\dfrac{L}{4\pi F}}$
- Comoving distance: $d_C(z) = d_H \displaystyle\int_0^z \frac{dz'}{E(z')}$
- Hubble distance: $d_H = \dfrac{c}{H_0}$



https://www.daviddarling.info/encyclopedia/C/cosmic_distance_ladder.html

Distance Modulus & Magnitudes

- Apparent & Absolute Magnitudes: neg. log-scale brightness
- Observed quantity: flux (in a certain wavelength range = band)

$$m_x = -2.5 \log_{10}\left(\frac{F_x}{F_{x,0}}\right) \qquad \mu = m - M = 5\mathrm{mag} \cdot \log_{10}\left(\frac{r}{10\mathrm{pc}}\right)$$

Star/Object	Apparent Magnitude (*m*)	Absolute Magnitude (*M*)	Distance modulus (*m* – *M*)	Distance
Sun	−26.832 mag	+4.84 mag	−31.57	1 AE
Sirius	−1.46 mag	+1.43 mag	−2.89	2.64 pc = 8.6 ly
Vega	+0.03 mag	+0.58 mag	−0.55	7.75 pc
SN Ia		-19.3 mag		
Milky Way		-20.8 mag		