Question 1

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   "John is not 6 feet tall, or he weighs less than 200 pounds."
2. $-1 < x \leq 4$.
   $x \leq -1$ or $x > 4$. ($\vee$ may be used instead of "or")

Question 2

Show that the following statements P and Q are logically equivalent:

$$P : \sim (\sim p \wedge q) \wedge (p \vee q) \wedge q$$

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$$Q : \ p \wedge q$$

p	q	P	Q
T	T	T	T
T	F	F	F
F	F	F	F
F	F	F	F

or,

$$P \equiv (q \to p) \wedge (p \vee q) \wedge q \equiv (q \to p) \wedge q \equiv p \wedge q = Q$$

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   "My car is in the repair shop and I can get to class."
2. "If Sara lives in Athens, then she lives in Greece."
   "Sara lives in Athens, but not in Greece."

Question 4

Rewrite the following statements in the form "If A then B":

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2. George's attaining age 35 is a necessary condition for his being president of the U.S.
   If (George is president of the U.S.), then (George has attained age 35).

Question 5

Is the following argument valid or invalid?

$$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \rightarrow r$$

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But then, $p \vee q$ cannot be true. Contradiction!

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Assume that it's invalid. Then,

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- $\sim q$ (by $q \rightarrow r$ and $\sim r$)

But then, $p \vee q$ cannot be true. Contradiction!
So, our assumption that the argument invalid is wrong, so the argument is valid.

Question 6

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   Here, $\mathbb{P}$ represents the set of all prime numbers.
2. There exists a triangle T such that the sum of the angles of T equals 200 degrees.
   Let $\mathbb{T}$ be the set of all triangles. "$\forall T \in \mathbb{T}$, the sum of the angles of $T \neq 200^\circ$."
   The negation is true, the sum of the angles on a triangle is 180° for all triangles.
   (Correction 29/8/12: Used more formal mathematical language.)

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Rewrite the following statements as quantified conditional statements.

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1. Squareness is a sufficient condition for rectangularity.
   For all polygons P, P is a square $\to$ P is a rectangle.
2. A product of two numbers is zero only if one of the numbers is zero.

$$\forall x, y \in \mathbb{R},\ xy = 0 \to x = 0 \lor y = 0.$$

(Correction 29/8/12: fixed the solution.)

Question 8(a-b)

Prove the following statements:

1. There exists an even integer n that can be written in two ways as a sum of two prime numbers.

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   Example:
   $$10 = 5 + 5 = 3 + 7.$$

2. For two integers r and s, there exists an integer k such that $22r + 18s = 2k$.
   For integers r and s, $k = 11r + 9s$ is an integer such that $22r + 18s = 2k$.

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2. For all integers a, b, and c, if $a|b$ and $b|c$, then $a|c$.

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2. For all integers a, b, and c, if $a|b$ and $b|c$, then $a|c$.
   If $a|b$ and $b|c$, then $b = ma$ and $c = nb$ for some $m, n \in \mathbb{Z}$.

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   If $a|b$ and $b|c$, then $b = ma$ and $c = nb$ for some $m, n \in \mathbb{Z}$.
   Then, $c = n(ma) = (nm)a$, so $a|c$ (since $nm \in \mathbb{Z}$).

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Prove or disprove the following statement: For all integers a and b, if $a|b$ and $b|a$, then $a = b$.

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$1| - 1$ and $-1|1$, but $1 \neq -1$, so the statement is false.

Question 10

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$$4m \equiv 4 \cdot 6 \equiv 24 \equiv 2 \mod 11.$$

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for some $k \in \mathbb{Z}$.

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If m is the square of an odd integer, then

$$m = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$$

for some $k \in \mathbb{Z}$. One of k and $k+1$ is even, so $k(k+1)$ is even, and $4k(k+1) = 8n$ for some $n \in \mathbb{Z}$.

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If m is the square of an odd integer, then

$$m = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$$

for some $k \in \mathbb{Z}$. One of k and $k+1$ is even, so $k(k+1)$ is even, and $4k(k+1) = 8n$ for some $n \in \mathbb{Z}$. So, $m = 8n + 1$, therefore $m \equiv 1 \mod 8$.

Question 12

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2. For all real numbers x and all integers m, $\lfloor x + m \rfloor = \lfloor x \rfloor + m$.

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   Let $n = \lfloor x \rfloor$, i.e. n is an integer such that $n \leq x < n + 1$.

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   Let $n = \lfloor x \rfloor$, i.e. n is an integer such that $n \leq x < n+1$.
   Then, $n+m$ is an integer s.t. $n+m \leq x+m < n+m+1$.

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   Let $n = \lfloor x \rfloor$, i.e. n is an integer such that $n \leq x < n+1$.
   Then, $n+m$ is an integer s.t. $n+m \leq x+m < n+m+1$.
   Therefore, $\lfloor x+m \rfloor = n+m = \lfloor x \rfloor + m$.