

1. Write the Cayley table for the rotations and reflections of a (non-square) rectangle. Call the identity ι , and call rotations r_i and reflections s_i . Hint: the group should have four elements.
(Trivia: This is called the ‘Klein four-group’.)
2. Consider the Cayley tables of \mathbb{Z}_6 and D_3 in Example J.2.1. Find the order of each element in each group, find the cyclic subgroups generated by each one.
3. (*Challenge*): For any nonempty set X , let $S(X)$ denote the set of all bijections from X to X . That is, $f \in S(X)$ iff $f : X \rightarrow X$ is a bijection.
 - (a) Show that $(S(X), \circ)$ is a group, where \circ represents function composition, i.e. for any $f, g \in S(X)$, $f \circ g$ is the function $(f \circ g)(x) = f(g(x))$. (You may use theorems from the course workbook.)
 - (b) Suppose X is finite, and let $n = |X|$. Find $|S(X)|$.