

# The SSR under Quadratic Rough Heston

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# Outline of this talk

- The skew-stickiness ratio (SSR)
- The quadratic rough Heston (QRH) model
- Simulating SPX and VIX under QRH.
- The SSR under quadratic rough Heston
- Unreasonably good fit quality on one day in history
- Why does QRH fit so well?

# Implied volatility

According to the definition of implied volatility  $\sigma_{BS}(k, T)$ , the market price of an option is given by

$$C(S, K, T) = C_{BS}(S, K, T, \sigma_{BS}(k, T))$$

where  $C_{BS}$  denotes the Black-Scholes formula and  $k = \log K/S$  is the log-strike.

# Updating European option prices

Market makers, when updating option prices using the Black-Scholes formula, typically consider two effects:

- The explicit spot effect

$$\frac{\partial C}{\partial S} \delta S$$

and

- The change in implied volatility conditional on a change in the spot

$$\frac{\partial C}{\partial \sigma} \mathbb{E} [\delta \sigma | \delta S].$$

# Estimating $\mathbb{E} [\delta\sigma(T)|\delta X]$

- ATM implied volatilities  $\sigma_t(T) = \sigma_{BS,t}(0, T)$  and stock prices are both observable.
- Market makers can estimate the second component using a simple regression:

$$\delta\sigma_t(T) = \beta_t(T) \frac{\delta S_t}{S_t} + \text{noise} =: \beta_t(T) \delta X_t + \text{noise}.$$

- Then

$$\beta_t(T) = \frac{\mathbb{E}_t [d \langle \sigma(T), X \rangle_t]}{\mathbb{E}_t [d \langle X \rangle_t]}.$$

# The skew-stickiness ratio

- For a given time to expiration  $T$ , we define the ATM volatility skew

$$\mathcal{S}_t(T) = \left. \frac{\partial}{\partial k} \sigma_{\text{BS}}(k, T) \right|_{k=0}.$$

- Bergomi [[Ber09](#), [Ber16](#)] calls

$$\mathcal{R}_t(T) = \frac{\beta_t(T)}{\mathcal{S}_t(T)}$$

the *skew-stickiness ratio* or *SSR*.

# The quadratic rough Heston model

The quadratic rough (QR) Heston model of [GJR20] may be written as

$$\begin{aligned}\frac{dS_t}{S_t} &= -\sqrt{V_t} dW_t, \\ V_t &= Y_t^2 + c,\end{aligned}\tag{1}$$

where  $c \geq 0$ , and

$$Y_t = \bar{Y} + \int_{-\infty}^t \kappa(t-s) \sqrt{V_s} dW_s = \bar{Y} - \int_{-\infty}^t \kappa(t-s) \frac{dS_s}{S_s} \tag{2}$$

is a weighted average of historical returns and  $\kappa(\cdot)$  a kernel function.

Then, for  $u > t$ ,

$$Y_u = y_t(u) + \int_t^u \kappa(u-s) \sqrt{V_s} dW_s \quad (3)$$

where

$$y_t(u) := \mathbb{E}_t[Y_u] = \bar{Y} + \int_{-\infty}^t \kappa(u-s) \sqrt{V_s} dW_s.$$

Then,  $y_t(u)$  is a martingale and

$$dy_t(u) = \kappa(u-t) \sqrt{V_t} dW_t. \quad (4)$$



## Variance of $Y_u$

Thus  $\mathbb{E}_t[Y_u] = y_t(u)$  and from (4) and Itô's isometry,

$$\text{var}_t[Y_u] = \int_t^u \xi_t(s) \kappa(u-s)^2 ds, \quad (5)$$

where  $\xi_t(s) := \mathbb{E}_t[V_s]$  for all  $s \geq t$ .

- Under QR Heston, we have the mean and variance of  $Y_u$  in (quasi-) closed form.

### Remark

The QR Heston model with  $c = 0$  and kernel  $\kappa(\tau) = \nu e^{-\lambda\tau}$  is an inverse gamma model with  $\rho = -1$ .

# The forward variance curve

From the model definition (1), for  $u > t$ ,  $V_u = y_u(u)^2 + c$  so applying Itô's Formula, we have

$$\begin{aligned}\xi_t(u) &= \mathbb{E}_t [y_u(u)^2] + c \\ &= y_t(u)^2 + \mathbb{E}_t \left[ \int_t^u d \langle y.(u) \rangle_s \right] + c \\ &= y_t(u)^2 + \int_t^u \xi_t(s) \kappa(u-s)^2 ds + c.\end{aligned}\tag{6}$$

Alternatively,

$$y_t(u)^2 = \xi_t(u) - \int_t^u \xi_t(s) \kappa(u-s)^2 ds - c,\tag{7}$$

- $y_t(u)$  may be easily imputed from the forward variance curve.

## $\xi_t(u)$ from $y_t(u)$

(6) is a Wiener–Hopf equation for  $\xi_t(u)$  whose solution may be written as

$$\xi_t(u) = y_t(u)^2 + c + \int_t^u K(u-s) [y_t(s)^2 + c] ds. \quad (8)$$

- The Laplace transform (denoted as  $\mathcal{L}$ ) of the resolvent kernel  $K$  is given by

$$\mathcal{L}[K] = \frac{\mathcal{L}[\kappa^2]}{1 - \mathcal{L}[\kappa^2]}. \quad (9)$$

# Admissibility condition on the kernel

- Non-negativity of the forward variance curve imposes a condition on the kernel.
- For example, in the case  $\xi_t(u) = \xi$ , we must have

$$\int_t^u \kappa(u-s)^2 ds \leq 1 - \frac{c}{\xi}.$$

- Thus, a necessary condition on the kernel is

$$\int_t^u \kappa(u-s)^2 ds \leq 1.$$

# Dynamics of forward variance

Applying Itô's Formula to (8) and using that  $\xi_t(u)$  is a martingale,

$$\begin{aligned} d\xi_t(u) &= 2 y_t(u) dy_t(u) + 2 \int_t^u y_t(s) dy_t(s) K(u-s) ds \\ &= 2 \left\{ y_t(u) \kappa(u-t) + \int_t^u y_t(s) \kappa(s-t) K(u-s) ds \right\} \sqrt{V_t} dW_t \\ &= -2 \left\{ \kappa(u-t) y_t(u) + \int_t^u \kappa(s-t) K(u-s) y_t(s) ds \right\} \frac{dS_t}{S_t}. \end{aligned} \tag{10}$$

# Choice of kernel

- We choose the gamma kernel.

$$\kappa(\tau) = \frac{\nu}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\lambda \tau}. \quad (11)$$

- With this choice, the QR Heston model has only four parameters:  $H$ ,  $\nu$ ,  $\lambda$ , and  $c$ .
- There is a closed-form expression for the resolvent kernel:

$$K(\tau) = \hat{\nu}^2 e^{-2\lambda\tau} \tau^{2H-1} E_{2H,2H}(\hat{\nu}^2 \tau^{2H}),$$

where  $E_{\bullet,\bullet}$  is the two-parameter Mittag-Leffler function.

# The QRH simulation scheme

- Recall the dynamics of  $y_{\bullet}(u)$  from (4):

$$dy_t(u) = \kappa(u - t) \sqrt{V_t} dW_t.$$

- The dynamics of  $y$  are formally identical to those of  $\xi$  in an affine forward variance (AFV) model.
  - Thus, we may use the HQE scheme of [Gat22] to simulate  $y$ .

# Simulating $Y$

- Following [Gat22] closely, we have from (3) that

$$Y_T = y_T(T) = y_t(T) + \int_t^T \kappa(T-s) \sqrt{V_s} dW_s. \quad (12)$$

- Wlog, let  $t = 0$ . Then, with more-or-less obvious notation,

$$Y_n = y_n + \sum_{k=1}^n \int_{(k-1)\Delta}^{k\Delta} \kappa(n\Delta - s) \sqrt{V_s} dW_s =: \hat{y}_n + u_n,$$

- The  $\mathcal{F}_{n-1}$ -adapted variable  $\hat{y}_n$  is given by

$$\hat{y}_n := \mathbb{E}[Y_n | \mathcal{F}_{n-1}] = y_n + \sum_{k=1}^{n-1} \int_{(k-1)\Delta}^{k\Delta} \kappa(n\Delta - s) \sqrt{V_s} dW_s,$$

- The martingale increment  $u_n$  is given by

$$u_n := \int_{(n-1)\Delta}^{n\Delta} \kappa(n\Delta - s) \sqrt{V_s} dW_s.$$



# The $X$ -process

- We also need to simulate the  $n$ th increment of the component of the log-stock price process  $X = \log S$ :

$$\chi_n := \int_{(n-1)\Delta}^{n\Delta} \sqrt{V_s} dW_s.$$

# Notation

- For convenience, define for every  $i, j \in \mathbb{N}$ ,

$$\mathcal{K}_i(\Delta) := \int_0^\Delta \kappa(s + i\Delta) ds$$

$$\mathcal{K}_{i,j}(\Delta) := \int_0^\Delta \kappa(s + i\Delta) \kappa(s + j\Delta) ds.$$

- For the gamma kernel (11),

$$\mathcal{K}_i(\Delta) = \frac{\nu}{\lambda^{H+1/2}\Gamma(\alpha)} [\Gamma(H + 1/2, i\lambda\Delta) - \Gamma(H + 1/2, (i + 1)\lambda\Delta)],$$

$$\mathcal{K}_{i,i}(\Delta) = \frac{\nu^2}{(2\lambda)^{2H}\Gamma(\alpha)^2} [\Gamma(2H, 2i\lambda\Delta) - \Gamma(2H, 2(i + 1)\lambda\Delta)],$$

where  $\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt$ .

# Computation of $\text{var} [u_n | \mathcal{F}_{n-1}]$

The conditional variance may be approximated by

$$\begin{aligned}\text{var} [u_n | \mathcal{F}_{n-1}] &= \text{var} [Y_n | \mathcal{F}_{n-1}] \\ &\approx [\theta \hat{y}_n^2 + (1 - \theta) Y_{n-1}^2 + c] K_0(\Delta) \\ &=: \bar{V}_n K_{0,0}(\Delta),\end{aligned}$$

with  $\theta := \frac{1}{2H+1}$  and

$$K_0(\Delta) := \int_0^\Delta K(s) ds, \quad (13)$$

where  $K$  is the resolvent kernel of  $\kappa^2$ , see (9).

# Simulation of $u_n$ and $\chi_n$

- As in [Gat22], we make the hybrid approximation  $u_n \approx \beta_{u\chi} \chi_n + \epsilon_n$ , where  $\beta_{u\chi} = \mathcal{K}_0(\Delta)/\Delta$ .
- We simulate  $\chi_n$  and  $\epsilon_n$  as

$$\chi_n \sim \sigma_\chi Z_\chi; \quad \epsilon_n \sim \sigma_\epsilon Z_\epsilon,$$

where  $Z_\chi$  and  $Z_\epsilon$  are two independent standard normal random variables, and

$$\sigma_\chi^2 = \bar{V}_n \Delta; \quad \sigma_\epsilon^2 = \bar{V}_n [\mathcal{K}_{0,0}(\Delta) - \mathcal{K}_0(\Delta)^2/\Delta],$$

- In the large  $n$  limit,  $\text{var}[u_n]$ ,  $\text{var}[\chi_n]$  and  $\text{cov}[u_n \chi_n]$  are matched.

# Simulation of $\hat{y}_n$

- For any  $j \in \mathbb{N}^*$ , let  $b_j^*$  be the root mean square average of the kernel  $\kappa$  over the  $j$ th interval<sup>1</sup>:


$$b_j^* = \sqrt{\frac{1}{\Delta} \int_{(j-1)\Delta}^{j\Delta} \kappa^2(s) ds} = \sqrt{\frac{1}{\Delta} \mathcal{K}_{j-1,j-1}(\Delta)}.$$

- At the  $n$ th step of the algorithm, we need  $\hat{y}_n = \mathbb{E}[y_n | \mathcal{F}_{n-1}]$ :

$$\hat{y}_n \approx y_n + \sum_{k=1}^{n-1} b_{n-k+1}^* \chi_k.$$

- Note the explicit path-dependence!

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<sup>1</sup>This choice ensures that quadratic variations are matched exactly. 

# Simulation of $V_n$ and $X_n$

Having now simulated  $\hat{y}_n$  and  $u_n$ , we simulate  $V_n$  and  $X_n = \log(S_n)$  as

$$V_n = Y_n^2 + c = (\hat{y}_n + u_n)^2 + c,$$

$$X_n = X_{n-1} - \frac{1}{4} (V_n + V_{n-1}) \Delta - \chi_n.$$

## Remark

At this point, we have simulated  $X$  and its quadratic variation, so we can, in principle, price all SPX derivatives, including variance swaps.

# SPX option valuation

To be even more explicit, we value a European call expiring at time  $T$  as

$$\begin{aligned}\mathbb{E}[(S_T - K)^+] &= \mathbb{E}[(S_0 e^{X_T} - K)^+] \\ &\approx \frac{1}{M} \sum_{j=1}^M (S_0 \exp\{X_{N,j}\} - K)^+, \end{aligned}$$

where

- $M$  is the number of Monte Carlo paths,
- $N = T/\Delta$  is the number of time steps.

# Simulation of VIX

- Let  $\delta = 30/365$  (30 days).
- Given  $y_T(u)$ ,  $u > T$ , we can show that, under quadratic rough Heston,

$$\text{VIX}_T^2 = \frac{1}{\delta} \int_0^\delta (y_T(T+u)^2 + c) \{1 + K_0(\delta - u)\} du, \quad (14)$$

where  $K_0(\tau) = \int_0^\tau K(s) ds$ .



# Simulation of the $y_T(u)$

- For any  $u > T$ ,

$$\begin{aligned}y_T(u) &= y(u) + \int_0^T \kappa(u-s) \sqrt{V_s} dW_s \\&\approx y(u) + \sum_{k=1}^m a_k(u) \int_{(k-1)\Delta}^{k\Delta} \sqrt{V_s} dW_s \\&= y(u) + \sum_{k=1}^m a_k(u) \chi_k,\end{aligned}$$

where the  $a_k(u)$  are chosen so as to match the variances of the  $y_T(u)$ .

# Final VIX simulation recipe

- Plugging into (14) gives

$$\begin{aligned} \text{VIX}_T^2 &= \frac{1}{\delta} \int_0^\delta (y_T(T+u)^2 + c) (1 + K_0(\delta - u)) du \\ &= \frac{1}{\delta} \int_0^\delta \left[ \left( y(T+u) + \sum_{k=1}^n a_k(T+u) \chi_k \right)^2 + c \right] \\ &\quad \times [1 + K_0(\delta - u)] du. \end{aligned}$$

- A numerical integral that may be easily computed.
  - For example, using the trapezoidal rule or a Gauss–Legendre quadrature.

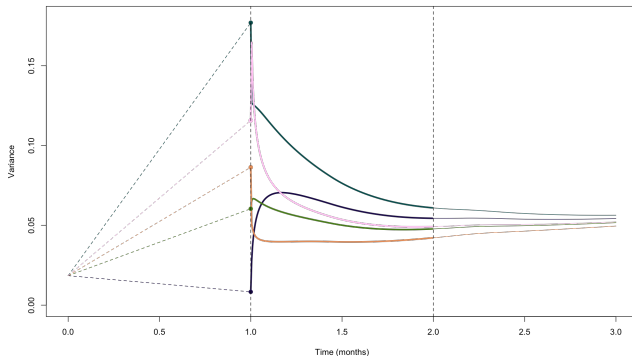
# Options on VIX: An example

- We consider a 1 month option on VIX.
- We use our 15-February 2023 parameters.
- We select 5 Monte Carlo paths and plot the forward variance curves as of time  $T = 1m$  corresponding to these paths.
- Recall that

$$VIX_T^2 = \frac{1}{\delta} \int_T^{T+\delta} \xi_T(u) du.$$

- Valuation depends on the forward variance curve between  $T = 1m$  and  $T = 2m$ .
- We show this graphically in Figure 1.

# Options on VIX: Graphical illustration



**Figure 1:** Five forward variance curves contributing to the valuation of a 1-month option on VIX.

# Estimation of the SSR

- Recall that the SSR is given by

$$\mathcal{R}_t(T) = \frac{\beta_t(T)}{\mathcal{S}_t(T)}$$

where

- $\beta_t(T)$  is the regression coefficient in

$$\delta\sigma_t(T) = \beta_t(T) \delta X_t + \text{noise},$$

- and  $\mathcal{S}_t(T)$  is the ATM skew.

# Estimation of the regression coefficient $\beta_t(T)$

- Recall that the QR Heston model reads:

$$dy_t(u) = \kappa(u - t) \sqrt{V_t} dW_t.$$

- We apply the finite difference methodology of Bourgey et al. [BDDM24].
- Thus, the regression coefficient is estimated as

$$\beta_t(T) \approx \frac{1}{h} \left\{ \sigma_t \left( T, y_t^h(\cdot) \right) - \sigma_t \left( T, y_t(\cdot) \right) \right\},$$

where  $h$  is a small parameter, and for  $t \leq u \leq T$ ,

$$y_t^h(u) := y_t(u) - h \kappa(u - t). \quad (15)$$

# Estimation of the ATM skew $\mathcal{S}_t(T)$

- Recall that the ATM skew

$$\mathcal{S}_t(T) = \left. \frac{\partial \sigma_{\text{BS}}(k, T)}{\partial k} \right|_{k=0}.$$

- We estimate the ATM skew using a trick from [BDMFP23].
- 

$$\frac{\partial \sigma_{\text{BS}}(k, T)}{\partial k} = \left. \frac{\Phi\left(-\frac{k}{\sqrt{w}} - \frac{\sqrt{w}}{2}\right) - \mathbb{P}(X_T \geq k)}{\sqrt{T}\phi\left(-\frac{k}{\sqrt{w}} - \frac{\sqrt{w}}{2}\right)} \right|_{w=T\sigma_{\text{BS}}^2(k, T)}$$

where  $X_T = \log(\frac{S_T}{S_0})$  and  $\phi$  is the p.d.f. of a standard normal distribution.

# Estimation of the SSR

- Finally, the SSR is estimated as

$$\mathcal{R}_t(T) \approx \frac{\beta_t(T)}{\mathcal{S}_t(T)}. \quad (16)$$

## Remark

The estimator (16) only requires only the ATM implied volatilities for two different forward volatility curves  $y_t(u)$ , and one estimate of the ATM skew.



# Calibrated parameters as of 15 February 2023

- The QR Heston model was calibrated to the SPX and VIX smiles for expiries  $T = 7, 14, 20, 28$  days as of 15 February 2023<sup>2</sup>.
- Calibrated parameters were:

$H$	$\nu$	$c$	$\lambda$
0.068	0.572	$8.1 \times 10^{-3}$	9.68

- $\xi_0(u)$  was estimated from SPX option data.
- We used 300 time steps and  $3 \times 10^5$  MC samples.
- The small parameter  $h = 0.01 \times \sqrt{\tau}$  where  $\tau$  is the maturity,

<sup>2</sup>SPX and VIX option prices were sourced from OptionMetrics via WRDS.

# Some SPX smiles as of 15 February 2023

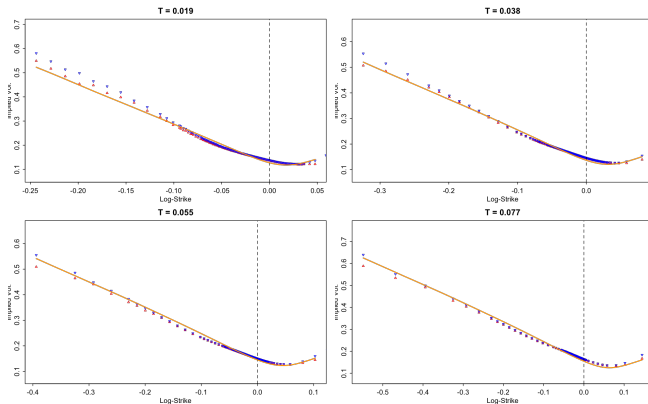
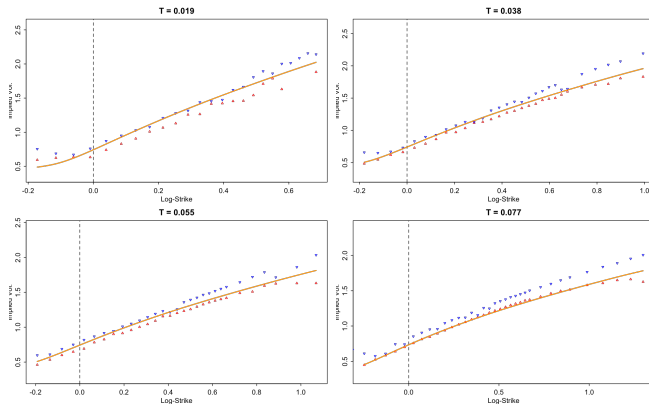


Figure 2: SPX smiles as of 15 February 2023. Bid and offer implied volatilities are in red and blue respectively; QR Heston fits generated using the supplied R-code are in orange.

# Some VIX smiles as of 15 February 2023



**Figure 3:** VIX smiles as of 15 February 2023. Bid and offer implied volatilities are in red and blue respectively; QR Heston fits generated using the supplied R-code are in orange.

# Path-dependence of the QR Heston model

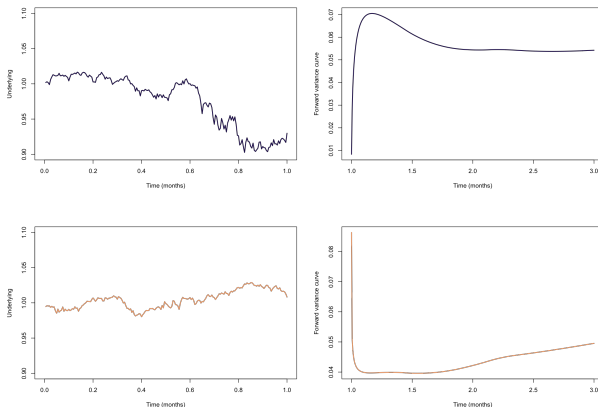
- From (10), changes in the forward variance curve are directly expressible in terms of changes in the underlying.
  - The QR Heston model is purely path-dependent in the language of [GL23].

- Explicitly, using  $dy_t(u) = \kappa(u - t) \sqrt{V_t} dW_t$ , we may write

$$y_t(u) = \bar{Y} + \int_{-\infty}^t \kappa(u - r) \sqrt{V_r} dW_r = \bar{Y} - \int_{-\infty}^t \kappa(u - r) \frac{dS_r}{S_r}.$$

- The forward volatility curve is thus a weighted average of historical stock returns.
- The fair values of SPX options, VIX options, and the SSR are all functionals of the forward volatility curve.

# Examples of path-dependence



**Figure 4:** Two price paths and corresponding forward variance curves. With our fitted parameters, the forward variance curve is most sensitive to the very recent past.

# SSR term structure as of 15 February 2023

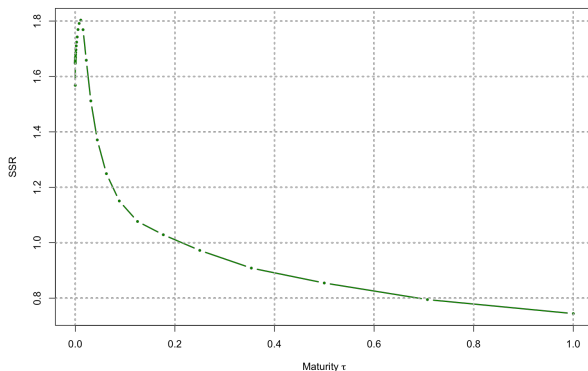


Figure 5: The SSR as a function of maturity  $\tau$  generated using the supplied R-code.

## Remarks on the fits

- Despite the impressive joint fits to short-dated SPX and VIX smiles, fits deteriorate for longer periods.
- The very short-time behavior of the SSR shown in Figure 5, where the SSR increases sharply before declining, is consistent with the general results derived in [FG24].
- $c > 0$  is really necessary.
  - Fits with  $c = 0$  are much less impressive.

# Why does the QR Heston model fit so well?

- In view of its extreme parsimony, it may at first seem surprising that the QR Heston model can generate such impressive fits.
- By taking a historical perspective and relating the QR Heston model to earlier models, we may begin to understand its strong fitting performance.



# QR Heston from Quadratic Hawkes

- The QR Heston model in (1) is a special case of the quadratic rough Heston model of [DJR21].
- This model, in turn, emerges as the continuous-time limit of a non-Markovian extension of the quadratic Hawkes model originally proposed in [BDB17].
  - Calibration of that model to intraday data yielded very impressive results.
- Thus the QR Heston model can be seen as a non-Markovian extension of a long-standing successful statistical model of financial markets.

# Markovian QR Heston

- The QR Heston model can also be seen as the non-Markovian extension of various realistic models in the classical stochastic volatility literature.
- Consider the QR Heston model in the Markovian case ( $H = 1/2$ ) with the gamma kernel (11):

$$\kappa(\tau) = \nu e^{-\lambda \tau},$$

and  $c = 0$ .

- In this case,  $\sigma_t = Y_t + \bar{Y}$  is the instantaneous volatility and (2) may be rewritten as

$$d\sigma_t = -\lambda(\sigma_t - \bar{Y}) dt + \nu \sigma_t dW_t, \quad (17)$$

# QR Heston and the Inverse Gamma model

- (17) are the dynamics of volatility in the Inverse Gamma model of [LLZ16].
- As noted in that paper, the Inverse Gamma model is a special case of various other models that have desirable properties consistent with empirical studies of stocks and their volatilities.
- For example, the Inverse Gamma model has an inverse gamma stable distribution, which was shown to be a good fit to the unconditional distribution of volatility by Bouchaud and Potters [BP03] and Sepp [SK12], amongst others.
  - In fact, this observation was one of the reasons that Sepp chose his particular model dynamics.

# QR Heston and the Double Lognormal model

- In terms of  $V_t = \sigma_t^2 = (Y_t + \bar{Y})^2$ , (17) may be written in the form

$$\begin{aligned}dV_t &= -\hat{\lambda} (V_t - V'_t) dt + 2\nu V_t dW_t, \\dV'_t &= -\lambda (V'_t - \bar{V}') dt + \nu V'_t dW_t,\end{aligned}$$

where

$$\hat{\lambda} = 2\lambda - \nu^2; \quad V'_t = \frac{2\lambda}{\hat{\lambda}} \bar{Y} \sigma_t; \quad \bar{V}' = \frac{2\lambda}{\hat{\lambda}} \bar{Y}^2.$$

- This is a special case of the Double Lognormal (DL) model of [Gat08]
  - DL generates positively sloping VIX smiles.
  - DL parameters are reasonable stable.

# Lifting the Double Lognormal model

- Viewing the gamma kernel as an infinite sum of exponential kernels with different characteristic timescales in the spirit of [AJ19], the QR Heston model may be seen as an infinite sum of Double Lognormal models.
- Accordingly, the QR Heston model inherits various properties of the Double Lognormal model such as positively sloped VIX smiles.

## Concluding remarks

- The Quadratic Rough Heston (QR) Heston model can successfully capture the joint behavior of SPX and VIX implied volatility smiles, whilst producing realistic values for the SSR.
- The QR Heston model with the gamma kernel is:
  - Extremely parsimonious with only four parameters.
  - Econometrically motivated.
  - Non-Markovian.
  - Purely path-dependent.
- The QRH scheme is efficient and easy to implement.
  - The QR Heston model is thus a viable candidate for practical trading applications.

# GitHub

- Last but not least, we have made R and Python codes plus Jupyter notebooks illustrating their usage available at <https://github.com/jgatheral/QuadraticRoughHeston>.

# References



Eduardo Abi Jaber.

Lifting the Heston model.

*Quantitative Finance*, 19(12):1995–2013, 2019.



Lorenzo Bergomi

Smile dynamics IV.

*Risk December*, pages 94–100, 2009.



Lorenzo Bergomi.

Stochastic Volatility Modeling.

*Chapman and Hall/CRC*, 2016.



Pierre Blanc, Jonathan Donier, and Jean-Philippe Bouchaud.

Quadratic Hawkes processes for financial prices.

*Quantitative Finance*, 17(2):171–188, 2017.



Jean-Philippe Bouchaud and Marc Potters.

*Theory of financial risk and derivative pricing: From statistical physics to risk management*.

Cambridge University Press, 2003.



Florian Bourgey, Stefano De Marco, Peter K Friz, and Paolo Pigato.

Local volatility under rough volatility.

*Mathematical Finance*, 33(4):1119–1145, 2023.





Florian Bourgey, Jules Delemotte, and Stefano De Marco.

Smile dynamics and rough volatility.

*SSRN:4911186*, 2024.



Florian Bourgey and Jim Gatheral.

The SSR under Quadratic Rough Heston.

*SSRN:4911186*, 2025.



Aditi Dandapani, Paul Jusselin, and Mathieu Rosenbaum.

From quadratic Hawkes processes to super-Heston rough volatility models with Zumbach effect.

*Quantitative Finance*, 21(8):1235–1247, 2021.



Peter Friz and Jim Gatheral.

Computing the SSR.

*Quantitative Finance*, 25(5) 2025.



Jim Gatheral.

Consistent modeling of SPX and VIX options.

In *Bachelier congress*, volume 37, pages 39–51, 2008.



Jim Gatheral.

Efficient simulation of affine forward variance models.

*Risk.net*, February, 2022.



Jim Gatheral, Paul Jusselin, and Mathieu Rosenbaum.

The quadratic rough Heston model and the joint S&P 500/VIX smile calibration problem.  
*Risk.net*, May 2020.



Julien Guyon and Jordan Lekeufack.

Volatility is (mostly) path-dependent.  
*Quantitative Finance*, 23(9):1221–1258, 2023.



Nicolas Langrené, Geoffrey Lee, and Zili Zhu.

Switching to non-affine stochastic volatility: A closed-form expansion for the inverse gamma model.  
*International Journal of Theoretical and Applied Finance*, 19(5):1650031, 2016.



Aitor Muguruza.

Not so Particular about Calibration: Smile Problem Resolved.  
*arXiv*:1909.13366, 2019.



Artur Sepp and Piotr Karasinski.

Beta stochastic volatility model.  
*Risk Magazine*, pages 66–71, October, 2012.