

# Too Proud to Stop: Regret in Dynamic Decisions\*

Philipp Strack<sup>†</sup>

Paul Viefers<sup>‡</sup>

## Abstract

Regret and its anticipation affect a wide range of decisions. Jobseekers reject offers waiting for an offer to match their best past offer; investors hold on to badly-performing stocks; and managers throw good money after bad projects. We analyze behavior of a decision maker with regret preferences in a dynamic context and show that regret agents have a disposition to gamble until a payoff matching the best past offer is received. Results from a lab experiment confirm that many subjects exhibit such behavior and are reluctant to stop below the past peak.

JEL-CLASSIFICATION: D03, C91

KEYWORDS: optimal stopping, dynamic behavior, regret.

---

\*This version: June 17, 2017. We thank Gabriel Carroll, Martin Cripps, Stefano DellaVigna, Paul Heidhues, Ulrike Malmendier, Markus Mobius, Ryan Oprea, Pietro Ortoleva, Klaus Schmidt, Andrew Schotter, Tomasz Strzalecki, and Georg Weizsäcker for helpful comments and discussions. We also like to thank seminar participants at Berkeley, Berlin, EEA Gothenburg, Florence, Laval, LMU Munich, Michigan, UC Davis, Santa Clara, and Stanford. This paper is part of Viefers' dissertation submitted to the Humboldt-University Berlin. Part of this paper was written while Viefers was a guest at Microsoft Research and their hospitality is gratefully acknowledged. We also thank the ERC (Starting Grant 263412) and the Fritz Thyssen Foundation for financial support.

<sup>†</sup>University of California Berkeley, Office 513, Evans Hall, Berkeley, 94720 California, USA. Email: pstrack@berkeley.edu.

<sup>‡</sup>University of Cologne, Universitätsstr. 22a, D-50931 Cologne, Germany. Phone: +49 (0)221 470 3418, Email: pviefers@uni-koeln.de.

# 1. Introduction

I should have computed the historical covariance of the asset classes and drawn an efficient frontier. Instead I visualized my grief if the stock market went way up and I wasn't in it or if it went way down and I was completely in it. My intention was to minimize my future regret, so I split my [pension] contributions 50/50 between bonds and equities.

—Harry Markowitz.

The feeling of having considered but passed on what later turned out to be a better opportunity affects a wide range of experiences. For example, jobseekers reject offers in the hope of getting a better one that never materializes; investors hold on to badly-performing stocks; and managers throw good money after bad projects.

In this paper we study the role of regret and counterfactual thinking in a stopping problem—both theoretically and experimentally. While regret theory is well-studied in static contexts (see [Bleichrodt and Wakker, 2015](#), for a recent survey), we are—to the best of our knowledge—the first to analyze a dynamic extension of anticipated regret in the sense of [Loomes and Sugden \(1982\)](#) to a stopping problem. In a stopping problem an agent observes a sequence of offers,  $X = (X_1, X_2, \dots)$ , which are the realizations of some stochastic process. After observing the  $t$ -th offer, the agent has to decide whether to continue and forgo the current offer or to stop and seize it. In the former case, she observes the next offer and faces the same decision again. In the latter case, the agent's decision to stop is irreversible and she receives a net payoff  $X_t - K$ , where  $K > 0$  is known and fixed. In considering which stopping rule is optimal, the agent has to weigh the immediate gains from stopping at  $X_t$ , against the loss of the option to stop at higher values in the future.

Stopping problems represent the most basic, yet truly dynamic decision-making problem and are used throughout finance, economics and statistics to model many different decision-making contexts, such as job search, irreversible investment, option pricing and market entry decisions. While stopping theory is widely applied, the literature on stopping behavior mostly considers agents with expected utility (EU) preferences. Under EU it is well-known that optimal stopping behavior is a cut-off rule: stop as soon as the payoff process hits a time-constant reservation level. Otherwise wait. We show that the cut-off result holds in our model for non-concave value functions or gain-loss utility, e.g. the famous S-shape utility of [Kahneman and Tversky \(1979\)](#), as long as the reference point does not change over time. From a theoretical perspective cut-off behavior is thus a very robust prediction. From an empirical perspective, it is often found that individuals'

decisions are influenced by the history of events, e.g. prior gains and losses, which would not lead to time-constant cut-off behavior.<sup>1</sup>

With this mind, we explore how the behavior of an agent affected by regret depends on the past peak of the payoff process. Toward that end, we extend the static anticipated regret preferences due to [Loomes and Sugden \(1982\)](#) to the context of optimal stopping. In a dynamic setting, the agent may not only regret past, but also future decisions, i.e. not having stopped later when the process reached higher levels. We consider both forms of regret and analyze whether they can explain behavior different from cut-off strategies. First, we prove that if regret over past and future decisions is equally strong, it induces the same preferences as EU and is thus not testable in our setup. Second, we show that if regret is only over potential past decisions, regret preferences are distinct from EU preferences. After not having stopped at the best past offer a regret agent will reconsider her plan and raise the ex-ante cut-off level to match the past maximum of the payoff process. This is an important finding, because it implies that in a random utility setting à la [Luce \(1959\)](#), our regret model becomes testable. We derive several testable predictions on the probability with which the agent stops after different sequences of offers. Extending our model to the case of stochastic choice is also an important step for future applied work, because it provides a sound theoretical foundation for the estimation of structural dynamic discrete-choice models with regret preferences. Structural models are widely used in the applied literature, but mostly assume the agent maximizes EU (see [Rust, 1987, 1994](#); [Aguirregabiria and Pedro, 2010](#), and citations therein).<sup>2</sup> Notably, as the regret agent is time-consistent, our predictions hold irrespective of whether the agent is sophisticated, naïve, or able to commit to her strategy.

We test our predictions through a lab experiment. Using the influential design by [Oprea et al. \(2009\)](#), we give each subjects the option to stop 65 different paths of a geometric random walk with random duration. An important change to the experimental design by [Oprea et al. \(2009\)](#) and most of the earlier experimental literature on optimal stopping (see *inter alia* [Rapoport and Tversky, 1966, 1970](#); [Kahan et al., 1967](#); [Schotter and Braunstein, 1981](#); [Cox and Oaxaca, 1989, 1992, 2000](#); [Seale and Rapoport, 1997](#); [Brown](#)

---

<sup>1</sup>For example, it is a well-established fact in the behavioral finance literature that individual investors have a disposition to sell winning stocks and ride losing stocks (e.g. [Shefrin and Statman, 1985](#); [Lakonishok and Smidt, 1986](#); [Weber and Camerer, 1998](#); [Grinblatt and Keloharju, 2000](#); [Ben-David and Hirshleifer, 2012](#)). [Gneezy and Potters \(1997\)](#); [Gneezy et al. \(2003\)](#); [Haigh and List \(2005\)](#); [Magnani \(2015\)](#) obtain similar results on the disposition effect in the laboratory. Regret and counterfactual thinking also well-documented in neuroscience, e.g. [Zeelenberg et al. \(1998, 2000\)](#); [Camille et al. \(2004\)](#); [Coricelli et al. \(2005\)](#).

<sup>2</sup>For example, [Fioretti et al. \(2017\)](#) made use of our results and fitted such a dynamic discrete-choice model in order to estimate structural regret parameters from their data.

et al., 2011), is that we not only recorded the level at which subjects stopped the process, but also the entire history of the process. This allows us to test a much richer set of behavioral hypotheses compared to earlier studies. For example, the data collected in the experiment by Oprea et al. (2009) only allows to test whether the level at which a subject stopped across rounds is consistent with EU, whereas we can also test whether the behavior within a round is consistent with EU.

We find two patterns which were mostly unnoticed in the prior literature by analyzing the tick-level data of our experiment: (i) subjects clearly do not use cut-off strategies (even within a round) and (ii) subjects are less likely to stop the process the further they are below its past maximum.

## 2. The Setting

Time is discrete and indexed by  $t \in \{0, 1, \dots\}$ . The agent observes a sequence  $X_0, X_1, \dots$  of realizations of a multiplicative binomial random walk. For a given starting value  $X_0 > 0$ , future values of  $X_t$  are drawn according to the transition rule

$$X_{t+1} = \begin{cases} h X_t & \text{with probability } p \\ \frac{1}{h} X_t & \text{with probability } 1 - p \end{cases}.$$

We call  $h > 1$  the step size and  $p \in (1/2, 1)$  the uptick probability. At the end of any period  $t$  there is a fixed exogenous probability  $1 - \delta \in (0, 1)$  that the game ends and the agent receives a payoff of zero. We denote by  $T \geq 0$  the random time the game ends, such that  $X_t = 0$  for  $t \geq T$ . At any time  $t < T$  before the game ended the agent observes the realization of the random walk  $X_t$  and decides whether to ‘continue’ or to ‘stop’.

If the agent chooses to stop in period  $t$ , she receives  $X_t$  minus a constant transaction cost  $K > 0$ , such that her material pay-off equals  $X_t - K$ . After an agent decided to stop, she continues to observe the realization of the process until the game ends in period  $T$ .

If the agent chooses to continue, the game ends with probability  $1 - \delta$  and the agent gets a payoff of zero. With probability  $\delta$ , the game does not end in period  $t$ , but period  $t + 1$  starts and the agent observes the next realization of the random walk  $X_{t+1}$ . Throughout the paper we assume that  $\delta(ph + (1 - p)h^{-1}) < 1$ , as otherwise the optimal stopping problem an expected value maximizer faces is not well defined (see Appendix A.1 for details).

### 3. Theories of Dynamic Behavior

In this section, we derive our theoretical predictions under different theories of dynamic behavior. It turns out that a common prediction is that agents' observed behavior will be indistinguishable from a cut-off strategy. A cut-off strategy, is a strategy which satisfies the following definition.

**Definition 1** (Cut-off Strategy). *The cut-off strategy  $\tau(b)$  prescribes that the agent stops at time  $t$  if the value of the process  $X_t$  exceeds the cut-off  $b$  and continues otherwise.*

If the agent uses the cut-off strategy  $\tau(b)$  she will stop at the time<sup>3</sup>

$$\tau(b, X) = \min\{t \geq 0 : X_t \geq b\} . \quad (1)$$

#### 3.1. Expected Utility

An expected utility (EU) agent evaluates outcomes according to the strictly increasing, but not necessarily concave, utility function  $u : [-K, \infty) \rightarrow \mathbb{R}$ . If the EU agent uses the continuation strategy  $\tau$  when the value of the process equals  $x$  at time  $t$  she receives a continuation utility of<sup>4</sup>

$$V^u(\tau, x) = \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K) + \mathbf{1}_{\{\tau \geq T\}} u(0)] . \quad (2)$$

Since these preferences over stopping times are invariant under additive translations of the utility  $u$ , we set  $u(0) = 0$ . We assume that the EU agent stops at time  $t$  with  $X_t = x$  whenever this is optimal<sup>5</sup>

$$u(x - K) = \sup_{\tau} V^u(\tau, x) . \quad (3)$$

Denote by  $U_t = \mathbf{1}_{\{t < T\}} u(X_t - K)$  the utility the agent experiences when she stops at time  $t$ . We denote by  $\mathcal{L}u(x)$  expected change in utility when stopping in period  $t + 1$  instead

<sup>3</sup>Formally, the realized stopping time  $\tau(b, X)$  depends on  $X = (X_1, X_2, \dots)$ , but as common in the literature we will sometimes omit this dependence in our notation when there arises no confusion.

<sup>4</sup> $\mathbf{1}_A$  denotes the indicator function that takes the value one on the event  $A$  and zero otherwise. We denote conditional expectations by  $\mathbb{E}_{t,x}[\cdot] = \mathbb{E}[\cdot | X_t = x, T > t]$ ,  $\mathbb{E}_x[\cdot] = \mathbb{E}_{0,x}[\cdot]$  and conditional probabilities by  $\mathbb{P}_{t,x}[\cdot] = \mathbb{P}[\cdot | X_t = x, T > t]$ ,  $\mathbb{P}_x[\cdot] = \mathbb{P}_{0,x}[\cdot]$ .

<sup>5</sup>In other words, we assume that the EU maximizer uses the minimal optimal stopping strategy—an assumption made in virtually the entire optimal stopping literature.

of  $t$  with  $X_t = x$

$$\begin{aligned}\mathcal{L}u(x) &= \mathbb{E}_{t,x}[U_{t+1} - U_t] \\ &= \delta [pu(xh - K) + (1 - p)u(xh^{-1} - K)] - u(x - K).\end{aligned}$$

The following (weak) assumption on the utility function will be useful to state some of our results:

**Assumption A1** (Single Crossing).  $\mathcal{L}u(x)$  changes its sign at most once in  $x$ .

Assumption A1 states that the agent's risk aversion vanishes sufficiently slowly in her wealth level such that if it is unattractive to wait and delay the stopping decision by a single period at some level of the process it is also unattractive at any higher level. It is for example satisfied for positive constant absolute and positive constant relative risk-aversion (Lemma 3 in the Appendix).

We define the EU cut-off  $b^u$

$$b^u = \min \left( \arg \max_{b \geq K} \left[ \left( \frac{K}{b} \right)^\alpha u(b - K) \right] \right), \quad (4)$$

where  $\alpha = \frac{1}{\log(h)} \log \left( \frac{1}{2p\delta} + \sqrt{\frac{1}{4p^2\delta^2} - \frac{1-p}{p}} \right) > 1$ .<sup>6</sup> Then the following proposition characterizes the behavior of an EU agent:

**Proposition 1** (Optimal Stopping under EU). (i) For every path of the process  $X$ , starting in  $X_0 \leq b^u$ , the agent stops at the cut-off stopping time  $\tau(b^u, X)$ . (ii) Under A1 this holds for any initial value  $X_0$ .

To get an intuition for Proposition 1, note that the strategy which never stops before the game ends guarantees a payoff of zero. It follows that the EU agent does not stop when  $X_t < K$  as this would yield a negative payoff. As we argue in the proof of Proposition 1,  $b^u$  is the smallest value of the process such that it is optimal to stop. Hence, the EU agent does not stop below  $b^u$ . Because, when starting at  $X_0 \leq b^u$ , the process cannot reach any value above  $b^u$  without visiting  $b^u$  before, the agent will stop the first time the process reaches  $b^u$ . In case  $X_0 > b^u$ , Assumption A1 ensures that it is optimal to stop at any higher value of the process and thus that the agent stops immediately for any initial value  $X_0 \geq b^u$ .<sup>7</sup>

<sup>6</sup>The minimum in Eq. 4 is taken over the set of maximizers.

<sup>7</sup>The proof of Proposition 1 reveals that a weaker assumption which is necessary and sufficient for the agent to stop at all initial values  $X_0 > b^u$  is that  $\mathcal{L}u(x)$  is non-positive for all  $x > b^u$ .

Proposition 1 allows us to dramatically simplify the problem of finding the optimal stopping time to one of finding an optimal cut-off point which solves Eq. 4. In our experiment we chose  $X_0 = K$  and as  $b^u \geq K$ , Assumption A1 is immaterial to our tests of expected utility. Our cut-off result for  $X_0 = K$  only requires the utility function  $u$  to be monotone, but not necessarily differentiable or concave. Therefore cut-off strategies are also optimal for gain-loss utility and S-shaped utility as in Kahneman and Tversky (1979), i.e. when  $u$  is kinked at a reference point  $r$  and the agent is risk-seeking below and risk-averse above.<sup>8</sup> Where this reference point lies is immaterial to our results, as long as  $r$  is determined a priori and constant.

While Proposition 1 establishes that the optimal strategy is the cut-off rule which stops once the process reaches  $b^u$  our next result establishes that if  $u$  is concave using any cut-off rule with a cut-off less than  $b^u$  is better than stopping immediately.

**Proposition 2.** *Let  $u$  be concave. Using a cut-off strategy  $\tau(b)$  with a cut-off  $b$  below the optimal EU cut-off  $b \leq b^u$  is better than stopping immediately, i.e. for all  $x < b \leq b^u$*

$$V^u(\tau(b), x) > u(x - K).$$

The proof of Proposition 2, derives a closed form solution for the value an EU maximizer assigns to any cut-off strategy and then argues that, for concave  $u$ , the gain from using a cut-off strategy is concave in the cut-off which implies that it is positive for all cut-offs below the optimal cut-off  $b^u$ . Proposition 2 will play an important role in the characterization of the behavior of regret averse agents.

## 3.2. Regret Preferences

In this section we present our model of dynamic regret. For a regret agent, the utility associated with the consequences of her action, is not solely a function of final wealth, but the difference between final wealth and the wealth attained by the ex-post optimal action. If in hindsight the action chosen by the agent turns out to be suboptimal, the agent feels regret. The disutility from regret  $R$  equals the difference between the utility she realized when stopping at time  $t$  and the utility she could have realized when stopping at the time which turned out to be *ex-post* optimal. Formally,

$$R = \left( \max_{s \in \mathcal{S}} U_s \right) - U_t, \quad (5)$$

---

<sup>8</sup>We preclude probability distortion. For a discussion of the stopping behavior of a prospect theory agent with probability distortion and naivete see Ebert and Strack (2015), with probability distortion and commitment see Xu and Zhou (2013) or without probability distortion see Henderson (2012).

where  $\mathcal{S}$  is the set of stopping times relative to which the agent evaluates her regret.

### Unanticipated Regret

If the agent does not anticipate future changes in regret, she will treat the regret term  $R$  as a constant penalty that has to be incurred irrespective her actions. From her perspective,  $R$  is independent of her continuation strategy, which makes the agent behave as if maximizing EU. As unanticipated regret induced the same preferences over stopping rules as EU, we know from the analysis done in section 3.1 that the agent will stop the first time the process crosses a time independent cut-off. Consequently, unanticipated regret cannot explain behavior different from cut-off strategies and we will restrict attention to anticipated regret.

### Anticipated Regret

Regret becomes behaviorally meaningful once we assume it is anticipated by the agent. We model the value of an agent who stops at time  $t$  as the weighted sum of physical utility  $U_t$  minus regret  $R$

$$(1 - \kappa)U_t - \kappa R = (1 - \kappa)U_t - \kappa \left[ \max_{s \in \mathcal{S}} U_s - U_t \right] = U_t - \kappa \max_{s \in \mathcal{S}} U_s. \quad (6)$$

Here,  $\kappa \in [0, 1)$  denotes the intensity of regret. Eq. 6 shows that the agent's objective may be equivalently viewed as the objective of an EU maximizer minus  $\kappa$  times the utility obtained under the ex-post optimal strategy. Obviously, regret preferences contain EU preferences for  $\kappa = 0$ .

There are two natural choices relative to which the agent could calculate her regret in our setting: *i*) the ex-post optimal decision, calculated when the game ended at time  $T$ , and *ii*) the ex-post optimal decision calculated at time  $\tau$ , when she stopped the process and accepted an offer. In the former case, regret is based on all potential past and future decisions  $\mathcal{S} = \{t \leq T\}$ , whereas in the latter case regret is realized immediately after stopping and based solely on foregone past opportunities  $\mathcal{S} = \{t \leq \tau\}$ . These two different notions of regret over past (stopping too late) and future decisions (stopping too early) resemble that of winner's (winning an auction with a too high bid) and loser's regret (not winning an auction due to a too low bid) introduced in Filiz-Ozbay and Ozbay (2007).

**Anticipated Regret over Past and Future Decisions.** Consider the first case where the agent feels regret relative to past and future decisions. Denote by  $S_t = \max_{r \leq t} X_r$  the maximal value of the process prior to period  $t$ . The ex-post optimal decision for the agent



is to stop when the process reaches its maximal value  $S_T$ , which would yield a utility of

$$\max_{s \leq T} U_s = u(S_T - K) .$$

Thus, the expectation of the regret value, given in Eq. 6, when the agent uses the strategy  $\tau$  simplifies to

$$\mathbb{E}[U_\tau] - \kappa \mathbb{E}[u(S_T - K)] .$$

As  $\mathbb{E}[u(S_T - K)]$  is a constant independent of the agent's strategy  $\tau$  the agent faces the same maximization problem as an EU maximizer. It follows that the model where the agent anticipates regret relative to past and future decisions, induces the same preferences over stopping times as EU, and thus does not yield different predictions.

**Proposition 3.** *For every path of the process  $X$ , the observed behavior of an agent who minimizes anticipated regret over past and future decisions is identical to that of an EU maximizer, i.e. given by  $\tau(b^u, X)$ .*

**Regret Only over Past Decisions.** Finally, we consider the situation where the agent feels regret only over past past decisions. The ex-post optimal decision for the agent is to stop in the period  $t \leq \tau$  when the process reached its maximal value. This would yield a utility of

$$\max_{t \leq \tau} u(X_t - K) = u(S_\tau - K) .$$

Thus, the expected regret value if the agent uses the continuation strategy  $\tau$  after a history where the value of the process equals  $x$  and the past maximum equals  $s$  is given by<sup>9</sup>

$$V^r(\tau, x, s) = \mathbb{E}_{x,s}[U_\tau - \kappa u(S_\tau - K)] . \quad (7)$$

The regret agent stops at time  $t$  with value  $X_t = x$  and past maximum  $s = S_t$  whenever this is optimal

$$u(x - K) - \kappa u(s - K) = \sup_{\tau} V^r(\tau, x, s) . \quad (8)$$

**Time Consistency** It is important to note that anticipated regret preferences in our model are time-consistent in the sense that a strategy which maximizes the regret functional given in Eq. 7 at time zero also maximizes it at every future point in time.

---

<sup>9</sup>We define  $\mathbb{E}_{t,x,s}[\cdot] = \mathbb{E}[\cdot \mid X_t = x, S_t = s, T > t]$  and  $\mathbb{E}_{x,s}[\cdot] = \mathbb{E}_{0,x,s}[\cdot]$ . Conditional probabilities  $\mathbb{P}_{t,x,s}[\cdot]$  are defined analogously.

**Proposition 4.** *Anticipated regret preferences are time-consistent.*

As there is no time inconsistency we do not need to specify whether it is possible for the agent to commit to a strategy, or whether the agent is sophisticated in anticipating her own future behavior as all predictions made by anticipated regret are independent of those assumptions.

**The Optimal Strategy** As the regret agent evaluates payoffs relative to the counterfactual payoff she could have obtained by stopping when the process was at its maximum, her optimal stopping decision does not only depend on the current value of the process  $X_t$ , but also on the past maximum  $S_t$ . Intuitively, when the past maximum is higher, the regret is higher, and as continuation entails the opportunity of reducing regret the incentive to continue is greater.<sup>10</sup>

**Theorem 1** (Optimal Strategy). *Assume  $u$  is concave and A1 holds. The regret agent stops if and only if*

- i)  $X_t = S_t$  and  $X_t \in [b^r, b^u]$  or
- ii)  $X_t \geq b^u$ .

The regret cut-off  $b^r \leq b^u$  is given by

$$b^r = \min\{x: h^{-\alpha}(1 - \kappa)u(xh - K) \leq (1 - \kappa h^{-\alpha})u(x - K)\}$$

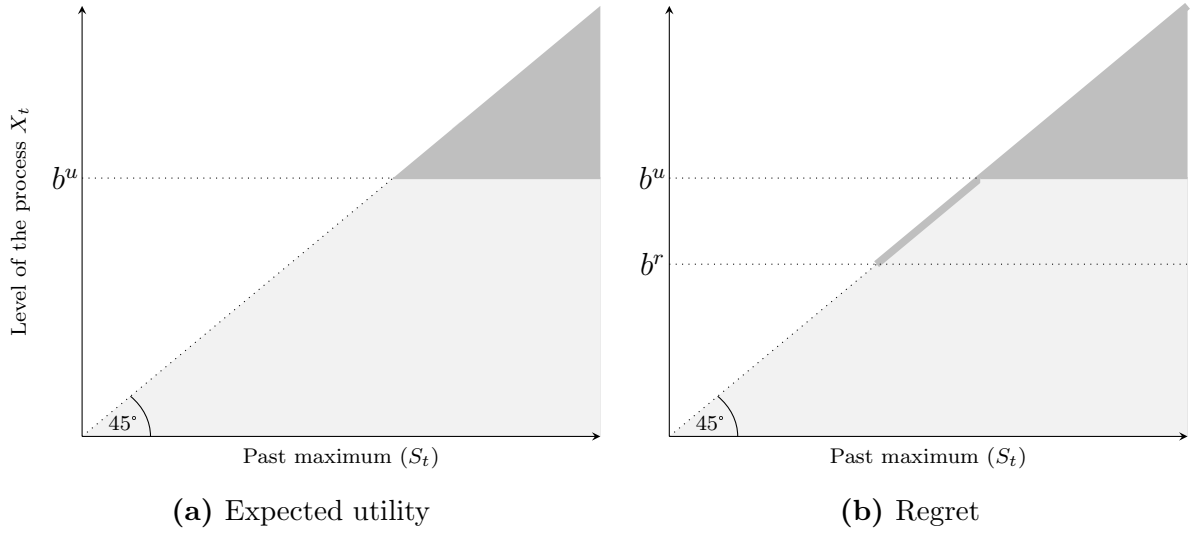
and non-increasing in the intensity of regret  $\kappa$ .

We plotted the optimal strategy of the regret agent in panel (b) of Figure 1. For comparison, we also plotted the optimal strategy of the EU agent in panel (a). The regret agent stops below the EU cut-off  $b^u$  if and only if she feels no regret  $X_t = S_t$  and the value of the process is above  $b^r$  or if the value of the process is above the EU cut-off.

The proof of Theorem 1 proceeds in several steps. First, the proof establishes that it can never be optimal to continue when the current value of the process is above the EU cut-off  $b^u$ . The reason is that the regret value (given in Eq. 7) is decreasing in the past maximum and the maximum only increases over time. Hence, the feeling of regret can make continuation only less attractive compared to the corresponding EU preference without regret and the regret agent always stops before the corresponding EU agent.

---

<sup>10</sup>We provide a closed form solution of the regret agent's value function  $V^r(x, s)$  in Lemma 9 in the Appendix.



**Figure 1:** This shows the subgame perfect cut-off of an EU agent (panel a) and regret agent (panel b). Regions shaded in light grey show points where the agent continues and regions shaded in dark grey show where it is optimal for the agent to stop.

In the second step of the proof we establish that below  $b^u$  the regret agent never stops when the process is not at its past maximum. The reason for this property is that when the current value of the process is below its past maximum and  $b^u$  the cut-off strategy which continues until the process reaches  $b^u$  or its past maximum is better than stopping when the agent feels no regret by Proposition 2. Note that the past maximum (and thus the regret) never changes when this strategy is used. This implies that the regret agent has the same preferences between this strategy and stopping immediately as the EU agent and thus never stops when the current value is below the past maximum.

It follows from the definition of  $b^r$  as the smallest point such that it is not optimal to wait for an up-tick that the regret agent never stops below  $b^r$ . In the final step we establish that the regret agent stops whenever the process is at its past maximum and between  $b^r$  and  $b^u$ . This is established recursively using that the value of waiting for an up-tick is monotone for concave  $u$ .

Theorem 1 implies that the agent stops the first time the process reaches a value greater  $b^r$  and thus her optimal behavior is indistinguishable from a cut-off rule:

**Proposition 5.** *For every path of the process  $X$ , starting in  $X_0 \leq b^r$ , the observed stopping time is given by the cut-off stopping time  $\tau(b^r, X)$ . Under Assumption A1 and for concave  $u$  this holds for any initial value  $X_0$ .*

Intuitively, even though the agent would behave differently from a cut-off rule after a history where the process started in  $X_0 < b^r$  and she did not stop the first time when  $X_t = S_t = b^r$  such histories never occur when the agent behaves optimally. This makes

it impossible to distinguish the regret agent's optimal behavior from a cut-off rule, and thus from the optimal behavior of an EU maximizer.

As  $b^r$  is smaller  $b^u$  the observed behavior of the regret agent equals the expected behavior of an EU agent with a lower cut-off, i.e. greater risk-aversion. This implies that an increase in regret has the same consequences for *optimal behavior* as an increase in risk-aversion. It is thus interesting to consider a setting where agents have a strictly positive probability to continue beyond the point where it was optimal to stop as otherwise the regret agent will never reach any history, where regret predicts behavior different from EU.

### 3.3. Stochastic Stopping Behavior

Observed individual choices in practice (in the lab and the field) often exhibit a strong stochastic component, i.e. identical repeated choices lead to different decisions, and there exists a vast literature surrounding the analysis of such stochastic choice behavior. For example, static discrete-choice models and their econometric analysis were introduced into economics by McFadden (1968, 1976), based on the earlier work on random utility maximization by Thurstone (1927); Luce (1959); Becker et al. (1963). Discrete-choice analysis was adapted to a dynamic context by Heckman (1981) and an application to stopping choices was developed by Rust (1987).<sup>11</sup>

In this section we will show that random choice behavior allows us to distinguish regret from EU preferences. Following the standard approach in that literature, we assume that the agent *randomly* chooses between stopping and continuation with a probability that depends on their relative utility value. That is, we introduce the possibility that in every period, both stopping and continuation are chosen by the agent with positive probability. More precisely, there exists an increasing function  $\Psi : \mathbb{R} \rightarrow [0, 1]$  such that the probability that the agent continues at a point  $(x, s)$  is given by

$$\mathbb{P}_{t,x,s} [\tau \neq t \mid \tau \geq t] = \Psi (\rho(x, s)) , \quad (9)$$

where the argument  $\rho : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$  of  $\Psi$  is the difference between the stopping and continuation value

$$\rho(x, s) = \sup_{\tau > t} V^r(\tau, x, s) - [u(x - K) - \kappa u(s - K)] . \quad (10)$$

---

<sup>11</sup>See McFadden (2000); Rust (1994); Aguirregabiria and Pedro (2010) for more detailed surveys of the literature.

Intuitively,  $\rho(x, s)$  measures the intensity with which the agent desires to continue, and the more she prefers continuation over stopping the more likely she continues.

This specification nests the two most common functional forms of random choice: Logit  $\Psi(\nu) = \frac{1}{1+e^{-\nu}}$  and Probit  $\Psi(\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\nu} e^{-z^2/2} dz$ . Choice probabilities of the logit-form can be shown to derive from random utility maximization model, where the agent chooses the action that yields highest utility, but perceives utility with an error that is Extreme Value Type I distributed (McFadden, 1981).

To simplify notation we denote by

$$\pi(x, s) = \Psi(\rho(x, s))$$

the probability that the agent continues when the current value of the process equals  $x$  and the past maximum equals  $s$ . The next result shows that the agent is more likely to continue if her current regret/past maximum of the process is high and more likely to stop if the current value of the process is high.

**Proposition 6** (Continuation Probabilities). *The probability of continuation  $\pi(x, s)$*

- i) decreases in the current value of the process  $x$  if  $\mathcal{L}u$  is non-increasing,*
- ii) increases in the past maximum  $s$ , and*
- iii) decreases in the intensity of regret  $\kappa$ .*

The fact that the agent is more likely to stop when the current value of the process is high is intuitive as the higher the value of the process the more the agent loses when she decides not to stop and the game ends. The assumption that  $\mathcal{L}u(x)$  is decreasing in  $x$  ensures that the agent's risk aversion does vanish so quickly in the agent's wealth that continuation becomes more attractive for higher values of the process. It is for example satisfied for positive constant relative risk-aversion.

The second part of Proposition 6 is what distinguishes regret from expected utility in a random choice environment and the heart of our identification strategy. In the proof of Proposition 6 we show that while the preferences of an expected utility maximizer are not affected by the past maximum of the process, a regret agent prefers to continue more strongly when the past maximum is higher and she is subject to higher regret. As the agent's behavior in a random choice model reacts to the intensity of preferences, this implies that the regret agent is more likely to continue. This yields the testable prediction that subjects stopping frequency should be decreasing in the past maximum under regret, and constant under EU. We test this in Section 5 and find that subjects are significantly

less likely to stop for high values of the past maximum, which is consistent with a regret, but not EU preferences.

Proposition 6 shows that when the current value of the process is higher the agent becomes less likely to continue. Note, that such an increase in the current value of the process also decreases the distance to the past maximum, and thus the regret which makes continuation less attractive. To isolate the effect of the current value of the process we will next consider how the stopping probability changes in the level of the process when the distance to the past maximum is kept fix.

**Assumption A2** (Decreasing Differences).

$u(\beta s' - K) - u(\beta s - K)$  is decreasing in  $\beta \in \mathbb{R}$  for all  $s < s'$ .

Assumption A2 is a strengthening of the assumption made in Proposition 6 and satisfied whenever the agent is sufficiently risk-averse.

**Lemma 1.** *A2 is satisfied if  $u$  is twice differentiable, the agent is risk-averse and the sum of relative risk-aversion and  $K$  times absolute risk-aversion exceeds one at every point*

$$\frac{x|u''(x)|}{u'(x)} + K \frac{|u''(x)|}{u'(x)} \geq 1.$$

If we impose Assumption A2 we get that the probability to continue increases in the current level of the process even if one keeps the distance to the past maximum and thus the regret fixed.

**Proposition 7.** *Let  $\mathcal{L}u(x)$  be decreasing in  $x$  and  $u$  satisfy A2, then the probability of continuation conditional on being  $n$  ticks below the past maximum  $\pi(x, xh^n)$  is decreasing in  $x$ .*

## 4. Relation to Other Models of Regret

The type of regret preferences we consider were introduced by Loomes and Sugden (1982) in a static one-shot setting. There is only one other theoretical study that we are aware of that considers such regret preferences in dynamic settings. Krämer and Stone (2012) study behavior induced by anticipated regret preferences in a two-period model. If payoffs are correlated, they show that regret might lead to an excess tendency to stick to the first period action or an excessive tendency to switch actions.

The earliest theoretical study of dynamic regret we are aware of is Hayashi (2009, henceforth HY). Hayashi considers min-max regret preferences in a dynamic setting,

with multiple priors. In his model the agent minimizes the weighted sum of the expected and the worst case regret. Our notion of regret preferences is different along several dimensions: First, agents in our model are not pure regret minimizer, but maximize expected utility minus regret and our model thus nests EU as a special case. Second, while we assume that the agent cares only about the expected regret, HY assumes that the agent's decisions are influenced by the regret in a worst case scenario. This difference is not merely of technical nature, but has important implications. HY shows that due to this worst-case approach, the agent is time-inconsistent and HYs analysis focus on questions around time-inconsistency, while time-inconsistency plays no role in our model.

Third, the set of counterfactuals relative to which an agent evaluates her payoffs differs. We consider both the case where an agent feels regret only over past decisions, as well as the case where regret is over past and future decisions. HY restricts attention to the latter case.

Another related paper is given by [Krestel \(2015\)](#). Krestel analyzes regret preferences in a stopping model, where the set of hypothetical decisions relative to which the agent evaluates her regret is chosen by an adversary in order to minimize the agent's value and the intensity of regret decreases over time. In our setting, the set of hypotheticals is exogenous and either the set of all past and future, or all past decisions. This leads to very different predictions, e.g. the model predicts that agents never stop when the value of the process is equal to its maximum, while we our model predicts they only stop at the maximum. While [Krestel \(2015\)](#) describes many convincing applications of his model, this prediction is not consistent with the behavior we observe in the lab.

A paper that develops a notion of regret very similar to ours in a *static* context is given by [Filiz-Ozbay and Ozbay \(2007\)](#). They consider behavior of a regret minimizer in first-price auctions and show that anticipated regret can explain overbidding in such auctions.

## 5. Testable Hypotheses & Lab Implementation

In this section, we translate our findings from section [3.1](#) and [3.2](#) into hypotheses about behavior. First, the prediction that subjects will use cut-off strategies implies that the agent always stops at the same level independently of the sequence of offers she observed.

**Hypothesis H1** (Constant Reservation Level). *For all realized sequences of offers  $X = (X_1, X_2, \dots) \neq (X'_1, X'_2, \dots) = X'$  the level at which the agent stops is the same*

$$X_{\tau(X)} = X'_{\tau(X')}.$$

Hypothesis H1 implies that the level at which subjects stop is same across different repetitions of the same stopping task. We also showed that subjects should behave consistent within a round.

**Hypothesis H2** (Stop at Maximum). *For all realized sequences of offers  $X$  the agent only stops if the process is at it's maximum*

$$X_\tau = S_\tau.$$

The third hypothesis is closely related and says that subjects should stop the first time the process reaches the level at which they finally decided to stop.

**Hypothesis H3** (Stop the First Time). *For all realized sequences of offers  $X$  the agent never stops at a value at which she decided to continue before.*

$$\tau = \inf\{t: X_t = X_\tau\}.$$

In the previous section 3.3, we derived further predictions about behavior in a model where choice is stochastic. Specifically, Propositions 6 and 7 give us the following hypothesis for  $\kappa > 0$

**Hypothesis H4** (Reluctance to Stop below Maximum). *The empirical frequency with which subjects stop at the level  $x$  given a past maximum of  $s = xh^n$  is*

(H4a) *increasing in  $x$ , and*

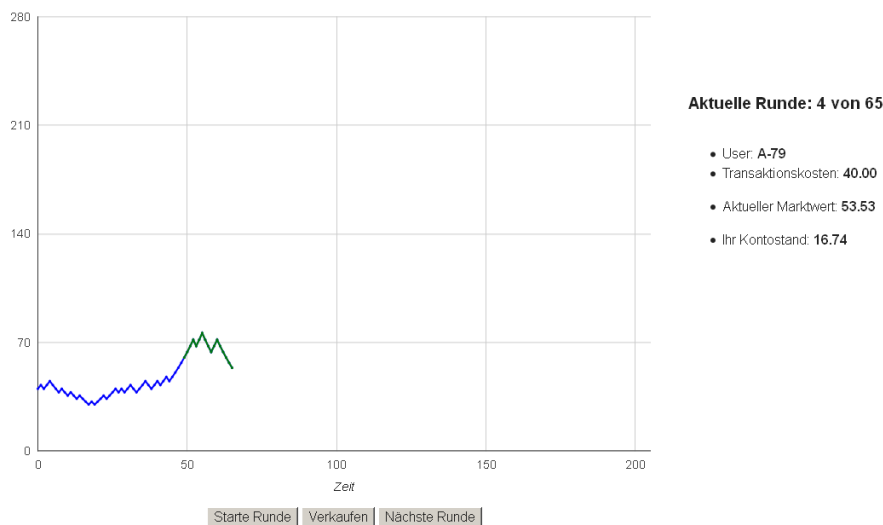
(H4b) *decreasing in  $s$ .*

## 5.1. Laboratory Implementation

Our lab experiment follows the well-established design of Oprea et al. (2009). Hence, we framed the stopping task as an asset-selling problem, where  $X_t$  is the market price for the asset. Subjects were explained that they own one unit of a fictitious asset and that they have the opportunity, but not the obligation, to sell it. The instructions then explained in detail the setting laid out in Section 2. All parameters were given in the instructions and their meaning explained to subjects. Their values are listed in table 1 in the Appendix. A translated version of the instructions can be found in a web appendix to this paper.<sup>12</sup> The experiment was computer-based and conducted at the lab of the

<sup>12</sup>We convinced ourselves that subjects had indeed understood (i) how payoffs are computed, (ii) that the increments of the process are iid and (iii) what is the risk that a round ends before the next period, through a questionnaire with control questions that subjects had to answer prior to the experiment (see web appendix). 95% of the time subjects answered our control questions correctly.

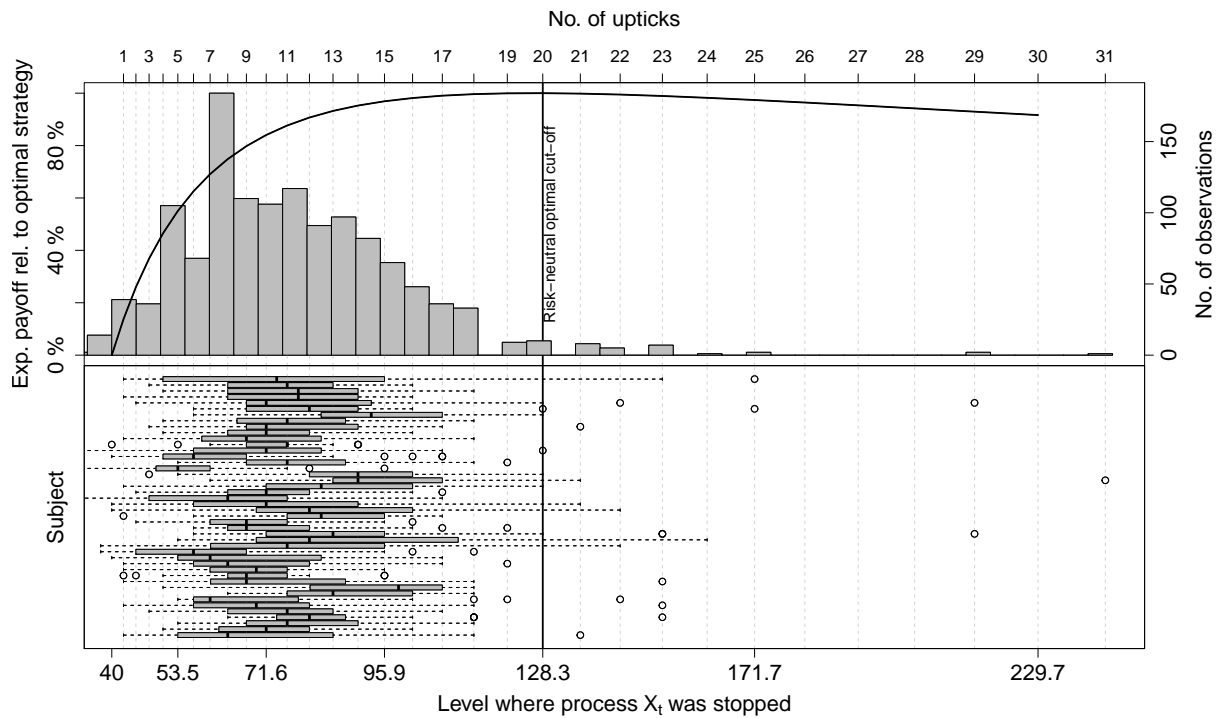




**Figure 2:** The main experimental screen (in German).

Technical University Berlin. Subjects played 65 rounds in which they had the option to sell their stock. In each round they observed a path of the market price in a diagram until the jump to zero (see Figure 2). Each second, there were two ticks of the process. Pressing the middle button labeled 'Sell', a subject sold the stock at the current value of the price process.<sup>13</sup> The paths of the 65 random walks were generated prior to the experiment and were the same for all subjects, only the order in which they were shown to subjects was random.

At the end of the experiment one round was randomly selected with equal probability to determine a subject's payoff. We paid subjects 0.15 times the number of points they had obtained in that round plus 10 Euro show-up fee.<sup>14</sup> We ran two sessions with 22 students each. Participants were recruited from the ORSEE pool of the TU and WZB.<sup>15</sup> The average duration of the overall experiment was 80 minutes, and the mean earnings for subjects was 12.30 Euros (median=10 Euros), where the minimum and the maximum payment were 10.00 Euros and 19.00 Euros respectively.



**Figure 3:** Line in the upper chart shows how the expected payoff from using different cut-offs changes as a percentage of the payoff under the optimal strategy (left scale), grey bars show a histogram of observed stopping decisions (right scale). Boxplots in bottom chart illustrate the variation in reservation levels at the subject level. The vertical line indicates the optimal cut-off for a risk neutral agent.

## 6. Empirical Results

We first inspect our data with respect to Hypotheses H1 to H3. For this, we plotted the observed stopping decisions in Figure 3. The top panel shows a histogram of the reservation levels across subjects in our sample. With respect to Hypothesis H1, this shows that there is a large variation in reservation levels across rounds. The boxplots in the bottom chart display a breakdown of the observed reservation levels for each subject separately. It shows that large part of the variation observed in the top chart is due to the fact that each subject individually varied levels a lot.

**Finding 1.** *Hypotheses H1 is not confirmed in our sample. Subjects vary their reservation levels substantially over different rounds of the same stopping task and do not appear to converge to a unique level.*

<sup>13</sup>Subjects continued to observe the process until termination. We could have given subjects the option to skip to the next round immediately after the stopping decision, but this provides an incentive to impatient subjects to stop early to reduce lab time.

<sup>14</sup>We rounded to the nearest Euro.

<sup>15</sup>See: <https://experimente.wzb.eu/>

This could be due to the gradual convergence of reservation levels. In that case, we would expect the variation in reservation levels to decrease with the number of rounds played. This is not what we find our data, instead reservation levels fluctuate rather unsystematically over rounds (see Table 2 in Appendix C). Given this, we also believe that learning only plays a minor role in explaining observed behavior.<sup>16</sup>

The experimental literature has largely focused on checking hypothesis H1 and our results replicate the overarching finding that subjects vary their reservation level across rounds and stop below the risk-neutral optimal cut-off. To the best of our knowledge, hypotheses H2 and H3 have received no, or much less attention in the extant literature. We measure deviations from H2 and H3 along two dimensions: (i) the number of ticks the stopping value is below the running maximum and (ii) the deviation in terms of *multiplicity*, i.e. the number of times a subject had seen her stopping value before stopping.

Table 3 in the Appendix shows the results in a simple contingency table. We observe only 326 out of 1279, i.e. roughly 25%, decisions that are consistent with cut-off decisions. The remaining 75% are not.

**Finding 2.** *For the large majority of decisions Hypotheses H2 and H3 do not hold. Only 25% of the time subjects stop the process at the running maximum the first time it is reached.*

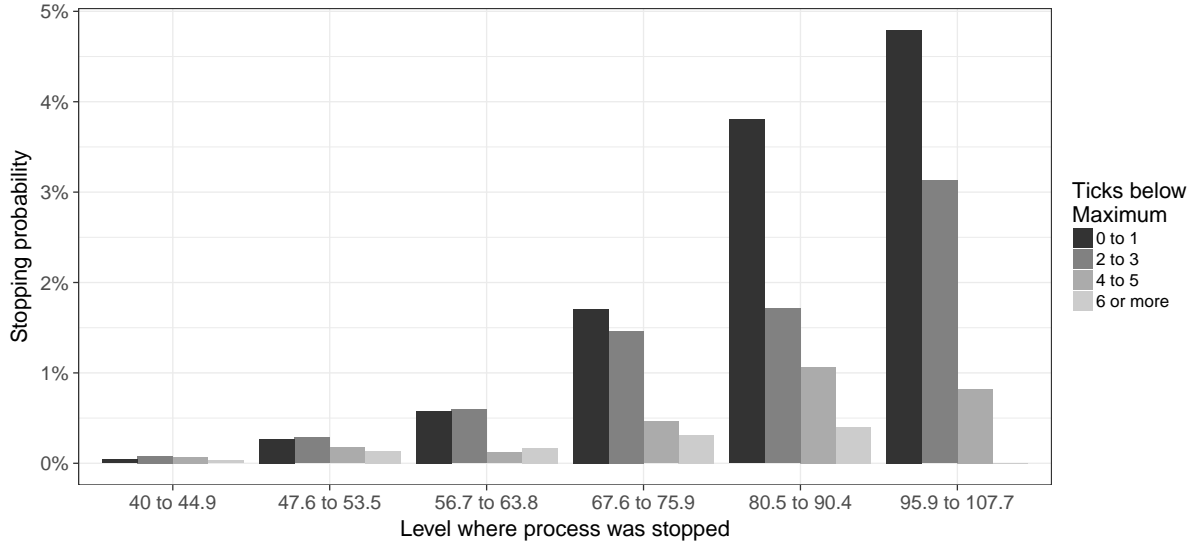
In light of these findings, subjects clearly do not play cut-off strategies. Instead they vary their reservation levels over different rounds of the same stopping task, do not behave time-consistently within rounds 75% of the time and visit the same level of the process on average three times before they eventually stop at it.

## 6.1. Regret Aversion

In this section we analyze our data with respect to Hypothesis H4, i.e. whether subjects are less likely to stop when the past maximum is high as we have shown is predicted by anticipated regret preferences. Toward that end, Figure 4 shows the empirical probability

---

<sup>16</sup>Finding that subjects' reservation levels do not settle to a constant level does not necessarily mean that their variation is entirely unsystematic. Hence, we estimated the model put forward by Oprea et al. (2009), where subjects' reservation level  $b^j$  in round  $j$  depends on forgone earnings in the previous round (see Oprea et al., 2009, for details). As Oprea et al. (2009) we find that subjects tend to increase (decrease) their reservation level after stopping too early (late). In our sample, however, the effect of stopping too late is not statistically significant. Following Oprea et al. (2009), we iteratively simulated stopping decisions using our fitted model, but found that the model only tracks the evolution of average reservation levels for the first 20 rounds and overshoots thereafter (see Fig. 1 in the Web Appendix).



**Figure 4:** Empirical stopping probabilities as a function of the level of the process  $x$  for different distances  $n$  to the maximum  $s$ .

to stop at a given level of the process conditional on reaching it and the distance to the past maximum.<sup>17</sup> The graph shows that on the aggregate level, there is a clear tendency in our sample for the stopping probability to (i) increase with the level of the process and (ii) decrease with the distance to the past maximum.<sup>18</sup>

To investigate this more formally, we estimate a hierarchical dynamic discrete-choice model (Rust, 1987, 1994). We assume a simple logit model, where the probability that agent  $i$  stops at a given point is a simple function of  $x$  and the distance to the past maximum<sup>19</sup>

$$1 - \pi_i(x, s) = \frac{1}{1 + \exp(-(\alpha_i + \beta_{i,1}x + \beta_{i,2}(s - x)))} . \quad (11)$$

Hypothesis H4 then is consistent with  $\beta_{i,1} > 0$  and  $\beta_{i,2} < 0$ . This gives us the likelihood

<sup>17</sup>To avoid overplotting, we have aggregated empirical frequencies over 3-tick intervals of  $X_t$  and over blocks of two ticks in terms of the distance to the past peak. We also restricted attention to the range from 40 to roughly 153 into which 99% of our observations fall (cf. histogram in Fig. 3).

<sup>18</sup>Figure 4 depicts the empirical frequency with which subjects stopped in one period, i.e. half a second. Note, that those probabilities grow exponentially for example if the per period probability to stop equals 4% then the probability to stop in 10 seconds equals  $1 - 0.96^{20} \approx 58\%$  and the expected stopping time equals  $\frac{1}{0.04} \times 0.5s = 12.5s$ .

<sup>19</sup>We make the assumption that the stopping or decision rule is a deterministic function in the eyes of a subject and that subjects do not anticipate deviations from the ex-ante plan. This is standard in the stochastic choice literature. If subjects anticipate deviations, this reduces their incentive to continue at any point and lowers their ex-ante cut-off level. Other commonly assumed sources behind the econometric error in our model, such as unobserved state variables (see e.g. Rust, 1996), seem less likely to play a major role in our controlled laboratory setting.

function  $\ell_i$  for each subject across all 65 rounds

$$\ell_i = \prod_{k=1}^{65} \left( 1 - \mathbf{1}_{\{\tilde{\tau}_{i,k} < T^{i,k}\}} \pi_i(X_{\tilde{\tau}_{i,k}}^{i,k}, S_{\tilde{\tau}_{i,k}}^{i,k}) \right) \prod_{r=0}^{\bar{t}} \pi_i(X_r^{i,k}, S_r^{i,k}), \quad (12)$$

where  $\tilde{\tau}_{i,k}$  is the observed stopping time of subject  $i$  in round  $k$ ,  $X^{i,k}$  the realized path,  $S^{i,k}$  its associated maximum process,  $T^{i,k}$  the random time the game ended,  $\bar{t} = \min\{\tilde{\tau}_{i,k}, T^{i,k}\}$ , and  $\gamma_i = (\alpha_i, \beta_{i,1}, \beta_{i,2})$  the vector of parameters for subject  $i$ . Taking the product across all subjects, gives us the final likelihood function of our model.

The following Proposition shows that there is a direct link between our theoretical results on stochastic choice (Section 3.3), our behavioral hypothesis H4, and our econometric model.

**Proposition 8.** *The natural log of the likelihood function (12) is given by*

$$\log(\ell_i) = \sum_s \sum_{x \leq s} n(x, s) \left[ f^i(x, s) \log \left( \frac{\pi_i(x, s)}{1 - \pi_i(x, s)} \right) + \log(1 - \pi_i(x, s)) \right],$$

where  $st(x, s)$  is the number of observed stopping decisions,  $ct(x, s)$  the number of observed continuation decisions,  $n(x, s)$  the total number of decisions observed, and  $f(x, s) = \frac{ct(x, s)}{n(x, s)}$  the continuation frequency.

While Propositions 6 and 7 make directed predictions about the continuation probability, Proposition 8 shows that observed behavior is identified in the likelihood only through their empirical counterpart: the observed continuation frequencies.

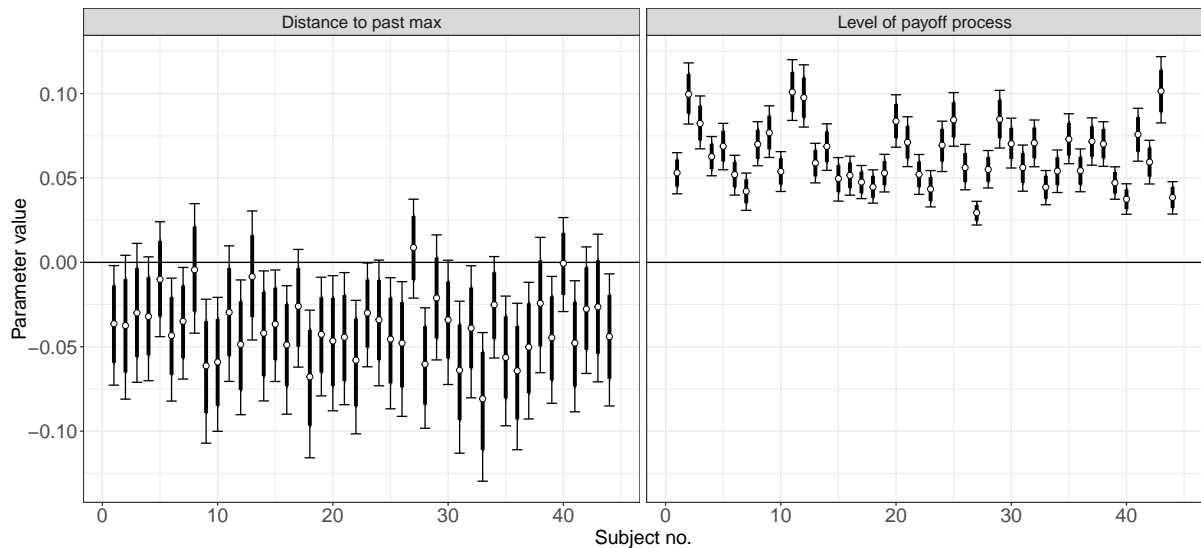
In order to take into account individual heterogeneity, our model treats subjects as belonging to a population of individuals with idiosyncratic parameters which come from a common population distribution

$$\begin{aligned} \alpha_i &\sim N(\mu_\alpha, \sigma_\alpha^2) ; \beta_{i,g} \sim N(\mu_{\beta_g}, \sigma_{\beta_g}^2), \quad g = 1, 2 \\ \mu_\alpha, \mu_{\beta_g} &\sim N(0, 1) \\ \sigma_\alpha, \sigma_{\beta_g} &\sim \text{Half-Cauchy}(0, 2) . \end{aligned}$$

We have deferred further methodological details of the estimation to Appendix B.<sup>20</sup>

Figure 5 shows the posterior means for  $\beta_{i,1}$  and  $\beta_{i,2}$  together with 80% and 95% posterior intervals. For all subjects we find that the likelihood to stop the process increases with its level ( $\beta_{i,1} > 0$ ). This is the expected result and confirms part (a) of hypothesis H4.

<sup>20</sup>Web Appendix B provides an alternative structural Bayesian estimation and shows that our results are robust to introducing the possibility that subjects are inattentive.



**Figure 5:** Shows the posterior means (hollow circles) for  $\beta_{i,1}$  (right) and  $\beta_{i,2}$  (left) across all 44 subjects together with 80% and 95% posterior intervals.

More interestingly, our model also confirms what we suspected from the pattern in Figure 4. The distance to the past maximum ( $s - x$ ) has a negative influence on the likelihood to stop, thus confirming part (b) of hypothesis H4. Even though posteriors for  $\beta_{i,2}$  are wider than for  $\beta_{i,1}$ , the bulk of the posterior mass is still well below zero for all but one subject. For 27 (36) out of 44 subjects the 95% (80%) posterior interval is entirely in the negative range. This is also reflected in the posterior of the population mean  $\mu_{\beta_2}$ , which has posterior mean  $-0.04$  and 95% posterior probability to lie within  $[-0.03, -0.05]$ .

**Finding 3.** *We find strong evidence for Hypothesis H4a in our sample: participants are more likely to stop the process for higher values of the payoff process. We also find evidence for Hypothesis H4b in our sample: for a given value of the payoff process, subjects are less likely to stop, the higher the past maximum.*

## 7. Conclusion

In this paper we sought to answer two questions: What do theories of dynamic behavior predict people to do in stopping problems and how well do these predictions fare when it comes to describing actual behavior? A classical result from the literature is that the optimal strategy of an agent maximizing EU is to wait for the payoff process to reach a given reservation level and then stop immediately. That is, (i) agents are predicted to have a unique reservation level and (ii) they should never stop at a point where they chose to continue before. We have shown that this prediction is, in our setup, not restricted to

risk-averse EU preferences, but extends to gain-loss or S-shape utility.

If and to which extent part (ii) of this prediction is satisfied by subjects in the laboratory has been largely unexplored and not recorded in the data in the extant literature. And it seems that a substantial deviation from what theory predicts has gone undetected. In our sample subjects do not stop the first time they reach their final stopping level in 75% of all observations and the majority of the decisions is irreconcilable with any of the considered theories assuming that choices are deterministic.

We have show that in a random choice setting, regret preferences lead to behavior which is testable, and different from behavior under EU and gain-loss utility. The regret agent becomes more likely to continue the higher the past maximum of the process, and thus the regret she feels. We find this to be a prevalent pattern in our data. The empirical frequency with which subjects stop the process at a given level conditional on reaching it, decreases uniformly with the distance to the past maximum. Estimating an econometric model confirms that most subjects reaction to the past maximum is statistically significant.

There are several avenues open for future research on this topic. While we have made a first attempt to provide empirical evidence for regret effects in the laboratory, it would be desirable to complement this with evidence from field data, e.g. using data on house sales or liquidation of financial assets. In a similar vein, it seems straightforward to design further experiments to provide more direct tests for regret effects in stopping tasks. Recently various papers were brought to our attention, which appear to have made some headway in this direction. For example, [Descamps et al. \(2016\)](#) find significant evidence for non-anticipated regret in an experiment where subjects decide not only when to stop, but also which of two options two pick. [Fioretti et al. \(2017\)](#) replicate our experiment and furthermore show that in line with our model of regret, subjects stop later when the outcomes after stopping are revealed.

## References

- Aguirregabiria, V. and Pedro, M. (2010). Dynamic discrete choice structural models: A survey. *Journal of Econometrics*, 156(1):38–67.
- Becker, G., DeGroot, M., and Marschak, J. (1963). Stochastic models of choice behavior. *Behavioral Science*, 8(1):41–55.
- Ben-David, I. and Hirshleifer, D. (2012). Are investors really reluctant to realize their losses? trading responses to past returns and the disposition effect. *Review of Financial Studies*, 25(8):2485–2532.

- Betancourt, M. and Girolami, M. (2015). Hamiltonian monte carlo for hierarchical models. In Upadhyay, S. K., Singh, U., Dey, D. K., and Loganathan, A., editors, *Current Trends in Bayesian Methodology with Applications*, chapter 4, pages 79–102. CRC Press.
- Bleichrodt, H. and Wakker, P. (2015). Regret Theory: A Bold Alternative to the Alternatives. *The Economic Journal*, 125:493–512.
- Brown, M., Flinn, C., and Schotter, A. (2011). Real-time search in the laboratory and the market. *The American Economic Review*, 101(2):948–974.
- Camille, N., Coricelli, G., Sallet, J., Pradat-Diehl, P., Duhamel, J.-R., and Sirigu, A. (2004). The involvement of the orbitofrontal cortex in the experience of regret. *Science*, 304(5674):1167–1170.
- Coricelli, G., Critchley, H. D., Joffily, M., O’Doherty, J. P., Sirigu, A., and Dolan, R. J. (2005). Regret and its avoidance: a neuroimaging study of choice behavior. *Nature neuroscience*, 8(9):1255–1262.
- Cox, J. and Oaxaca, R. (1989). Laboratory experiments with a finite-horizon job-search model. *Journal of Risk and Uncertainty*, 2(3):301–329.
- Cox, J. and Oaxaca, R. (1992). Direct tests of the reservation wage property. *Economic Journal*, 102(415):1423–32.
- Cox, J. and Oaxaca, R. (2000). Good news and bad news: Search from unknown wage offer distributions. *Experimental Economics*, 2(3):197–225.
- Descamps, A., Massoni, S., and Page, L. (2016). Knowing when to stop and make a choice, an experiment on optimal sequential sampling.
- Ebert, S. and Strack, P. (2015). Until the bitter end: On prospect theory in the dynamic context. *American Economic Review*, 105(4):1618–1633.
- Filiz-Ozbay, E. and Ozbay, E. Y. (2007). Auctions with anticipated regret: Theory and experiment. *American Economic Review*, 97(4):1407–1418.
- Fioretti, M., Vostroknutov, A., and Coricelli, G. (2017). Structural estimation of past and future regret preferences in an optimal stopping experiment.
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., and Rubin, D. B. (2013). *Bayesian Data Analysis*. Chapman & Hall/CRC, 3rd edition.
- Gelman, A. and Rubin, D. B. (1992). Inference from Iterative Simulation using Multiple Sequences. *Statistical science*, pages 457–472.



- Gneezy, U., Kapteyn, A., and Potters, J. (2003). Evaluation periods and asset prices in a market experiment. *The Journal of Finance*, 58(2):821–838.
- Gneezy, U. and Potters, J. (1997). An experiment on risk taking and evaluation periods. *The Quarterly Journal of Economics*, 112(2):631–645.
- Grinblatt, M. and Keloharju, M. (2000). The investment behavior and performance of various investor types: a study of finland’s unique data set. *Journal of Financial Economics*, 55(1):43–67.
- Haigh, M. S. and List, J. A. (2005). Do professional traders exhibit myopic loss aversion? an experimental analysis. *The Journal of Finance*, 60(1):523–534.
- Hayashi, T. (2009). Stopping with anticipated regret. *Journal of Mathematical Economics*, 45(7):479–490.
- Heckman, J. (1981). Statistical models for the analysis of discrete panel data. In Manski, C. and McFadden, D., editors, *Structural Analysis of Discrete Data*, pages 114–178. MIT Press, Cambridge.
- Henderson, V. (2012). Prospect theory, liquidation, and the disposition effect. *Management Science*, 58(2):445–460.
- Hoffman, M. D. and Gelman, A. (2014). The no-u-turn sampler: Adaptively setting path lengths in hamiltonian monte carlo. *The Journal of Machine Learning Research*, 15(1):1593–1623.
- Kahan, J., Rapoport, A., and Jones, L. (1967). Decision making in a sequential search task. *Attention, Perception & Psychophysics*, 2(8):374–376.
- Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, pages 263–291.
- Krähmer, D. and Stone, R. (2012). Regret in dynamic decision problems. *University of Bonn Working Paper*.
- Krestel, C. (2015). Optimal Stopping With Regret: When To Stop If Nature Is Malevolent. *Working Paper*.
- Lakonishok, J. and Smidt, S. (1986). Volume for winners and losers: Taxation and other motives for stock trading. *The Journal of Finance*, 41(4):951–974.
- Loomes, G. and Sugden, R. (1982). Regret theory: An alternative theory of rational choice under uncertainty. *The Economic Journal*, 92(368):805–824.

- Luce, R. D. (1959). *Individual choice behavior: A theoretical analysis*. New York: Wiley.
- Magnani, J. (2015). Testing for the disposition effect on optimal stopping decisions. *American Economic Review*, 105(5):371–75.
- McFadden, D. (1968). The Revealed Preferences of a Public Bureaucracy. Dept. of Economics, Univ. of California, Berkeley.
- McFadden, D. (1976). The revealed preferences of a public bureaucracy: Empirical evidence. *The Bell Journal of Economics and Management Science*, 7:55–72.
- McFadden, D. (1981). Econometric Models of Probabilistic Choice. In Manski, C. and McFadden, D., editors, *Structural Analysis of Discrete Data*, pages 198–272. Cambridge: MIT Press.
- McFadden, D. (2000). Economic choices. Nobel Lecture.
- Neal, R. (2011). MCMC using Hamiltonian dynamics. In Brooks, S., Gelman, A., Jones, G. L., and Meng, X.-L., editors, *Handbook of Markov Chain Monte Carlo*, pages 116–162. Chapman and Hall/CRC.
- Oprea, R., Friedman, D., and Anderson, S. (2009). Learning to wait: A laboratory investigation. *Review of Economic Studies*, 76(3):1103–1124.
- Peskir, G. and Shiryaev, A. (2006). *Optimal stopping and free-boundary problems*. Birkhäuser Basel.
- Rapoport, A. and Tversky, A. (1966). Cost and accessibility of offers as determinants of optional stopping. *Psychonomic Science*, 4:145.
- Rapoport, A. and Tversky, A. (1970). Choice behavior in an optional stopping task. *Organizational Behavior and Human Performance*, 5(2):105–120.
- Rust, J. (1987). Optimal replacement of gmc bus engines: An empirical model of harold zurcher. *Econometrica*, pages 999–1033.
- Rust, J. (1994). Structural estimation of markov decision processes. In Engle, R. E. and McFadden, D., editors, *Handbook of Econometrics*, volume 4, pages 3082–3143. North-Holland, Amsterdam.
- Rust, J. (1996). Estimation of dynamic structural models, problems and prospects: discrete decision processes. In *Advances in econometrics: Sixth world congress*, volume 2, pages 119–170. Cambridge Univ Pr.

- Schotter, A. and Braunstein, Y. (1981). Economic search: An experimental study. *Economic Inquiry*, 19(1):1–25.
- Seale, D. and Rapoport, A. (1997). Sequential decision making with relative ranks: An experimental investigation of the "secretary problem". *Organizational Behavior and Human Decision Processes*, 69(3):221–236.
- Shefrin, H. and Statman, M. (1985). The disposition to sell winners too early and ride losers too long: Theory and evidence. *The Journal of Finance*, 40(3):777–790.
- Stan Development Team (2016a). Rstan: the r interface to stan. r package version 2.14.1.
- Stan Development Team (2016b). Rstan: the r interface to stan, version 2.14.0.
- Thurstone, L. L. (1927). A law of comparative judgment. *Psychological review*, 34(4):273.
- Weber, M. and Camerer, C. F. (1998). The disposition effect in securities trading: An experimental analysis. *Journal of Economic Behavior & Organization*, 33(2):167–184.
- Xu, Z. Q. and Zhou, X. Y. (2013). Optimal stopping under probability distortion. *The Annals of Applied Probability*, 23(1):251–282.
- Zeelenberg, M., Van Dijk, W. W., Manstead, A. S., and vanr de Pligt, J. (2000). On bad decisions and disconfirmed expectancies: The psychology of regret and disappointment. *Cognition & Emotion*, 14(4):521–541.
- Zeelenberg, M., van Dijk, W. W., Van der Pligt, J., Manstead, A. S., Van Empelen, P., and Reinderman, D. (1998). Emotional reactions to the outcomes of decisions: The role of counterfactual thought in the experience of regret and disappointment. *Organizational behavior and human decision processes*, 75(2):117–141.

## A. Mathematical Appendix

We denote by  $\mathcal{X} = \{h^k X_0 : k \in \mathbb{Z}\}$  the set of possible states of the process  $X_t$ .

### A.1. Justification for $\delta(ph + (1-p)h^{-1}) \geq 1$

The expected gain from stopping in period  $t+1$  instead of period  $t$ , if  $X_t = x$  equals

$$\delta (\mathbb{E}[X_{t+1} | X_t = x] - K) - (X_t - K) = x (\delta (ph + (1-p)h^{-1}) - 1) + (1 - \delta)K.$$

If  $\delta(ph + (1-p)h^{-1}) \geq 1$  this gain in expected payoff is positive for all  $x \in \mathcal{X}$  and an expected value maximizing agent never stops. The payoff from never stopping  $\tau = \infty$  equals zero as the agent never stops before the deadline  $\mathbb{P}[\tau < T] = 0$ . Thus no optimal strategy exists.

### A.2. Proofs

The following lemma allows us to derive the EU value of any cut-off strategy and the optimal EU cut-off.

**Lemma 2** (Probability to Stop before the Deadline). *When using the cut-off strategy  $\tau(b)$  as a continuation strategy from a given level  $X_t = x$ , the probability of stopping before the game ends,  $\tau(b) < T$ , is given by*

$$\mathbb{P}_{0,x}[\tau(b) < T] = \min \left\{ 1, \left( \frac{x}{b} \right)^\alpha \right\}.$$

where  $\alpha$  is given by  $\alpha = \frac{1}{\log(h)} \log \left( \frac{1}{2p\delta} + \sqrt{\frac{1}{4p^2\delta^2} - \frac{1-p}{p}} \right) > 1$ .

**Proof of Lemma 2.** For all  $x < b$ , the probability of reaching the level  $b$  from period-0 perspective, is equal to the probability of reaching the next period,  $T > 1$ , times the expected probability of reaching  $b$  from period-1 perspective, i.e.

$$\begin{aligned} \mathbb{P}_{0,x}[\tau(b) < T] &= \mathbb{P}_{0,x}[T > 1] \mathbb{E}_{0,x}[\mathbb{P}_{1,X_1}[\tau(b) < T]] \\ &= \delta(p \mathbb{P}_{1,xh}[\tau(b) < T] + (1-p) \mathbb{P}_{1,xh^{-1}}[\tau(b) < T]) \\ &= \delta(p \mathbb{P}_{0,xh}[\tau(b) < T] + (1-p) \mathbb{P}_{0,xh^{-1}}[\tau(b) < T]). \end{aligned}$$

To simplify notation define  $\psi_b(x) = \mathbb{P}_{0,x}[\tau(b) < T]$ . By definition  $\psi_b$  is a solution to the difference equation

$$\psi_b(x) = \begin{cases} 1 & \text{for all } x \geq b \\ \delta (p\psi_b(xh) + (1-p)\psi_b(xh^{-1})) & \text{for all } x < b \end{cases}, \quad (13)$$

taking values in  $[0, 1]$ . If we have two solutions  $\psi_b, \hat{\psi}_b$  of Eq. 13 it holds that

$$\begin{aligned} |\psi_b(x) - \hat{\psi}_b(x)| &= \mathbf{1}_{\{x < b\}} \delta \left| p(\psi_b(xh) - \hat{\psi}_b(xh)) + (1-p)(\psi_b(xh^{-1}) - \hat{\psi}_b(xh^{-1})) \right| \\ &\leq \delta \sup_{z < b} |\psi_b(z) - \hat{\psi}_b(z)|. \end{aligned} \quad (14)$$

As  $\psi_b(x), \hat{\psi}_b(x)$  lie between zero and one, the supremum of the differences  $\sup_z |\psi_b(z) - \hat{\psi}_b(z)|$  exists and is bounded by one. Taking the supremum over  $x$  in Eq. 14 yields

$$\sup_z |\psi_b(z) - \hat{\psi}_b(z)| \leq \delta \sup_z |\psi_b(z) - \hat{\psi}_b(z)|.$$

As  $\delta < 1$  it follows that  $\sup_z |\psi_b(z) - \hat{\psi}_b(z)| = 0$  and thus Eq. 13 can have at most one solution taking values in  $[0, 1]$ . Guessing the solution of (13) to be of the form  $\psi_b(x) = \mathbf{1}_{\{x < b\}} (\frac{x}{b})^\alpha + \mathbf{1}_{\{x \geq b\}}$  gives  $1 = \delta(ph^\alpha + (1-p)h^{-\alpha})$ . Substituting  $z = h^\alpha$  yields the quadratic equation

$$\begin{aligned} 0 &= \delta(pz + (1-p)z^{-1}) - 1 \\ \Rightarrow 0 &= z^2 - \frac{z}{\delta p} + \frac{1-p}{p} \Rightarrow z = \frac{1}{2\delta p} \pm \sqrt{\frac{1}{4\delta^2 p^2} - \frac{1-p}{p}}. \end{aligned}$$

This equation has two solutions  $z_1, z_2$ . Let  $z_1$  be the larger solution. As  $z \mapsto \delta(pz + (1-p)z^{-1}) - 1$  is negative for  $z = 1$  and becomes positive when  $z \rightarrow 0$  or  $z \rightarrow \infty$  it follows that  $z_2 < 1 < z_1$ .

For the smaller solution,  $z_2 < 1$ , it follows that  $\alpha = \log(z_2)/\log(h) < 0$ . Hence, the resulting function  $\psi(x) = \mathbf{1}_{\{x < b\}} (\frac{x}{b})^\alpha + \mathbf{1}_{\{x \geq b\}}$  is decreasing in  $x$  and takes values outside  $[0, 1]$ . This leads to a contradiction and shows that  $z = \frac{1}{2\delta p} + \sqrt{\frac{1}{4\delta^2 p^2} - \frac{1-p}{p}}$  and  $\alpha > 1$ .  $\square$

### A.2.1. Constant Absolute and Relative Risk Aversion

In this section we show that the following lemma holds:

**Lemma 3.** *Assumption A1 is satisfied if  $u$  exhibits positive constant absolute or positive constant relative risk aversion.*

*Proof.* Constant Absolute Risk Aversion: Let  $u(x) = -\frac{1}{\theta} \exp(-\theta x)$ , i.e. assume the agent has constant absolute risk-aversion of  $\theta$ . The expected change in utility from waiting one more period at  $x$  equals

$$\begin{aligned} \mathcal{L}u(x-K) &= \delta(pu(xh-K) + (1-p)u(xh^{-1}-K)) + (1-\delta)u(0) - u(x-K) \\ &= \delta u(x-K) \left[ \left( p \frac{u(xh-K)}{u(x-K)} + (1-p) \frac{u(xh^{-1}-K)}{u(x-K)} \right) + (1-\delta) \frac{u(0)}{u(x-K)} - 1 \right] \\ &= -\frac{e^{-\theta(x-K)}}{\theta} \left[ \delta(pe^{-\theta x(h-1)} + (1-p)e^{-\theta x(h^{-1}-1)}) + (1-\delta)e^{\theta(x-K)} - 1 \right] \end{aligned}$$

We will show that the second part is monotone increasing in  $x$ . Taking derivatives of the term in square brackets gives

$$\delta(-\theta(h-1)pe^{-\theta x(h-1)} + \theta(1-h^{-1})(1-p)e^{\theta x(1-h^{-1})}) + (1-\delta)\theta e^{\theta(x-K)}$$

As  $e^{-\theta x(h-1)} < 1$  and  $e^{\theta x(1-h^{-1})}, e^{\theta(x-K)} > 1$  a lower is given by

$$\begin{aligned} &\geq \theta [\delta(-(h-1)p + (1-h^{-1})(1-p)) + (1-\delta)] \\ &= \theta [-\delta(hp + h^{-1}(1-p)) + 1] > 0, \end{aligned}$$

Where the last step follows as  $hp + h^{-1}(1-p) < 1$  from the assumption that  $\delta(ph + (1-p)h^{-1}) < 1$ . Consequently  $\mathcal{L}u$  changes its sign at most once.

Constant Relative Risk Aversion: Let  $u(x) = \frac{(x+K)^\theta - K^\theta}{\theta}$ . The expected change in utility from waiting one more period at  $x$  equals

$$\begin{aligned} \mathcal{L}u(x-K) &= \frac{\delta}{\theta}(p(xh)^\theta - K^\theta + (1-p)(xh^{-1})^\theta - K^\theta) - \frac{1}{\theta}(x^\theta - K^\theta) \\ &= \frac{1}{\theta} \left\{ \delta [p(xh)^\theta + (1-p)(xh^{-1})^\theta] - x^\theta + (1-\delta)K^\theta \right\} \\ &= \frac{1}{\theta} x^\theta \left( \delta [ph^\theta + (1-p)h^{-\theta}] - 1 \right) + \frac{1}{\theta} (1-\delta)K^\theta. \end{aligned}$$

As  $p > 1/2$  for all  $\theta \geq 0$

$$\frac{\partial}{\partial \theta} (ph^\theta + (1-p)h^{-\theta}) = p \log(h)h^\theta - (1-p) \log(h)h^{-\theta} \geq p \log(h)(h^\theta - h^{-\theta}) \geq 0.$$

Thus,  $ph^\theta + (1-p)h^{-\theta} < ph^\alpha + (1-p)h^{-\alpha} = \frac{1}{\delta}$  for all  $\theta < \alpha$ , by definition of  $\alpha$ . As  $\frac{1}{\theta}x^\theta$  is increasing in  $x$ , this completes the proof.  $\square$

### A.2.2. Expected Utility

As a consequence of Lemma 2 the EU from using the cut-off strategy  $\tau(b)$  as a continuation strategy from  $x \leq b$ , equals

$$\begin{aligned} V^u(\tau(b), t, x) &= \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau(b) < T\}} u(X_{\tau(b)} - K)] = \mathbb{P}_{t,x} [\tau(b) < T] u(b-K) \\ &= \min \left\{ 1, \left( \frac{x}{b} \right)^\alpha \right\} u(b-K). \end{aligned} \tag{15}$$

Therefore the optimal EU cut-off is given by

$$b^u = \min \left( \arg \max_b \left[ \min \left\{ 1, \left( \frac{X_0}{b} \right)^\alpha \right\} u(b-K) \right] \right).$$

To establish Proposition 1 it remains for us to show that the cut-off strategy which stops at

$b^u$  is optimal for the EU agent.

**Proof of Proposition 1.** (i) Define  $\tilde{b}$  as the lowest value of the process for which stopping is weakly optimal

$$\tilde{b} = \min\{x: \sup_{\tau} V^u(\tau, x) = u(x - K)\}.$$

First, note that the strategy which never stops before  $T$  guarantees a payoff of zero. It follows that the EU maximizer does not stop when  $X_t < K$  and as a consequence  $\tilde{b} > K$ . By definition, the decision maker does not stop below  $\tilde{b}$ , as the some continuation strategy yields a strictly higher payoff than stopping for all values of the process lower than  $\tilde{b}$ . Furthermore, by definition it is optimal to stop at  $\tilde{b}$  and hence for every initial value of the process  $x \leq \tilde{b}$  we have that the optimal strategy of the EU agent is the cut-off strategy which stops at  $\tilde{b}$

$$V^u(\tau(\tilde{b}), x) = \sup_{\tau} V^u(\tau, x).$$

As we defined  $b^u$  as the optimal cut-off when the initial value of the process is  $K < \tilde{b}$  this implies that  $b^u = \tilde{b}$ .

(ii) To simplify notation let us denote by  $w : \mathcal{X} \rightarrow \mathbb{R}$  the continuation value from using the cut-off strategy which stops at  $b^u$

$$w(x) = V^u(\tau(b^u), x) = \begin{cases} \left(\frac{x}{b^u}\right)^{\alpha} u(b^u - K) & \text{for } x \leq b^u \\ u(x - K) & \text{for } x > b^u \end{cases}.$$

By the dynamic programming principle (cf. Peskir and Shiryaev, 2006, Theorem 1.11),  $\tau(b^u)$  is an optimal strategy if and only if the function  $w(x)$  satisfies the dynamic programming equation for all  $x \in \mathcal{X}$

$$0 = \max\{\mathcal{L}w(x), u(x - K) - w(x)\}. \quad (16)$$

Here  $\mathcal{L}w(x)$  is defined analogous to  $\mathcal{L}u(x)$  by  $\mathcal{L}w(x) = \mathbb{E}_{t,x} [\mathbf{1}_{\{T > t+1\}} w(X_{t+1}) - w(X_t)]$ .

First, we verify that (16) is satisfied for  $x = b^u$ . By definition  $w(b^u) = u(b^u - K)$  and thus, it remains to prove that  $\mathcal{L}w(b^u) \leq 0$

$$\begin{aligned} \mathcal{L}w(b^u) &= \mathbb{E}_{t,b^u} [\mathbf{1}_{\{T > t+1\}} w(X_{t+1}) - w(X_t)] \\ &= \delta \left[ p w(b^u h) + (1 - p) w(b^u h^{-1}) \right] - u(b^u - K) \\ &= \delta \left[ p u(b^u h - K) + (1 - p) \left( \frac{b^u h^{-1}}{b^u} \right)^{\alpha} u(b^u - K) \right] - u(b^u - K) \\ &= u(b^u - K) \left[ \delta \left( p \frac{u(b^u h - K)}{u(b^u - K)} + (1 - p) h^{-\alpha} \right) - 1 \right]. \end{aligned}$$

By definition of  $b^u$  we have that

$$\left(\frac{K}{b^u}\right)^\alpha u(b^u - K) \geq \left(\frac{K}{b^u h}\right)^\alpha u(h b^u - K) \Rightarrow u(b^u - K) \geq h^{-\alpha} u(h b^u - K).$$

This implies that  $u(h b^u - K)/u(b^u - K) \leq h^\alpha$ . As  $h$  satisfies  $\delta(p h^\alpha + (1-p)h^{-\alpha}) = 1$  the expected change  $\mathcal{L}w(b^u)$  is less or equal to zero

$$\mathcal{L}w(b^u) \leq u(b^u - K) (\delta [p h^\alpha + (1-p) h^{-\alpha}] - 1) = 0.$$

Second, we verify that (16) is satisfied for  $x > b^u$ . As  $b^u$  is the lowest point at which it is weakly optimal to stop there is no gain from delaying the stopping decision by a single period  $\mathcal{L}u(b^u) \leq 0$ . By definition  $w(x) = u(x - K)$  for all  $x \geq b^u$ . By the single crossing property of  $\mathcal{L}u$  we thus have  $\mathcal{L}w(x) = \mathcal{L}u(x) \leq 0$  for all  $x > b^u$  and (16) is satisfied for all  $x > b^u$ .

Finally, we show that (16) is satisfied for all  $x < b^u$ . Simple algebra shows that  $\mathcal{L}w(x) = 0$  for  $x < b^u$  and as we established in (i) that  $w(x) > u(x - K)$  for  $x < b^u$  (16) holds.  $\square$

The following definition is useful to establish Proposition 2. Let  $\Gamma^u : \mathcal{X} \rightarrow \mathbb{R}$  be the expected gain in utility from continuing until the process reaches  $xh$  instead of stopping at  $x$  when the current value of the process equals  $x$

$$\Gamma^u(x) = V^u(\tau(xh), x) - u(x - K) = h^{-\alpha} u(xh - K) - u(x - K). \quad (17)$$

The following result characterizes the expected utility from using a cut-off strategy  $\tau(b)$  using  $\Gamma^u$ .

**Lemma 4** (Expected Payoff of a Cut-off Strategy). *Let  $n \geq 1$ . The EU from using the cut-off strategy  $\tau(xh^n)$  instead of stopping at  $x$  is given by*

$$V^u(\tau(xh^n), x) = u(x - K) + \sum_{j=1}^n h^{-(j-1)\alpha} \Gamma^u(xh^{j-1}).$$

**Proof of Lemma 4.** We show the result inductively using the fact that once the agent reaches  $xh^{n-1}$  the continuation value is given by the expected value of waiting for one uptick

$$\begin{aligned} V^u(\tau(xh^n), x) &= \mathbb{E}_{0,x} [U_{\tau(xh^n)}] = \mathbb{E}_{0,x} [\mathbf{1}_{\{\tau(xh^{n-1}) < T\}} V^u(\tau(xh^n), xh^{n-1})] \\ &= \mathbb{E}_{0,x} [\mathbf{1}_{\{\tau(xh^{n-1}) < T\}} (\Gamma^u(xh^{n-1}) + u(xh^{n-1} - K))] \\ &= V^u(\tau(xh^{n-1}), x) + \mathbb{P}_{0,x} [\tau(xh^{n-1}) < T] \Gamma^u(xh^{n-1}) \\ &= V^u(\tau(xh^{n-1}), x) + h^{-(n-1)\alpha} \Gamma^u(xh^{n-1}). \end{aligned} \quad (18)$$

The result follows inductively in combination with the fact that  $V^u(\tau(x), x) = u(x - K)$ .  $\square$



**Proof of Proposition 2.** It follows as a special case of Lemma 5 (below) by setting  $\kappa = 0$  that  $\Gamma^u(x)$  is decreasing in  $x$ . As it is optimal to continue until the process reaches  $b^u$  one tick below  $b^u$  we have that  $\Gamma^u(h^{-1}b^u) > 0$ . As  $\Gamma^u$  is decreasing it follows that  $\Gamma^u(x) > 0$  for all  $x < b^u$ . Then by Lemma 4 for  $b = xh^n < b^u$

$$V^u(\tau(b), x) = u(x - K) + \sum_{j=1}^n h^{-(j-1)\alpha} \Gamma^u(xh^{j-1}) > u(x - K). \quad \square$$

### A.2.3. Proofs for Regret

**Proof of Proposition 4.** To show that anticipated regret preferences are time-consistent we need to show that no future self wants to deviate from a plan that is optimal for an earlier self. Consider the self at time  $t$  when the current value of the process equals  $x = X_t$  and the past maximum equals  $s = S_t$ . Let  $\tau$  be the continuation strategy which is optimal for the time  $t$  self, i.e. for all  $\tau'$

$$V^r(\tau, x, s) \geq V^r(\tau', x, s).$$

Let  $\hat{t} \geq t$  be the first times at which the future self is at least  $\epsilon > 0$  better off by deviating to some other strategy  $\hat{\tau}$ .

Suppose, that the time  $t$  self changes its continuation strategy after time  $\hat{t}$  from  $\tau$  to  $\hat{\tau} \geq \hat{t}$ . The law of iterated expectations yields that this change would increase time  $t$  selves value which contradicts that the strategy  $\tau$  was optimal in the first place.

$$\begin{aligned} V^r(\tau, x, s) &= \mathbb{E}_{t,x,s}[U_\tau - \kappa S_\tau] \\ &= \mathbb{E}_{t,x,s} \left[ \mathbf{1}_{\{\tau < \hat{t}\}} (U_\tau - \kappa S_\tau) + \mathbf{1}_{\{\tau \geq \hat{t}\}} (U_\tau - \kappa S_\tau) \right] \\ &= \mathbb{E}_{t,x,s} \left[ \mathbf{1}_{\{\tau < \hat{t}\}} (U_\tau - \kappa S_\tau) + \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T > \hat{t}\}} \mathbb{E}_{\hat{t}, X_{\hat{t}}, S_{\hat{t}}} [U_\tau - \kappa S_\tau] - \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T \leq \hat{t}\}} \kappa S_{\hat{t}} \right] \\ &= \mathbb{E}_{t,x,s} \left[ \mathbf{1}_{\{\tau < \hat{t}\}} (U_\tau - \kappa S_\tau) + \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T > \hat{t}\}} V^r(\tau, X_{\hat{t}}, S_{\hat{t}}) - \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T \leq \hat{t}\}} \kappa S_{\hat{t}} \right] \\ &\leq \mathbb{E}_{t,x,s} \left[ \mathbf{1}_{\{\tau < \hat{t}\}} (U_\tau - \kappa S_\tau) + \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T > \hat{t}\}} V^r(\hat{\tau}, X_{\hat{t}}, S_{\hat{t}}) - \mathbf{1}_{\{\tau \geq \hat{t} \text{ and } T \leq \hat{t}\}} \kappa S_{\hat{t}} \right] \\ &= \mathbb{E}_{t,x,s} \left[ \mathbf{1}_{\{\tau < \hat{t}\}} (U_\tau - \kappa S_\tau) + \mathbf{1}_{\{\tau \geq \hat{t}\}} (U_{\hat{\tau}} - \kappa S_{\hat{\tau}}) \right] \\ &= V^r(\mathbf{1}_{\{\tau < \hat{t}\}} \tau + \mathbf{1}_{\{\tau \geq \hat{t}\}} \hat{\tau}, x, s). \end{aligned}$$

As the strategy in which the agent continues after time  $\hat{t}$  by using the strategy  $\hat{\tau}$  is feasible it follows that for any  $\epsilon > 0$ , the probability of the event  $\hat{t} < T$  must be zero, i.e. there is no history after which a future self can gain  $\epsilon$  by deviating, as otherwise the above inequality is strict which contradicts the optimality of  $\tau$ .  $\square$

Before proving Theorem 1 we show several auxiliary results. To derive the optimal strategy of the regret agent, let us denote by  $\Gamma : \mathbb{R}_+ \rightarrow \mathbb{R}$  the expected change in regret value from waiting until the process reaches the level  $xh$  or the game ends instead of stopping at  $x$  when

the current maximum of the process equals  $x$  equals the current value of the process:

$$\Gamma(x) = V^r(\tau(xh), x, x) - [u(x - K) - \kappa u(x - K)] . \quad (19)$$

Using Lemma 2  $\Gamma(\cdot)$  can be explicitly calculated and equals

$$\Gamma(x) = h^{-\alpha}(1 - \kappa)u(xh - K) - (1 - \kappa h^{-\alpha})u(x - K) . \quad (20)$$

The following single crossing assumption on  $\Gamma(\cdot)$  will be useful to characterize the optimal regret strategy explicitly.

**Assumption A3** (Single Crossing under Regret).  $\Gamma(\cdot)$  changes its sign at most once,

$$\Gamma(x) \leq 0 \Rightarrow \Gamma(x') < 0 \text{ for all } x < x' .$$

Assumption A3 is a weak assumption on  $u$  and is satisfied whenever  $u$  is concave, i.e. the agent is risk-averse.

**Lemma 5.** *If  $u$  is concave,  $\Gamma$  is strictly decreasing.*

**Proof of Lemma 5.** The change of  $\Gamma$  in  $x$  equals

$$\begin{aligned} \Gamma(xh) - \Gamma(x) &= (h^{-\alpha}(1 - \kappa)u(xh^2 - K) - (1 - \kappa h^{-\alpha})u(xh - K)) \\ &\quad - (h^{-\alpha}(1 - \kappa)u(xh - K) - (1 - \kappa h^{-\alpha})u(x - K)) \\ &= (1 - \kappa) h^{-\alpha} (u(xh^2 - K) - u(xh - K)) \\ &\quad - (1 - \kappa h^{-\alpha}) (u(xh - K) - u(x - K)) \end{aligned} \quad (21)$$

As  $u$  is concave, we know that

$$\frac{u(xh - K) - u(x - K)}{x(h - 1)} \geq \frac{u(xh^2 - K) - u(xh - K)}{xh(h - 1)}$$

Multiplying by both sides  $x(h - 1)$  and as  $\alpha > 1$  it follows

$$\begin{aligned} u(xh - K) - u(x - K) &\geq h^{-1} (u(xh^2 - K) - u(xh - K)) \\ &> h^{-\alpha} (u(xh^2 - K) - u(xh - K)) , \end{aligned}$$

As  $1 - \kappa \leq 1 - \kappa h^{-\alpha}$  Eq. (21) is negative. Consequently,  $\Gamma$  is strictly decreasing.  $\square$

We next show that the regret agent always stops if the current value of the process is above  $b^u$ .

**Lemma 6.** *Assume A1 holds. For every past maximum  $S_t$  the regret agent stops in period  $t$  if  $X_t \geq b^u$ .*

**Proof of Lemma 6.** As show in Proposition 1 we know that it is optimal to stop for an expected utility maximizer if  $X_t \geq b^u$ . From the fact that it is optimal for an EU maximizer to stop above  $b^u$ , we know that any strategy which continues at a point  $x = b^u$  must yield a change in utility which is negative in expectation

$$\sup_{\tau > t} \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] < u(x - K).$$

From the monotonicity of the maximum process  $S_t$  and the utility function  $u$  we may further conclude that  $u(S_\tau - K) \geq u(S_t - K)$  for all stopping times  $\tau > t$ . Hence,

$$\sup_{\tau > t} \mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K) - \kappa u(S_\tau - K)] < u(x - K) - \kappa u(s - K) \quad (22)$$

and it is optimal to stop at  $b^u$ .  $\square$

Our next result shows that the agent never stops below  $b^u$  when the process is not at it's past maximum.

**Lemma 7.** *Let  $u$  be concave. The continuation strategy which stops when the process reaches it's past maximum  $s$  or  $b^u$  is better than stopping immediately, i.e. for every current value of the process  $x < s$*

$$V^r(\tau(\min\{s, b^u\}), x, s) > u(x - K) - \kappa u(s - K).$$

**Proof of Lemma 7.** The regret value when using the strategy  $\tau(\min\{s, b^u\})$  is given by

$$V^r(\tau(\min\{s, b^u\}), x, s) = \mathbb{E}_{0,x,s} [U_{\tau(\min\{s, b^u\})} - \kappa u(S_\tau - K)].$$

As the strategy always stops before  $s + 1$  is reached it follows that  $S_\tau = s$  and hence

$$V^r(\tau(\min\{s, b^u\}), x, s) = \mathbb{E}_{0,x,s} [U_{\tau(\min\{s, b^u\})}] - \kappa u(s - K).$$

As  $\min\{s, b^u\} \leq b^u$  it follows from Proposition 2 that

$$\mathbb{E}_{0,x,s} [U_{\tau(\min\{s, b^u\})}] > u(x - K)$$

which completes the proof.  $\square$

Our next result shows that the incentive to continue increases in the past maximum.

**Lemma 8** (Reluctance to Regret). *For every current value  $x$  of the process the value of continuing with the strategy  $\tau$  minus the value of stopping immediately*

$$V^r(\tau, x, s) - [u(x - K) - \kappa u(s - K)]$$

is i) non-decreasing in the past maximum, ii) increases strictly from  $s = x$  to  $s = x + 1$  if  $\tau > 0$ , and iii) non-increasing in the intensity of regret  $\kappa$ .

**Proof of Lemma 8.** As the physical utility under any strategy is unaffected by changes in the past maximum  $s$  we can ignore it and focus on how the regret is affected by the past maximum for a fixed strategy  $\tau$ . The increase in the utility of having stopped at the optimal time  $u(S_\tau - K) - u(S_t - K)$  when the agent uses the continuation strategy  $\tau$  after a point in time  $t$  where the past maximum equals  $s = S_t$  is given by

$$\begin{aligned} u(S_\tau - K) - u(s - K) &= \mathbf{1}_{\{S_\tau > s\}} [u(S_\tau - K) - u(s - K)] \\ &= \mathbf{1}_{\{\max\{X_0, \dots, X_\tau\} > s\}} [u(\max\{X_0, X_1, \dots, X_\tau\} - K) - u(s - K)] \\ &= \min \{u(\max\{X_0, X_1, \dots, X_\tau\} - K) - u(s - K), 0\} . \end{aligned} \quad (23)$$

The right-hand-side of Eq. 23 decreases in  $s$  and  $\max\{X_t, \dots, X_\tau\}$  is independent of  $s$ . It therefore follows that for a given continuation strategy  $\tau$  the gain from continuing  $V^r(\tau, x, s) - V^r(0, x, s)$  is non-decreasing in the past maximum  $s$ .

Note that when  $s$  increases from  $s = x$  to a higher value, the right-hand-side of Eq. 23 decreases strictly for a set of events which has positive probability under any continuation strategy  $\tau > 0$  that does not stop immediately and thus the gain from continuing  $V^r(\tau, x, s) - V^r(0, x, s)$  decreases strictly.

Finally, as the increase in regret is non-negative by Eq. 23 it follows that the value of continuing is non-increasing in  $\kappa$ .  $\square$

**Proof of Theorem 1.** By Lemma 6 the agent stops if the current value of the process  $x$  is weakly greater the EU cut-off  $b^u$ . We are thus only left to consider the case where  $x < b^u$ . As a consequence of Lemma 7 the agent never stops when the current value of the process  $x$  is below it's past maximum. It thus only remains to show that the agent stops if the current value of the process is equal its past maximum  $x = s$  and weakly greater  $b^r$  and continues if  $x = s < b^r$ . By definition of  $b^r$  it is optimal to wait for an uptick at  $x = s < b^r$ . It remains to show that it is optimal to stop when  $x = s \geq b^r$ .

We prove this result by induction over  $x$ . Induction start: By Lemma 6 it is optimal to stop at  $x = s = b^u$ . Induction step: Consider a history where the current value of the process is  $x$  equal to it's past maximum and it is optimal to stop once the process (and it's maximum) reaches  $xh$ . If it is optimal to continue after such a history the strategy which stops only if the process reaches  $xh$  is an optimal continuation strategy as it is never optimal to stop when the process is below it's maximum by Lemma 7. The change in payoff from waiting for this uptick is given by

$$\Gamma(x) = V^r(\tau(xh), x, x) - (1 - \kappa)u(x - K) .$$

Note, that  $\Gamma(b^r) \leq 0$  by definition of  $b^r$  and  $\Gamma(x)$  is negative for  $x \geq b^r$  by the the single crossing

property of  $\Gamma$ . Hence, it is weakly optimal to stop when  $x = s \geq b^r$ . Finally, that  $b^r$  decreases weakly in the intensity of regret follows immediately from Lemma 8 *iii*).  $\square$

#### A.2.4. Proofs for Stochastic Choice

To show the next results some additional notation will be useful. Fix a continuation strategy  $\tau$  for the situation when the process starts in  $x$  at time  $t$ . Let us make explicit the starting value  $x$  of the process at time  $t$  by writing  $X_q^x$  for all  $q \geq t$ . If the process starts at a different value  $y$  the agent can still calculate the counter-factual value the process would have had if started in  $x$

$$X_q^x = X_q^y \frac{x}{y}. \quad (24)$$

Thus,  $\tau$  is an admissible strategy even if the process is started in  $y$  instead of  $x$ , i.e. the time at which the agent stops when using the strategy  $\tau$  is independent of the initial value of the process.

**Proof of Proposition 6.** Part i): We first prove that for any continuation strategy  $\tau$  continuation is more attractive at  $x$  than at  $y > x$  if the agent is regret neutral  $\kappa = 0$ . Using the law of iterated expectations we can rewrite the expected value of continuing optimally as the sum over incentives to delay the stopping decision by one period

$$\begin{aligned} V^u(\tau, x) - u(x - K) &= \mathbb{E}_{t,x} [\mathbf{1}_{\{\tau < T\}} u(X_\tau - K)] - u(x - K) \\ &= \mathbb{E}_{t,x} \left[ \sum_{q=t}^{\tau-1} \mathbf{1}_{\{\tau < q\}} u(X_{q+1} - K) - \mathbf{1}_{\{\tau < q-1\}} u(X_q - K) \right] \\ &= \mathbb{E}_{t,x} \left[ \sum_{q=t}^{\tau-1} \mathbf{1}_{\{\tau < q-1\}} \mathbb{E}_{q, X_q} [\mathbf{1}_{\{\tau < q\}} u(X_{q+1} - K) - u(X_q - K)] \right] \\ &= \mathbb{E}_{t,x} \left[ \sum_{q=t}^{\tau-1} \mathbf{1}_{\{\tau < q-1\}} \mathcal{L}u(X_q) \right]. \end{aligned}$$

Hence, it follows from the monotonicity of  $\mathcal{L}u$  that the value of continuing with any strategy  $\tau$  is decreasing in  $x$  if the agent maximizes EU.

In the second step we show that for any fixed strategy  $\tau$  the increase in regret is monotone increasing in  $x$ . By (24) the maximal value of the process when started in  $y$  instead of  $x$  at time  $t$  is given by

$$\max\{s, X_t^y, X_{t+1}^y, \dots, X_\tau^y\} = \max\{s, \frac{y}{x} \max\{X_t^x, X_{t+1}^x, \dots, X_\tau^x\}\}. \quad (25)$$

Thus, the maximal value is non-decreasing in  $y$  for any realization of the process. Consequently, the additional regret incurred after time  $t$  increases in the initial value of the process for any fixed strategy  $\tau$ . Taking the expectation and the supremum over strategies  $\tau > t$  yields that

$\rho(x, s)$  is decreasing in  $x$ .

Part ii): By Lemma 8, for every current value  $x$  of the process the value of continuing with the strategy  $\tau$  minus the value of stopping immediately

$$V^r(\tau, x, s) - [u(x - K) - \kappa u(s - K)]$$

is non-decreasing in the past maximum  $s$  and hence the stopping probability

$$\pi(x, s) = \Psi \left( \sup_{\tau} V^r(\tau, x, s) - [u(x - K) - \kappa u(s - K)] \right)$$

is non-decreasing in the past maximum  $s$ .

Part iii): Follows, by the same argument from Lemma 8 iii). □

**Proof of Proposition 7.** Fix a continuation strategy  $\tau$ . We first show monotonicity of

$$V^u(\tau, x) - u(x - K) - \kappa \mathbb{E}_{t,x,s} [u(S_{\tau} - K) - u(s - K)]$$

in  $x$ . First, note that by Lemma 8  $V^u(\tau, x) - u(x - K)$  is decreasing in  $x$ . By (24) the maximal value  $S_{\tau}^y$  of the process when started in  $y$  instead of  $x$  with a past maximal value of  $s \frac{y}{x}$  instead of  $s$  at time  $t$  is given by

$$\begin{aligned} S_{\tau}^y &= \max\{s \frac{y}{x}, X_t^y, X_{t+1}^y, \dots, X_{\tau}^y\} = \max\{s \frac{y}{x}, \frac{y}{x} \max\{X_t^x, X_{t+1}^x, \dots, X_{\tau}^x\}\} \\ &= \frac{y}{x} \max\{s, X_t^x, X_{t+1}^x, \dots, X_{\tau}^x\} = \frac{y}{x} S_{\tau}^x. \end{aligned}$$

Thus, we have that the change in regret when the process is started in  $y$  with a maximal value  $s \frac{y}{x} > 1$  equals

$$\begin{aligned} u(S_{\tau}^y - K) - u(s \frac{y}{x} - K) &= u\left(\frac{y}{x} S_{\tau}^x - K\right) - u\left(\frac{y}{x} s - K\right) \\ &\leq u(S_{\tau}^x - K) - u(s - K). \end{aligned}$$

The last steps follows from Assumption A2. □

**Proof of Lemma 1.** The result follows from the assumption that the sum of relative and

absolute risk aversion are bounded from below and  $u'' < 0$

$$\begin{aligned}
\frac{\partial}{\partial \beta} (u(\beta s' - K) - u(\beta s - K)) &= \frac{\partial}{\partial \beta} \int_s^{s'} \beta u'(\beta z - K) dz \\
&= \int_s^{s'} u'(\beta z - K) + z \beta u''(\beta z - K) dz \\
&= \int_s^{s'} u'(\beta z - K) + (z\beta - K)u''(\beta z - K) + K u''(\beta z - K) dz \\
&= \int_s^{s'} u'(\beta z - K) \left[ 1 - (\beta z - K) \frac{|u''(\beta z - K)|}{u'(\beta z - K)} - K \frac{|u''(\beta z - K)|}{u'(\beta z - K)} \right] dz \\
&\leq 0
\end{aligned}$$

□

**Lemma 9** (Regret Value). Assume  $u$  is concave and A1 holds.  $V^r(x, s)$  is given by

$$V^r(x, s) = \begin{cases} u(x - K) - \kappa u(s - K) & \text{for } x \geq b^u \\ \left(\frac{x}{c_b}\right)^\alpha [u(c_b - K) - \kappa u(\max\{c_b, s\} - K)] \\ \quad - \kappa \sum_{z \in \mathcal{X} \cup [x, c_b)} \left(\frac{x}{z}\right)^\alpha (1 - h^{-\alpha}) u(\max\{z, s\} - K) & \text{for } x < b^u \end{cases}$$

where  $c_b = \max\{b^r, \min\{s, b^u\}\}$ .

*Proof.* Finally, we characterize the regret value. First, we derive the probability that the maximum of the process is at least  $y \in \mathcal{X}$

$$\mathbb{P}[S_T \geq y \mid X_t = x, S_t = s] = \begin{cases} 1 & \text{if } s \geq y \\ \mathbb{P}[\tau(y) < T \mid X_t = x] & \text{if } s < y \end{cases}$$

Hence, we have that the probability that the maximum of the process is exactly  $y \in \mathcal{X}$  for all  $s < y$  equals

$$\begin{aligned}
\mathbb{P}[S_T = y \mid X_t = x, S_t = s] &= \mathbb{P}[S_T \geq y \mid X_t = x, S_t = s] - \mathbb{P}[S_T \geq y h \mid X_t = x, S_t = s] \\
&= \left(\frac{x}{y}\right)^\alpha - \left(\frac{x}{y h}\right)^\alpha = \left(\frac{x}{y}\right)^\alpha (1 - h^{-\alpha}) .
\end{aligned}$$

Let  $b = x h^m$ . Given the regret functional derived in Eq. 7, the expected value of using the

cut-off strategy  $\tau(b)$  equals

$$\begin{aligned}
V^r(\tau(b), x, s) &= \mathbb{E}_{t,x,s} [\mathbf{1}_{\{\tau(b) < T\}} u(X_{\tau(b)} - K) - \kappa u(S_{\tau(b)} - K)] \\
&= \mathbb{P}[\tau(b) < T \mid X_t = x] (u(b - K) - \kappa u(\max\{s, b\} - K)) \\
&\quad - \kappa \sum_{i=0}^m \mathbb{P}[S_T = xh^i \mid X_t = x, S_t = s] u(xh^i - K) \\
&= \left(\frac{x}{b}\right)^\alpha u(b - K) - \kappa \sum_{i=0}^{m-1} \left(\frac{x}{xh^i}\right)^\alpha (1 - h^{-\alpha}) \max\{u(s - K), u(xh^i - K)\} \\
&\quad - \kappa \left(\frac{x}{b}\right)^\alpha \max\{u(s - K), u(b - K)\}. \quad \square
\end{aligned}$$

**Proof of Proposition 8.** For this, let  $st(x, s)$  be the number of observed stopping decisions,  $ct(x, s)$  the number of observed continuation decisions,  $n(x, s)$  the total number of decisions observed at the value  $x$  with a past maximum of  $s$ , and  $f(x, s) = \frac{ct(x, s)}{n(x, s)}$ .

$$\begin{aligned}
\log(\ell_i) &= \log \left[ \prod_k \prod_{r=0}^{\min\{\hat{\tau}_{i,k}, T^{i,k}\}} (\Psi \circ \rho)(X_r^{i,k}, S_r^{i,k}) (1 - \mathbf{1}_{\{\hat{\tau}_{i,k} < T^{i,k}\}} (\Psi \circ \rho)(X_{\hat{\tau}_{i,k}}^{i,k}, S_{\hat{\tau}_{i,k}}^{i,k})) \right] \\
&= \log \left[ \prod_{s \in \mathcal{X}} \prod_{x \leq s} (\Psi \circ \rho)(x, s)^{ct(x, s)} (1 - (\Psi \circ \rho)(x, s))^{st(x, s)} \right] \\
&= \sum_{s \in \mathcal{X}} \sum_{x \leq s} ct(x, s) \log(\Psi \circ \rho)(x, s) + st(x, s) \log(1 - (\Psi \circ \rho)(x, s)) \\
&= \sum_{s \in \mathcal{X}} \sum_{x \leq s} n(x, s) \left[ \frac{ct(x, s)}{n(x, s)} \log(\Psi \circ \rho)(x, s) + \left(1 - \frac{ct(x, s)}{n(x, s)}\right) \log(1 - (\Psi \circ \rho)(x, s)) \right] \\
&= \sum_{s \in \mathcal{X}} \sum_{x \leq s} n(x, s) \left[ f(x, s) \log \left( \frac{(\Psi \circ \rho)(x, s)}{1 - (\Psi \circ \rho)(x, s)} \right) + \log(1 - (\Psi \circ \rho)(x, s)) \right] \quad \square
\end{aligned}$$

## B. Methodological details on econometric model

We model our data hierarchically, i.e. in two layers or levels: (i) the subjects level and (ii) the population level. Hierarchical models have the advantage to allow model coefficients to be different for each subject, but still share some similarities across the whole group. This helps to regularize subject-level parameter estimates in the light of the observations available for the whole group of subjects. Conceptually, this acts like a population-level prior on the subject-level characteristics with parameters that are themselves estimated (Gelman et al., 2013).

Using the exponential of the subject-specific log-likelihood given in Proposition 8, we arrive



the complete likelihood across all subjects as

$$L(Y, f \mid \gamma) = \prod_{i=1}^{44} \ell_i(Y^i, f^i \mid \gamma_i) . \quad (26)$$

where we write  $Y^i$  to denote the lattice of points  $(x, s)$  visited by subject  $i$  across all rounds with  $Y = \{Y^i\}_{i=1}^{44}$ . We use  $f^i$  to denote the associated observed continuation frequency with  $f = \{f^i\}_{i=1}^{44}$ , and  $\gamma = \{\gamma_i\}_{i=1}^{44}$  with  $\gamma_i = (\alpha_i, \beta_{i,1}, \beta_{i,2})$  to denote the subject-specific parameters.

On the one hand, our model assumes that parameters in  $\gamma_i$  can vary for each subject. On the other hand, subjects are at the same time treated as belonging to a population of similar individuals. Therefore, idiosyncratic parameters come from a common population distribution

$$\alpha_i \sim N(\mu_\alpha, \sigma_\alpha^2) ; \beta_{i,g} \sim N(\mu_{\beta_g}, \sigma_{\beta_g}^2), \quad g = 1, 2 \quad (27)$$

$$\mu_\alpha, \mu_{\beta_g} \sim N(0, 1) \quad (28)$$

$$\sigma_\alpha, \sigma_{\beta_g} \sim \text{Half-Cauchy}(0, 2) . \quad (29)$$

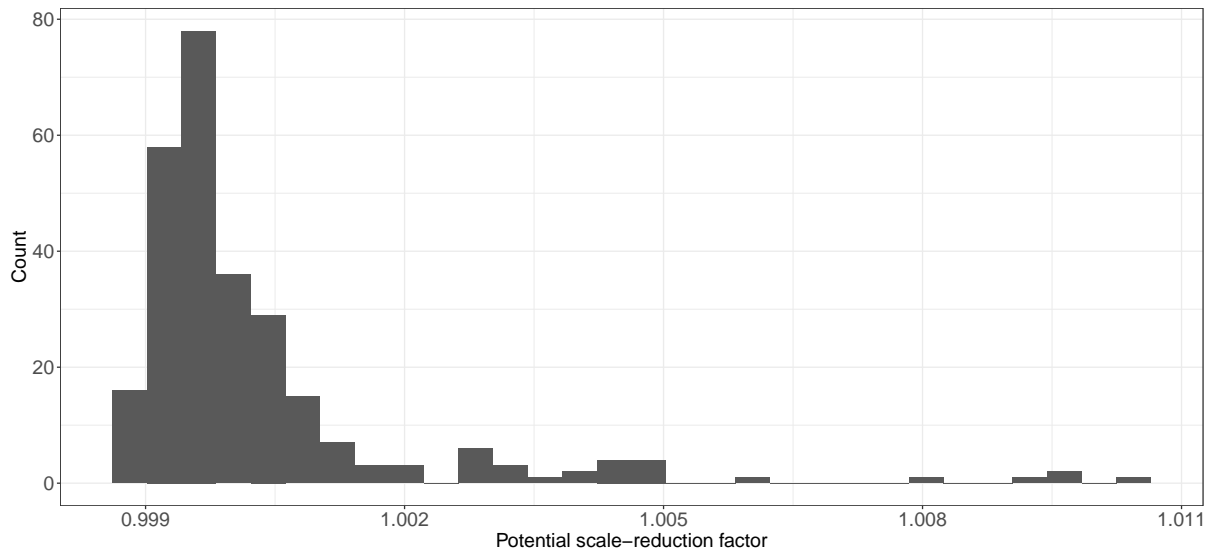
It is instructive to note that our priors are centered around zero and thus shrink the posterior of the population-level parameters  $\mu_\alpha$  and  $\mu_\beta$  to zero. Therefore, our posterior estimates for the effect sizes are conservative.

Since now the parameters in  $\gamma$  dependent on  $\mu = (\mu_\alpha, \mu_{\beta_1}, \mu_{\beta_2})$  and  $\sigma = (\sigma_\alpha, \sigma_{\beta_1}, \sigma_{\beta_2})$ , the full-fledged posterior distribution reads

$$p(\gamma, \mu, \sigma \mid Y, f) = p(\mu, \sigma) p(\gamma \mid \mu, \sigma) L(Y, f \mid \gamma, \mu, \sigma) . \quad (30)$$

We fit the model to our data using the open-source software Stan through its R interface RStan (Stan Development Team, 2016a,b). This software implements a posterior simulation algorithm (Hamiltonian Monte Carlo or HMC) that has proven to be particularly suited to estimate hierarchical models (Neal, 2011; Hoffman and Gelman, 2014; Betancourt and Girolami, 2015). We ran four chains with 1,500 iterations each. We discard the first 750 iterations as a warmup sample and base any inference on the combined second halves of all four chains.

As with any MCMC algorithm, the convergence of the Markov chain to its target distribution (30) is important to check. For this we computed the potential scale-reduction factor (PSRF) due to Gelman and Rubin (1992). The PSRF is a measure for each scalar parameter that compares the variation of the sampled values within and between chains. If the within and between variation is the same, the chains are said to have converged to the same target distribution. In that case  $\hat{R} \approx 1$ . In practice it is suggested to that for every parameter for which  $\hat{R} < 1.1$ , the chain can be considered to have converged. Figure 6 depicts a histogram of all PSRFs of all unknowns in our model. Importantly, the log-posterior density is also an unknown that can be evaluated for each iteration and that is important to check whether it converges to the same



**Figure 6:** Histogram of potential scale-reduction factors for all unknowns in our model.

target across four chains. It is therefore included in the figure. We do not find any unknown in our model, for which the threshold of 1.1 is exceeded and thus conclude that our simulation algorithm successfully converged to its target distribution.

## C. Tables

**Table 1:** Parameters for the binomial random walk in the experiment.

Cost $K$	Stepsize $h$	Uptick prob. $p$	Exp. prob. $1 - \delta$
40	1.06	52 %	0.7 %

**Table 2:** Average variance of reservation levels across subjects.

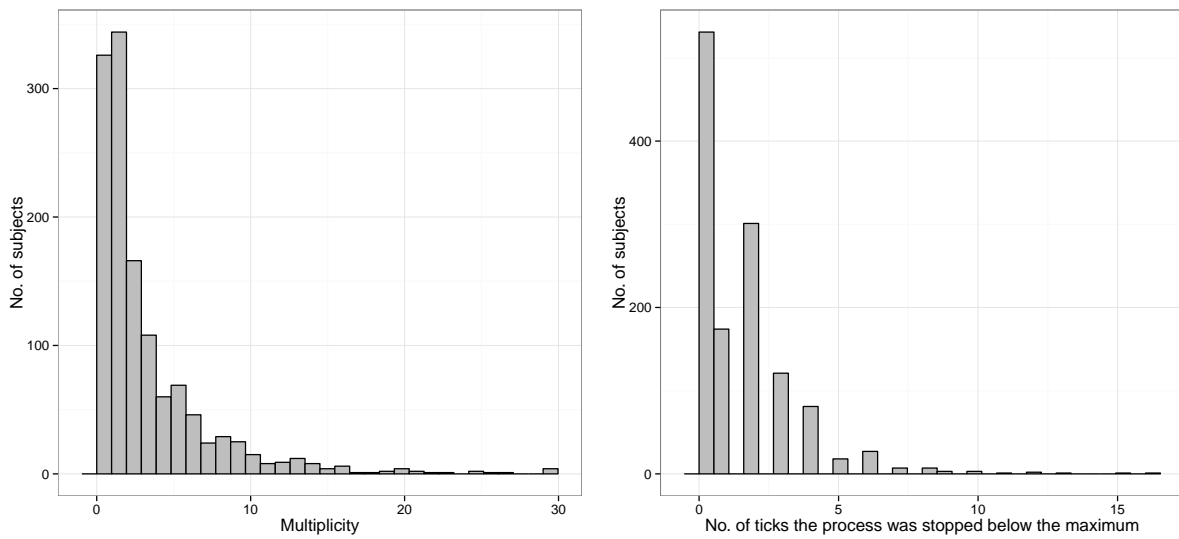
Rounds	1-5	1-15	1-25	1-35	1-45	1-55	1-65
Variance	270.07	566.03	612.07	582.97	507.15	494.71	467.18

**Table 3:** Contingency table for observed decisions.

	$X_\tau = S_\tau$	$X_\tau < S_\tau$	No. of obs.
stopped first time	326 (25%)	0 (0%)	326 (25%)
not stopped first time	205 (16%)	748 (58%)	953 (75%)
No. of obs.	531 (42%)	748 (58%)	1279 (100%)

*Notes:* Decisions in the upper left cell are time-consistent.  $X_t$  denotes the value of the process and  $S_t = \max_{s \leq t} X_s$ . Percentage of total observations in parentheses.

## D. Figures



**Figure 7:** The empirical distribution of the multiplicity of stopped values for all subjects (left) and the number of ticks subjects stopped below the previous maximum (right).

WEB APPENDIX TO: TOO PROUD TO STOP: REGRET IN DYNAMIC DECISIONS  
—NOT INTENDED FOR PUBLICATION—

Paul Viefers\*

Philipp Strack†

A DETAILS IN OPREA ET AL.’S GRADUAL CONVERGENCE MODEL

In this section, we follow-up the idea put forward by Oprea, Friedman & Anderson (2009) that subjects adapt their reservation levels in response to forgone earnings. Following Oprea et al. (2009), we therefore estimated a model on the pooled data, where subjects use a cut-off strategy  $\tau(b^j)$  in every round  $j$  and adapt their reservation level  $b^j$  in response to forgone earnings in the previous round. More specifically, Oprea et al. assume that the reservation level  $b^j$  in round  $j$  follows a simple linear model, which makes the difference in reservation levels between round  $j$  and  $j - 1$  a linear function of previous losses

$$b^j = b^{j-1} + K [\delta_E \mathbf{1}_{\{\tau^{j-1} < T\}} + \delta_L \mathbf{1}_{\{\tau^{j-1} \geq T\}}] (S_{\tau^{j-1}}^{j-1} - b^{j-1}) . \quad (\text{A.1})$$

The parameters  $\delta_E$  and  $\delta_L$  measure an individual’s sensitivity to a loss that stems from stopping below  $S_j$  and from not having stopped before the deadline, respectively.

We use the same estimation method as Oprea et al. (2009). That is, we obtain an estimate of  $\delta_E$  for each two consecutive rounds a subject stopped by setting  $b^j = X_{\tau^j}^j$  and solving for  $\delta_E$  in (A.1). For each block of consecutive rounds a subject did not stop, we may use the two adjacent reservation levels to estimate  $\delta_L$  from the losses suffered due to not stopping (see Oprea et al., 2009, for further details). In Table 1 we follow Oprea et al. and report the median of the by-subject median of  $\delta_E$  and  $\delta_L$ . Our estimates are qualitatively similar to that of Oprea et al.. The estimate for  $\delta_E$  implies that subjects increase their reservation level, if in the previous round they observed that after stopping they could have stopped at higher values. The estimate for  $\delta_L$  implies that subjects reduce their reservation level, if in the previous round the process jumped to zero before they stopped and they missed the opportunity to get a positive payoff. In our sample the adjustment to the latter is not statistically significant. This is actually a common finding of most of the literature on regret and counterfactual thinking, i.e. that people experience more regret over outcomes that stem from action than from equally miserable outcomes that stem from inaction (see e.g. Kahneman & Tversky, 1982; Gilovich & Medvec, 1994; Gilovich, Medvec & Kahneman, 1998; Coricelli, Critchley, Joffily, O’Doherty, Sirigu & Dolan, 2005; Summers & Duxbury, 2007).

To inspect how much variation between rounds can be explained by this model, we took the first observed reservation level for each subject and iteratively forecasted their reservation levels for all remaining rounds using the estimated model. We have plotted the average forecasted reservation levels across all 44 subjects in Figure 1. The plot shows that the model has limited explanatory power in our sample. In the first 20 rounds, the adaption model tracks the development of reservation levels reasonably well, but it clearly overshoots thereafter.

---

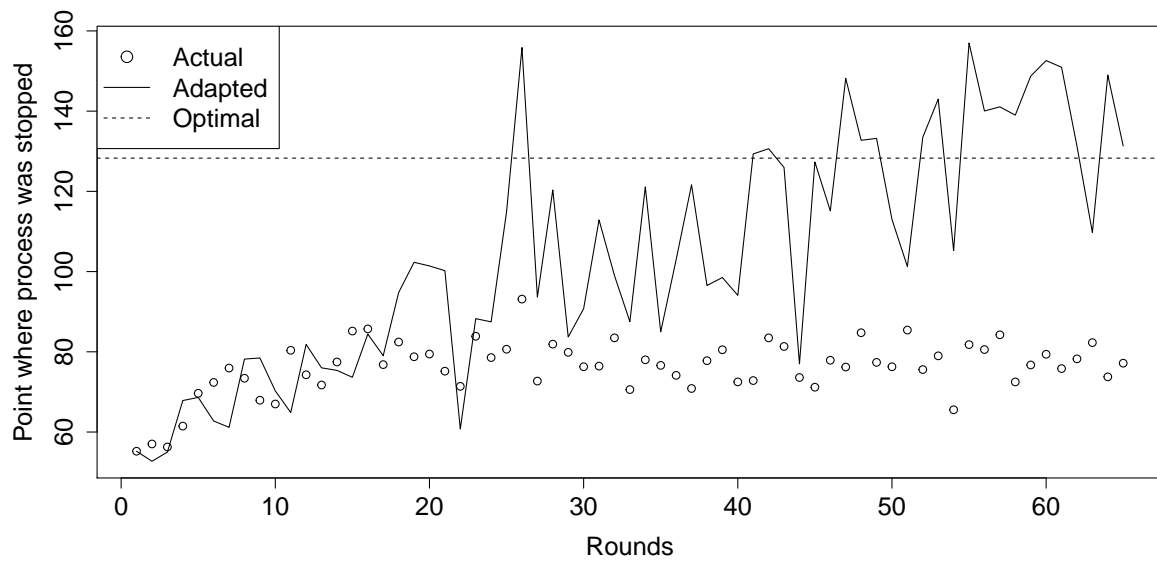
\*University of Cologne, Universitätsstr. 22a, D-50931 Köln, Germany (email: paulviefers@gmail.com).

†University of California Berkeley, Office 513, Evans Hall, Berkeley, 94720 California, USA (philipp.strack@gmail.com).

**Table 1:** Estimated effects of losses on subsequent stopping choices.

Parameter	Oprea et al.	This study
$\delta_E \times 1,000$	0.5486***	1.3873**
$\delta_L \times 1,000$	-0.9185***	-1.1227

*Notes:* Median estimates and  $p$ -value for the Wilcoxon signed-rank test that the distribution is centered around zero: \*\*\*  $p \leq 0.01$ , \*\*  $p \leq 0.05$  and \*  $p \leq 0.1$ .



**Figure 1:** Shows simulation results from adaptive learning model versus actual choices.

## B DETAILS ON STRUCTURAL DISCRETE CHOICE MODEL

This section elaborates on the econometric details behind the dynamic discrete choice model laid out in the main text. To estimate the model, we assume a specific functional form for  $\Psi$  and the utility  $u$  underlying  $\rho$ . For  $u$  we assume power utility of the form<sup>1</sup>

$$u(x - K) = \begin{cases} \frac{K}{\theta} \left[ \left( \frac{x}{K} \right)^\theta - 1 \right] & \text{for } \theta \neq 0, \\ K \ln \left( \frac{x}{K} \right) & \text{for } \theta = 0. \end{cases}$$

With this specification for  $(\Psi \circ \rho)_i(x, s)$ , we can set up the same likelihood as for the reduced-form model in the main text.

**Inattention.** Because we do not want our results to depend on the discretization of time in our experiment, we set  $\Psi(x) = (1 - w)\Phi_\sigma(x) + w$  where  $\Phi_\sigma$  denotes the normal cdf with mean zero and variance  $\sigma^2$  and  $w$  denotes the probability that the agent is inattentive, i.e. continues automatically, in period  $t$ .<sup>2</sup> Without the possibility for the agent to be inattentive, the more ticks we squeeze into one second, the more our results depend on the reaction times of subjects.<sup>3</sup> Since the model will mistake any delay in response to be a deliberate decision, this will entail an attenuation of subjects' risk-aversion  $\theta$ .<sup>4</sup> With this specification, the probability of continuation becomes

$$\mathbb{P}_{t,x,s} [\tilde{\tau}_{i,k} \neq t \mid \tilde{\tau}_{i,k} \geq t, T > t, \gamma] = (\Psi \circ \rho)_i(x, s) = (1 - w_i) [\Phi_{\sigma_i}(\rho(x, s \mid \theta_i, \kappa_i))] + w_i,$$

and thus  $\gamma_i = [\theta_i, \kappa_i, \sigma_i, w_i]$ .<sup>5</sup>

We fit two models for each subject: one EU model for which we restrict  $\kappa = 0$  and one Regret model for which we estimate  $\kappa \in [0, 1]$ . Given the observed decisions  $\tilde{\tau}_i$  of subject  $i$  and a prior distribution  $p(\gamma_i)$  the joint posterior is proportional to prior times likelihood

$$p(\gamma_i \mid \tilde{\tau}_i) \propto \ell(\tilde{\tau}_i \mid \gamma_i) p(\gamma_i), \quad (\text{B.1})$$

where we have omitted the conditioning on the observables  $X^{i,k}$ ,  $S^{i,k}$  and  $T^{i,k}$ . In both models we assume that the parameters are a priori independent, i.e. we set  $p(\gamma_i) = p(\theta_i)p(\kappa_i)p(w_i)p(\sigma_i)$  in the regret model. For  $\kappa$  and  $w$  we have little to no substantive prior knowledge and thus set an uninformative uniform prior over the unit interval, for  $\theta$  we set a normal prior that was calibrated to match the empirical findings of Holt & Laury (2002) and for  $\sigma$  we set a scaled inverse chi-squared prior with scale and degrees of freedom equal to one

$$\theta_i \sim \text{Normal}(0.7, 0.3); \kappa_i \sim \text{Uniform}[0, 1]; \sigma_i^2 \sim \text{Inv-}\chi^2(1); w_i \sim \text{Uniform}[0, 1]. \quad (\text{B.2})$$

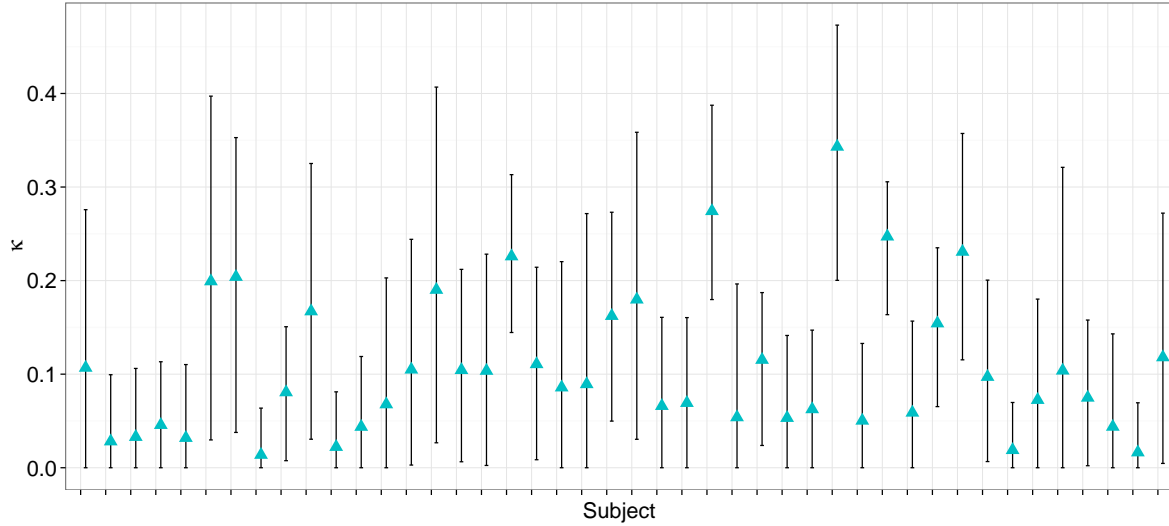
<sup>1</sup>This form for the utility function ensures that (i)  $u(0) = 0$  and (ii) that utility is well-defined for values  $x < K$ .

<sup>2</sup>An intuitive interpretation of  $w$  comes in terms of the expected delay between two periods the agent deliberates. Given our assumptions, this delay is geometrically distributed with mean  $\frac{1}{2(1-w)} - \frac{1}{2}$ .

<sup>3</sup>We thus assume that the inattention level is independent of the cost of being inattentive. We could extend this into a rational inattention model by assuming that  $w$  is a monotone function of the cost of making a mistake  $|x| = |\rho(x, s)|$  or the cost of being inattentive  $\max\{0, -x\} = \max\{0, -\rho(x, s)\}$ .

<sup>4</sup>It has been documented in the psychology literature that for stimuli which arrive in rapid succession, the human brain demands a refractory period to process the available information. If this refractory period is longer than the time between two stimuli, an informational overload entails and response to current stimuli is delayed due to mental capacity being occupied by past stimuli (see e.g. Craik, 1947, 1948; Smith, 1969; Kahneman, 1973). Moreover, each individual tick itself did rarely alter material payoffs from stopping much, i.e. it might not warrant perfect attention in the eyes of a subject.

<sup>5</sup>In order to calculate  $(\Psi \circ \rho)_i(x, s)$  at every point we need a closed-form expression for  $\rho_i(x, s)$  and in particular the regret value  $V^r$ .



**Figure 2:** Plot shows the posterior means and 90% credible intervals for  $\kappa$ .

**Table 2:** Summary statistics for posterior means across subjects.

	Expected utility ( $\kappa = 0$ )			Regret			
	Rel. Risk Aversion $1 - \theta$	Noise Level $\sigma$	Attentional Lapse $\frac{1}{2(1-w)} - \frac{1}{2}$	Rel. Risk Aversion $1 - \theta$	Regret Aversion $\kappa$	Noise Level $\sigma$	Attentional Lapse $\frac{1}{2(1-w)} - \frac{1}{2}$
Min.	-0.15	1.61	0.64s.	-0.27	0.01	1.13	0.64s.
1st Qu.	0.13	3.48	1.67s.	0.04	0.05	3.11	2.00s.
Median	0.25	4.20	4.50s.	0.15	0.09	3.74	4.05s.
Mean	0.26	4.50	2.83s.	0.18	0.11	3.98	3.07s.
3rd Qu.	0.33	5.50	5.75s.	0.27	0.16	4.66	6.64s.
Max.	0.76	8.72	12.00s.	0.57	0.35	7.73	16.17s.

In the EU model all expressions involving  $\kappa$  drop out of the equations above.

The joint posterior density is not of any known form and there is no direct way to sample from it. We therefore simulate the posterior of the parameters using a Markov Chain Monte Carlo algorithm Metropolis, Rosenbluth, Rosenbluth, Teller & Teller (1953).

**Estimation.** We adopt the notation of the main text, i.e. we denote by  $\tilde{\tau}_{i,k}$  the observed stopping time by subject  $i$  in round  $k$ ,  $X^{i,k}$  the realized path,  $S^{i,k}$  its associated maximum process,  $T^{i,k}$  the random time the game ended, and by  $\gamma_i$  the vector of parameters for subject  $i$ . For each round we model the stopping decision of a subject as a series of continuation decisions that culminate either in the decision to stop ( $\tilde{\tau}_{i,k} \leq T_{i,k}$ ) or the end of the round ( $\tilde{\tau}_{i,k} > T_{i,k}$ ). Given the observed decisions  $\tilde{\tau}_i$  of subject  $i$  and a prior distribution  $p(\gamma_i)$  the joint posterior is proportional to prior times likelihood

$$p(\gamma_i | \tilde{\tau}_i) \propto \ell(\tilde{\tau}_i | \gamma_i) p(\gamma_i) . \quad (\text{B.3})$$



To estimate the model, we assume that  $\theta$ ,  $\kappa$ ,  $w$  and  $\sigma$  are all *a priori* independent

$$p(\gamma_i) = p(\theta_i)p(\kappa_i)p(w_i)p(\sigma_i) . \quad (\text{B.4})$$

For  $\kappa$  and  $w$  we set an uninformative uniform prior over the unit interval  $U[0, 1]$ , for  $\theta$  we set a normal prior that was calibrated to match the empirical findings of Holt & Laury (2002) (see also further details in the next section) and for  $\sigma$  we set a scaled inverse chi-squared prior with scale and degrees of freedom equal to one

$$\theta \sim \mathcal{N}(0.7, 0.3) ; \kappa \sim U[0, 1] ; \sigma^2 \sim \text{Inv-}\chi^2(1) ; w \sim U[0, 1] \quad (\text{B.5})$$

The joint posterior density is not of any known form and thus there is no direct way to sample from it. We therefore first find the posterior modes of (B.3) with a standard hill-climbing algorithm (BFGS) and take these as starting values for a Markov Chain Monte Carlo algorithm (random walk Metropolis-Hastings) to simulate the full joint posterior (Metropolis et al., 1953). The Metropolis-Hastings algorithm iterates over the followings steps:

1. Step 0 (Initialization): Choose a starting values  $\gamma^{(0)}$  and a *candidate-generating* or *proposal* density  $Q(\theta^{(s+1)} | \gamma^{(s)})$ .
2. Step 1 (Proposal): For iteration  $s$  generate a proposal  $\hat{\gamma}$  from  $Q$  based on  $\gamma^{(s-1)}$ .
3. Step 2 (Accept/Reject): Accept the current draw, i.e. set  $\gamma^{(s)} = \hat{\gamma}$ , with probability

$$\alpha = \min \left\{ 1, \frac{p(\hat{\gamma} | X)}{p(\gamma^{(s-1)} | X)} \right\} , \quad (\text{B.6})$$

or reject the candiate with probability  $1 - \alpha$ , i.e. set  $\gamma^{(s)} = \gamma^{(s-1)}$ .

4. Step 3 (Iterate): Go back to Step 2.

The algorithm uses the fact that we can evaluate the posterior at every point. It thus starts from a given point in the parameter space and randomly draws a candidate from  $Q$  in the vicinity of the starting point. In our case it draws from a normal distribution centered at the current position. More specifically, in step 2 of iteration  $s$  we generate a candidate according to the following rule

$$\begin{bmatrix} \hat{\theta} \\ \hat{\kappa} \\ \hat{\sigma} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} \theta^{(s-1)} \\ \kappa^{(s-1)} \\ \sigma^{(s-1)} \\ w^{(s-1)} \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix} ; \quad \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{bmatrix} \sim \mathcal{N}(0, \Sigma) . \quad (\text{B.7})$$

Hence the name *random walk* Metropolis-Hastings. Initially we set the covariance matrix  $\Sigma$  equal to the inverse negative Hessian matrix of the parameter vector at the posterior mode. To facilitate convergence the matrix  $\Sigma$  is adapted during iteration. After the first 100 iterations, we adapt it every 100 iterations and its diagonal elements are set to

$$\Sigma = \frac{0.5s - 1}{0.5s} \text{diag}(\widehat{\text{Var}}(\theta^{(0.5s: s)}), \widehat{\text{Var}}(\kappa^{(0.5s: s)}), \widehat{\text{Var}}(\sigma^{(0.5s: s)}), \widehat{\text{Var}}(w^{(0.5s: s)}))$$

where  $(a : b) = a, \dots, b$  and

$$\widehat{\text{Var}}(\theta^{(0.5s: s)}) = \frac{1}{0.5s - 1} \sum_{i=0.5s}^s (\theta^{(i)} - \bar{\theta})^2 \quad (\text{B.8})$$

and  $\bar{\theta}$  denotes the sample mean of  $\theta^{(i)}$  over iterations  $i = 0.5s, \dots, s$ .<sup>6</sup> Adapting  $\Sigma$  is useful, because if  $\Sigma$

---

<sup>6</sup>Haario, Saksman & Tamminen (2001) for details. Their algorithm is implemented in the R package `MHadaptive` (Chivers, 2012).

is such that proposed jumps through the parameter space are excessively large, this will result in a high rejection rate and thus inefficient sampling. On the other hand, if  $\Sigma$  is such that  $Q$  only produces small jumps through the parameter space, the sampler will be slow mixing and not explore the parameters space sufficiently. We stop adaption of the proposal density after the first half of iterations to make sure the algorithm converges to a stationary target distribution.

In Step 2 the candidate drawn from  $Q$  is accepted with probability one if it has higher posterior probability. If the candidate has lower posterior probability, it is accepted with probability  $\alpha$  equal to the ratio between the posterior probability of the candidate and the current position. If a candidate is accepted the algorithm continues to sample from there in its next iteration. If not, the the algorithm samples form the same position again.

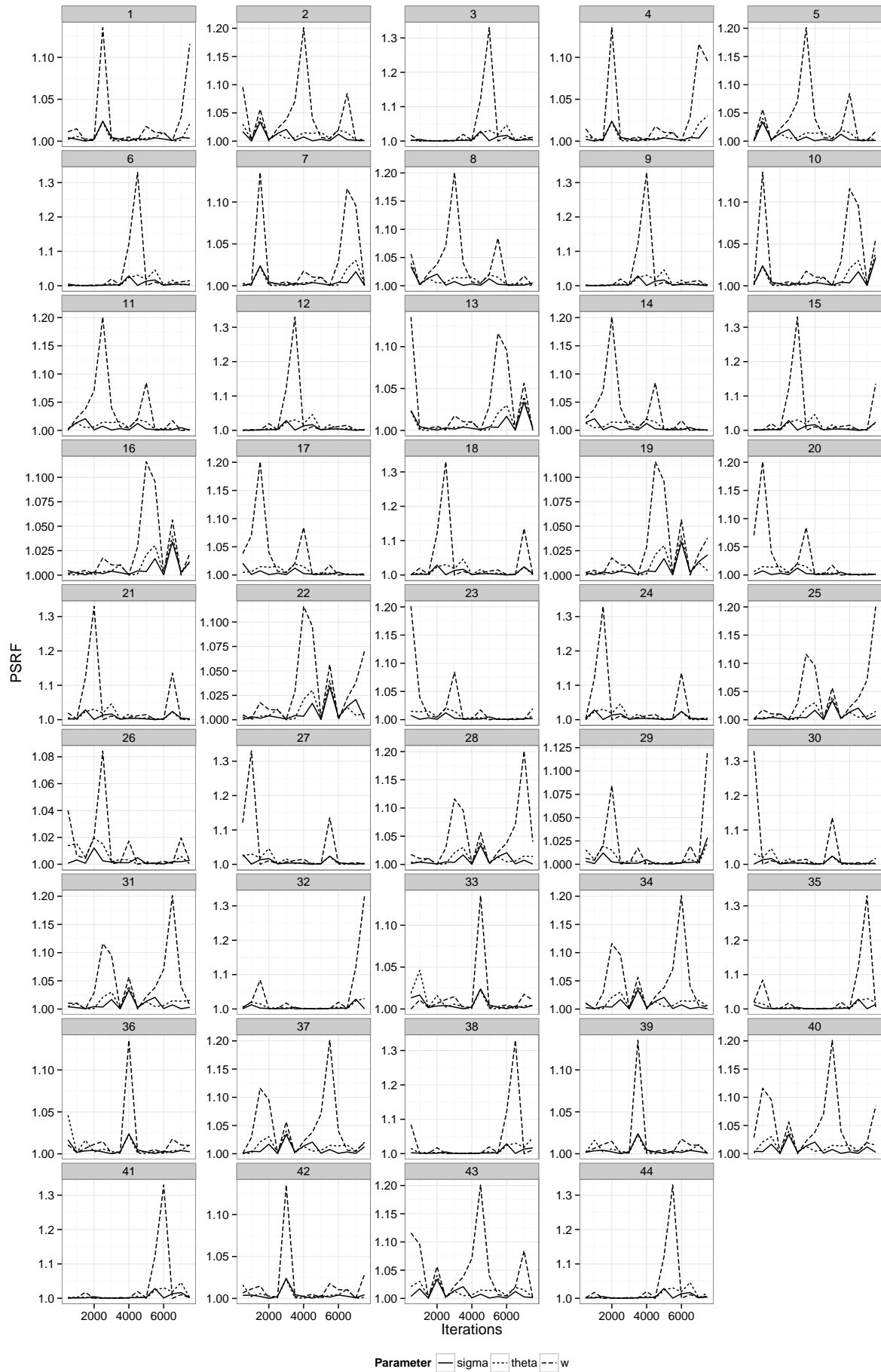
To avoid taking possibly many samples from a low-probability region in the beginning, i.e. in an irrelevant region, we first determine the posterior mode of (B.3) with a standard hill-climbing algorithm and start the sampler from there to facilitate convergence.<sup>7</sup> We generate 10,000 draws from the posterior and discard the first 2,500 as burn-in or training sample.

## B.1 CONVERGENCE DIAGNOSTICS

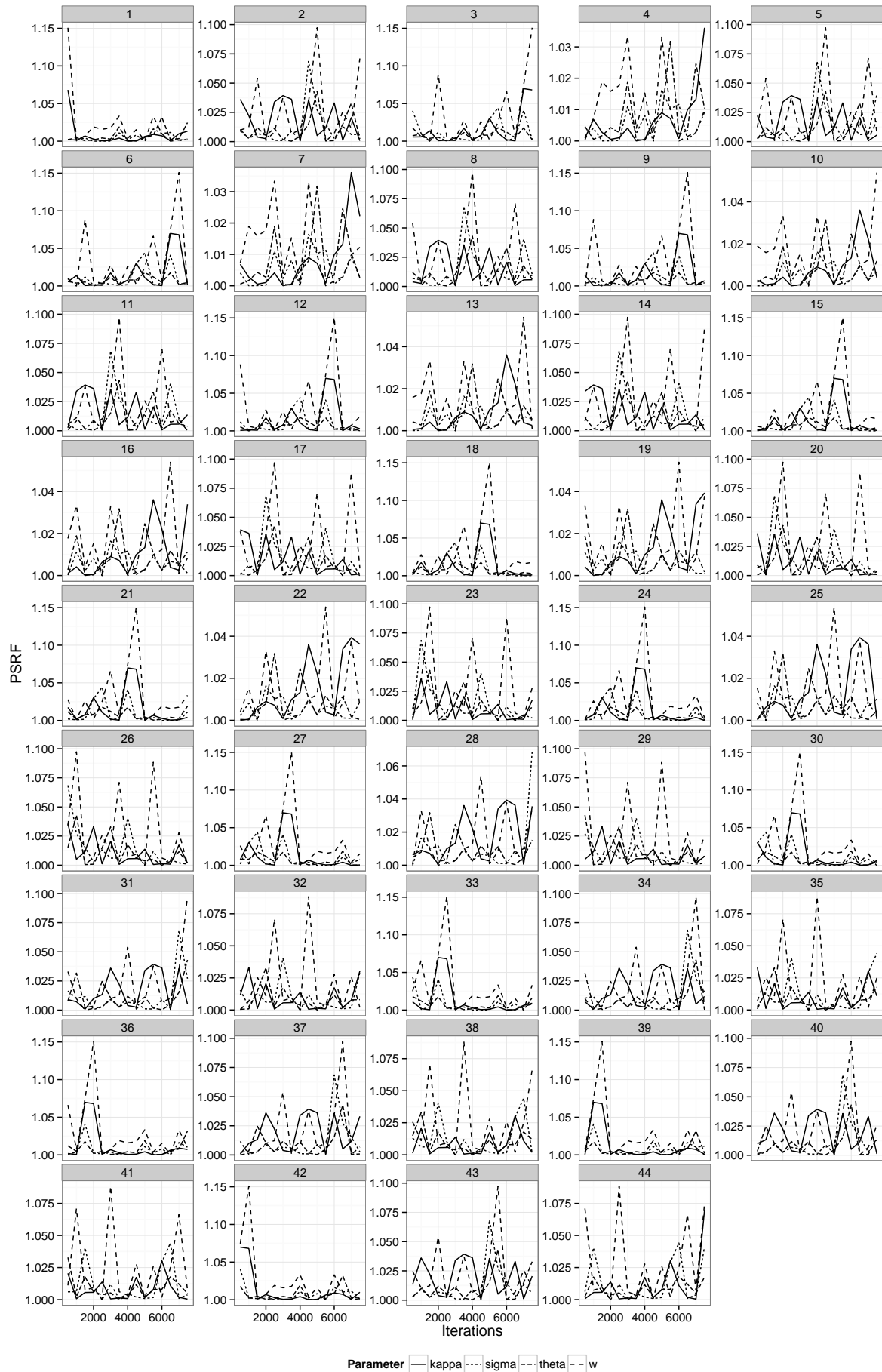
To gauge whether the Markov chain generated through the Metropolis-Hastings algorithm has converged to its stationary distribution and in order to convince ourselves that the results do not depend on the starting values, we follow Gelman & Rubin (1992) and computed the potential scale-reduction factor (PSRF)  $\hat{R}$  for each parameter (also see Gelman, Carlin, Stern, Dunson, Vehtari & Rubin, 2013, for details). For this we generated a second Markov chain with 10,000 iterations, which was started at a random point in the parameter space. We draw overdispersed starting values for the chain from a multivariate normal centered at the posterior mode and with covariance matrix equal to twice the inverse negative Hessian. The PSRF is a measure for each scalar parameter that compares the variation of the sampled values within and between chains. If the within and between variation is the same, the chains are said to have converged to the same target distribution. In that case  $\hat{R} \approx 1$ . The resulting PSRFs for the EU and Regret model are plotted in Figure 3 and 4. We have discarded the frist 2,500 warmup iterations from the plot and calculated the PSRF for the remaining 7,500 iterations on our sample recursively by succesively adding blocks of 500 iterations. The plot shows that the PSRFs have approached unity after the warmup sample of 2,500 iterations and from there on fluctuate only within the range from 1 to 1.3.

---

<sup>7</sup>We use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm implemented in general-purpose optimization R function `optim`.



**Figure 3:** Potential scale reduction factors over blocks of iterations for the EU model.

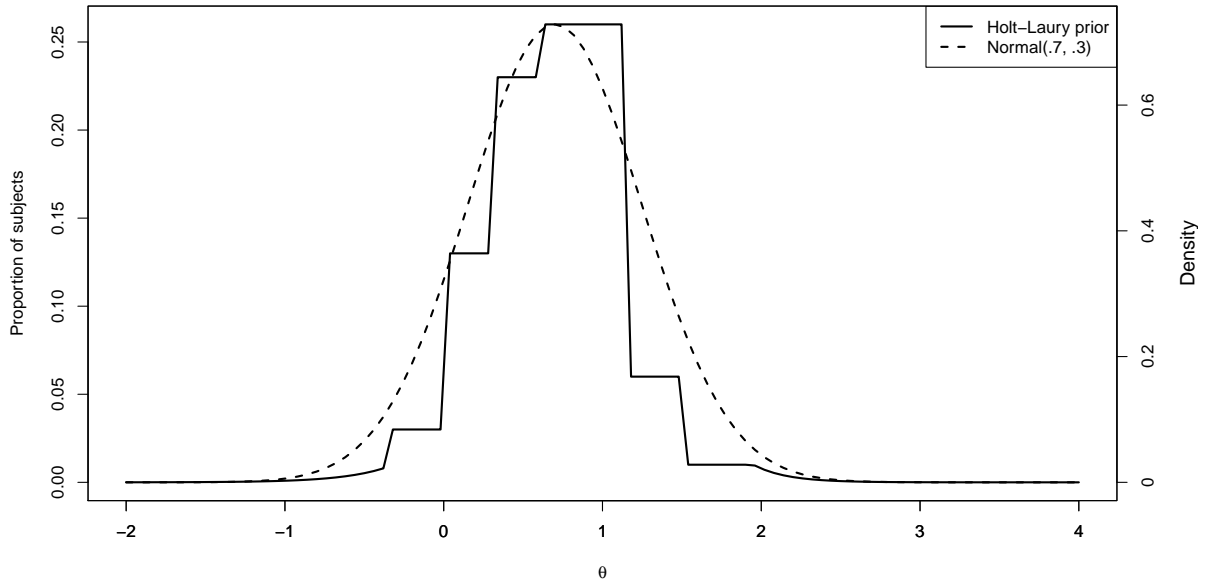


**Figure 4:** Potential scale reduction factors over blocks of iterations for the Regret model.

## B.2 PRIOR SENSITIVITY ANALYSIS

For the degree of risk aversion  $\theta$ , there exists substantial prior evidence on the range of reasonable values for it in experimental settings. To illustrate how we used this information to elicit a prior on  $\theta$ , we have plotted the normal prior with mean equal to 0.7 and standard deviation 0.3 together with the data presented in the study by Holt & Laury (see Holt & Laury, 2002, Table 3) in Figure 5.<sup>8</sup> One can see that a suitably scaled normal distribution with mean 0.7 and standard deviation 0.3 matches the reported step function reasonably well. For  $\kappa$  and  $w$  we have less clear evidence on their reasonable range from comparable studies. We therefore adopt the Principle of Insufficient Reason and set a uniform prior. Similarly for  $\sigma$ , where the scaled inverse chi-square prior is a standard prior for variances.

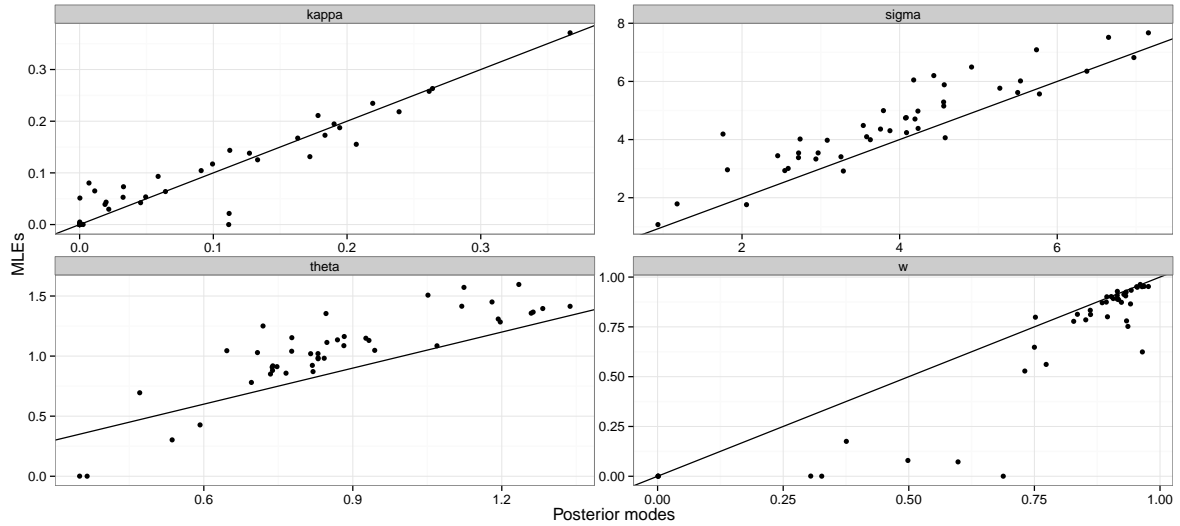
How much is our posterior driven by the choice of the prior? To show that the prior we set is inconsequential, we compare the modes of the resulting posterior with the modes of the likelihood function. If the prior exerts much influence, these should be very different. Figure 6 shows the posterior modes under the mentioned prior against the maximum likelihood estimates. We see that the point estimates for  $\theta$  and  $\sigma$  tend to increase, showing that the Holt-Laury prior attenuates our estimate for  $\theta$ . It should be noted that since regret aversion is a substitute for risk aversion, we expect a prior that pulls  $\theta$  towards a smaller value (indicating higher risk aversion) also to shrink our estimate for  $\kappa$ . Even though, on average there is no discernible difference between the MLE and the posterior modes for  $\kappa$ , in the range between 0 to 0.1 the prior seems to shrink values closer to zero. Similarly, there are some estimates for  $w$  that decrease notably, but again there seems to be no undue influence of the prior on the posterior. From this we conclude that the prior is only of minor importance in our model and that our results are primarily driven the information encoded in the data.



**Figure 5:** Figure shows the distribution of the parameter  $\theta$  as reported in Holt & Laury (2002) (left scale) together with the  $\mathcal{N}(0.7, 0.3)$  distribution (right scale).

Table 2 and Figure 2 provide a summary of our results. We find that subjects are on average mildly risk averse, albeit slightly less under the regret model. For the regret intensity  $\kappa$ , we find that (i)

<sup>8</sup>The results in Holt & Laury are only reported for bins of different size and also require to define how the mass for values greater than their most extreme values is distributed. We decided to let the density decay exponentially for  $\theta > 0.97$  and  $\theta < -0.95$ .

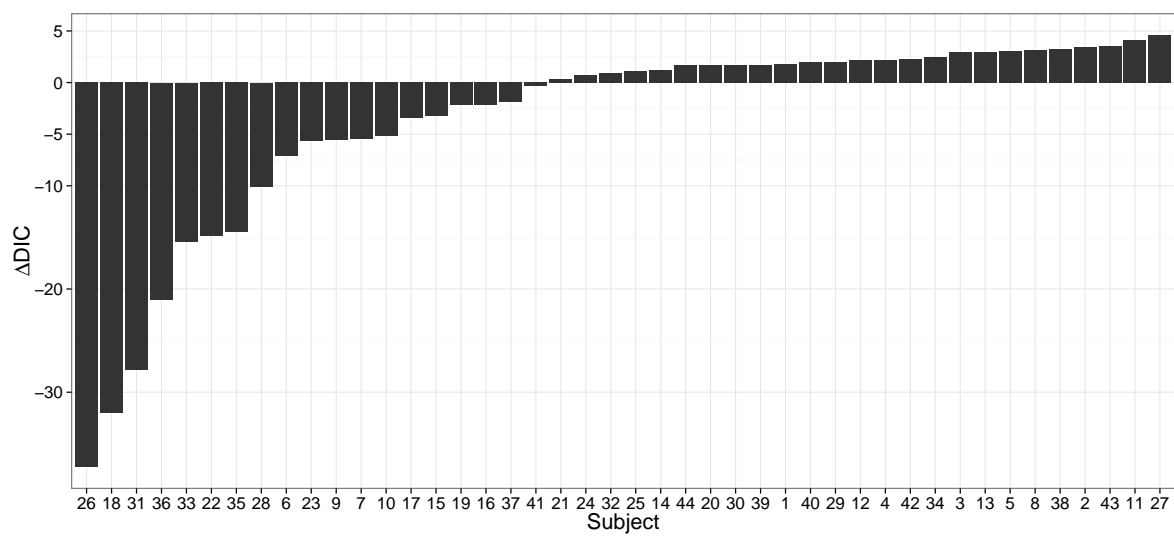


**Figure 6:** Comparison between posterior modes and maximum likelihood estimates.

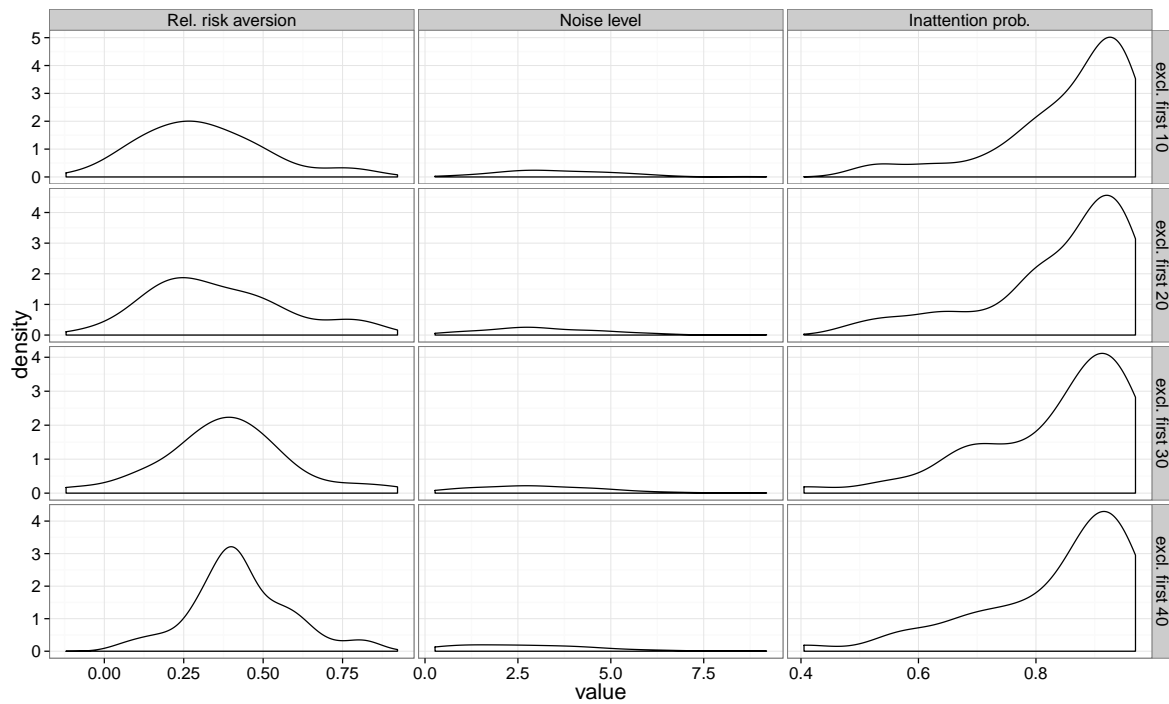
for roughly 35% of our subjects the 90% credible interval does not contain the zero and that (ii) the posterior mean for  $\kappa$  can be as high as 0.35, with an average value of 0.1. In terms of attentional lapse, the posterior mean for  $w$  implies that subjects in our sample pay attention every 4.62 seconds in the EU model and every 4.1 seconds in the regret model. This seems to be a plausible amount of inattention in our experiment, where the process evolves relatively fast. In order to perform a formal model comparison, we computed the deviance information criterion (DIC) from the Metropolis output (Spiegelhalter, Best, Carlin & Van Der Linde, 2002). Models with smaller DIC are preferred and because the DIC rises in the effective number of parameters  $p_D$  (see e.g. Spiegelhalter et al., 2002), this also means that more parsimonious models are preferred. In Figure 7 we plotted the DIC of the regret minus that of the EU model. Differences in the DIC less than five are considered negligible, whereas differences larger than five are considered substantial and larger than 10 to be decisive.

### B.3 ECONOMETRIC RESULTS WHEN EARLY ROUNDS ARE EXCLUDED

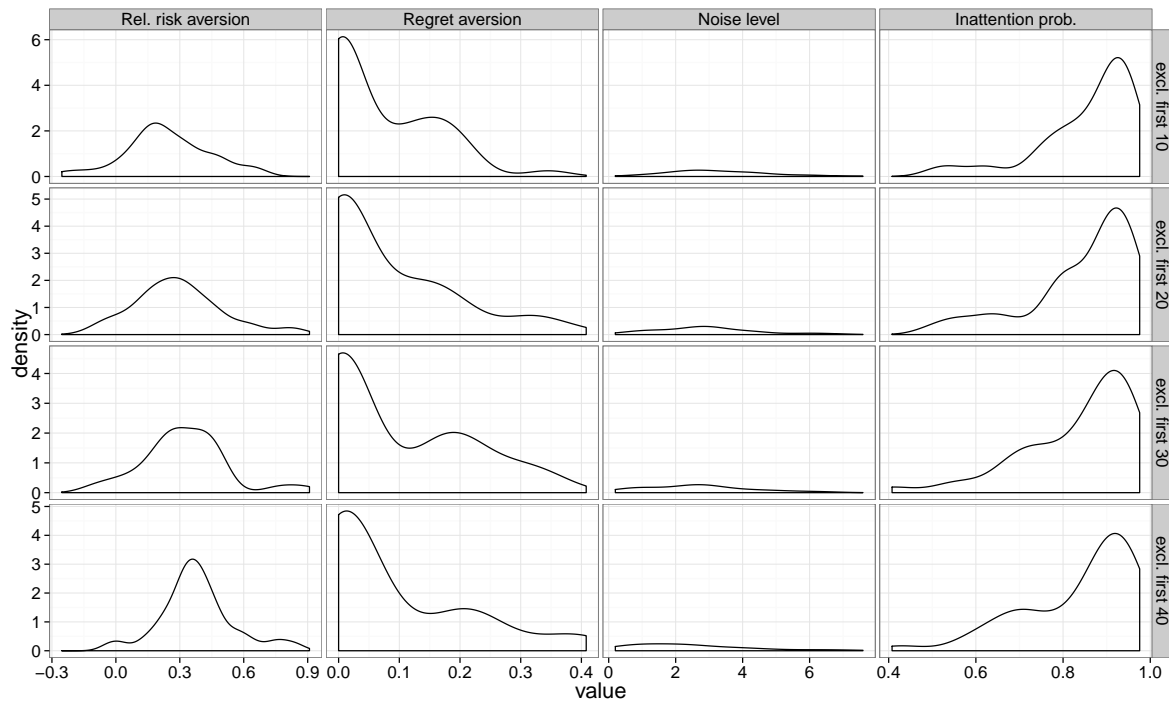
We have re-estimated our model to find the posterior modes after excluding the first 10, 20, 30, or 40 rounds of play for each subject.



**Figure 7:** Difference between the deviance information criterion of our models. Negative values indicate evidence in favor the regret model.



**Figure 8:** Density estimates for the posterior modes under the EU model across all subjects in our sample.



**Figure 9:** Density estimates for the posterior modes under the Regret model across all subjects in our sample.



# Welcome!

---

Please read the following instructions carefully.

**Please remain seated during the whole experiment. Do not communicate with any other participant and remain silent.**

**Should you encounter any problems or have questions related to the experiment, please raise your hand and a lab instructor will come to your place.**

**After the experiment is over, please remain seated. Do not log off the computer experiment by yourself, but please wait until you have been paid by the lab administrator. Payment will begin as soon as the last participant has completed the experiment.**

Today's experiment consists of 65 rounds in which you will have to make one decision each. After you have completed the experiment, you will be paid a show-up fee of 10 EUR plus the number of points that you have earned during one of the 65 rounds.

Your total payment will be determined as:

$$\text{Total EUR} = 0,15 * (\text{no. of points earned in one round}) + 10,00 \text{ EUR show-up fee.}$$

Which of the 65 rounds becomes pay-off relevant, will be determined by the computer at the end of the experiment. The computer will draw one of the numbers from 1 to 65 with equal probability (1/65). The number of points you earned in the drawn round is then inserted into the formula above to determine your final payoff..

## The basic setting

In this experiment you will have to decide if, and if yes, at which point in time, you want to sell a fictitious stock that you own. You will observe the market price for your stock in a diagram and you are able to sell it at any point during a round. The market price of your stock will not be constant over the round, but will be fluctuating randomly. Details about how it will fluctuate can be found below.

In case you decide at some point in time to sell your stock, you will earn the market price minus transaction costs of 40 ECU.

That is, your earnings from the sale of the stock will be market price minus cost.

The experiment will be computer-based. The screen in front of you will display the market price of your stock in each round and also some other useful information about the current round. The main experimental screen is depicted in the figure below:

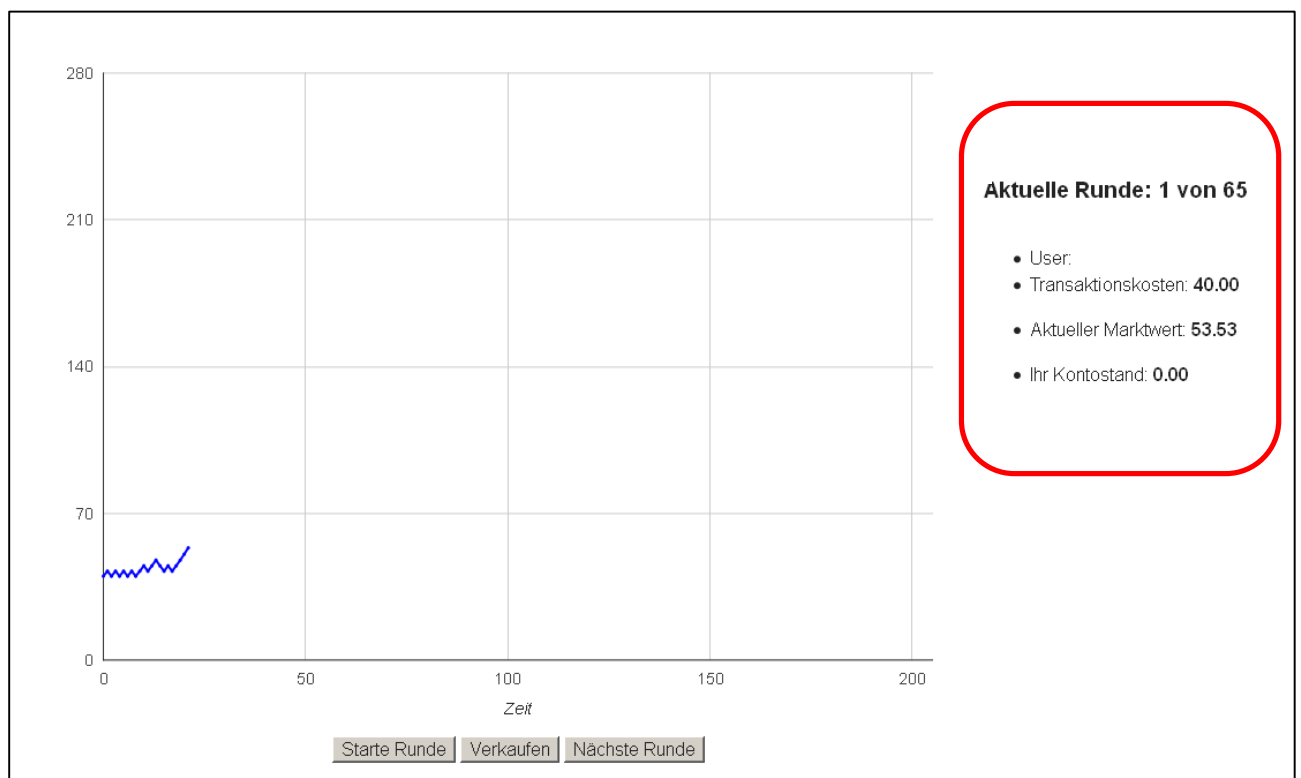


Figure 1

## Details

As you can see in figure 1, there are three buttons available at the bottom of the screen.

In order to start a round, please press the button labeled „start round“. Once you have clicked that button, the computer will begin to display the evolution of the market price over time. Time is depicted on the horizontal axis. The market price in ECU will be depicted on the vertical axis.

The market price will be depicted as a jagged blue line in the diagram. Every second there will be two ticks. The starting value for the line will always be 40. From there the market price will increase or decrease by 6% for every up- or downtick. For example, suppose the current value of the market price is 100. The next value then either is  $100 \cdot 1.06 = 106$  or  $100 \cdot (1/1.06) \approx 94.34$ . The current market price is the rightmost value at the leading edge of the jagged line (e.g. in fig. 1 the current value is 53.53 ECU).

The probability that the market value will increase always is 52% irrespective of what happened before. That is, the probability for the market price to increase never changes, neither within, nor between rounds.

Apart from diagram with the evolution of the market price, the screen shows further information about current round. You can see them in the red box at the right of the screen.

Once a round has been started, the button labeled „Sell“ is available to you. When you click on it, you sell the stock at the current market price. This also means, however, you will have to incur transaction cost of 40 ECU. For example, suppose you sell the stock in the situation depicted in figure 1. Then you would earn the current value of 53.53 ECU, but would have to deduct 40 ECU. Thus, your earnings equal  $53.53 - 40 = 13.53$  ECU. In order to illustrate that you have sold your stock, the jagged line will turn green once you sold the stock. If you decide to sell at a market value below 40, you will make a loss. Should the computer after the experiment select a round where you sold at a value below 40 ECU, your loss will be deducted from your show-up fee. If you do not decide to sell the stock before the end of a round, your earnings will be zero.

The duration of every round is totally random. After every tick, the computer determines whether the round ends before the next tick. This happens with a constant probability of 0.7%. You will notice the end of a round, when the jagged line ceases to continue scroll rightwards and no new values are being depicted. .

The end of a round also means you will not be able to sell the stock anymore. As we have mentioned before, in case you have not decided to sell before the end of the round, your earnings will be zero.

After a round has ended, you will be able to skip to the next round by clicking on the button labeled “Next round”. Before the end of a round, you will not be able to abort a round and move to the next round, but will have to wait for the end of a round.

## Summary

To summarize:

- You will have to make selling decisions over 65 rounds.
- In every round you will have the option to sell your stock at the current market price.
- The market price evolves randomly over time. In every second there will be two
- The probability for an uptick is always 52%. The probability for a downtick always is  $100\% - 52\% = 48\%$ .
- The probability that the round will end and you do not have the option to sell the stock is 0.7% before every tick.
- Instead of selling the stock at the current market price, you always have the option to postpone selling to future periods. You can be lucky and the price increases further. You may be unlucky and the price decreases.

# Questionnaire

---

Prior to the experiment we would like you to the following questions

**Question 1:**

Recall the experiment we just described. If the current value of your stock at which you decide to sell it is 75ECU, what will you earn from selling it?

Answer: \_\_\_\_\_ ECU.

**Question 2:**

Recall the experiment we just described. Suppose your are at the beginning of a round. The current value of your stock is equal to 40 as always. Now the market price increased in the very first period. What is the probability it will increase again in the next period?

Answer: \_\_\_\_\_ %.

**Question 3:**

Recall the experiment we just described. Suppose that since the beginning of the round the market price has changed five times (i.e. you have seen five ticks until now). What is the chance in percent that the round will terminate and you will not see a sixth tick?

Answer: \_\_\_\_\_ %.

**In case you do not have further questions, please switch on the computer screen now and log into the experiment using your seat number and confirm by clicking ,Login'. Please note that the login is case sensitive and that you have to include the minus sign between the initial and you number.**

**You seat number is: A-16.**

**The experiment will begin with the first round after your login.**

## REFERENCES

- Chivers, C. (2012). *MHadaptive: General Markov Chain Monte Carlo for Bayesian Inference using adaptive Metropolis-Hastings sampling*. R package version 1.1-8.
- Coricelli, G., Critchley, H. D., Joffily, M., O'Doherty, J. P., Sirigu, A., & Dolan, R. J. (2005). Regret and its avoidance: a neuroimaging study of choice behavior. *Nature neuroscience*, 8(9), 1255–1262.
- Craik, K. J. (1947). Theory of the human operator in control systems i. the operator as an engineering system. *British Journal of Psychology*, 38(2), 56–61.
- Craik, K. J. (1948). Theory of the human operator in control systems ii. man as an element in the control system. *British Journal of Psychology*, 38(3), 142–148.
- Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian Data Analysis* (3rd ed.). Chapman & Hall/CRC.
- Gelman, A. & Rubin, D. B. (1992). Inference from Iterative Simulation using Multiple Sequences. *Statistical science*, 457–472.
- Gilovich, T. & Medvec, V. H. (1994). The temporal pattern to the experience of regret. *Journal of Personality and Social Psychology*, 67(3), 357.
- Gilovich, T., Medvec, V. H., & Kahneman, D. (1998). Varieties of regret: A debate and partial resolution. *Psychological Review*, 105(3), 602.
- Haario, H., Saksman, E., & Tamminen, J. (2001). An adaptive metropolis algorithm. *Bernoulli*, 7(2), 223–242.
- Holt, C. & Laury, S. (2002). Risk aversion and incentive effects. *The American Economic Review*, 92(5), 1644–1655.
- Kahneman, D. (1973). *Attention and Effort*. Prentice-Hall.
- Kahneman, D. & Tversky, A. (1982). The psychology of preferences. *Scientific American*.
- Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., & Teller, E. (1953). Equation of state calculations by fast computing machines. *Journal of Chemical Physics*, 21, 1087–1092.
- Oprea, R., Friedman, D., & Anderson, S. (2009). Learning to wait: A laboratory investigation. *Review of Economic Studies*, 76(3), 1103–1124.
- Smith, M. C. (1969). The effect of varying information on the psychological refractory period. *Acta Psychologica*, 30, 220–231.
- Spiegelhalter, D. J., Best, N. G., Carlin, B. P., & Van Der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64(4), 583–639.
- Summers, B. & Duxbury, D. (2007). Unraveling the disposition effect: The role of prospect theory and emotions. *Available at SSRN 1026915*.