2048

TD Learning

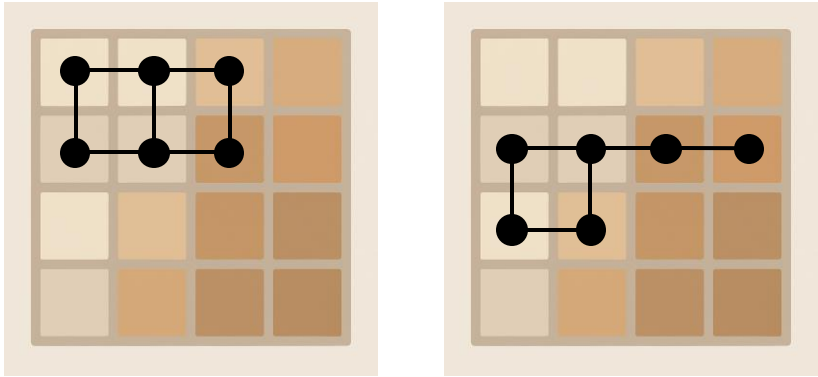
- TD(0) reinforcement learning algorithm
- Agent aims to maximize score
- Decision-making driven by evaluation of “afterstates”
- State space too large to track all possible states
  - Requires approximation: n-tuple networks

M. Szubert and W. Jaśkowski, "Temporal difference learning of N-tuple networks for the game 2048," 2014 IEEE Conference on Computational Intelligence and Games, Dortmund, Germany, 2014, pp. 1-8, doi: 10.1109/CIG.2014.6932907.

N-Tuple Networks

- *n*-tuple: set of *n* tile positions
- *m* number of *n*-tuples are used to approximate a state's value
- 8x6 tuple network was used for this agent

- Example 2x6 tuple network:



M. Szubert and W. Jaśkowski, "Temporal difference learning of N-tuple networks for the game 2048," 2014 IEEE Conference on Computational Intelligence and Games, Dortmund, Germany, 2014, pp. 1-8, doi: 10.1109/CIG.2014.6932907.
K. Matsuzaki, "Systematic selection of N-tuple networks with consideration of interinfluence for game 2048," 2016 Conference on Technologies and Applications of Artificial Intelligence (TAAI), Hsinchu, Taiwan, 2016, pp. 186-193, doi: 10.1109/TAAI.2016.7880154.

TDL Results

- Training data: 40,000 episodes
- Results from playing 2,500 games:
  - Average score: 28,619
  - Reached 8192 tile: **0.04%**
  - Reached 4096 tile: **9.52%**
  - Reached 2048 tile: **60.64%**

Cumulative Gameplay Statistics:

# of Games	Time	Mean Score	Rate of Highest Tile
100	68.34	28037.36	4096: 10.00%, 2048: 48.00%, 1024: 27.00%
200	137.00	28329.46	4096: 10.00%, 2048: 48.50%, 1024: 28.50%
300	201.96	27735.84	4096: 9.00%, 2048: 48.67%, 1024: 27.33%
400	269.41	27656.40	4096: 9.25%, 2048: 47.50%, 1024: 28.75%
500	338.87	27791.74	4096: 9.40%, 2048: 47.00%, 1024: 29.00%
600	413.84	28324.83	4096: 9.50%, 2048: 49.17%, 1024: 28.00%
700	483.09	28318.33	4096: 9.00%, 2048: 50.14%, 1024: 27.86%
800	552.79	28320.72	4096: 8.62%, 2048: 51.00%, 1024: 27.38%
900	619.44	28194.64	4096: 8.67%, 2048: 50.44%, 1024: 27.33%
1000	687.26	28129.24	4096: 8.70%, 2048: 50.30%, 1024: 27.30%
1100	754.63	28061.04	4096: 8.55%, 2048: 50.45%, 1024: 27.73%
1200	821.66	28003.81	4096: 8.58%, 2048: 49.92%, 1024: 28.17%
1300	892.15	28076.50	4096: 9.00%, 2048: 49.62%, 1024: 27.85%
1400	959.88	28026.26	4096: 8.64%, 2048: 50.29%, 1024: 27.64%
1500	1034.75	28214.10	4096: 8.87%, 2048: 50.33%, 1024: 27.80%
1600	1107.74	28328.28	8192: 0.06%, 4096: 8.81%, 2048: 50.94%
1700	1182.62	28491.13	8192: 0.06%, 4096: 8.88%, 2048: 51.47%
1800	1250.19	28435.90	8192: 0.06%, 4096: 8.94%, 2048: 51.28%
1900	1324.59	28569.65	8192: 0.05%, 4096: 9.11%, 2048: 51.63%
2000	1389.93	28444.75	8192: 0.05%, 4096: 9.05%, 2048: 51.30%
2100	1456.38	28377.94	8192: 0.05%, 4096: 8.95%, 2048: 51.33%
2200	1533.99	28545.99	8192: 0.05%, 4096: 9.27%, 2048: 51.27%
2300	1600.79	28470.02	8192: 0.04%, 4096: 9.09%, 2048: 51.35%
2400	1671.80	28487.01	8192: 0.04%, 4096: 9.21%, 2048: 51.17%
2500	1751.11	28619.47	8192: 0.04%, 4096: 9.48%, 2048: 51.12%

Experimental TD Learning

- Used an 8x6 Tuple network combined with a 10x8 Tuple Network.
  - Accounting for all 8 symmetries, this allows for 144 features per state evaluation.
- Decision making based on the "afterstates" and a reward system that prioritizes certain game strategies.
- Implements game heuristics to improve scoring and tile building.
- Implements epsilon-greedy exploration to balance exploitation of learned knowledge.

Training Data Results

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+-------------------------------------------------------------------------+
| Statistics from last 100 episodes:                                      |
|-------------------------------------------------------------------------|
| Episodes   | Time       | Mean Score     | Mean Max Tile  | Max Tile       |
|------------|------------|----------------|----------------|----------------|
|        100 |     127.71 |       12470.36 |         840.96 |           2048 |
|        200 |     134.65 |       13150.42 |         952.32 |           4096 |
|        300 |     170.10 |       13471.36 |         979.20 |           2048 |
|        400 |     155.03 |       13439.69 |         913.92 |           2048 |
|        500 |     139.52 |       13504.05 |         960.00 |           2048 |
|        600 |     146.11 |       13666.75 |         983.04 |           2048 |
|        700 |     134.98 |       13547.58 |         862.72 |           2048 |
|        800 |     148.68 |       13518.41 |         885.76 |           2048 |
|        900 |     129.95 |       13391.86 |         878.08 |           2048 |
|       1000 |     140.02 |       13417.06 |         952.32 |           2048 |
Model saved to td_agent_episode_ft1000.pkl
|       1100 |     229.88 |       13365.61 |         849.92 |           2048 |
|       1200 |     140.99 |       13372.94 |         906.24 |           2048 |
|       1300 |     147.44 |       13337.07 |         904.96 |           2048 |
|       1400 |     138.97 |       13331.15 |         906.24 |           2048 |
|       1500 |     140.50 |       13347.34 |         960.00 |           2048 |
|       1600 |     135.92 |       13323.12 |         862.72 |           2048 |
|       1700 |     143.50 |       13365.30 |         961.28 |           2048 |
|       1800 |     155.60 |       13406.88 |         968.96 |           4096 |
|       1900 |     138.99 |       13399.47 |         880.64 |           2048 |
|       2000 |     139.17 |       13397.06 |         888.32 |           2048 |
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Model saved to td_agent_episode_ft63000.pkl
|      63100 |     469.50 |       16387.31 |        1233.92 |           2048 |
|      63200 |     190.02 |       16388.07 |        1105.92 |           2048 |
|      63300 |     189.24 |       16388.80 |        1126.40 |           2048 |
|      63400 |     159.97 |       16390.18 |        1123.84 |           2048 |
|      63500 |     198.38 |       16392.75 |        1164.80 |           4096 |
|      63600 |     203.48 |       16396.58 |        1282.56 |           4096 |
|      63700 |     205.91 |       16400.37 |        1228.80 |           2048 |
|      63800 |     191.12 |       16401.54 |        1136.64 |           4096 |
|      63900 |     207.22 |       16406.15 |        1280.00 |           4096 |
|      64000 |     191.19 |       16407.14 |        1118.72 |           2048 |
Model saved to td_agent_episode_ft64000.pkl
|      64100 |     469.31 |       16409.83 |        1218.56 |           4096 |
|      64200 |     200.87 |       16412.89 |        1244.16 |           4096 |
|      64300 |     210.44 |       16417.66 |        1272.32 |           4096 |
|      64400 |     189.48 |       16418.38 |        1113.60 |           4096 |
|      64500 |     155.98 |       16418.73 |        1100.80 |           2048 |
|      64600 |     202.31 |       16422.03 |        1218.56 |           2048 |
|      64700 |     199.21 |       16424.59 |        1159.68 |           2048 |
|      64800 |     207.20 |       16428.86 |        1285.12 |           4096 |
|      64900 |     192.42 |       16430.39 |        1162.24 |           2048 |
|      65000 |     201.90 |       16433.38 |        1244.16 |           4096 |
Model saved to td_agent_episode_ft65000.pkl
|      65100 |     469.14 |       16436.98 |        1213.44 |           2048 |
|      65200 |     205.53 |       16441.05 |        1300.48 |           4096 |
|      65300 |     159.65 |       16442.30 |        1187.84 |           2048 |
|      65400 |     191.69 |       16443.26 |        1103.36 |           2048 |
|      65500 |     201.65 |       16446.08 |        1190.40 |           2048 |
|      65600 |     195.04 |       16447.64 |        1107.20 |           2048 |
|      65700 |     205.07 |       16451.40 |        1280.00 |           2048 |
|      65800 |     161.50 |       16453.01 |        1128.96 |           2048 |
|      65900 |     201.07 |       16456.36 |        1246.72 |           4096 |
|      66000 |     212.26 |       16461.77 |        1351.68 |           4096 |
Model saved to td_agent_episode_ft66000.pkl
|      66100 |     491.67 |       16464.13 |        1228.80 |           2048 |
|      66200 |     208.86 |       16468.04 |        1269.76 |           4096 |
|      66300 |     198.00 |       16470.09 |        1172.48 |           2048 |
|      66400 |     209.83 |       16475.13 |        1336.32 |           4096 |
|      66500 |     174.93 |       16479.25 |        1264.64 |           2048 |
|      66600 |     196.14 |       16480.84 |        1136.64 |           2048 |
|      66700 |     201.91 |       16483.59 |        1195.52 |           4096 |
|      66800 |     206.06 |       16486.40 |        1213.44 |           2048 |
|      66900 |     207.55 |       16490.35 |        1264.64 |           4096 |
|      67000 |     192.83 |       16490.42 |        1111.04 |           4096 |
```

Testing Data Results

Cumulative Gameplay Statistics:											
# of Games	Time	Mean Score	Rate of Highest Tile								
100	78.51	15324.68	2048:	20.00%,	1024:	47.00%,	512:	28.00%			
200	344.84	15813.62	2048:	20.00%,	1024:	51.50%,	512:	22.50%			
300	578.52	15619.55	2048:	17.67%,	1024:	53.00%,	512:	25.00%			
400	810.48	15609.61	2048:	18.25%,	1024:	52.00%,	512:	25.50%			
500	882.47	15437.46	2048:	16.80%,	1024:	54.00%,	512:	25.20%			
600	1136.53	15445.07	2048:	17.00%,	1024:	53.17%,	512:	25.83%			
700	1391.71	15213.09	2048:	16.29%,	1024:	52.43%,	512:	27.00%			
800	1460.06	15263.72	2048:	16.12%,	1024:	53.25%,	512:	26.75%			
900	1673.70	15272.36	2048:	16.11%,	1024:	53.44%,	512:	26.67%			
1000	1898.24	15196.13	2048:	15.50%,	1024:	54.10%,	512:	26.60%			
1100	1977.11	15251.00	2048:	15.55%,	1024:	54.09%,	512:	26.73%			
1200	2212.09	15230.67	4096:	0.08%,	2048:	15.42%,	1024:	54.17%			
1300	2487.67	15285.47	4096:	0.08%,	2048:	15.54%,	1024:	54.62%			
1400	2709.23	15286.67	4096:	0.07%,	2048:	15.71%,	1024:	54.43%			
1500	2790.18	15243.83	4096:	0.07%,	2048:	15.60%,	1024:	54.40%			
1600	3044.02	15260.45	4096:	0.06%,	2048:	15.62%,	1024:	54.31%			
1700	3118.48	15267.60	4096:	0.06%,	2048:	15.41%,	1024:	54.88%			
1800	3361.58	15264.10	4096:	0.06%,	2048:	15.33%,	1024:	54.83%			
1900	3619.28	15225.50	4096:	0.05%,	2048:	15.21%,	1024:	55.05%			
2000	3869.30	15293.38	4096:	0.05%,	2048:	15.35%,	1024:	55.55%			
2100	3952.73	15348.43	4096:	0.05%,	2048:	15.33%,	1024:	55.95%			
2200	4193.72	15332.26	4096:	0.05%,	2048:	15.18%,	1024:	56.18%			
2300	4268.99	15340.89	4096:	0.04%,	2048:	15.09%,	1024:	56.43%			
2400	4542.37	15416.46	4096:	0.04%,	2048:	15.33%,	1024:	56.50%			
2500	4778.27	15450.15	4096:	0.04%,	2048:	15.44%,	1024:	56.44%			
2600	5044.45	15440.53	4096:	0.04%,	2048:	15.27%,	1024:	56.77%			
2700	5296.70	15439.64	4096:	0.07%,	2048:	15.19%,	1024:	56.70%			
2800	5376.52	15440.13	4096:	0.07%,	2048:	15.21%,	1024:	56.61%			
2900	5656.95	15450.34	4096:	0.07%,	2048:	15.28%,	1024:	56.48%			
3000	5884.24	15461.49	4096:	0.07%,	2048:	15.40%,	1024:	56.37%			
11000	28767.91	16403.41	4096:	0.26%,	2048:	19.78%,	1024:	53.92%			
11100	29201.92	16426.41	4096:	0.27%,	2048:	19.87%,	1024:	53.90%			
11200	29300.08	16423.57	4096:	0.27%,	2048:	19.85%,	1024:	53.90%			
11300	29799.49	16430.95	4096:	0.27%,	2048:	19.88%,	1024:	53.93%			
11400	29883.65	16433.87	4096:	0.26%,	2048:	19.90%,	1024:	53.99%			
11500	30255.05	16446.67	4096:	0.26%,	2048:	19.97%,	1024:	53.95%			
11600	30628.03	16462.34	4096:	0.26%,	2048:	20.02%,	1024:	54.00%			
11700	30708.49	16463.84	4096:	0.26%,	2048:	20.01%,	1024:	54.09%			
11800	31195.55	16460.35	4096:	0.25%,	2048:	19.98%,	1024:	54.10%			
11900	31606.22	16467.27	4096:	0.25%,	2048:	20.01%,	1024:	54.13%			
12000	31696.11	16471.45	4096:	0.26%,	2048:	20.00%,	1024:	54.14%			
12100	32155.74	16491.41	4096:	0.27%,	2048:	20.06%,	1024:	54.16%			
12200	32605.85	16496.35	4096:	0.27%,	2048:	20.09%,	1024:	54.11%			
12300	32700.74	16503.19	4096:	0.28%,	2048:	20.08%,	1024:	54.17%			
12400	33178.35	16524.04	4096:	0.28%,	2048:	20.16%,	1024:	54.18%			
12500	33610.80	16542.70	4096:	0.30%,	2048:	20.21%,	1024:	54.16%			
12600	33697.53	16545.05	4096:	0.29%,	2048:	20.22%,	1024:	54.13%			
12700	34178.30	16554.95	4096:	0.31%,	2048:	20.23%,	1024:	54.19%			
12800	34640.04	16566.86	4096:	0.31%,	2048:	20.26%,	1024:	54.20%			
12900	35073.15	16580.60	4096:	0.32%,	2048:	20.30%,	1024:	54.18%			
13000	35182.47	16589.27	4096:	0.32%,	2048:	20.31%,	1024:	54.21%			
13100	35737.78	16607.31	4096:	0.31%,	2048:	20.37%,	1024:	54.20%			
13200	35826.00	16611.90	4096:	0.31%,	2048:	20.40%,	1024:	54.16%			
12500	33610.80	16542.70	4096:	0.30%,	2048:	20.21%,	1024:	54.16%			
12600	33697.53	16545.05	4096:	0.29%,	2048:	20.22%,	1024:	54.13%			
12700	34178.30	16554.95	4096:	0.31%,	2048:	20.23%,	1024:	54.19%			
12800	34640.04	16566.86	4096:	0.31%,	2048:	20.26%,	1024:	54.20%			
12900	35073.15	16580.60	4096:	0.32%,	2048:	20.30%,	1024:	54.18%			
13000	35182.47	16589.27	4096:	0.32%,	2048:	20.31%,	1024:	54.21%			
13100	35737.78	16607.31	4096:	0.31%,	2048:	20.37%,	1024:	54.20%			
13200	35826.00	16611.90	4096:	0.31%,	2048:	20.40%,	1024:	54.16%			
13300	36299.40	16619.69	4096:	0.32%,	2048:	20.44%,	1024:	54.12%			
13400	36821.28	16640.30	4096:	0.32%,	2048:	20.48%,	1024:	54.17%			
13500	37243.74	16649.44	4096:	0.33%,	2048:	20.45%,	1024:	54.25%			
13600	37351.18	16659.59	4096:	0.32%,	2048:	20.52%,	1024:	54.17%			
13700	37804.32	16672.00	4096:	0.34%,	2048:	20.53%,	1024:	54.18%			
13800	38304.18	16680.23	4096:	0.33%,	2048:	20.59%,	1024:	54.11%			
13900	38398.38	16690.62	4096:	0.34%,	2048:	20.65%,	1024:	54.01%			
14000	38766.14	16693.85	4096:	0.34%,	2048:	20.66%,	1024:	53.99%			

Experimental TDL Results

- Training data: 112,000 episodes
- Results from playing 14,000 games:
  - Average score: **16,693**
  - Reached 4096 tile: **0.34%**
  - Reached 2048 tile: **21%**
  - Reached 1024 tile: **74.99%**

Expectimax

- Simulates and ranks all legal moves
- Runs expectimax search alternating between max nodes and chance nodes
- Caching speeds up repeated states
- Rewards empty tiles, higher tiles in corners, merge potential
- 2048 tile odds increase significantly using depth 4 and above

Expectimax Results

Depth 2:

- 230 games played:
  - Average score: 8321
  - Reached 2048: 0.87%
  - Reached 1024: 19.57%
  - Reached 512: 76.53%

Depth 3:

- 100 games played:
  - Average score: 12569
  - Reached 2048: 4%
  - Reached 1024: 54%
  - Reached 512: 95%

Depth 4:

- 50 games played:
  - Average score: 18543
  - Reached 2048: 30%
  - Reached 1024: 48%
  - Reached 512: 20%

Expectimax 2048 Tile