(1) Método de EULER IMPLÍCITO

$$\boxed{y_{k+1} = y_k + f(t_{k+1}, y_{k+1}) * h}$$

Aplicação ao oscilador harmónico simples (OHS)

$$\begin{cases} v_{k+1} = v_k + f(t_{k+1}, y_{k+1}, v_{k+1}) * h \\ y_{k+1} = y_k + v_{k+1} * h \end{cases} \Leftrightarrow$$

$$\begin{cases} v_{k+1} = v_k - w^2 y_{k+1} * h \\ y_{k+1} = y_k + v_{k+1} * h \end{cases} \Leftrightarrow \text{(OHS)}$$

1) Método 1 → implementar um ciclo

$$\begin{cases} \text{---} \\ y_{k+1} = y_k + (v_k - w^2 y_{k+1} * h) * h \end{cases}$$

$$\begin{cases} \text{---} \\ y_{k+1} = y_k + v_k * h - w^2 h^2 y_{k+1} \end{cases} \Leftrightarrow$$

$$\begin{cases} \text{---} \\ (1 + w^2 h^2) y_{k+1} = y_k + v_k * h \end{cases}$$

$$\begin{cases} \text{---} \\ y_{k+1} = \dfrac{y_k + v_k * h}{1 + w^2 h^2} \end{cases}$$

$$\begin{cases} y_{k+1} = \dfrac{y_k + v_k * h}{1 + w^2 h^2} \\ v_{k+1} = v_k - w^2 y_{k+1} * h \end{cases}$$

Pode ser programado!

Implementar por ex: c/ ciclo For.

Método Euler-Implícito (2)

1) Método 2 → LINSOLVE

$$\begin{cases} v_{k+1} = v_k - w^2 y_{k+1} * h \\ y_{k+1} = y_k + v_{k+1} * h \end{cases} \quad (OHS) \iff$$

$$\begin{cases} v_{k+1} + w^2 y_{k+1} * h = v_k \\ y_{k+1} - v_{k+1} * h = y_k \end{cases} \iff$$

$$\begin{cases} y_{k+1} - v_{k+1} * h = y_k \\ w^2 h \, y_{k+1} + v_{k+1} = v_k \end{cases}$$

na forma matricial →

$$A z = b$$

$$\underbrace{\begin{bmatrix} 1 & -h \\ w^2 h & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{k+1} \\ v_{k+1} \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} y_k \\ v_k \end{bmatrix}}_{b}$$

(2) Método de CRANK – NICOLSON

$$\boxed{y_{k+1} = y_k + \frac{h}{2}\left[f(t_k, y_k) + f(t_{k+1}, y_{k+1})\right]}$$

Aplicação ao oscilador harmónico simples (OHS)

$$\begin{cases} v_{k+1} = v_k + \left[f(t_k, y_k) + f(t_{k+1}, y_{k+1})\right] * h/2 \\ y_{k+1} = y_k + \left[(v_k + v_{k+1})\right] * h/2 \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} y_{k+1} = y_k + (v_k + v_{k+1}) * h/2 \\ v_{k+1} = v_k - w^2 (y_k + y_{k+1}) * h/2 \end{cases}$$

$\Leftrightarrow$

$\Leftrightarrow$

2) Método 1 – implementar um ciclo

$\Leftrightarrow$

$$\begin{cases} v_{k+1} = v_k - w^2\left[y_k + y_k + (v_k + v_{k+1})\frac{h}{2}\right] h/2 \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} v_{k+1} = v_k - w^2\left(2y_k + v_k\, h/2 + v_{k+1}\, h/2\right) h/2 \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} v_{k+1} + \frac{w^2h^2}{4} v_{k+1} = v_k - \frac{\cancel{2} w^2 h}{\cancel{2}} y_k - \frac{w^2h^2}{4} v_k \end{cases}$$

Método Crank-Nicolson

$$\Leftrightarrow \begin{cases} \text{—} \\ v_{k+1}\left(1+\frac{w^2h^2}{4}\right) = -w^2h\,y_k + \left(1-\frac{w^2h^2}{4}\right)v_k \end{cases}$$

$$\begin{cases} \text{—} \\ v_{k+1} = \dfrac{-w^2h\,y_k + \left(1-\frac{w^2h^2}{4}\right)v_k}{1+\frac{w^2h^2}{4}} \end{cases}$$

$$\begin{cases} v_{k+1} = \dfrac{-w^2h\,y_k + \left(1-\frac{w^2h^2}{4}\right)v_k}{1+\frac{w^2h^2}{4}} \\ y_{k+1} = y_k + \left[(v_k + v_{k+1})\right] * h/2 \end{cases}$$

Pode ser programado (Ciclo For ou While).

2) Método 2 — LINSOLVE

(4)

$$\begin{cases} y_{k+1} = y_k + (v_k + v_{k+1}) * h/2 \\ v_{k+1} = v_k - w^2(y_k + y_{k+1}) * h/2 \end{cases} \Leftrightarrow$$

$$\begin{cases} y_{k+1} - v_{k+1} * h/2 = y_k + v_k\,h/2 \\ \underbrace{v_{k+1} + w^2 y_{k+1}\,h/2}_{k+1} = \underbrace{v_k - \frac{w^2h}{2}y_k}_{k} \end{cases} \Leftrightarrow$$

$$\begin{cases} y_{k+1} - \frac{h}{2} * v_{k+1} = y_k + h/2\,v_k \\ w^2\frac{h}{2}y_{k+1} + v_{k+1} = -\frac{w^2h}{2}y_k + v_k \end{cases}$$

na forma matricial $\longrightarrow$

$$\underbrace{\begin{bmatrix} 1 & -\frac{h}{2} \\ \omega^2 \frac{h}{2} & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{k+1} \\ v_{k+1} \end{bmatrix}}_{=\,z} = \underbrace{\begin{bmatrix} y_k + h/2 \, v_k \\ -\frac{\omega^2 h}{2} y_k + v_k \end{bmatrix}}_{b}$$

z → incógnita

b → é conhecido da iteração anterior!

Linsolve!