LQR Stabilization and Visualization
AA 516 Final

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March 18, 2025

Abstract

The goal of this project was to develop an LQR controller for the RCAM aircraft that will maintain a trim condition when the aircraft is affected by external disturbances (such as a wind gust). Once the controller was made, I also developed a visualization framework in FlightGear to get a more intuitive understanding of how the model is affected by external disturbances. Finally, the LQR controller is compared to an airplane with no active control methods (i.e. it is only passively stable due to the dynamics of the model). Code for this project can be found here.

LQR Control

LQR control is an optimal control method generally used to follow a trajectory while optimizing over some variable (typically control effort). The general form is given in 1.

$$\begin{aligned} \min_{u} \quad & \int_{0}^{\infty} \left(\mathbf{x}(t)^{\top} \mathrm{Q} \mathbf{x}(t) + \mathbf{u}(t)^{\top} \mathrm{R} \mathbf{u}(t) \right) \, \mathrm{dt} \\ \text{s.t.} \quad & \dot{\mathbf{x}}(t) = \mathrm{A}\mathbf{x}(t) + \mathrm{B}\mathbf{u}(t) \end{aligned} \tag{1}$$

For our aircraft model which we have been working on this entire quarter, it is desirable to determine the state and control that leaves the aircraft in a steady-state or a trim-state. Since our aircraft model is non-linear, we linearize the model about the trim-state to obtain the A and B dynamics matrices. Next, $\mathbf{x}(t)$ and $\mathrm{u}(t)$ are measured by their difference from the trim state as $\Delta\mathbf{x}(t)$ and $\Delta\mathbf{u}(t)$. We denote the trim state and control as $\bar{\mathbf{x}}$ and $\bar{\mathbf{u}}$ respectively, and for our case are constant. So the state used in the LQR formulation is given by $\Delta\mathbf{x}(t) = \mathbf{x}(t) - \bar{\mathbf{x}}$. While the control input, derived from the LQR problem and applied to the aircraft is given by $\mathbf{u}(t) = \Delta\mathbf{u}(t) + \bar{\mathbf{u}}$.

The LQR solution in continuous time can be found by solving the Algebraic Ricatti Equation. In MATLAB, there is already a built in function. This provides the solution to the Algebraic Ricatti Equation, along with the matrix K. The optimal control for the system is easily found in 2.

$$\Delta\mathbf{u}(t) = -\mathrm{K}\Delta\mathbf{x}(t) \tag{2}$$

One useful property of the RCAM model is that the longitudinal and lateral/directional states can be decoupled from each other. Essentially, changes in the longitudinal direction will not effect the lateral/directional direction and vice versa. We can leverage this in the controller by using separate LQR controllers for the longitudinal and lateral direction states. The longitudinal controller will mainly control pitch while the lateral/directional controller will mainly control roll and yaw. I noticed similar performance for the two methods, but in general, aircraft control systems decouple the longitudinal and lateral/direction directions.

The implementation for this project, in Simulink, can be found on the GitHub in the RCAM_model.slx file.

FlightGear Visualization

Next, I visualized the aircraft state in FlightGear. This required the addition of the Navigation Equations and Geodetic Position Equations to the Simulink model by following the lecture videos for Week 10. I also included the ability to control the aircraft with a joystick. I also installed a 757-200 model in FlightGear, as the RCAM model has similar performance to the 757.

LQR vs. Passive Stabilization

Finally, I compared the response of the model to different disturbances when it had the LQR controller enabled vs. when the aircraft was only being passively stabilized (due to the natural stability of the system). Disturbances are interjected into the system as additional control surface deflections. They are added after the control for the aircraft is saturated to ensure that an unlimited disturbance can be placed on the aircraft model. Since the model is sufficiently actuated (control inputs can influence all 6 degrees-of-freedom), any disturbance can be modeled through control surface deflections. Shown below in Figure 1 is a comparison between active LQR control and passive stability. For this case, the aileron, stabilizer and rudder were all disturbed beyond the saturation limit for the control surfaces. For the LQR controller, Q and R are defined as $10 \cdot I$ and I respectively. This provides an order of magnitude more weight on the state, ensuring the airplane returns to the trim condition quickly. Also note that there was no control effort to follow the heading angle ψ. ψ can be controlled if desired. Videos corresponding to the figures can be found here.

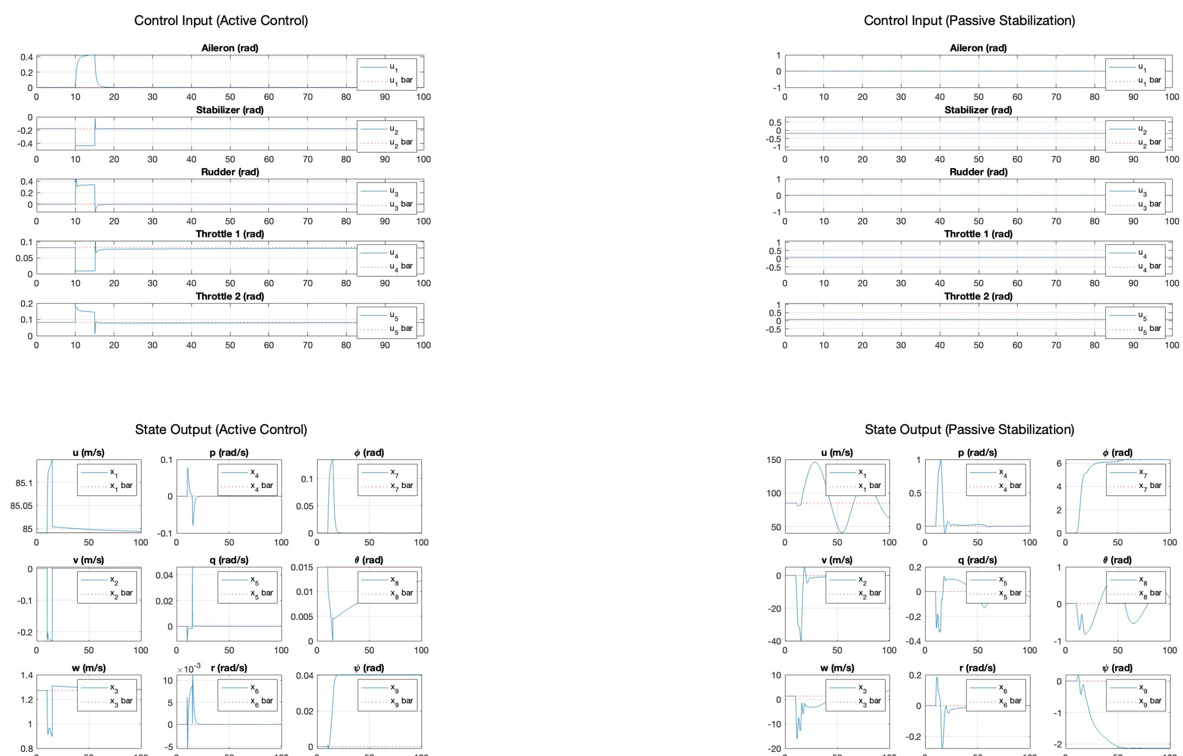

Figure 1: Active control (left) vs. passive stabilization (right). We can see that the LQR controller keeps the airplane far closer to trim. The passive stabilization remains far from trim, initially spinning out of control before recovering.