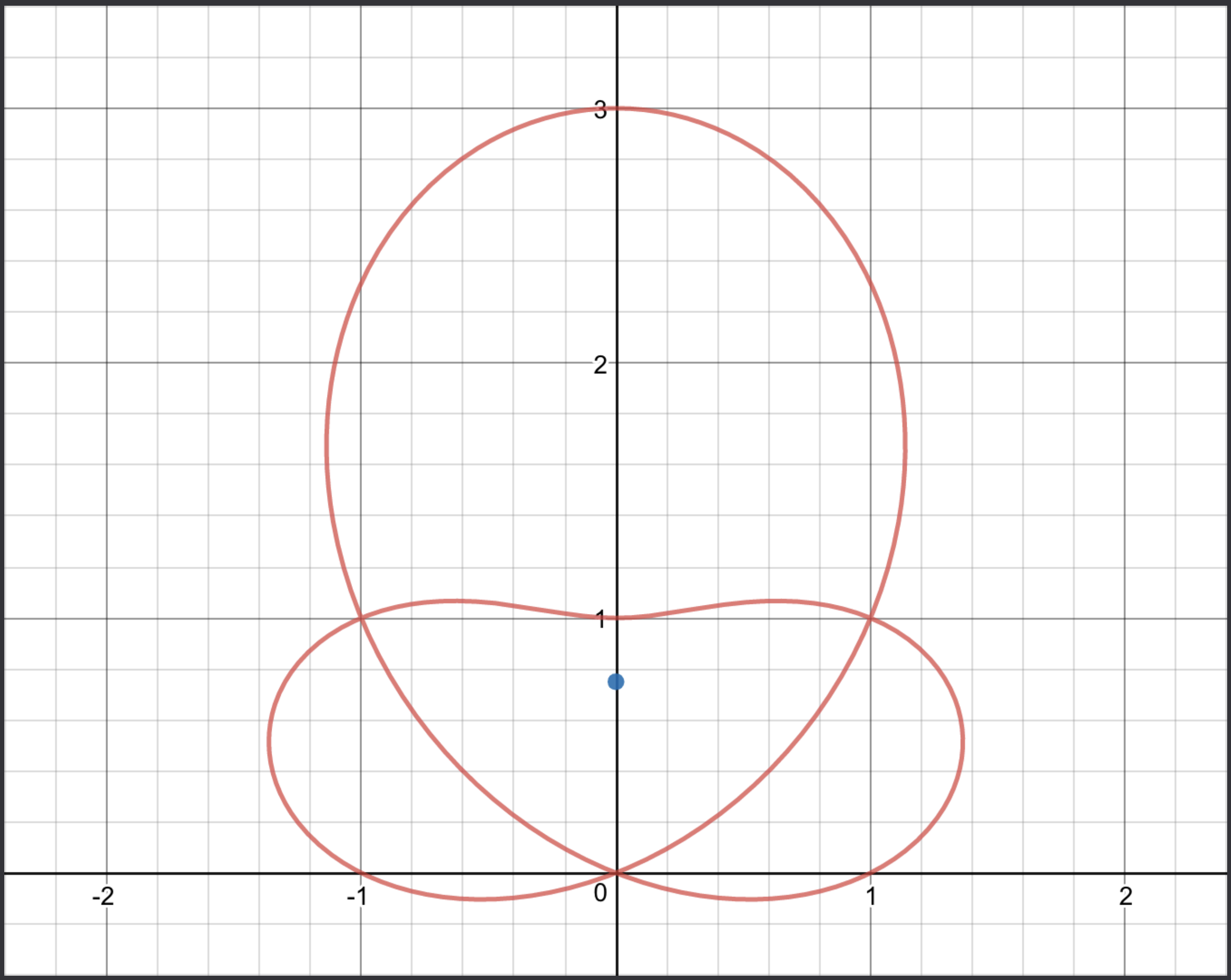
solutions

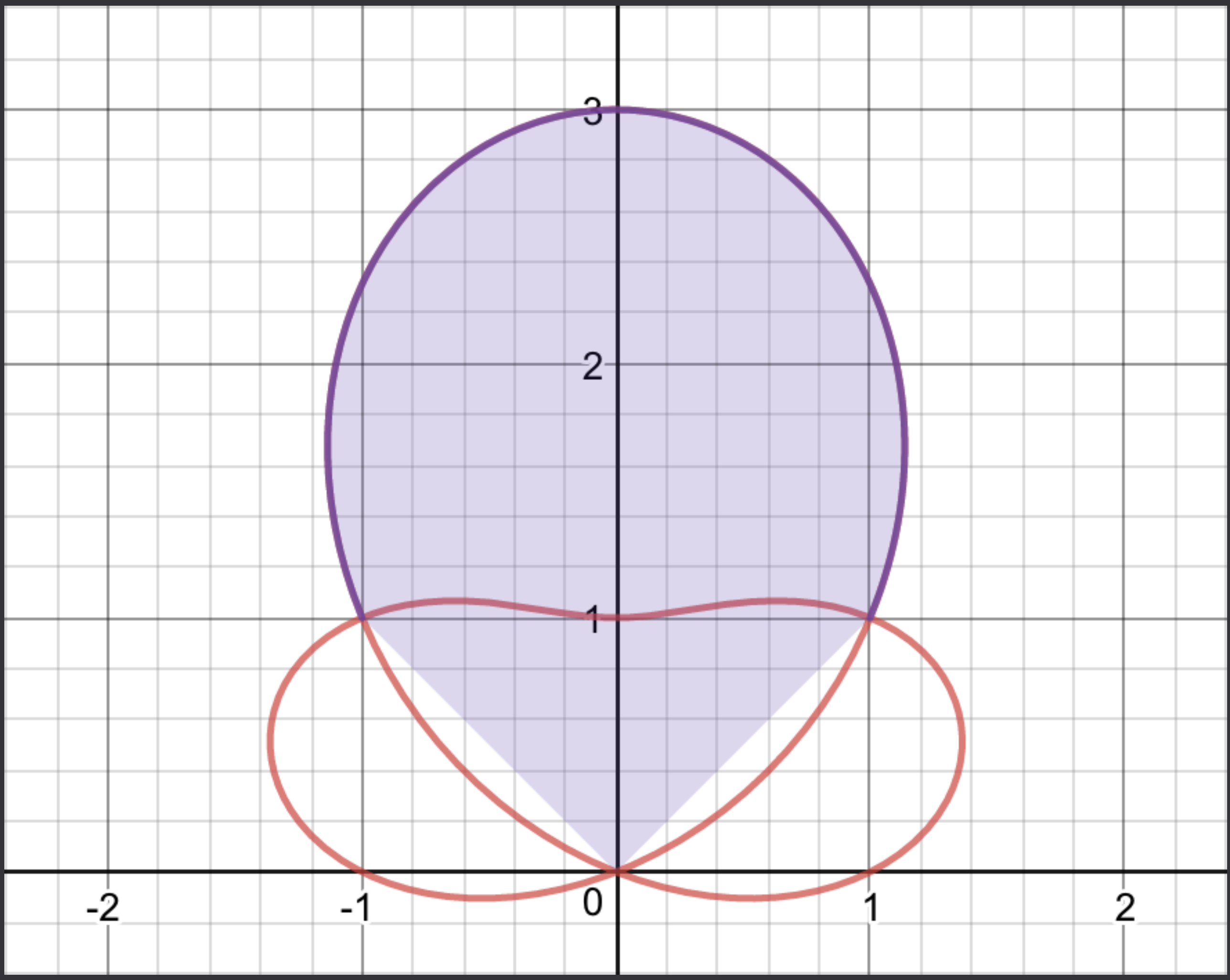
a)

by drawing the graph:

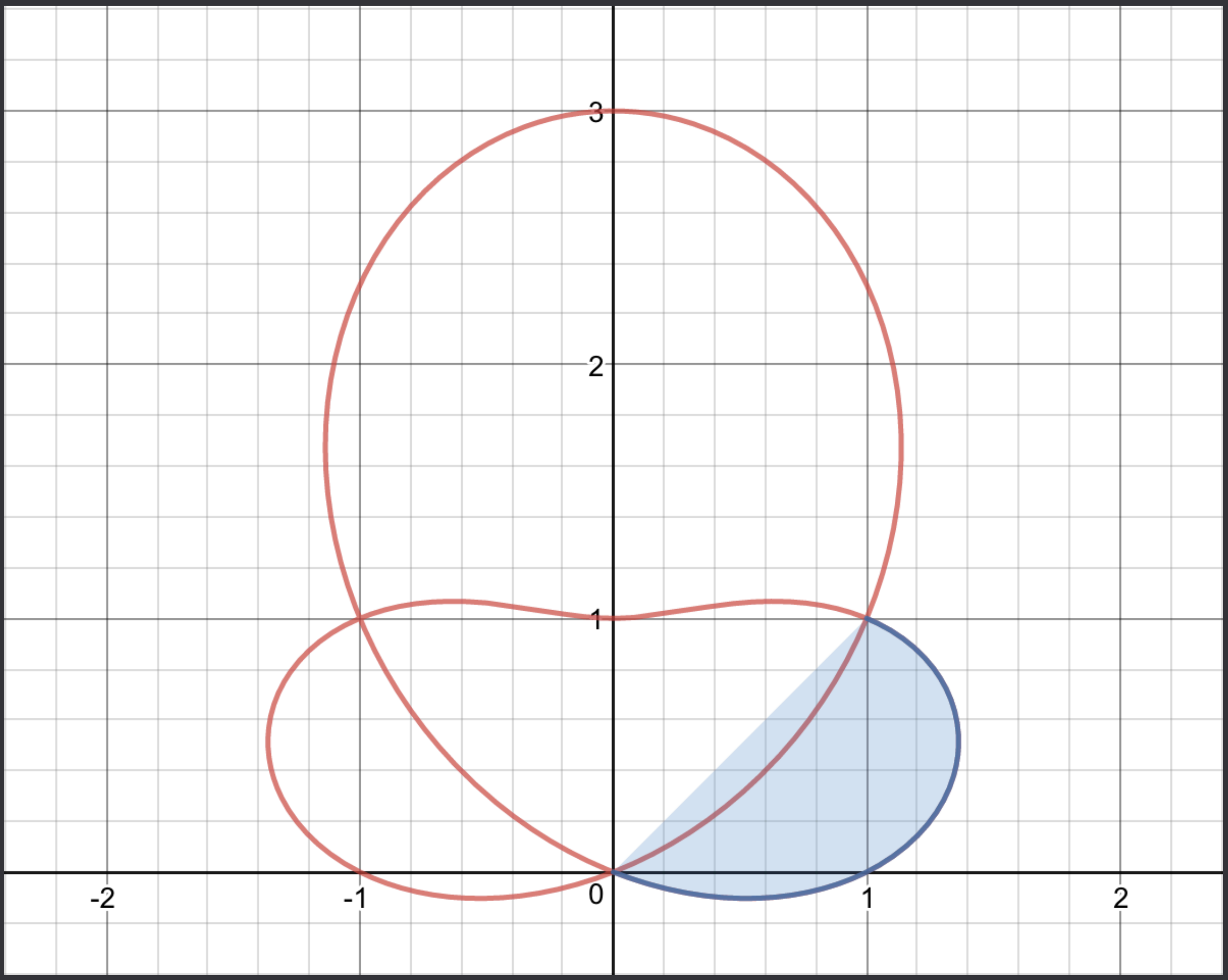


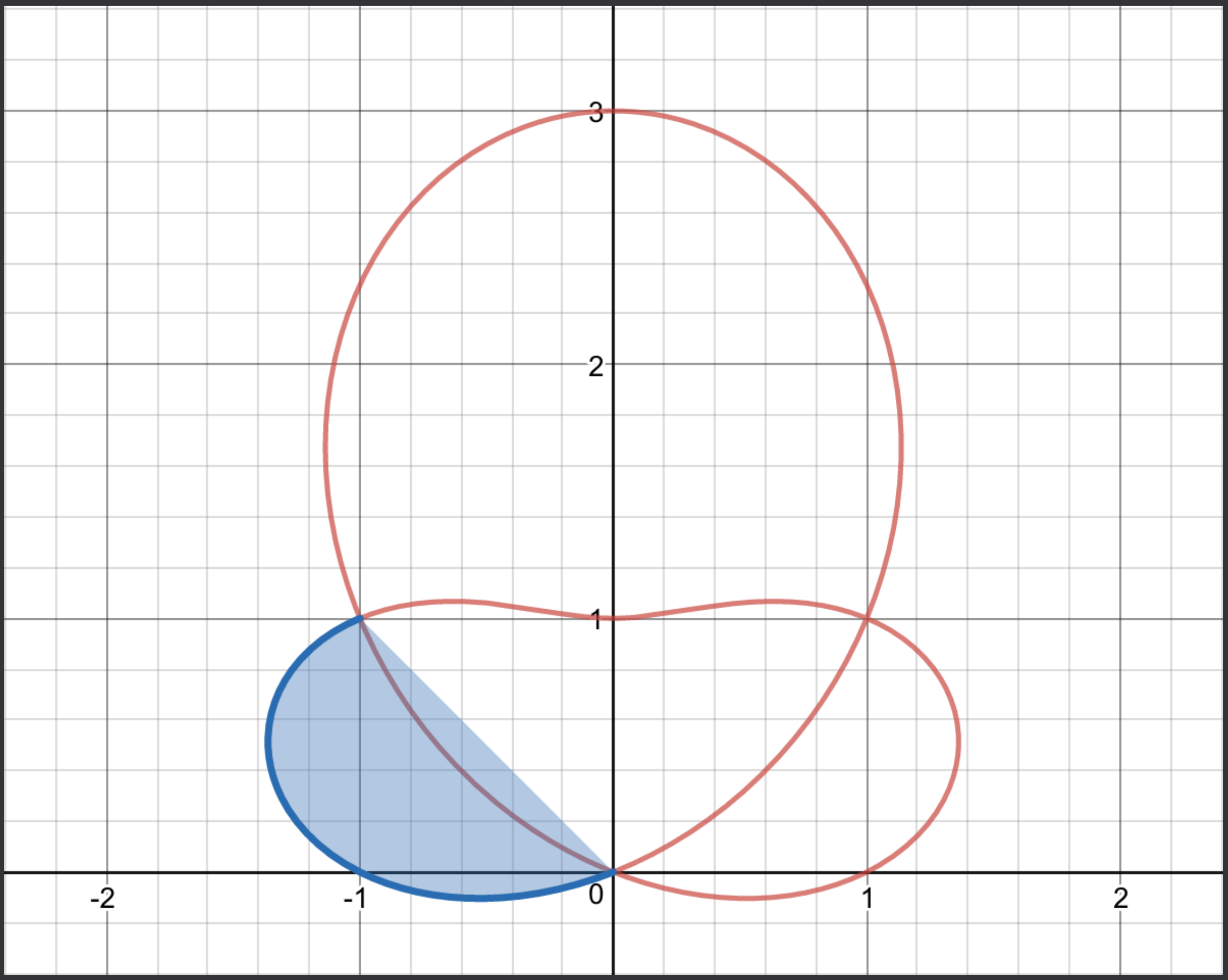
we can see that it will take multiple integrals to find this area. if we find the area by integrating from 0 to 2π, then we will double count the middle region.

to find the area, we will find the areas of the following regions:



3
2
1
-2
-1
0
1
2


3
2
1
-2
-1
0
1
2



by observing these graphs, we see that the points of intersection that are $(1,1)$ and $(1,-1)$. in polar, these are points at $(\frac{\pi}{4}, \sqrt{2})$ and $(\frac{3\pi}{4}, \sqrt{2})$

by observing the graphs, we can see that regions 2 and 3 are identical, just flipped, so we only need to find the area of region two and double it to get the area of 2 and 3 together. additionally, we need to notice that region one is actually the interval from $\frac{5\pi}{4} \to \frac{7\pi}{4}$.

with this in mind, we find the areas of these regions with:

$$A_1 = \int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}} \frac{1}{2}(2\sin\theta + \cos 2\theta)^2 \, d\theta$$

$$A_2 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}(2\sin\theta + \cos 2\theta)^2 \, d\theta$$

$$A_3 = A_2$$

so, total area A is:

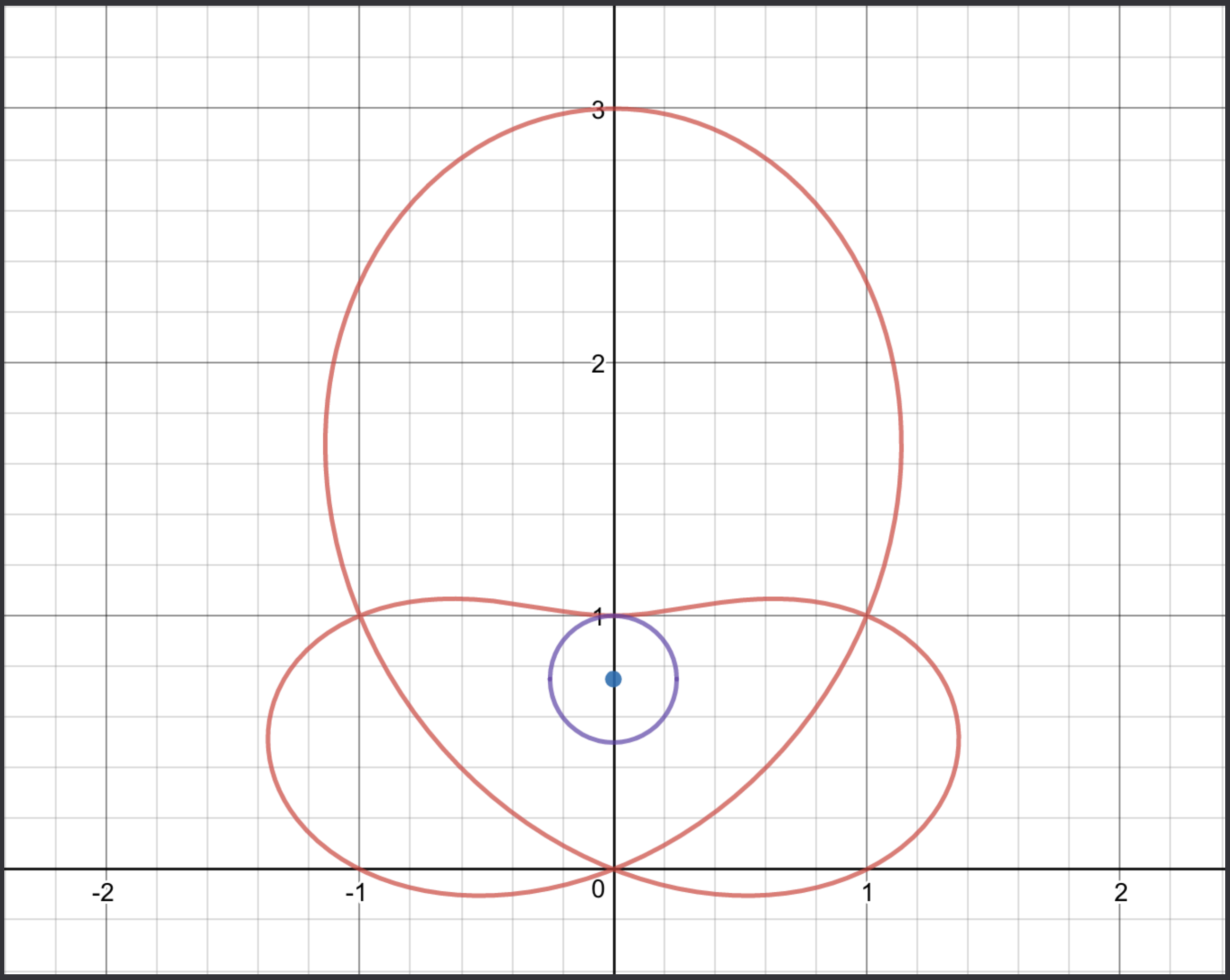
$$\int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}} \frac{1}{2}(2\sin\theta + \cos 2\theta)^2 \, d\theta + 2 \cdot \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}(2\sin\theta + \cos 2\theta)^2 \, d\theta$$

evaluating this with a calculator gives:

$$A \approx 6.776$$

b)

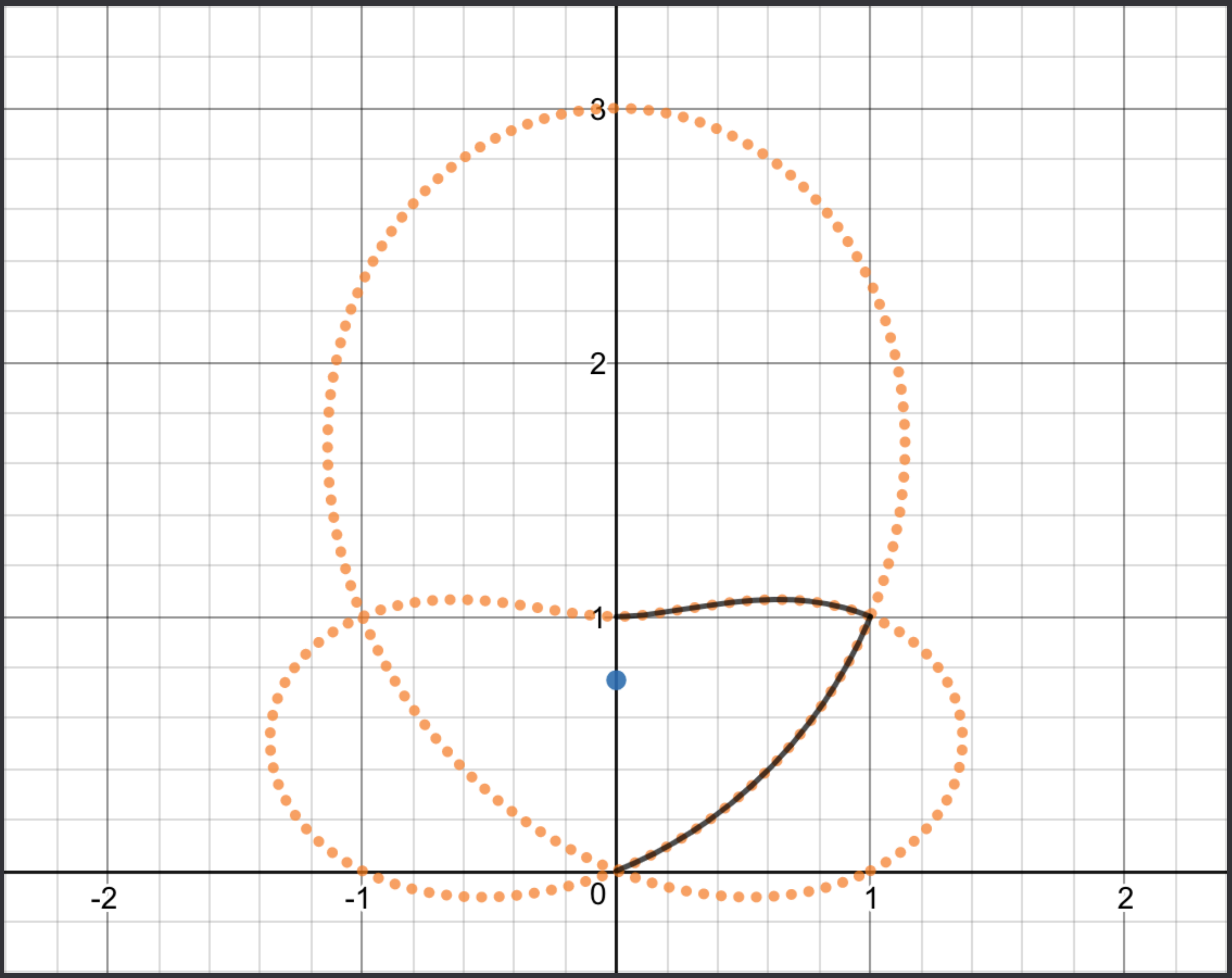
by inspecting the graph, one can see that the closest point to $(0, \frac{3}{4})$ on the dock is $(0, 1)$. since the segment drawn between these two points is perpendicular to the dock, it shows that this must be the closest point on that section of the dock, and there is no other section of the dock that comes this close. see this graph:



note: this is not going to be an important concept on the test, but it's important to solving part c.

c)

by inspecting the graph, it is clear that the most efficient path from $(0, 0)$ to $(0, 1)$ is the following:



this path is composed of two sections. the first one is fairly difficult to find; since it goes from $(0,0)$ to $(1,1)$, we know it ends at $\frac{5\pi}{4}$, (remember, this section of the loop has negative values of r) but to find the θ for which the graph crosses the origin, we must solve the equation:

$$2\sin(\theta) + \cos(2\theta) = 0$$

this is very difficult to solve, so we will use a graphing calculator, and we find that the angle we want is $\theta = 3.5164$. sadly, there is no easy way to express this, so we will leave it in this form. the second section is much simpler, going from $(1,1)$ to $(0,1)$. you can see that this goes from $\frac{\pi}{4}$ to $\frac{\pi}{2}$.

with this in mind, we find $r' = 2\cos(\theta) - 2\sin(2\theta)$

and use this to evaluate the following integrals:

$$S_1 = \int_{3.5164}^{\frac{5\pi}{4}} \sqrt{(2\sin(\theta) + \cos(2\theta))^2 + (2\cos(\theta) - 2\sin(2\theta))^2}\, d\theta$$

$$S_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{(2\sin(\theta) + \cos(2\theta))^2 + (2\cos(\theta) - 2\sin(2\theta))^2}\, d\theta$$

by finding $S_1 + S_2$, we arrive at the total length of this path. this is an incredibly complex set of integrals, please for the love of god just use a calculator.

we find $S_1 = 1.45198$, $S_2 = 1.01282$ so:

$$S_{total} \approx 2.46480$$

since chloe can run 1 unit per second, she can make it to the point in 2.46480 seconds, so she will be able save her mom!

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remember, these are tricky! just because you can't get some of them doesn't mean you aren't prepared for the test. even if you don't get it though, there's something valuable in walking through these solutions and trying to understand each part of this. these solutions are far from perfect, so don't be worried if you don't understand. feel free to text me at (203)-318-0628 if you have any questions!